LAB 2: Joint Space Motion Control Lab: Control of a 6-DoF Serial Manipulator

Michele Focchi and Octavio Villarreal
Edited by Thiago Boaventura

The goals of this assignment are:

- learning the basic procedure to design a motion controller in the joint space for a manipulator in free-motion (i.e. not in contact)

- analyze the advantages/disadvantages of decentralized/ centralized approaches (i.e. feedback linearization)

- implement the interaction with the environment with a compliant contact model

Tools that we are going to be using in this lab (all open-source):

- Python programming (2.7)[1]

- Robot Operating System (ROS)[2]

- Pinocchio Library for Rigid Body Dynamics Algorithms[3]

- Locosim[4]

In this assignment, we will simulate the behavior of a 6-DoF anthropomorphic robot manipulator (UR5, see Fig. 1) under the action of controllable joint torques. Initially, we will design some reference trajectories to track, defined at the joint level. Then, we will design several control laws with increasing complexity evaluating their impact on the accuracy and the tracking errors at the joint. The loop will be always closed at the joint level.

0   Preliminaries

To test the RVIZ visualization and have a grasp on the kinematic of the UR5 robot, try to move the joints with the sliders, running:

```
roslaunch visualize.launch robot_name:=ur5 test_joints:=true
```
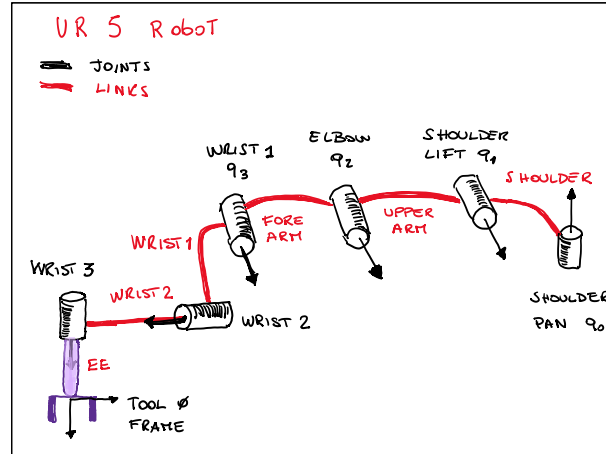
---

[1]https://docs.python.org/2.7/
[2]https://www.ros.org/
[3]https://github.com/stack-of-tasks/pinocchio
[4]https://github.com/mfocchi/locosim

Figure 1: Kinematics of the UR5 Robot.

1 Decentralized control (Lecture G0)

1.1 - Sinusoidal Reference generation: Generate a sinusoidal reference for the 2nd (Shoulder Lift) and 5th (Wrist 2) joint with frequency 1.0 Hz and 1.5 Hz and amplitude $0.2rad$ and 0.4 rad, respectively, starting from their initial configurations in q_0. Keep the other joints at q_0. After 4.0 s stop the sine and give a constant reference during 1.0 s. Note that you need to generate consistent references for velocity and acceleration.

1.2 - Step Reference generation: generate a -0.4 rad, 0.5 rad step reference change step at $t = 2.0s$ for the 2nd (Shoulder Lift) and 5th (Wrist 2) joint, respectively, starting from their initial configuration in q_0. Keep the other joints at q_0.

1.3 - Joint PD control: Implement a Joint PD control law with proportional $K_p = 300 \ Nm/rad$ and derivative $K_d = 20 \ Nms/rad$ gains for all joints. Use the step reference generated in (1.2). Plot the reference and the actual joint position for all joints and see that a tracking error is present both during the transient and at steady state. Explain why there is almost no tracking error in the 5th joint. Is there any overshoot in 2nd joint?

1.4 - Joint PD control – high gains: Using the same PD control law, increase the gains to $K_p = 600 \ Nm/rad$ and $K_d = 30 \ Nms/rad$. Use the step reference generated in (1.2). See that the tracking error is reduced by almost half (i.e., from 0.2 to 0.1 rad on 2nd joint). Play around with the value of K_d. See that the system becomes unstable if you increase too much K_d (e.g. for $K_d > 33$) because the phase margin is going to zero. Now remember that the digital implementation of the controller introduces delays. Try to reduce the sampling time dt to 0.0001 s and see what is the maximum value of K_d you can set (i.e. 300) before the robot gets unstable.

1.5 - Joint PD control critical damping: Until now, we did not take into account the

fact that the inertia changes with the joint configuration, therefore a fixed value of K_d is not suitable to obtain the same transient shape at different joint configurations. Set the proportional gain back to a constant value $K_p = 300 \ Nm/rad$, but update the derivative gain K_d at each loop in order to achieve a *critical damping* for all joints. We explained that the PD control can be seen as equivalent to a $2nd$ order spring-mass-damper system with stiffness K_p and damping K_d. Therefore you can use the inertia seen by each joint (i.e., the elements $m_{i,i}$ on the diagonal of the inertia matrix) and set $K_{d,i} = 2\sqrt{(K_{p,i}m_{i,i})}$ to achieve a critical damping response (natural damping ratio $\zeta = 1$). Use the step generated in (1.2) as a reference and see that the overshoot disappears in 2nd joint, but there is a steady-state error due to gravity. Using the sinusoidal reference in (1.1) check that the variation in the damping of the 2nd joint is much bigger that for the 5th joint, because the inertia seen by a joint is dependent on the motion of all subsequent joints and more proximal joints have more joints that are moving after them.

1.6 - Joint PD control + Gravity Compensation: Now, let us add a gravity compensation term to compensate the gravity forces at the joints. Compute this term with Pinocchio library at each loop and notice it is changing with the configuration. Give the step reference generated in (1.2) as input to the closed loop system. Check there is no longer a tracking error at steady state, but it is still present during the transient. What are these errors due to? Check also the tracking with the sinusoidal reference generated in (1.1).

1.7 - Joint PD + gravity + Feed-Forward term: At this point, let's see the impact of adding a feed-forward term to the PD control (advice: use the inertia seen by each joint). This will improve the tracking error for a time-varying reference signal, so use the sinusoidal reference generated in (1.1). See that the tracking error is significantly improved for the joints that are moving. Now, check the tracking of the 1st joint with the step reference (1.2), and notice that there are still disturbances coming from the motion of the other joints. These disturbances are due to the *inertial couplings* and cannot be dealt with a *decentralized* approach.

2   Centralized control (Lecture G1)

2.1 - Inverse Dynamics (Computed Torque): To effectively compensate for the inertial couplings, we can compensate them by linearizing the (non-linear) dynamics of the robot via a state feedback called feed-back linearization. This approach, that is valid for any nonlinear system, takes the name of *Inverse Dynamics*(Computed Torque). The intuition is based on cancelling the nonlinear terms and decoupling the dynamics of each link. The basic idea of feedback linearization is to construct a transformation which exactly linearizes the nonlinear system after a suitable state space change of coordinates.

One can then design a second stage or outer-loop control in the new coordinates to satisfy the traditional control design specifications such as tracking, disturbance rejection, etc. Implement the inverse dynamics (computed Torque) algorithm at the joint space. Keep the same K_p, K_d gains as in (4) and add a Feed-forward acceleration term $\ddot{q}^d$.

$$u = M(q)(\ddot{q}^d + K_p(q^d - q) + K_d(\dot{q}^d - \dot{q}) + h(q, \dot{q}))$$

Observe that the tracking errors now are always zero for the joints that are not moving, the inertial couplings are properly compensated. Now plot also the tracking of joint velocity. Why there is a tracking error at the beginning?

2.2 - Inverse Dynamics (Computed Torque) – initial velocity: Change the initial value of the joint velocities to be consistent with the initial reference velocity, and see the tracking error is zero also at the beginning, for all joints.

2.3 - Inverse Dynamics (Computed Torque) – low gains: Inverse dynamics completely decouple the system into a set of pure double integrators with the new input v:

$$\ddot{q}_i = v_i \quad i = 1 \ldots n$$

then the transient of each joint can be tuned separately for each joint, setting the gains in the respective control law:

$$v_i = \ddot{q}_i^d + K_{p,i}(q_i^d - q_i) + K_{d,i}(\dot{q}_i^d - \dot{q}_i) \quad i = 1 \ldots n$$

Note that to achieve critical damping now you need to set $k_{d,i} = 2\sqrt{(K_{p,i})}$ because inertia is unitary. In the absence of external disturbances, verify that the tracking is perfect, even if you decrease the K_p, K_d gains. Reduce K_p to $1/5^{t}h$ ($K_p = 60$) and observe the tracking is still perfect both for the joints in motion and not in motion. This is different with respect to the decentralized approach where decreasing the gain makes the influence of the inertial coupling more apparent and prominent.

2.4 - Inverse Dynamics (Computed Torque) – External force: Now try to add an external (3D) force applied at the end-effector $F_{ext} = [0, 0, 50]N$ (i.e. along the Z direction), starting at $t = 3s$ (hint: add $J^T F_{ext}$ to τ). Check than now, tracking errors appear on all joints due to the external disturbances that affect all the joints though J^T. Restore K_p back tp 300 and see that the tracking error reduces to $1/5^{th}$ because we have linearized the system. Play with different values of the K_p gain shows that with higher values of the gain the error is reduced.

2.5 - Inverse Dynamics (Computed Torque) – uncertainty in the cancellation: In real systems, we always have uncertainty in our model due to modeling limitations of

CAD drawing, non idealities, unmodeled dynamics, compliance of the structure, etc. This means that the matrix $\hat{M}$ and $\hat{h}$ we use in the inverse dynamics will be slightly different from the real ones M and h, leading to a not perfect cancellation. Add a 10% parametric variation (uncertainty) to scale the elements of M and h to obtain $\hat{M}$ and $\hat{h}$ to be used in the inverse dynamics scheme. Evaluate the degradation in tracking performances due to a not perfect cancellation, with the sinusoidal reference (1.1).

2.6 - Inverse Dynamics (Desired states): Now remove the parametric scaling from M and h but use the desired values of the states $\dot{q}^d$, q^d instead of the actual values $\dot{q}$, q to compute them. During the transient some coupling start to appear because there is always a small difference between q^d and q, so not perfect cancellation. This worsens significantly at the end of the sinusoids when there is not a smooth derivative of the reference that causes a bigger tracking error.

2.7 - Unilateral Compliant contact: To complete this lab, we want to consider an interaction of the robot with the environment happening at the end-effector. Model the contact using a compliant model with linear springs $K_{env} \in \mathbb{R}^{3 \times 3}$ and dampers $D_{env} \in \mathbb{R}^{3 \times 3}$ and insert the resulting contact force $F_{env} \in R^3$ into the free-motion dynamics via the J^T. Consider the contact *unilateral* and for now allow the force to be generated only along the Z direction (contact normal). Note you need also to compute the end-effector velocity for the damping term (i.e. use $\dot{p} = J\dot{q}$).

$$\begin{cases} F_{env} = n^T(K_{env}(p_0 - p) - D_{env}\dot{p}) & \text{if} \quad n^T(p_0 - p) > 0 \\ F_{env} = 0 & \text{if} \quad n^T(p_0 - p) < 0 \end{cases}$$

The approach we used to integrate the acceleration is not suitable for stiff dynamic equations. Try to increase the stiffness K_{env} until the limit where the integration scheme becomes unstable. Notice that the end-effector is sliding at the contact in the horizontal direction because we projected F_{env} along the contact normal. Try to use the full 3D force computed by the contact model:

$$\begin{cases} F_{env} = K_{env}(p_0 - p) - D_{env}\dot{p} & \text{if} \quad n^T(p_0 - p) > 0 \\ F_{env} = 0 & \text{if} \quad n^T(p_0 - p) < 0 \end{cases}$$

Notice that the contact does not move in the horizontal direction, like if the end-effector was clamped on the floor and internal motions appear on the joints that comply with the contact constraint.

2.8 - Coulomb friction: However, our contact point is not "glued" to the ground and there is a limit on the tangential force that depends on the friction coefficient μ (that depends on both contacting surfaces), beyond which the end-effector starts to slip (Coulomb

friction model). Incorporate a friction cone constraint by clamping appropriately the tangential forces using a friction coefficient $\mu = 0.8$:

$$\begin{cases} F_{env,x} = \mu F_{env,z} & \text{if} \quad F_{env,x} \geq \mu F_{env,z} \\ F_{env,x} = -\mu F_{env,z} & \text{if} \quad F_{env,x} \leq -\mu F_{env,z} \\ F_{env,y} = \mu F_{env,z} & \text{if} \quad F_{env,y} \geq \mu F_{env,z} \\ F_{env,y} = -\mu F_{env,z} & \text{if} \quad F_{env,y} \leq -\mu F_{env,z} \end{cases}$$

Notice that, for $\mu = 0.8$, the friction forces are still high enough to keep the contact in the horizontal direction and no slippage occurs. Reduce the friction coefficient to $\mu = 0.2$ and observe that a slippage starts to occur.