P	Q	$P\mid Q$	$\neg(P\wedge Q)$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

$$P\to Q \equiv \neg(P\wedge\neg Q) \equiv \neg P\vee\neg\neg Q \equiv \neg P\vee Q .$$

$$\neg P \equiv P\mid P$$

$$P\vee Q \equiv \neg(\neg P\wedge\neg Q) \equiv (P\mid P)\mid(Q\mid Q)$$

$$P\to Q \equiv P\mid(Q\mid Q)$$

$$P\wedge Q \equiv \neg\underbrace{\neg(P\wedge Q)} \Rightarrow \neg(P\mid Q) \Rightarrow (P\mid Q)\mid(P\mid Q)$$

$$P\mid(Q\mid Q)$$

$$\neg(\underbrace{\neg P}_{X}\wedge\underbrace{\neg Q}_{Y}) \equiv \neg\neg P\vee\neg\neg Q \equiv P\vee Q$$

$$X\mid Y \Rightarrow (P\mid P)\mid(Q\mid Q)$$

$$\hookrightarrow \neg(X\wedge Y)$$

$$\neg(\neg P\wedge\neg Q) \equiv P\vee Q .$$

$$P\mid(P\mid Q).$$

$$\neg(P\wedge\neg(P\wedge Q))$$

$$\neg P\vee(\neg P\vee\neg Q)$$

$$X\mid Y \equiv \neg(X\wedge Y)$$

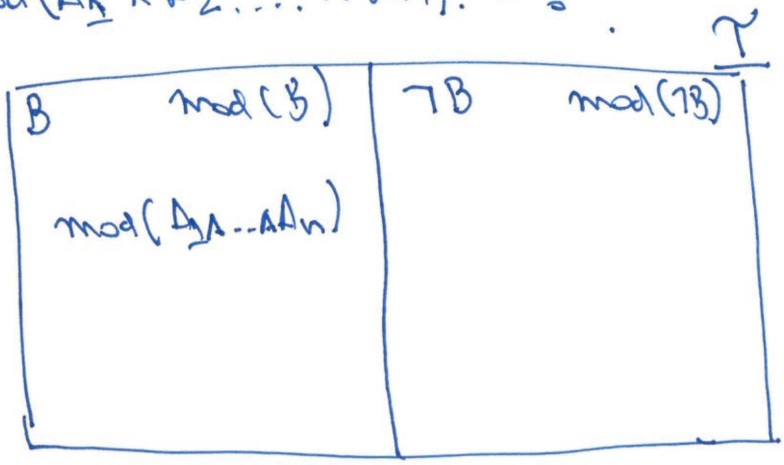
$$(P\mid P)\mid(Q\mid Q)$$

$$\neg(\neg P\wedge\neg Q) \equiv P\vee Q$$

②

$A_1, A_2, \dots, A_n \models B$.

$mod(A_1 \wedge A_2 \dots \wedge A_n) = ?$



$mod(A_1 \wedge \dots \wedge A_n \wedge \neg B) = ?\ \emptyset$

$\{A_1, A_2, \dots, A_n, \neg B\}$ is inconsistent!

$A, \neg A \models B$? Yes

if every interpretation that satisfies $A \wedge \neg A$

also satisfies B!

This is because no interpretation satisfies

$A \wedge \neg A \Rightarrow mod(A \wedge \neg A) = \emptyset \subseteq mod(B)$!

$P \vee Q, \neg P \vDash Q$? ③.

$\downarrow$ tautology.

P	Q	$P \vee Q$	$\neg P$	$(P \vee Q) \wedge \neg P$	$((P \vee Q) \wedge \neg P) \to Q$	$(P \vee Q) \wedge \neg P \wedge \neg Q$
0	0	0	1	0	1	0
0	1 ✓	1	1	1	1	0
1	0	1	0	0	1	0
1	1	1	0	0	1	0

$\downarrow$ contradiction

$$P \vee Q \equiv \neg P \to Q.$$

$$\frac{\neg P}{Q\ ?}$$

VALID!

$(P \wedge \neg P) \to Q \Rightarrow$ tautology.

$\neg((P \wedge \neg P) \to Q) \Rightarrow$ contradiction.

Premises $\qquad$ Conclusion. $\qquad$ (4)

$$\underbrace{P,\ P \to Q,\ Q \to R,\ (P \wedge R) \to S}_{\text{Premises}} \vdash \underbrace{S}_{\text{Conclusion}}\ ?$$

Proof

1.	P	data.
2.	$P \to Q$	data.
3.	$Q \to R$	data.
4.	$(P \wedge R) \to S$	data.
5.	Q	$\to E$, 1., 2.
6.	R	$\to E$, 5., 3.
7.	$P \wedge R$	$\wedge I$, 1., 6.
8.	S	$\to E$, 7., 4.

$P,\ P \to Q \vDash P \qquad (P \wedge (P \to Q)) \to P$

$\to E$ "modus ponens"

$A \wedge B \vdash B \wedge A$. ? $\qquad A \wedge B \vDash B \wedge A$

1.	$A \wedge B$	data.
2.	A	from $\wedge E$, 1.
3.	B	from $\wedge E$, 1.
4.	$B \wedge A$	from $\wedge I$, 2., 3.

$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$.

$\neg(P \wedge Q) \vDash \neg P \vee \neg Q$

and

$\neg P \vee \neg Q \vDash \neg(P \wedge Q)$

$\neg(P \wedge Q) \vdash \neg P \vee \neg Q$

$\neg P \vee \neg Q \vdash \neg(P \wedge Q)$

$$(A \vee B) \to C \vdash \underbrace{(A \to C)}_{X} \wedge \underbrace{(B \to C)}_{Y} \quad ?$$

$$\frac{X, \; Y}{X \wedge Y} \wedge I$$

$$\frac{\text{premises}}{A \to C} \to I \quad \text{if} \quad \frac{\text{premises}, A}{C}$$

$\to (A \vee B) \to C$

1. $(A \vee B) \to C$ data.
2. $A \to C$ from subcomputation.

2.1	A	assume.	$\subseteq$
2.2	$A \vee B$	$\vee I$, 2.1.	
2.3	C	$\to E$, 2.2, 1.	

3. $B \to C$.

3.1	B	assume	$\subseteq$
3.2	$A \vee B$	$\vee I$, 3.1.	
3.3	C	$\to E$, 3.2, 1.	

4. $(A \to C) \wedge (B \to C)$ $\wedge I$, 2., 3.

$\neg I \Rightarrow$

$$\frac{X \to Y, \; X \to \neg Y}{\neg X}$$

$$\frac{(A \wedge \neg A) \to Y, \; A \wedge \neg A \to \neg Y}{\neg \underbrace{(A \wedge \neg A)}_{X}}$$

$(A \land \neg A) \vDash A$.

$$\frac{A \land \neg A}{A} \land E \qquad \frac{A \land \neg A}{\neg A} \land E.$$

1. $(A \land \neg A) \to A$. from subcomp.

1.1.	$A \land \neg A$	assume.	$\underline{\underline{A}}$
1.2	A	from 1.1, $\land E$	

2. $(A \land \neg A) \to \neg A$ from subcomp

2.1	$A \land \neg A$	assume	$\neg A$.
2.2	$\neg A$	from 2.1, $\land E$.	

3. $\neg(A \land \neg A)$ from 1, 2, $\neg I$.

$$\frac{P \quad P \to Q}{Q}$$

"modus ponens"

$\to E$

$$\frac{\neg Q \quad P \to Q}{\neg P} \Rightarrow \frac{P \to Q}{\neg Q \to \neg P}$$

"modus tollens"

$$\begin{array}{l} C \models A \\ \neg C \models B \end{array} \Longrightarrow \models A \vee B$$

either	or
$v(C) = 1$	$v(C) = 0$.
Since $C \models A$: $v(A) = 1$.	$v(\neg C) = 1$.
and hence: $v(A \vee B) = 1$.	$v(B) = 1$. (Since $\neg C \models B$)
	$v(A \vee B) = 1$.

Conclusion: if $C \models A$ and $\neg C \models B$, then $\models A \vee B$