Spatial Update

5-22

Posterior distribution for weights

$$\boldsymbol{Y}_i \sim N_d(\boldsymbol{\mu}, \boldsymbol{\Lambda}), \boldsymbol{\epsilon}_i \sim N_d(\mathbf{0}, \boldsymbol{\Lambda}), \left(\frac{\boldsymbol{\epsilon}_i}{\sqrt{w_i}} \middle| w_i\right) \sim N_d(\mathbf{0}, w_i \boldsymbol{\Lambda})$$

$$w_i \sim \text{Gamma}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$$

$$p(w_i | \boldsymbol{Y}_i, \boldsymbol{\mu}_i, \Lambda) \propto |w_i \boldsymbol{\Lambda}|^{\frac{1}{2}} \exp\left(-\frac{(\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T w_i \boldsymbol{\Lambda} (\boldsymbol{y}_i - \boldsymbol{\mu}_i)}{2}\right) \times w_i^{\frac{\nu}{2}-1} \exp\left(-\frac{\nu}{2} w_i\right)$$

$$= w_i^{\frac{d}{2}} |\boldsymbol{\Lambda}|^{\frac{1}{2}} \exp\left(-\frac{w_i (\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Lambda} (\boldsymbol{y}_i - \boldsymbol{\mu}_i)}{2}\right) \times w_i^{\frac{\nu}{2}-1} \exp\left(-\frac{\nu}{2} w_i\right)$$

$$= w_i^{\frac{d+\nu}{2}-1} |\boldsymbol{\Lambda}|^{\frac{1}{2}} \exp\left(-w_i \frac{\nu + (\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Lambda} (\boldsymbol{y}_i - \boldsymbol{\mu}_i)}{2}\right)$$

$$w_i | \boldsymbol{y}_i, \boldsymbol{\mu}_i, \boldsymbol{\Lambda} \sim \text{Gamma}\left(\frac{d+\nu}{2}, \frac{\nu + (\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Lambda} (\boldsymbol{y}_i - \boldsymbol{\mu}_i)}{2}\right)$$

Normal (left) vs. *t*-distributed (right) error model

$$\boldsymbol{y}_i = \sum_{k=1}^{q} \boldsymbol{\mu}_k I(z_i = k) + \boldsymbol{\epsilon}_i$$

$$\boldsymbol{\epsilon}_i \sim N_d(\mathbf{0}, \boldsymbol{\Lambda}^{-1})$$

Priors:

$$\boldsymbol{\mu}_k \sim N_d(\boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0^{-1})$$

$$\boldsymbol{\Lambda} \sim W_d\left(\alpha, \text{diag}\,(\beta)_d^{-1}\right)$$

Posteriors:

$$\boldsymbol{\mu}_k \sim N\left((\boldsymbol{\Lambda}_0 + n_k\boldsymbol{\Lambda})^{-1}\left(\boldsymbol{\Lambda}_0\boldsymbol{\mu}_0 + \sum_{i:\{z_i=k\}} \boldsymbol{y}_i\right), (\boldsymbol{\Lambda}_0 + n_k\boldsymbol{\Lambda})^{-1}\right)$$

$$\boldsymbol{\Lambda} \sim W_d\left(n + \alpha, \left(\text{diag}(\beta)_d + \sum_i (\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T(\boldsymbol{y}_i - \boldsymbol{\mu}_i)\right)^{-1}\right)$$

Likelihood:

$$\boldsymbol{y}_i | z_i, \boldsymbol{\mu}, \Lambda \sim N\left(\sum_{k=1}^{q} \boldsymbol{\mu}_k I(z_i = k), \boldsymbol{\Lambda}^{-1}\right)$$

$$\boldsymbol{y}_i = \sum_{k=1}^{q} \boldsymbol{\mu}_k I(z_i = k) + \boldsymbol{\epsilon}_i / \sqrt{w_i}$$

$$\boldsymbol{\epsilon}_i \sim N_d(\mathbf{0}, \boldsymbol{\Lambda}^{-1})$$

Priors:

$$\boldsymbol{\mu}_k \sim N_d(\boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0^{-1})$$

$$\boldsymbol{\Lambda} \sim W_d\left(\alpha, \text{diag}\,(\beta)_d^{-1}\right)$$

$$w_i \sim \text{Gamma}\left(\frac{\nu}{2}, \frac{\nu}{2}\right)$$

Posteriors:

$$\boldsymbol{\mu}_k \sim N\left(\left(\boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda} \sum_{i:\{z_i=k\}} w_i\right)^{-1}\left(\boldsymbol{\Lambda}_0\boldsymbol{\mu}_0 + \sum_{i:\{z_i=k\}} w_i\boldsymbol{y}_i\right), \left(\boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda} \sum_{i:\{z_i=k\}} w_i\right)^{-1}\right)$$

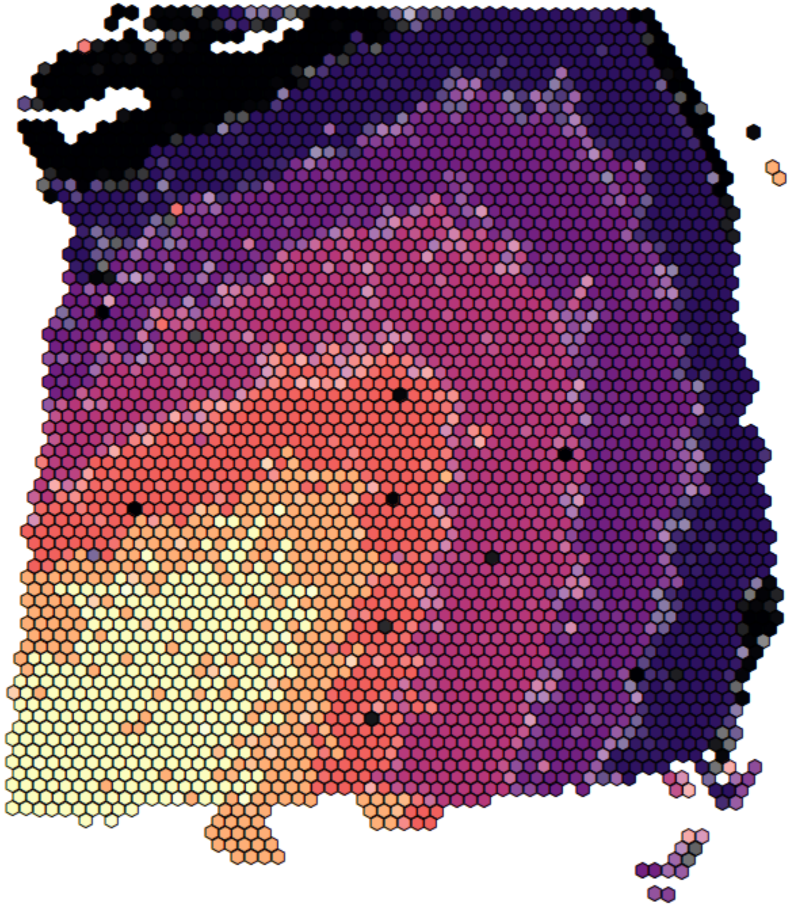
$$\boldsymbol{\Lambda} \sim W_d\left(\sum_i w_i + \alpha, \left(\text{diag}(\beta)_d + \sum_i w_i(\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T(\boldsymbol{y}_i - \boldsymbol{\mu}_i)\right)^{-1}\right)$$

$$w_i \sim \text{Gamma}\left(\frac{d+\nu}{2}, \frac{\nu + (\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T\boldsymbol{\Lambda}(\boldsymbol{y}_i - \boldsymbol{\mu}_i)}{2}\right)$$

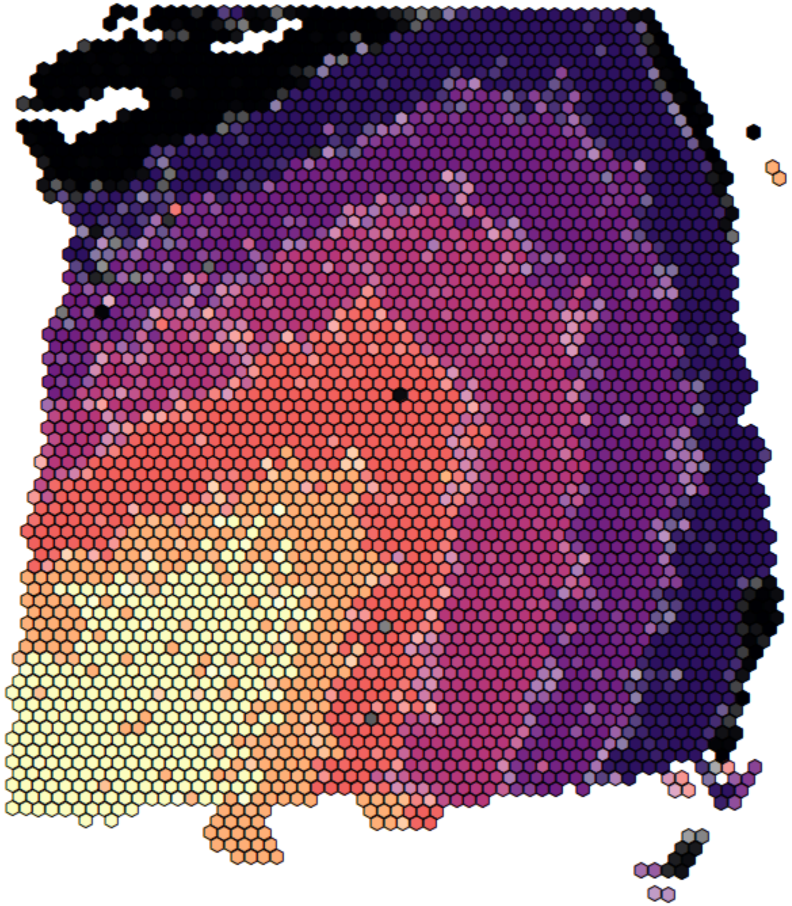
Likelihood:

$$\boldsymbol{y}_i | z_i, \boldsymbol{\mu}, \boldsymbol{\Lambda} \sim T_\nu\left(\sum_{k=1}^{q} \boldsymbol{\mu}_k I(z_i = k), \boldsymbol{\Lambda}^{-1}\right)$$

Normal (left) vs. *t*-distributed (right) error model



ARI with truth = 0.469
6 min runtime



ARI with truth = 0.478
7 min runtime

ARI normal vs t = 0.873

ARI as a function of chain length

ARI	Truth	Kmeans	Mode of 1:500 (21sec)	Mode of 1:1000 (42s)	Mode of 1000:2000 (84s)	Mode of 1000:10000 (420s)
Truth	1	0.281	0.480	0.481	0.479	0.479
Kmeans		1	0.459	0.458	0.457	0.458
Mode of 1:500			1	0.984	0.959	0.967
Mode of 1:1000				1	0.967	0.979
Mode of 1000:2000					1	0.981
Mode of 1000:10000						1

EEE vs VVV model

- Prior

$$\mathbf{\Lambda} \sim W_d\left(\alpha, \text{diag}\,(\beta)_d^{-1}\right)$$

- Posterior

$$\mathbf{\Lambda} \sim W_d\left(n + \alpha, \left(\text{diag}(\beta)_d + \sum_i (\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T (\boldsymbol{y}_i - \boldsymbol{\mu}_i)\right)^{-1}\right)$$

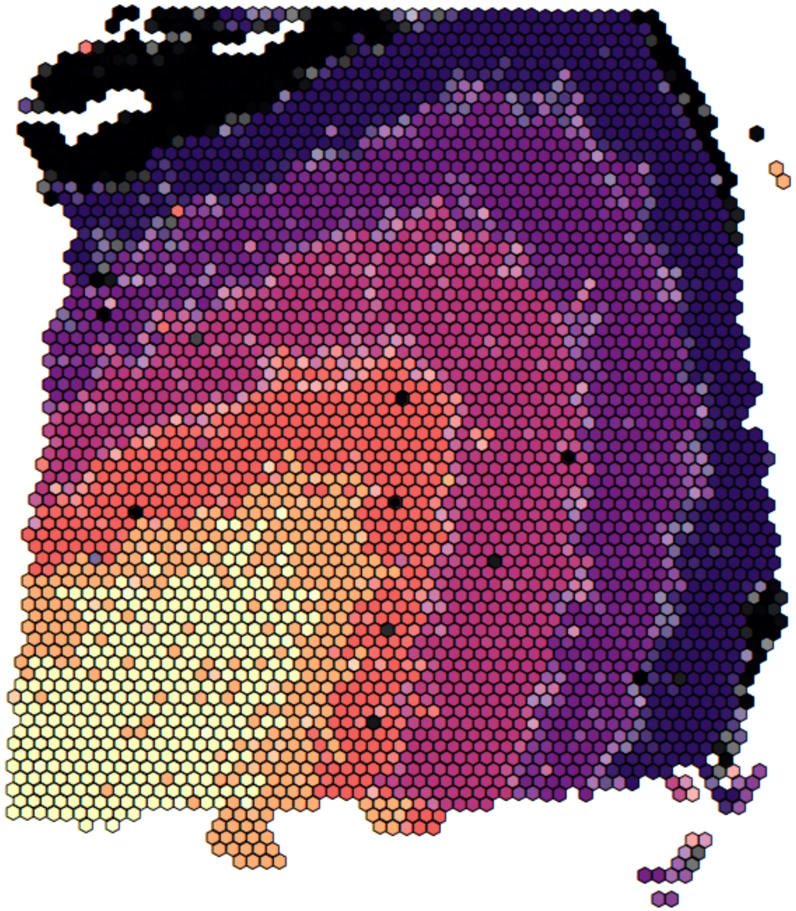
- Prior

$$\mathbf{\Lambda}_k \sim W_d\left(\alpha, \text{diag}\,(\beta)_d^{-1}\right)$$

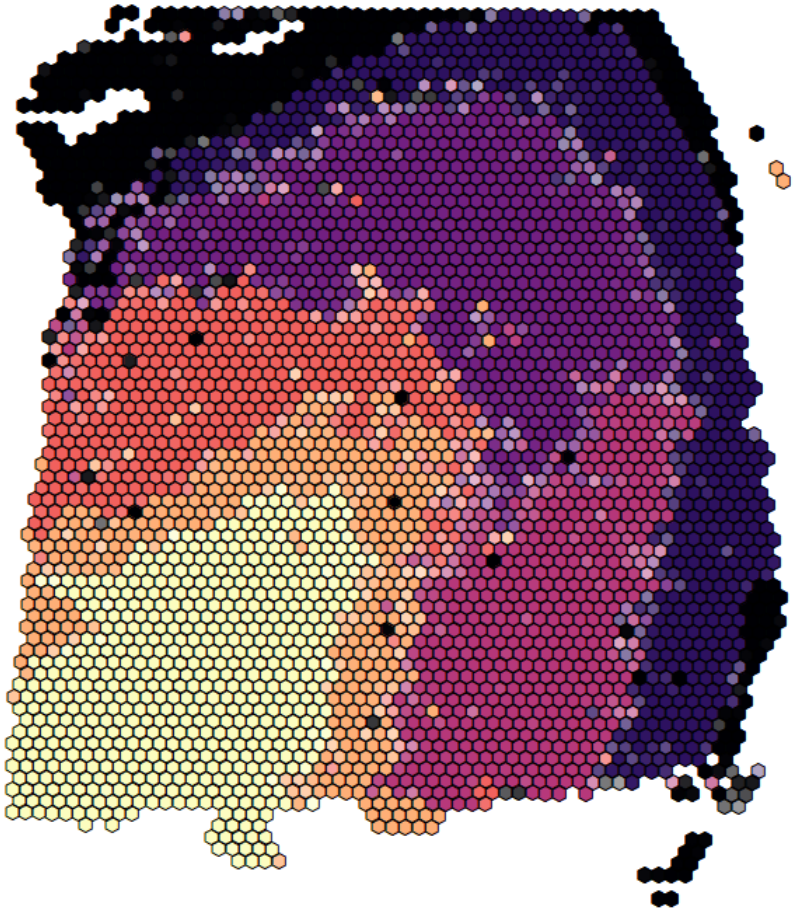
- Posterior

$$\mathbf{\Lambda}_k \sim W_d\left(n_k + \alpha, \left(\text{diag}(\beta)_d + \sum_{i:\{z_i = k\}} (\boldsymbol{y}_i - \boldsymbol{\mu}_i)^T (\boldsymbol{y}_i - \boldsymbol{\mu}_i)\right)^{-1}\right)$$

EEE Gaussian model (left) vs. VVV Gaussian model (right)



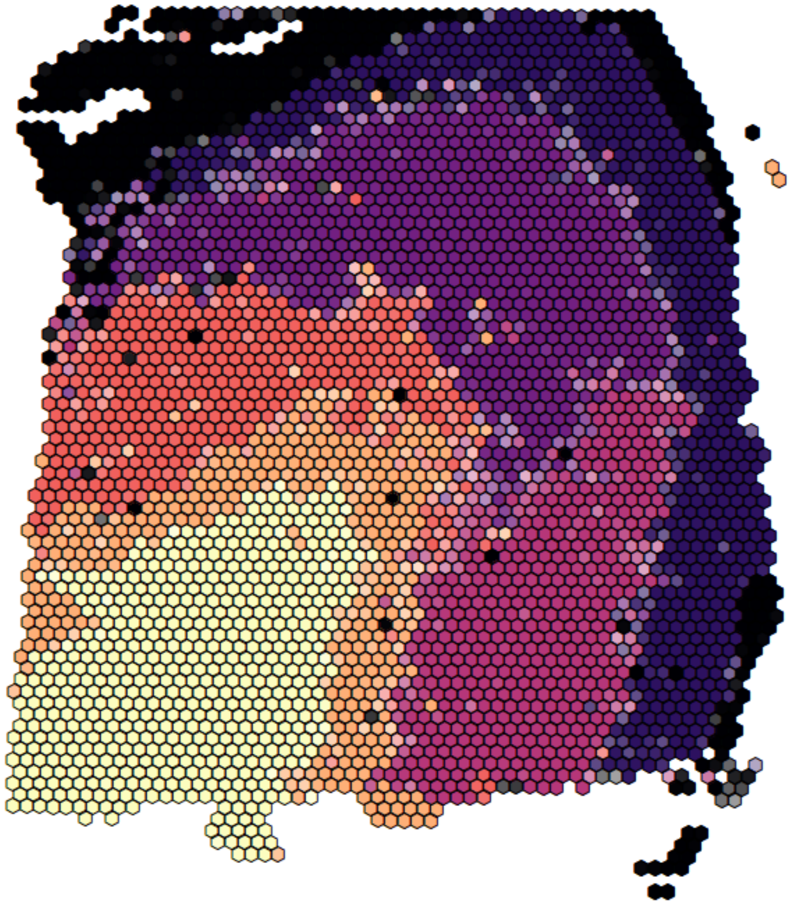
ARI with truth = 0.469

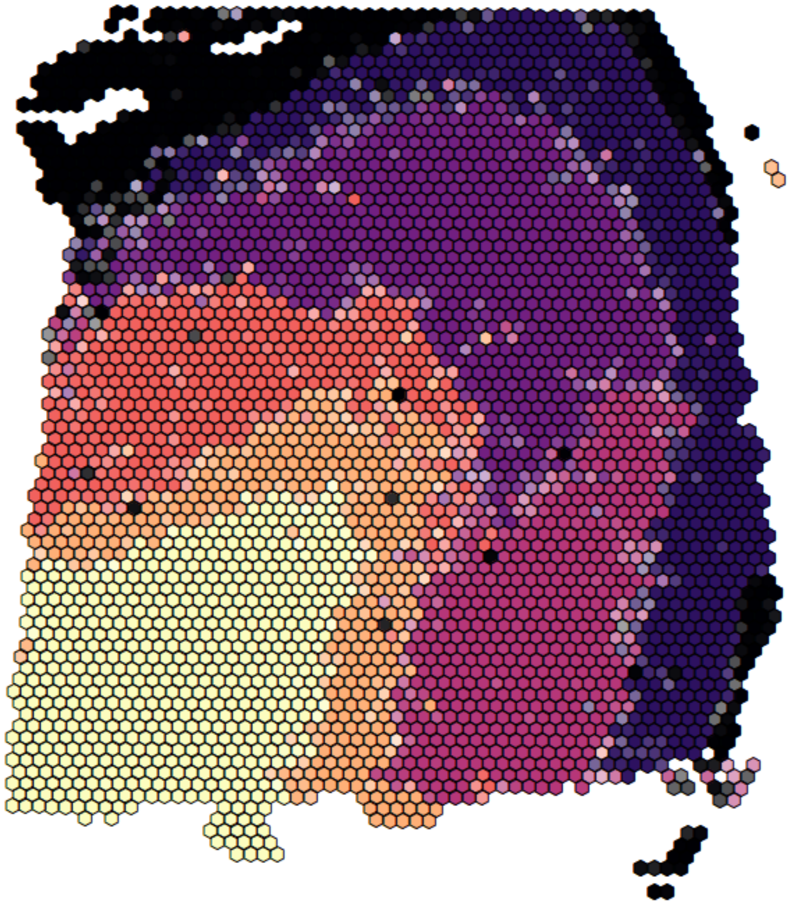


ARI with truth = 0.349

ARI EEE vs VVV = 0.430

VVV Gaussian model (left) vs. VVV *t* model (right)





ARI with truth = 0.349

ARI with truth = 0.367

ARI normal vs T = 0.898