Quiz 1-2

HUL315: Econometric Methods

Indian Institute of Technology Delhi
Semester-I: 2025–26
SET ACB

Name: ______________________ **Entry Number:** __________________

Select the most appropriate choice among the alternatives for the questions below

1. (3 points) For X and Y, two jointly distributed random variables, which of the following statements is false?

   A. If the covariance between X and Y is 0, then X and Y are independent.

   B. If X and Y are independently distributed, then the conditional distribution of Y given X equals the marginal distribution of Y.

   C. If the covariance between X and Y is 0, then X and Y are uncorrelated.

   D. If X and Y are independently distributed, then the joint density of X and Y is the product of their marginal densities.

2. (3 points) Which of the following statements are *false*?

   A. If the distribution of a random variable is symmetric to the mean, then the mean, median, and mode are the same.

   B. If the correlation between X and Y is r, then the correlation coefficient between the standardized values of X and Y is also r.

   C. If X and Y are independent, then $Var(Y|X) = Var(Y)$.

   D. If $Cov(X, Y) = 0$, then $\mathbb{E}(Y|X) = \mathbb{E}(Y)$.

3. (4 points) Let X be a discrete random variable with the probability mass function

$$P(X = n) = \begin{cases} -\frac{2c}{n} & \text{if } n = -1, -2 \\ d & \text{if } n = 0 \\ cn & \text{if } n = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

   where c and d are positive real numbers. If $P(|X| \leq 1) = \frac{3}{4}$, then $\mathbb{E}(X)$ is

   A. $\frac{1}{12}$

   B. $\frac{1}{6}$

   C. $\frac{1}{3}$

   D. $\frac{1}{2}$

4. (3 points) If X and Y are two random variables with $\mathbb{E}(X) = 10$, $\mathbb{E}(Y) = 5$, and they are independent, what is $\mathbb{E}(X^2Y)$?

   A. 50

B. Cannot be determined without knowing the variance of X.

C. 250

D. 100

5. (3 points) Which of the following describes a distribution with a kurtosis greater than 3?

   A. The distribution is symmetric.

   B. The distribution has a long right tail.

   C. The distribution has heavy tails and is 'leptokurtotic'.

   D. The distribution has a long left tail.

6. (4 points) Let X be a random variable with the probability density function

$$f(x) = \begin{cases} \alpha x^2 & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$$

   A new random variable is defined as $Y = X^2$. What is the expected value of Y, $\mathbb{E}(Y)$?

   A. 2α

   B. $\frac{16}{3}$

   C. $\frac{12}{5}$

   D. $\frac{8}{3}$

7. (3 points) If X and Y are independent random variables, and $Var(X) = 4$ and $Var(Y) = 9$, what is $Var(X+Y)$?

   A. 13

   B. Cannot be determined without knowing the covariance of σ_{XY}.

   C. 2.5

   D. 36

8. (4 points) Let X and Y be two random variables. The joint p.d.f. is given by $f(x,y) = k(x+y)$ for $x, y \in [0,1]$ and 0 otherwise. What is the value of k?

   A. $\frac{1}{6}$

   B. $\frac{1}{4}$

   C. $\frac{1}{2}$

   D. 1

9. (3 points) Which of the following statements are *false*?

   A. If the distribution of a random variable is symmetric to the mean, then the mean, median, and mode are the same.

B. If the correlation between X and Y is r, then the correlation coefficient between the standardized values of X and Y is also r.

C. If X and Y are independent, then $Var(Y|X) = Var(Y)$.

D. If $Cov(X, Y) = 0$, then $\mathbb{E}(Y|X) = \mathbb{E}(Y)$.

10. (3 points) Let X be a random variable with the PDF $f(x) = k(1 - x)$ for $x \in [0, 1]$ and 0 otherwise. What is the variance of X, $Var(X)$?

    A. $\frac{1}{18}$

    B. $\frac{1}{3}$

    C. $\frac{1}{6}$

    D. $\frac{1}{2}$

11. (3 points) Let $\{Y_1, Y_2, ..., Y_n\}$ be a random sample from a population with mean μ and variance σ^2. The sample mean is $\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$. A new estimator for the mean is given by $\hat{\mu} = \frac{1}{n-1}\sum_{i=1}^{n} Y_i$. What is the bias of this new estimator?

    A. 0

    B. $\frac{\mu}{(n-1)}$

    C. $\frac{\mu}{n}$

    D. $\frac{n\mu}{(n-1)}$

12. (4 points) The joint p.d.f. of two random variables X and Y is given by $f(x, y) = e^{-(x+y)}$ for $x, y \geq 0$ and 0 otherwise. What is the conditional expectation of X given $Y = y$, $\mathbb{E}(X|Y = y)$?

    A. 1

    B. e^{-y}

    C. 0.5

    D. y

13. (4 points) Let X be a random variable with the PDF $f(x) = k(2 - x)$ for $x \in [0, 2]$ and 0 otherwise. What is the value of the median of X?

    A. 1

    B. $2 - \sqrt{2}$

    C. $2 + \sqrt{2}$

    D. 0.5

14. (3 points) Let X and Y be independent random variables with $\mathbb{E}(X) = 1$ and $\mathbb{E}(Y) = 2$. Let $W = X + Y$ and $Z = X - Y$. What is $Cov(W, Z)$?

A. -1

B. Cannot be determined without knowing the variances of X and Y.

C. 1

D. 0

15. (3 points) For X and Y, two jointly distributed random variables, which of the following statements is false?

A. If the covariance between X and Y is 0, then X and Y are independent.

B. If X and Y are independently distributed, then the conditional distribution of Y given X equals the marginal distribution of Y.

C. If the covariance between X and Y is 0, then X and Y are uncorrelated.

D. If X and Y are independently distributed, then the joint density of X and Y is the product of their marginal densities.

16. (4 points) Consider a population with a Bernoulli distribution where $P(Y = 1) = 0.78$. Which of the following statements best describes the sampling distribution of the sample mean $\bar{Y}$ for a large *i.i.d.* sample of size n?

A. The sampling distribution of $\bar{Y}$ is approximately normal only if the probability of success $P(Y = 1)$ is close to 0.5.

B. By the Central Limit Theorem, the sampling distribution of $\bar{Y}$ is approximately normal, centered at the population mean ($\mu_Y = 0.78$).

C. The variance of the sampling distribution of $\bar{Y}$ is constant, regardless of the sample size n.

D. By the Central Limit Theorem, the sampling distribution of $\bar{Y}$ converges to the same Bernoulli distribution as the population.

17. (3 points) An econometrician tests H_0: policy has no effect vs. H_1: policy has a positive effect, using $\alpha = 0.05$. The p-value is 0.065, so they fail to reject the null hypothesis. What is the most precise conclusion?

A. There is a 6.5% chance that the null hypothesis is true.

B. Failing to reject H_0 avoids the risk of a Type I error.

C. The p-value indicates the policy is very likely to have no effect.

D. Failing to reject H_0 incurs the risk of a Type II error (a false negative).

18. (3 points) Suppose the significance level is set at $\alpha = 0.05$ and the observed p-value is 0.07. Which of the following statements is correct?

A. We reject H_0 because the p-value is smaller than 0.1.

B. We fail to reject H_0 because the observed p-value (0.07) is greater than the chosen significance level (0.05).

C. The null hypothesis is true because the p-value exceeds 0.05.

D. The test is inconclusive; p-values cannot be compared to α.

19. (4 points) Suppose $\{Y_i\}_{i=1}^n$ are i.i.d. random variables with $\mathbb{E}[Y_i] = \mu_Y$ and $\text{Var}(Y_i) = \sigma^2 < \infty$. Two estimators of μ_Y are defined as:

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i, \qquad \tilde{Y} = \sum_{i=1}^{n} w_i Y_i, \quad w_i > 0, \quad \sum_{i=1}^{n} w_i = 1.$$

Which of the following statements about their large-sample properties is correct?

A. Both $\bar{Y}$ and $\tilde{Y}$ are unbiased, but only $\tilde{Y}$ is consistent.

B. Both $\bar{Y}$ and $\tilde{Y}$ are unbiased and consistent, but $\tilde{Y}$ is asymptotically more efficient.

C. Both $\bar{Y}$ and $\tilde{Y}$ are unbiased and consistent, but $\bar{Y}$ has the minimum variance among all such linear unbiased estimators.

D. $\bar{Y}$ is unbiased but inconsistent, while $\tilde{Y}$ is biased but consistent.

20. (3 points) In the context of hypothesis testing, what does a very small p-value (e.g., 0.001) indicate?

A. Strong evidence that the null hypothesis is true.

B. Strong evidence against the null hypothesis, since such extreme data are unlikely if H_0 is true.

C. Proof that the alternative hypothesis is true.

D. No information; p-values cannot be used for inference.

21. (3 points) Suppose we are testing $H_0 : \mu = 50$ vs. $H_1 : \mu \neq 50$ using the test statistic $Z = \dfrac{\overline{Y} - 50}{\sigma/\sqrt{n}}$ with known variance σ^2. If the true mean is $\mu = 50 + \delta$, which of the following best describes how the power of the test behaves as $n \to \infty$?

A. Power converges to α, the size of the test.

B. Power converges to 0.5, regardless of δ.

C. Power converges to 1 for any fixed $\delta \neq 0$.

D. Power oscillates and does not converge.

22. (4 points) Let $\{Y_i\}_{i=1}^n$ be i.i.d. $\mathcal{N}(\mu, \sigma^2)$ with known σ^2. A 95% confidence interval for μ is given by

$$\left[\overline{Y} - 1.96 \cdot \frac{\sigma}{\sqrt{n}},\ \overline{Y} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}\right].$$

If instead the variance is unknown and we replace σ by the sample standard deviation $S = \sqrt{\dfrac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$, how does the probability that the interval contains the true mean behave as $n \to \infty$?

A. It becomes strictly less than 95% for all large n.

B. It becomes strictly greater than 95% for all large n.

C. It converges to 95%, since the t-distribution converges to the standard normal.

D. It oscillates indefinitely and does not converge.

23. (3 points) A student runs a two-sided test and obtains a p-value of 0.03. Which of the following interpretations is correct?

    A. There is a 3% probability that the null hypothesis is true.

    B. If the null hypothesis were true, the probability of observing data as extreme as (or more extreme than) the sample is 3%.

    C. The alternative hypothesis is true with 97% probability.

    D. Repeating the experiment 100 times, the null would be rejected exactly 3 times.

24. (3 points) Suppose we test $H_0 : \mu = 50$ against $H_1 : \mu \neq 50$ at significance level $\alpha = 0.05$. If we reduce the significance level to $\alpha = 0.01$ while keeping the sample size fixed, what is the effect on the probability of a Type II error (β)?

    A. It decreases, since the rejection region becomes larger.

    B. It increases, since the rejection region becomes smaller.

    C. It remains the same, since β does not depend on α.

    D. It oscillates between 0 and 1 as α decreases.

25. (3 points) Which of the following is the most precise definition of a p-value in hypothesis testing?

    A. The probability that the null hypothesis is true, given the observed data.

    B. The probability, under the assumption that the null hypothesis is true, of obtaining a test statistic at least as extreme as the one observed.

    C. The smallest significance level α at which we fail to reject the null hypothesis.

    D. The long-run frequency with which the null hypothesis is true.

26. (4 points) Let $\{Y_i\}_{i=1}^n$ be i.i.d. random variables with mean μ_Y and variance $\sigma_Y^2 < \infty$. Which of the following is the statement of the Central Limit Theorem (CLT) for the sample mean $\bar{Y}$?

    A. $\bar{Y} \sim N(\mu_Y, \sigma_Y^2)$ for any sample size n.

    B. $\sqrt{n}\,\frac{\bar{Y}-\mu_Y}{\sigma_Y} \xrightarrow{d} N(0,1)$ **as** $n \to \infty$**.**

    C. $\bar{Y} \sim t_{n-1}$ for large n.

    D. $\bar{Y}$ converges almost surely to $N(\mu_Y, \sigma_Y^2)$.

27. (3 points) Let $\{Y_i\}_{i=1}^n$ be i.i.d. $\mathcal{N}(\mu, \sigma^2)$ with known σ^2. A 95% confidence interval for μ is given by

$$\left[\overline{Y} - 1.96 \cdot \frac{\sigma}{\sqrt{n}},\ \overline{Y} + 1.96 \cdot \frac{\sigma}{\sqrt{n}}\right].$$

If instead the variance is unknown and we replace σ by the sample standard deviation $S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \bar{Y})^2}$, how does the probability that the interval contains the true mean behave as $n \to \infty$?

A. It becomes strictly less than 95% for all large n.

B. It becomes strictly greater than 95% for all large n.

C. It converges to 95%, since the t-distribution converges to the standard normal.

D. It oscillates indefinitely and does not converge.

28. (3 points) $\bar{Y}$ is the sample mean defined for an *i.i.d.* random sample $\{Y_i\}_{i=1}^n$ often used to estimate the population mean μ_Y. Consider a second estimator, $\tilde{Y}$, which uses unequal weights ($w_i > 0$, $\sum w_i = 1$)

$$\tilde{Y} = \sum_{i=1}^{n} w_i Y_i.$$

Which of the following statement is correct?

A. $\tilde{Y}$ is generally biased for μ_Y unless all the weights are equal.

B. $\tilde{Y}$ is unbiased for μ_Y, but its variance is larger than $\bar{Y}$.

C. $\tilde{Y}$ is consistent only if the weights are unequal.

D. $\tilde{Y}$ always has a smaller variance than $\bar{Y}$.

29. (3 points) Suppose $Y_i \sim \text{Bernoulli}(p)$ with $p = 0.6$. What happens to the variance of the sample mean $\bar{Y}$ as the sample size n increases?

A. It remains constant at $p(1-p)$.

B. It decreases at the rate $\frac{1}{n}$, i.e. $\text{Var}(\bar{Y}) = \frac{p(1-p)}{n}$.

C. It increases linearly with n.

D. It oscillates depending on whether n is even or odd.

30. (4 points) In a two-sided t-test with $n-1$ degrees of freedom, the observed test statistic is $t_{\text{obs}} = 2.5$. Which of the following correctly describes how the p-value is computed?

A. $P(T \geq 2.5)$ where $T \sim N(0,1)$.

B. $P(T \leq 2.5)$ where $T \sim t_{n-1}$.

C. $2 \cdot P(T \geq 2.5)$ where $T \sim t_{n-1}$.

D. $P(T \geq |2.5|)$ where $T \sim \chi^2_{n-1}$.