2-person Stackelberg pricing games

GROUP 36

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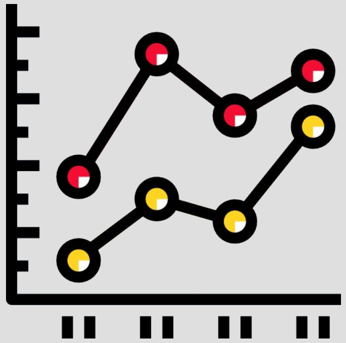
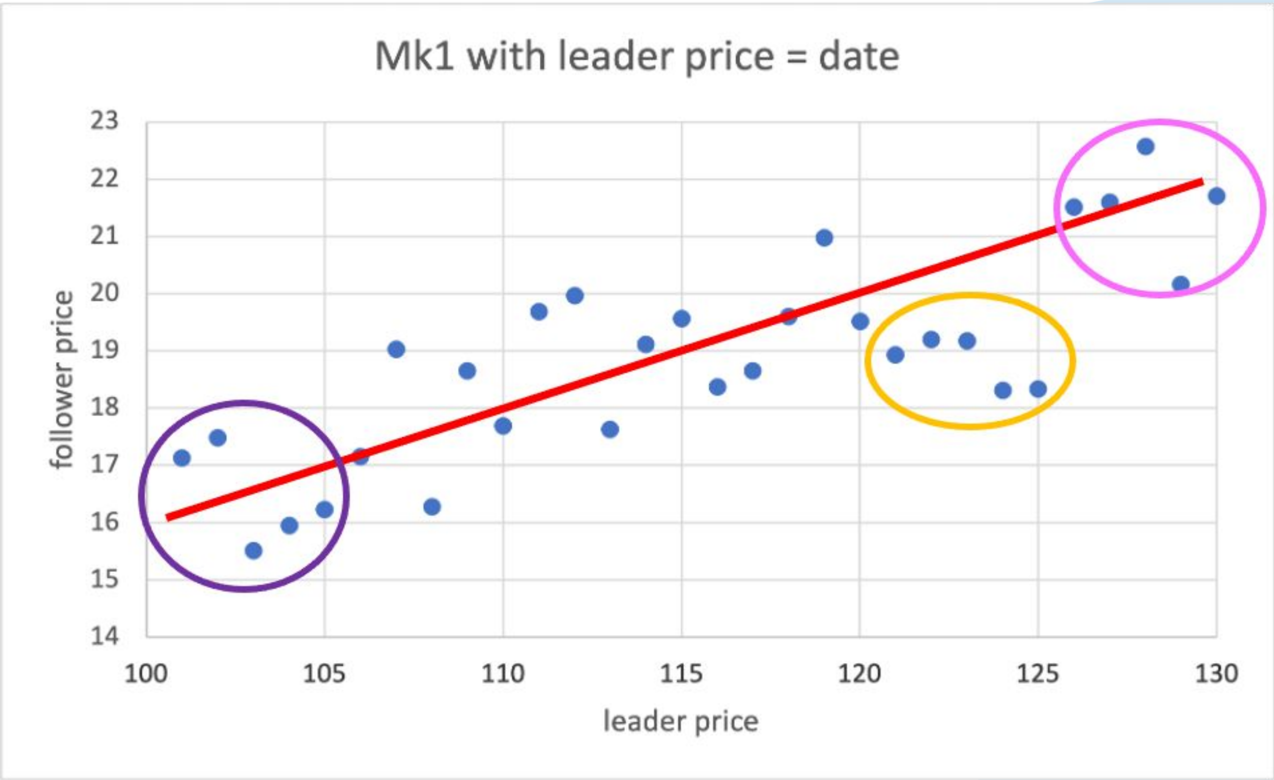
Analyze the behavior of the three followers

- Modify SimpleLeader.java

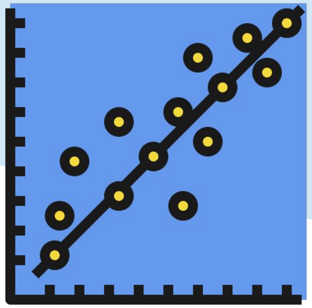
- Set the leader price to be **date**
- Plot for follower price against leader price

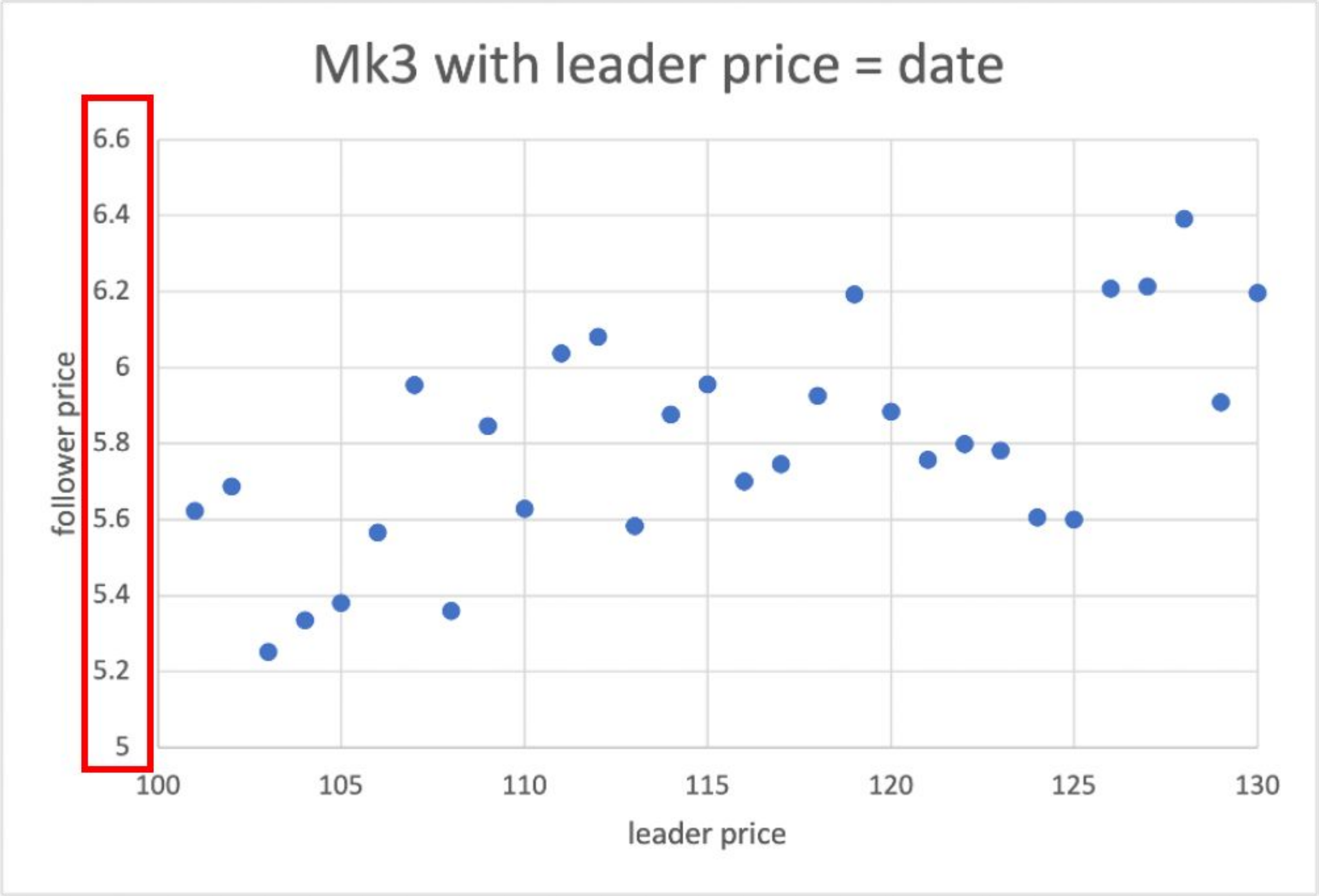
Linear or Non-Linear



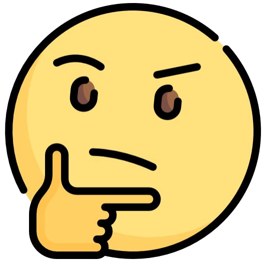


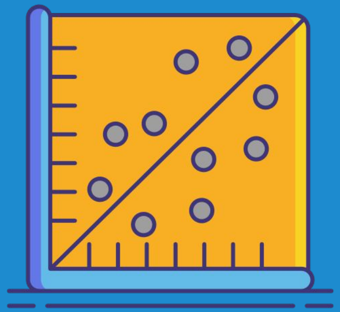
Mk1 and Mk2 are both linear





Mk3 could be
non-linear





Linear Approaches

Goal: Find the follower reaction function

- ALL HISTORICAL DATA
- MOVING WINDOW
- MODIFIED MOVING WINDOW
- WEIGHTED LEAST SQUARE WITH FORGETTING FACTOR

Best leader strategy given follower reaction

Given follower reaction

$$u_F = \hat{a} + \hat{b}u_L$$

Substitute into Leader profit equation

$$\begin{aligned}&(u_L - c_L)S_L(u_L, u_F)\\&= (u_L - 1)(2 - u_L + 0.3u_F)\\&= (u_L - 1)(2 - u_L + 0.3(\hat{a} + \hat{b}u_L))\end{aligned}$$

Group common terms

$$\begin{aligned}&2u_L - {u_L}^2 + 0.3\hat{a}\,u_L + 0.3\hat{b}\,{u_L}^2 - 2 + u_L - 0.3\hat{a} - 0.3\hat{b}\,u_L\\&= (0.3\hat{b} - 1)\,{u_L}^2 + (3 + 0.3\hat{a} - 0.3\hat{b})\,u_L + (-2 - 0.3\hat{a})\end{aligned}$$

Make 1st derivative equal to zero and solve the leader price

$$\frac{\partial}{\partial u_L}(0.3\hat{b} - 1)\,{u_L}^2 + (3 + 0.3\hat{a} - 0.3\hat{b})\,u_L + (-2 - 0.3\hat{a}) = 0$$

$$2(0.3\hat{b} - 1)u_L + (3 + 0.3\hat{a} - 0.3\hat{b}) = 0$$

$$\boxed{u_L = -\frac{(3 + 0.3\hat{a} - 0.3\hat{b})}{2(0.3\hat{b} - 1)}}$$

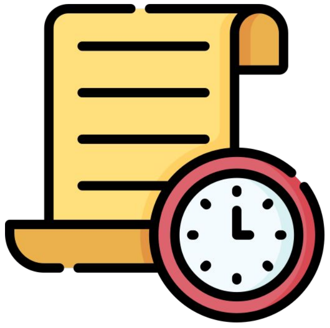
(Referenced from *Lecture03*)

All Historical Data Approach

- Use all the historical data to do the estimation

$$\sum_{t=1}^{T}\{y(t) - [a* + b* x(t)]\}^2 = min_{\{\hat{a},\hat{b}\}} \sum_{t=1}^{T}\{y(t) - [\hat{a} + \hat{b}\, x(t)]\}^2$$

In the equation x(t) is the leader price at day t
And y(t) is the follower price at day t



Convert the minimization problem to maximization problem.
Set the 1st order partial derivative equal to 0 to solve this problem.

$$\boxed{\hat{a}* = \frac{\sum_{t=1}^{T} x^2(t) \sum_{t=1}^{T} y(t) - \sum_{t=1}^{T} x(t) \sum_{t=1}^{T} x(t)y(t)}{T\sum_{t=1}^{T} x^2(t) - (\sum_{t=1}^{T} x(t))^2}}$$

$$\boxed{\hat{b}* = \frac{T\sum_{t=1}^{T} x(t)y(t) - \sum_{t=1}^{T} x(t) \sum_{t=1}^{T} y(t)}{T\sum_{t=1}^{T} x^2(t) - (\sum_{t=1}^{T} x(t))^2}}$$

It is checked that the 2nd order partial derivative is negative, so the solution found is indeed optimal.

(Referenced from *Lecture04 Slide16-20*)

Moving Window Approach

- Similar to All Historical Data Approach
- Use a window to loop only the recent data
- For example: a window size of 30 will just loop the data of past 30 days

(Referenced from *Lecture06 Slide 6-7*)

Modified Moving Window Approach

- The other term $\lambda(\theta - \theta_T^*)^\tau(\theta - \theta_T^*)$ added
- The past information is considered in θ_{T+1}^*

$$\theta_{T+1}^* = \min_\theta \left\{ \lambda(\theta - \theta_T^*)^\tau(\theta - \theta_T^*) + \sum_{t=T+1-T_w}^{T+1} \{y(t) - \hat{R}[X(t), \theta]\}^2 \right\}$$

- Balance remembering and new information using λ

(Referenced from *Lecture06 Slide 8*)

Modified Moving Window Approach

- A new formula can be got by simplifying the formula

$$\min_{\{\hat{a},\hat{b}\}} \sum_{t=T+1-T_w}^{T+1} \{y(t) - [\hat{a} + \hat{b}\,x(t)]\}^2 + \lambda(\hat{a} - a)^2 + \lambda(\hat{b} - b)^2$$

assume that the response function of the follower is

$$\hat{R}(X) = \hat{a} + \hat{b}x$$

Modified Moving Window Approach

- By applying the first order partial derivative equal to zero and solving linear equations, we could get formulas:

$$\hat{a} = \frac{t_1 n_2 - t_2 n_1}{n_2 m_1 - n_1 m_2},$$

$$\hat{b} = \frac{t_1 m_2 - t_2 m_1}{n_1 m_2 - n_2 m_1}$$

and

$$m_1 = (T_w + \lambda)$$

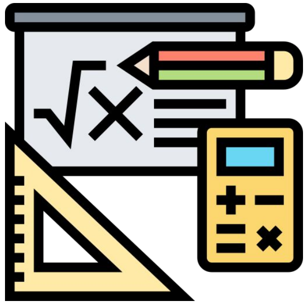
$$n_1 = \sum_{t=T+1-T_w}^{T+1} \{x(t)\}$$

$$t_1 = \sum_{t=T+1-T_w}^{T+1} \{y(t)\} + \lambda a$$

$$m_2 = \sum_{t=T+1-T_w}^{T+1} x(t)$$

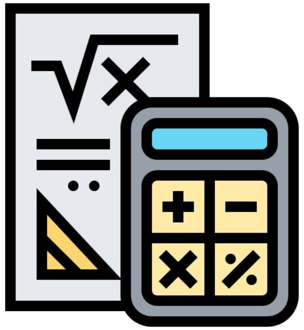
$$n_2 = (\sum_{t=T+1-T_w}^{T+1} \{x^2(t)\} + \lambda)$$

$$t_2 = \sum_{t=T+1-T_w}^{T+1} \{y(t)x(t)\} + \lambda b$$



Modified Moving Window Approach

- It has been verified that the above parameters are indeed minimising the errors



Weighted Least Square with Forgetting Factor Approach

- A Forgetting Factor is introduced to discount old data
- The more recent data will be fitted better than the older data

$$\theta^*_{T+1} = \arg\min_{\theta} \sum_{t=1}^{T+1} \lambda^{T+1-t} \left\{ y(t) - \hat{R}[X(t), \theta] \right\}^2$$

*In this equation, $\hat{R}(X, \theta^*_T)$ is the weighted least square

(Referenced from *Lecture06 Slide 9-10*)

Weighted Least Square with Forgetting Factor Approach

$$\hat{a} = \frac{n_1 m_1 - n_2 n_3}{n_1^2 - m_2 n_2}$$

$$\hat{b} = \frac{n_1 n_3 - m_1 m_2}{n_1^2 - m_2 n_2}$$

$$m_1 = \sum_{t=1}^{T+1} \lambda^{T+1-t}\, x(t)\, y(t)$$

$$m_2 = \sum_{t=1}^{T+1} \lambda^{T+1-t}$$

$$n_1 = \sum_{t=1}^{T+1} \lambda^{T+1-t}\, x(t)$$

$$n_2 = \sum_{t=1}^{T+1} \lambda^{T+1-t}\, x^2(t)$$

$$n_3 = \sum_{t=1}^{T+1} \lambda^{T+1-t}\, y(t)$$

Formula is deduced by calculating first order partial derivative equal to zero

- Old data are still be used but plays a less important (discount) role than new data

- Smoothly balance the old and new data

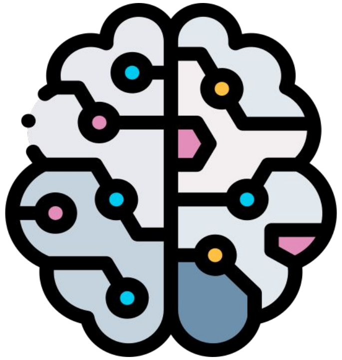
(Referenced from *Lecture06 Slide 9-10*)

Non-Linear Approach

Machine Learning Approach

Goal: Find a price for the leader to get maximum profit

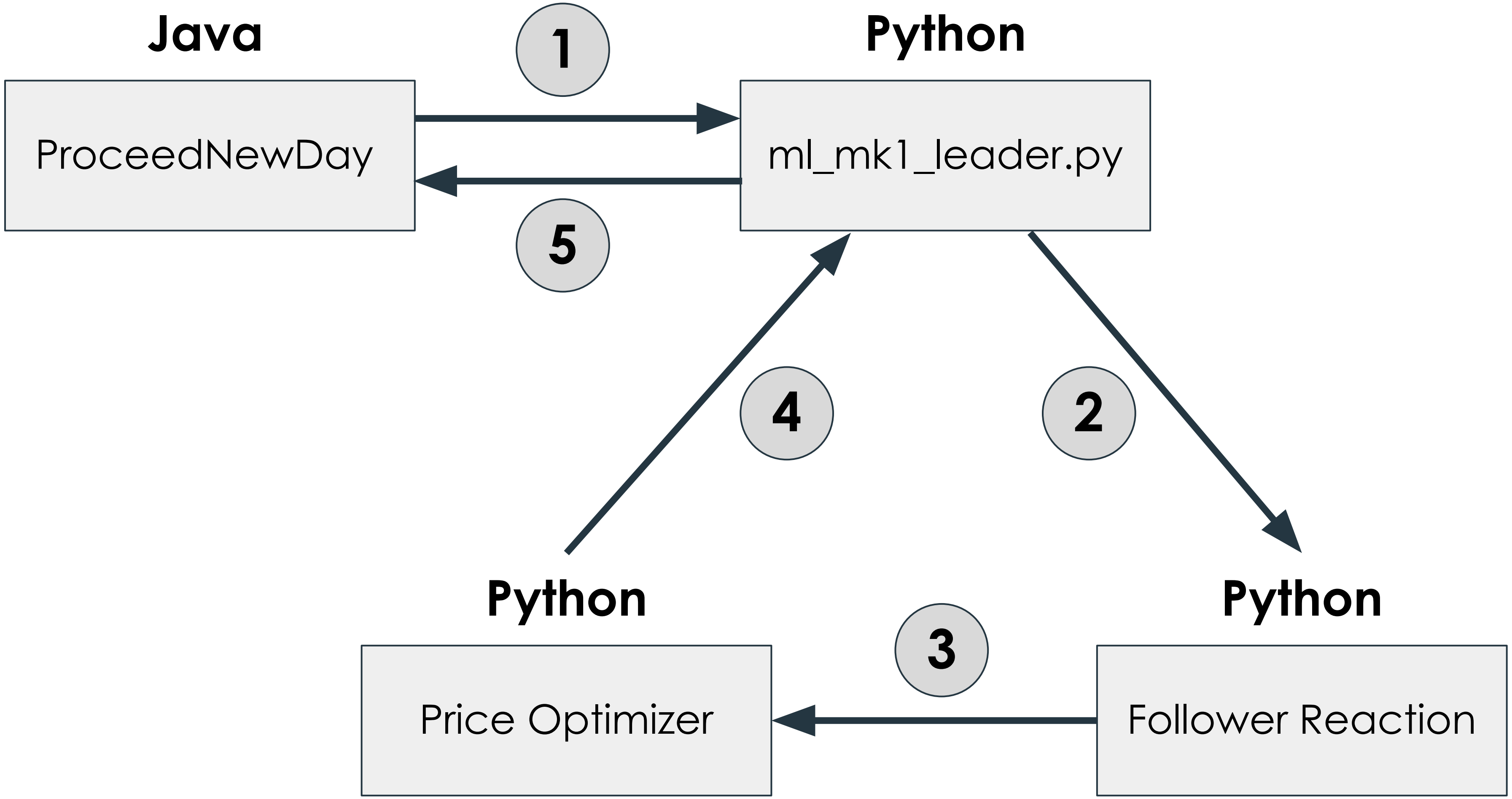
Machine Learning Approach

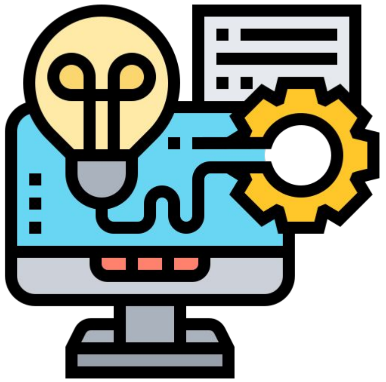


1. Follower Reaction Model
   a. linear + relu layers
   b. Mean Square Error + SGD + Reduce-on-plateau
   c. Many models are experimented and the best is selected
2. Price Optimizer Model
   a. Freeze the follower reaction model
   b. profit = (leader - 1) * (2 - leader + 0.3 * follower)
   c. Loss = -(profit)

https://pytorch.org/docs/stable/optim.html#torch.optim.lr_scheduler.ReduceLROnPlateau

Machine Learning Approach





Results and Evaluation

	All historical data	Moving window Size = 100	Modified moving window Size = 30 Lambda = 0.95	Weighted least square with forgetting factor Lambda = 0.99	Machine Learning	Best approach
Mk1	**17.5571613311768**	17.5557403564453	17.5533294677734	17.5550231933594	17.5550460815429	All historical data
Mk2	16.9564590454102	16.9558563232422	**16.9571857452393**	16.9558162689209	16.9468994140625	Modified moving window
Mk3	19.4882831573486	19.4883308410645	19.4879131317139	19.4882564544678	**19.4884433746337**	Machine Learning

Analysis and Conclusion

- All historical has most contextual information
- Modified moving window deals better with date related information
- Machine learning can model complex function

	Best approach	Analysis
Mk1	All historical data	**Linear** and **not depend** on date
Mk2	Modified moving window	**Linear** and **depend** on date
Mk3	Machine Learning	**Non-linear**

References

- Lecture03
- Lecture04 Slide16-20
- Lecture06 Slide 6-10
- https://pytorch.org/docs/stable/optim.html#torch.optim.lr_scheduler.ReduceLROnPlateau

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Thank you !

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