Chapter 4
Linked Lists

4.1 Singly Linked lists Or Chains

The representation of simple data structure using an array and a sequential mapping has the property:

- **Successive nodes of the data object are stored at fixed distance apart.**
- **This makes it easy to access an arbitrary node in O(1).**

Disadvantage of sequential mapping:

Insertion and deletion of arbitrary elements is expensive.

For example:

Insert "GAT" into or delete "LAT" from

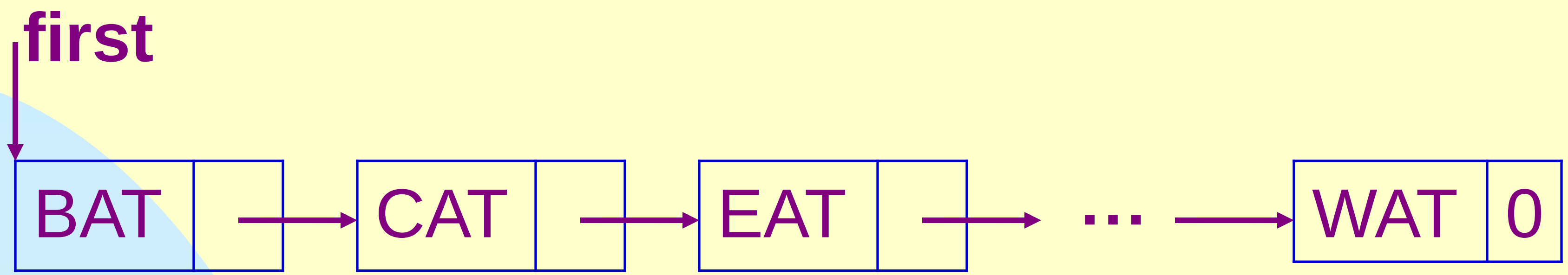
(BAT, CAT, EAT, FAT, HAT, JAT, LAT, MAT, OAT, PAT, RAT, SAT, TAT,VAT,WAT)

need data movement.

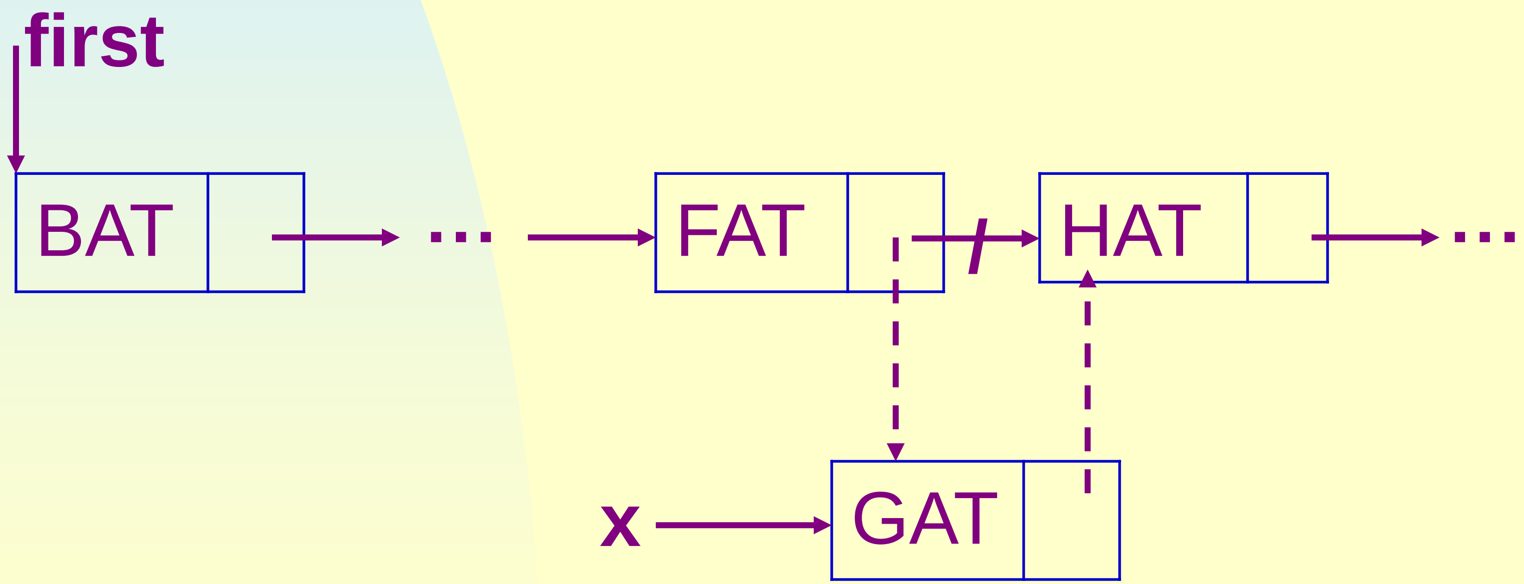
Solution---linked representation:

items of a list may be placed anywhere in the memory.

Associated with each item is a point (link) to the next item.



In linked list, insertion (deletion) of arbitrary elements is much easier:



The above structures are called singly linked lists or chains in which each node has exactly one pointer field.

- list elements are stored, in memory, in an arbitrary order
- explicit information (called a link) is used to go from one element to the next

Memory Layout

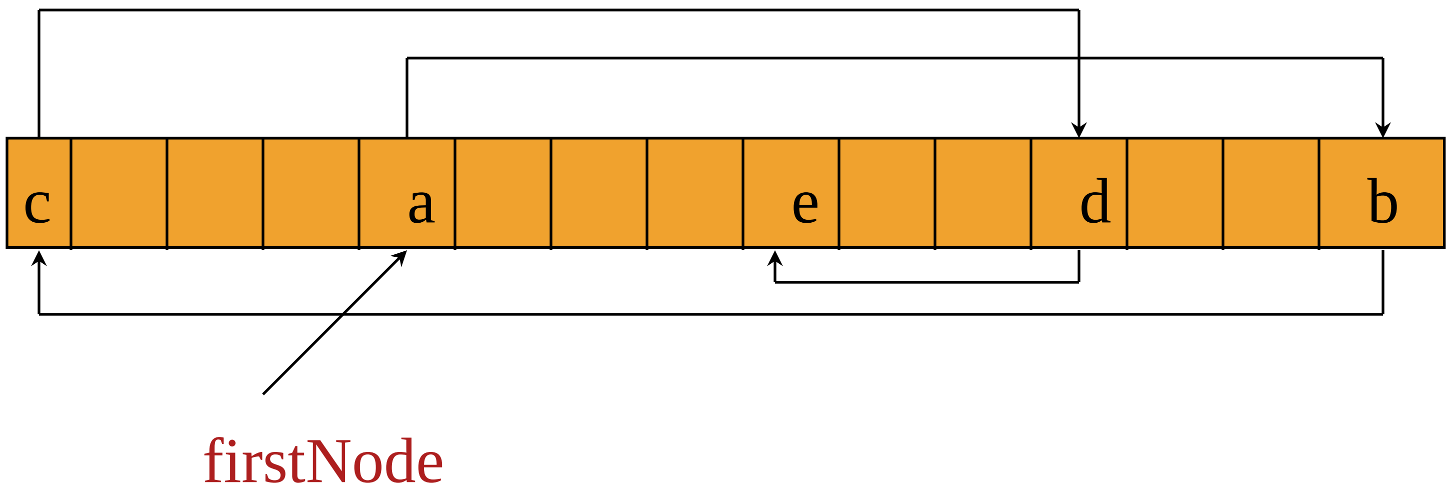
Layout of L = (a,b,c,d,e) using an array representation.

a	b	c	d	e										

A linked representation uses an arbitrary layout.

c				a				e			d			b

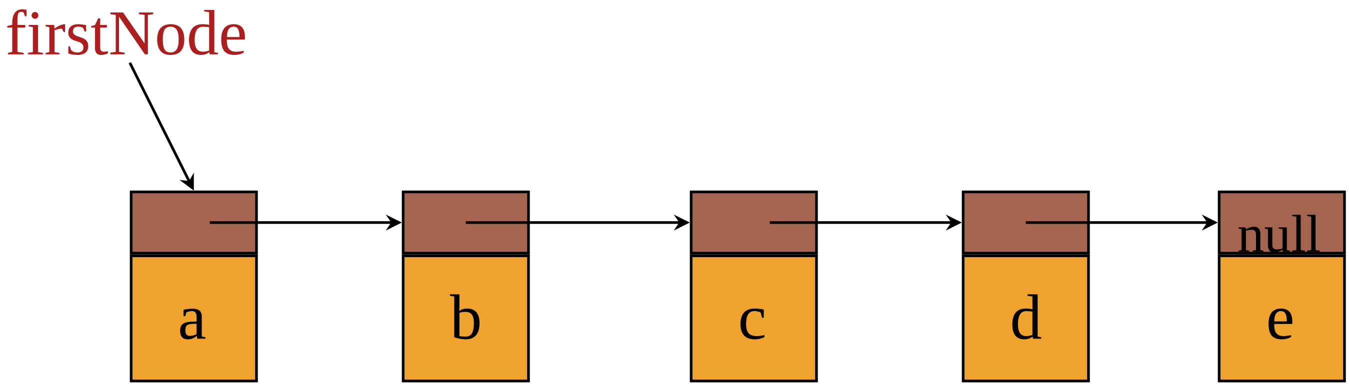
Linked Representation



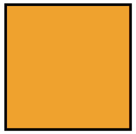
pointer (or link) in e is null

use a variable firstNode to get to the first element a

Normal Way To Draw A Linked List

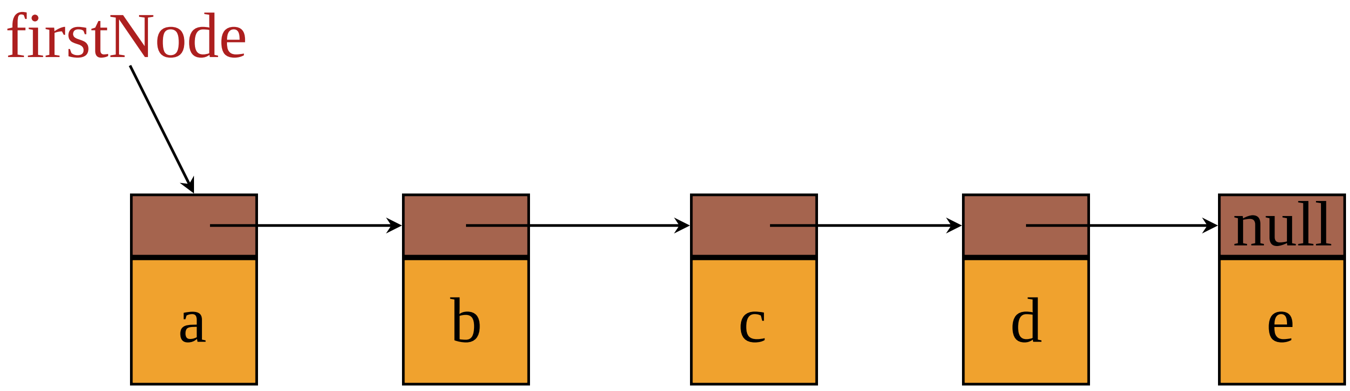


link or pointer field of node



data field of node

Chain



- A chain is a linked list in which each node represents one element.
- There is a link or pointer from one element to the next.
- The last node has a null pointer.

4.2 Representing Chains in C++

4.2.1 Defining A node in C++

```
class ChainNode {
private:
    int data;
    ChainNode *link;
};
```

4.2 Representing Chains in C++

4.2.2 Designing a Chain Class in C++

Attempt 1:

ChainNode *f;

f→data;

will cause a compiler error because a private data member cannot be accessed from outside of the object.

Attempt 2:

Define public member functions to directly access the data members.

This is not a good solution, however, because this solution allows one to read and change these data members from anywhere in the program.

4.2 Representing Chains in C++

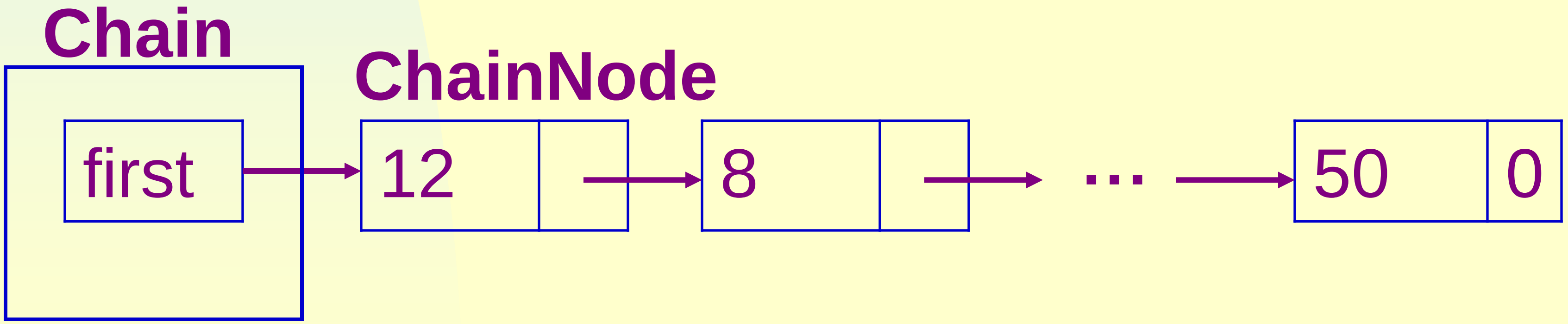
4.2.2 Designing a Chain Class in C++

The ideal solution would only permit those functions that perfor list manipulation operations (like inserting a node into or deleting a node from a chain) access to the data members of the ChainNode.

Attempt 3:
Using a composite of two classes: ChainNode and Chain

Definition: a data object of Type A HAS-A data object of Type B if A conceptually contains B or B is a part of A.

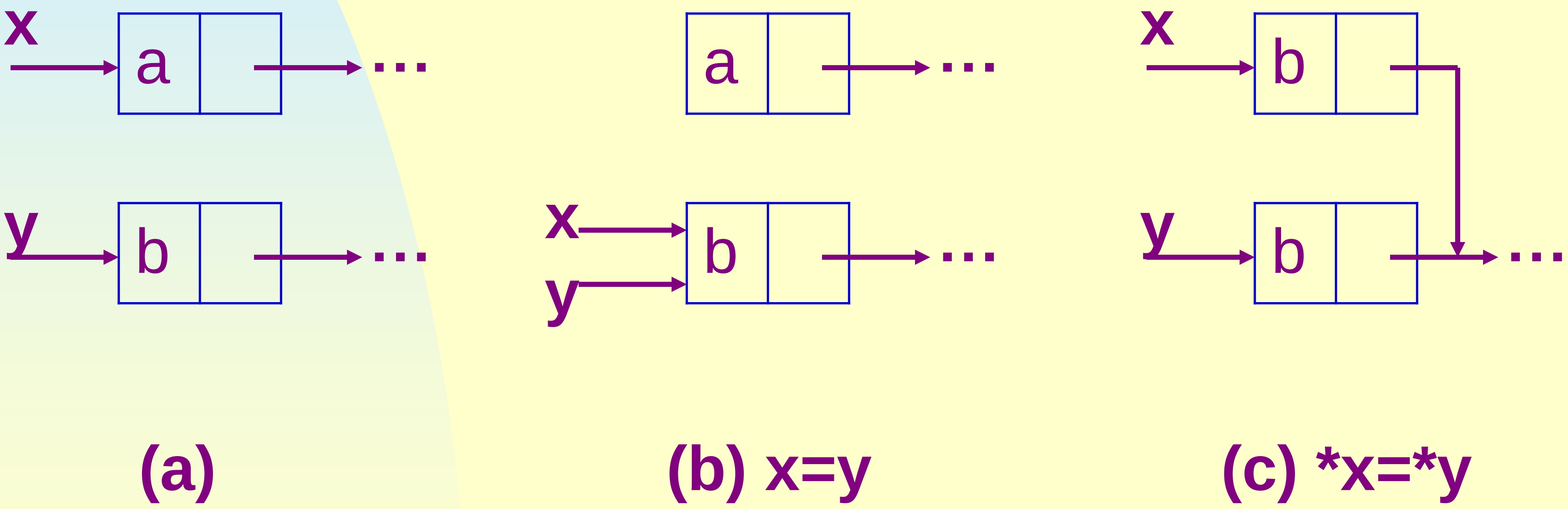
Chain HAS-A ChainNode.



```
class Chain;  // forward declaration
class ChainNode {
friend class Chain;
 // to make functions of Chain be able to
// access private data members of ChainNode
Public:
    ChainNode(int element = 0, ChainNode* next = 0)
        {data = element; link = next;}
private:
    int data;
    ChainNode *link;
};
class Chain {
public:
    // Chain manipulation operations
…
private:
    ChainNode *first;
};
```

4.2.3 Pointer manipulation in C++

Null pointer constant 0 is used to indicate no node.



4.2.4 Chain manipulation Operations

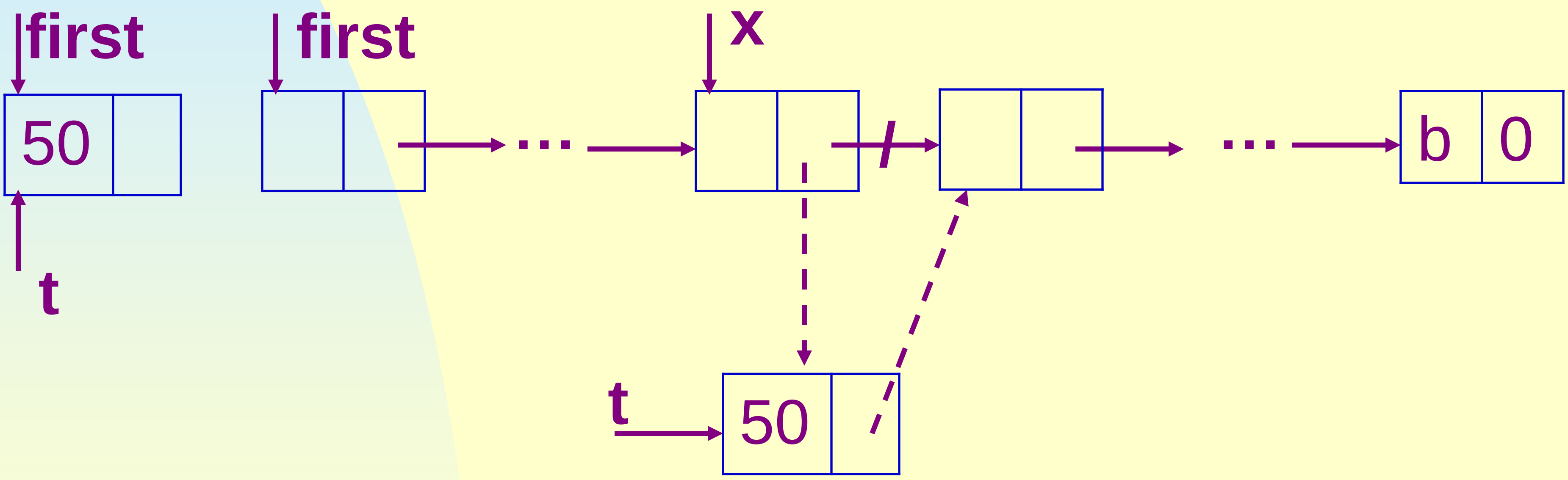
Two classes: ChainNode and Chain. ChainNode is defined as:

```
class ChainNode{
Friend class Chain;
public:
     ChainNode(int element=0,ChainNoded *next=0)
      {data=element; link=next;}
private:
   int data;
   ChainNode *link;
};
```

The access pointer first, which points to the first node in the chain, is a private data member of chain.

4.2.4 Chain manipulation Operations

Example 4.3: insert a node with data field 50 following the node x.



(a) first=0

(b) first !=0

```
void Chain::Insert50 (ChainNode *x)
{
    if (first)
        // insert after x
        x→link = new ChainNode(50, x→link);
    else
        // insert into empty chain
        first = new ChainNode(50);
}
```

Example 4.4: deletes node x from the chain. Let y point to the node (if any) that precedes x, and let y==0 iff x==first.

```
void Chain::Delete (ChainNode *x, ChainNode *y)
{
    if (x==first) first=first->link;
    else y->link=x-link;
    delete x}
```

Exercises: P183-1,2

4.3 The Template Class Chain

We shall enhance the chain class of the previous section to make it more reusable.

4.3.1 Implementing Chains with Templates

```
template <class T> class Chain;  // forward declaration
template <class T>
class ChainNode {
friend class Chain<T>;
public:
   ChainNode(T element, ChainNode* next = 0)
      { data = element; link = next;}
private:
   T data;
   ChainNode *link;
};
```

```
template <class T>
class Chain {
public:
    Chain() { first=0;}; // constructor initializing first to 0
    // Chain manipulation operations
…
private:
    ChainNode<T> *first;
};
```

A empty chain of integers intchain would be defined as:

```
    Chain<int> intchain ;
```

4.3.2 Chain Iterators

A container class is a class that represents a data structure that contains or stores a number of data objects. Objects can usually be added to or deleted from a container. **(p.130)**

An iterator is an object that is used to access the elements of a container class one by one. **(p.185)**

Why we need an iterator?

Consider the following operations that might be performed on a container class C, all of whose elements are integers:

(1) Output all integers in C.

(2) Obtain the sum, maximum, minimum, mean, median of all integers in C.

(3) Obtain the integer x from C such that f(x) is maximum.

……

These operations have to be implemented as member functions of C to access its private data members.

Consider the container class Chain<T>, there are, however, some drawbacks:

(1) All operations of Chain<T> should preferably be independent of the type of object to which T is initialized. However, operations that make sense for one instantiation of T may not for another instantiation.

(2) The number of member functions of Chain<T> can become quite large, making the class definition rather unwieldy.

(3) Even if it is acceptable to add member functions, the user would have to learn how to sequence through the elements of container class.

These suggest that container class be equipped with iterators that provide systematic access the elements of the object.

User can employ these iterators to implement their own functions depending upon the particular application.

Typically, an iterator is implemented as a nested class of the container class.

C++ Iterators

In C++, an iterator is a pointer to an element of an object. As the name suggests, an iterator permits you to go (or iterate) through the elements of the object one by one.

To simplify iterator development and categorization of generic iterator-based codes, the C++ STL defines five categories of iterators: input, output, forward, bidirectional and random access.

A forward Iterator for Chain

A forward Iterator class for Chain may be implemented as in the next slides, and it is required that ChainIterator be a public nested member class of Chain.

```
class ChainIterator {
public:
   // typedefs required by C++  for a forward iterator omitted

   // constructor
   ChainIterator(ChainNode<T>* startNode = 0)
      { current = startNode; }

   // dereferencing operators
   T& operator *() const { return current→data;}
   T* operator →() const { return &current→data;}
```

```
// increment
ChainIterator& operator ++() // preincrement
  {
    current = current→link;
    return *this;
  }
ChainIterator& operator ++(int) // postincrement
  {
    ChainIterator old = *this;
    current = current→link;
    return old;
  }
```

```
    // equality testing
    bool operator !=(const ChainIterator right) const
       { return current != right.current; }
    bool operator == (const ChainIterator right) const
       { return current == right.current; }

    private:
       ChainNode<T>* current;
};
```

Additionally, we add the following public member functions to Chain:

```
ChainIterator begin() {return ChainIterator(first);}
ChainIterator end() {return ChainIterator(0);}
```

We may initialize an iterator object yi to the start of a chain of integers y using the statement:

```
Chain<int>::ChainIterator yi = y.begin();
```

And we may sum the elements in y using the statement:

```
sum = accumulate(y.begin(), y.end(), 0);
// note sum does not require access to private members
```

Write an algorithm to print all data of a Chain.

Write an algorithm to print all data of a Chain using the iterator mechanism.

Exercises: P194-3, 4

4.3.3 Chain Operations

Operations provided in a reusable class should be enough but not too many.

Normally, include: constructor, destructor, operator=, operator==, operator>>, operator<<, etc.

A chain class should provide functions to insert and delete elements.

Another useful function is reverse that does an "in-place" reversal of the elements in a chain.

To insert efficiently at the end of a chain, we add a private member last to Chain<T>, which points to the last node in the chain.

Write an algorithm to find the last node of a Chain

InsertBack

```
template <class T>
void Chain<T>::InsertBack(const T& e)
{
   if (first) { // nonempty chain
      last→link = new ChainNode<T>(e);
      last =last→link;
    }
   else first = last= new ChainNode<T>(e);
}
```

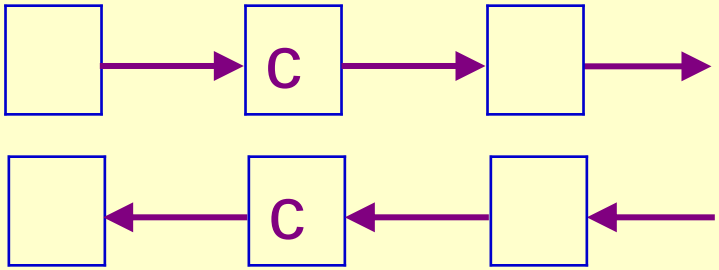
The complexity: O(1).

Concatenate

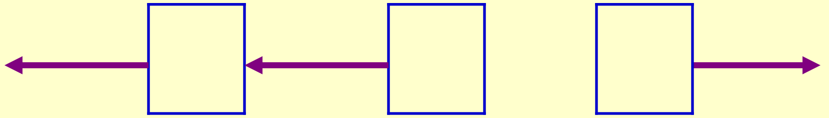
```
template <class T>
void Chain<T>::Concatenate(Chain<T>& b)
{ // b is concatenete to the end of *this
    if (first)
        { last→link = b.first; last = b.last;}
    else
        { first = b.first; last = b.last; );}
    b.first = b.last = 0;
}
```

The complexity: O(1).

Reverse




```
template <class T>
void Chain<T>::Reverse()
{ // make (a_1,.., a_n) becomes (a_n,.., a_1).
    ChainNode<T> *current = first, *previous = 0;
    while (current) {
        ChainNode<T> *r = previous; // r trails previous
        previous = current;
        current = current→link;
        previous→link = r;
    }
    first = previous;
}
```



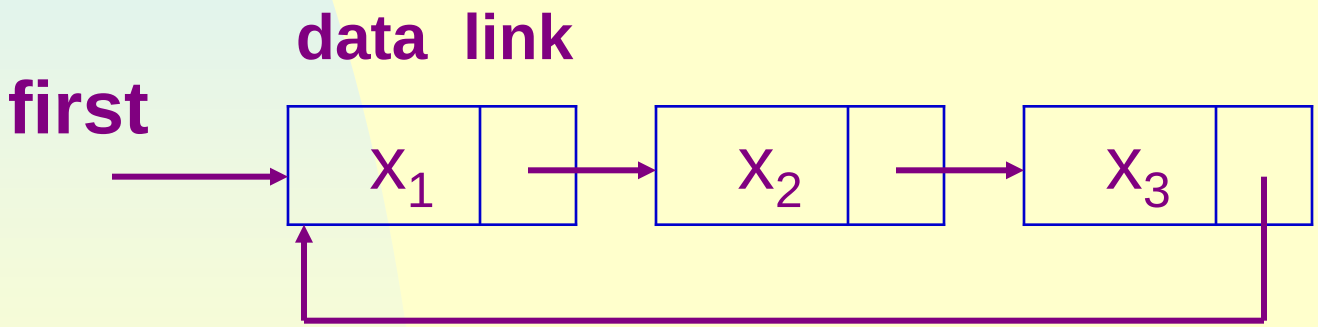
For a chain with $m \geq 1$ nodes, the computing time of Reverse is $O(m)$.

Write an algorithm to construct a Chain from an Array.

Exercises: P184-6

4.4 Circular Lists

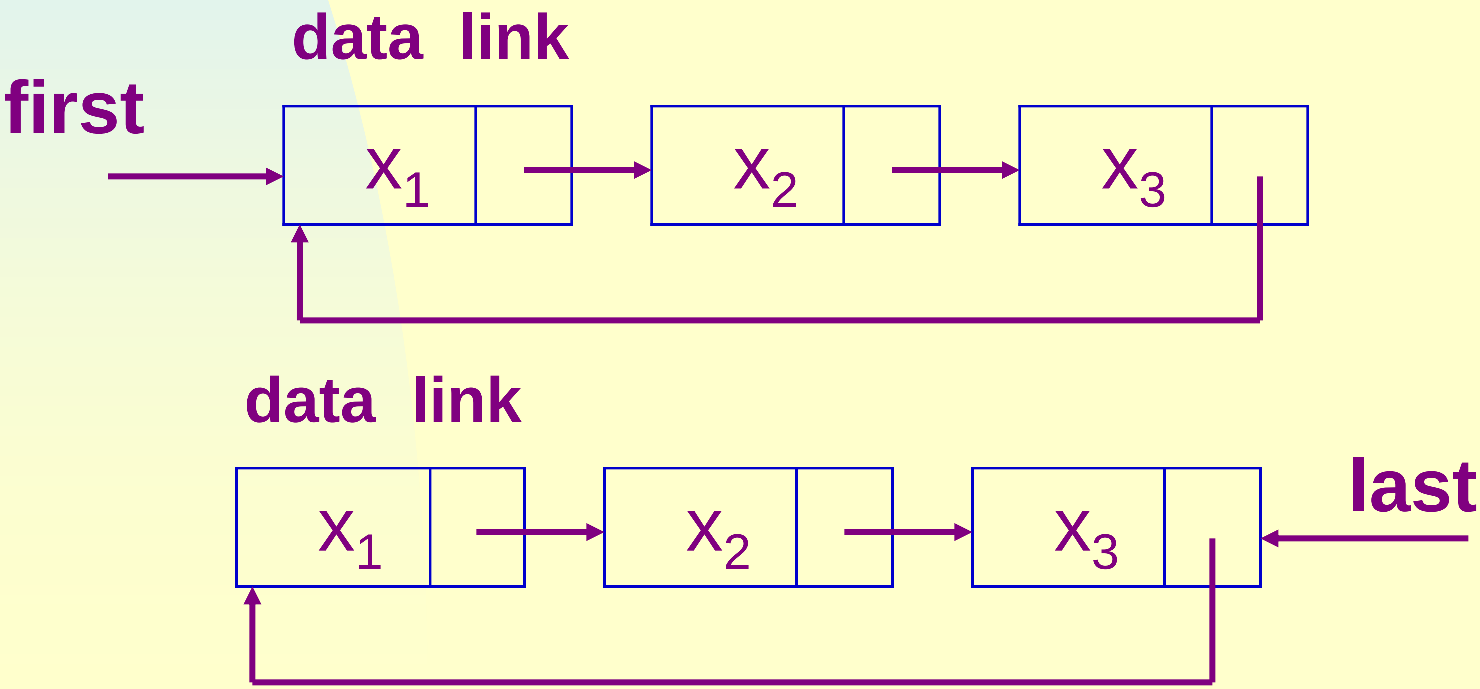
A circular list can be obtained by making the link field of the last node points to the first node of a chain.



Consider inserting a new node at the front of the list of Figure 4.14 (p.195)

We need to change the link field of the node containing x_3, which requires that we move down the entire length of the list until we find the last node.

It is more convenient if the access pointer points to the last rather than the first.



Now we can insert at the front in O(1):

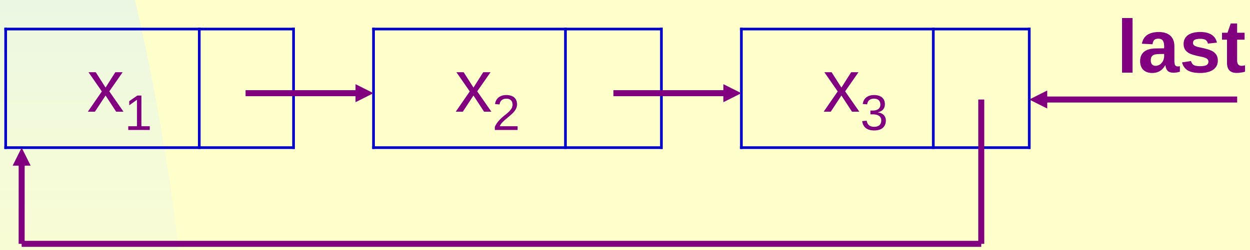
```
template <class T>
void CircularList<T>::InsertFront(const T& e)  //Assume Circularlist
//contains the private data member last that points to the last node.
{ // insert the element e at the "front" of the circular list *this,
  // where last points to the last node in the list.
    ChainNode<T>* newNode = new ChainNode<T>(e);
    if (last) { // nonempty list
        newNode→link =
                                        last→link;
        last→link =
                                       newNode;
     }
    else {
        last = newNode; newNode→link = newNode;
        }
}
```

To insert at the back,

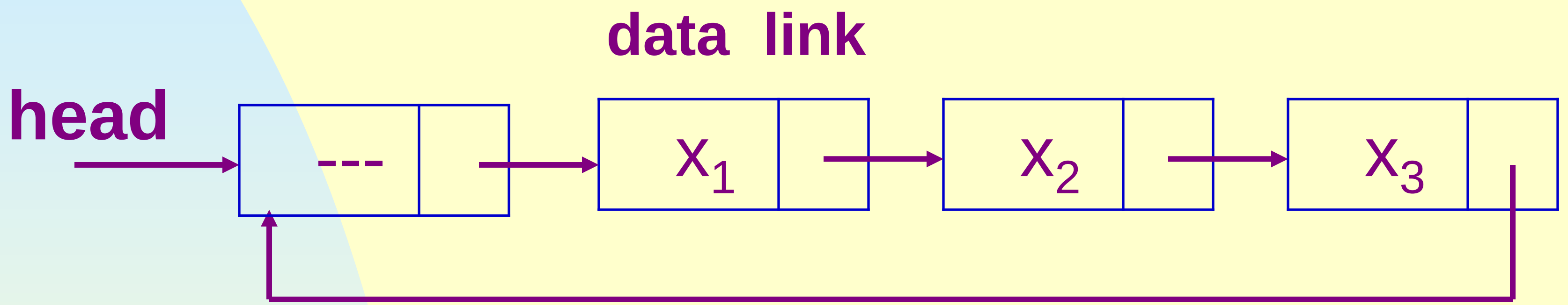
we only need to add the statement

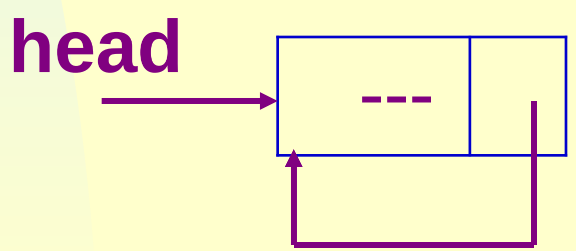
last = newNode;

to the if clause of InsertFront, the complexity is still O(1).



To avoid handling empty list as a special case introduce a dummy head node:.





empty circular list

4.5 Available Space lists

- The time of destructors for chains and circular lists is **linear** in the length of the chain or list.
- Tt may be reduced to O(1) if we maintain our own chain of free nodes. When a new node is needed, we may examine our chain of free nodes.
- Let av be a static class member of CircularList<T> of type ChainNode<T>* that points to the first node in our chain (the available space list) of nodes that have been "deleted."
- Initially, av = 0; only when the av list is empty do we need use new.

We shall now use CircularList<T>::GetNode instead of using new:

```
template <class T>
ChainNode<T>* CircularList<T>::GetNode( )
{ //provide a node for use
    ChainNode<T> * x;
    if (av) { x = av; av = av→link;}
    else x = new ChainNode<T>;
    return x;
}
```

And we use CircularList<T>::RetNode instead of using delete:

```
template <class T>
 void CircularList<T>::RetNode(ChainNode<T>*& x)
{ // free the node pointed to by x
    x→link = av;
    av = x;
    x = 0;
}
```

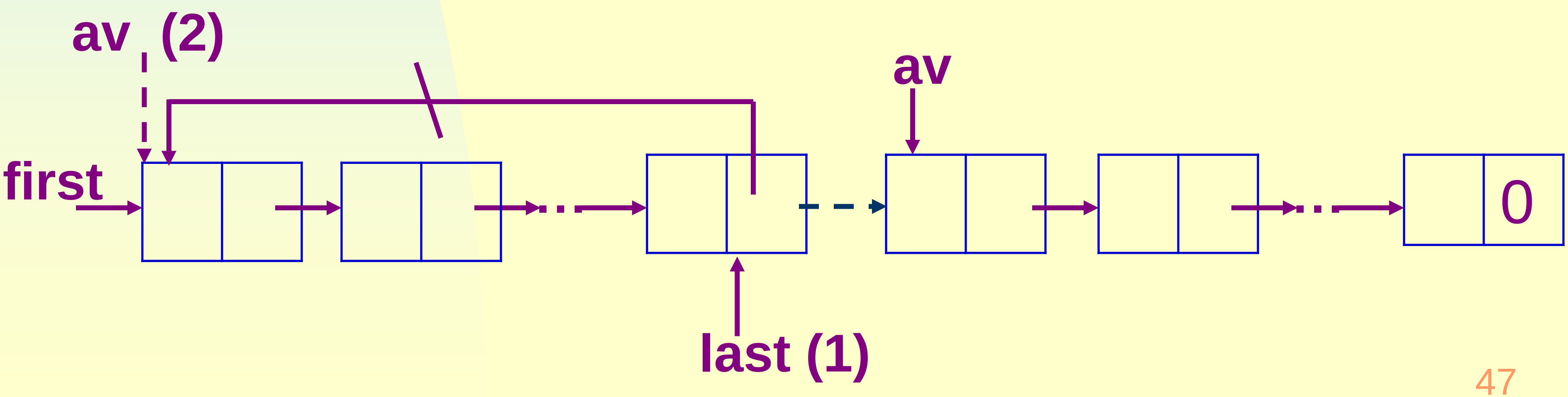
A circular list may be destructed in O(1):

```
template <class T>
void CircularList<T>::~CircularList()
{ // delete the circular list.
    if (last) {
        ChainNode <T> * first = last→link;
        last→link = av;   // (1)
        av = first; // (2)
        last = 0;
    }
}
```

As shown in the next slide:

A circular list may be deleted in O(1):

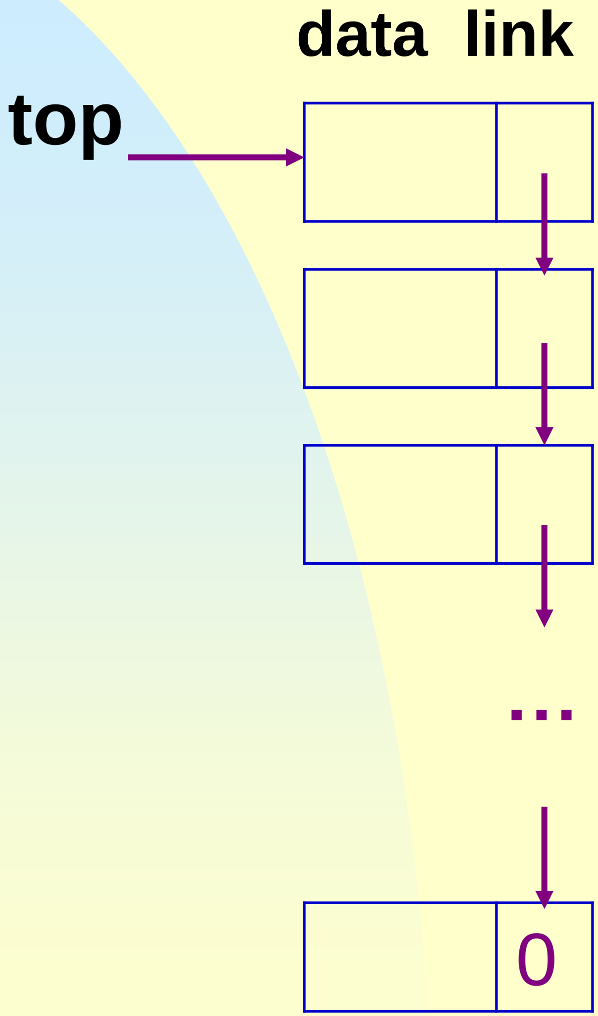
```
template <class T>
void CircularList<T>::~CircularList()
{ // delete the circular list.
  if (last) {  ChainNode <T> * first = last→link;
        last→link = av;   // (1)
        av = first; // (2)
        last = 0;
   }
}
```



A chain may be deleted in O(1) if we know its first and last nodes:

```
template <class T>
Chain<T>::~Chain()
{ // delete the chain
   if (first) {
      last→link = av;
      av = first;
      first = 0;
   }
}
```

4.6 Linked Stacks and Queues



linked stack

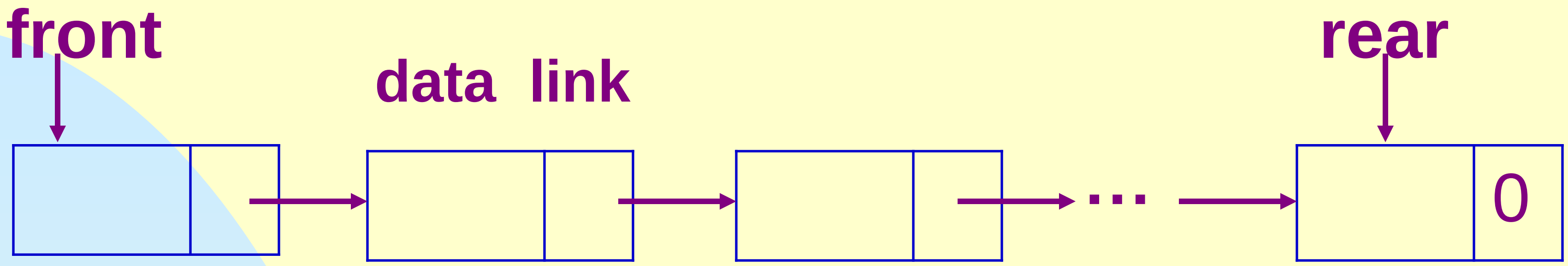
Assume the LinkedStack class has been declared as friend of ChainNode<T>.

```
template <class T>
class LinkedStack {
public:
   LinkedStack() { top=0;} // constructor initializing top to 0
   // LinkedStack manipulation operations
…
private:
   ChainNode<T> *top;
};
```

```
template <class T>
void LinkedStack<T>::Push(const T& e) {
    top = new ChainNode(e, top);
}

template <class T>
void LinkedStack<T>::Pop()
{ // delete top node from the stack.
    if (IsEmpty()) throw "Stack is empty. Cannot delete.";
    ChainNode<T> * delNode = top;
    top = top→link;
    delete delNode;
}
```

The functions IsEmpty, Front and Rear are easy to implement, and are omitted.



linked queue

Assume the LinkedQueue class has been declared as friend of ChainNode<T>.

```
template <class T>
class LinkedQueue {
public:
   LinkedQueue() { front=rear=0;} // constructor initializing front and rear to 0
   // LinkedQueue manipulation operations
…
private:
   ChainNode<T> *front;
   ChainNode<T> *rear;
};
```

```
template <class T>
void LinkedQueue<T>::Push(const T& e) {
   if(IsEmpty()) front=rear= new ChainNode(e, top);
}

template <class T>
void LinkedQueue<T>::Pop()
{ // delete first element in queue.
   if (IsEmpty()) throw "Queue is empty. Cannot delete.";
   ChainNode<T> * delNode = front;
   front = front→link;
   delete delNode;
}
```

The functions IsEmpty and Top are easy to implement, and are omitted.

Exercises: P201 1-2

4.7 Polynomials

4.7.1 Polynomial Representation

$$A(x) = a_m x^{em} + a_{m-1} x^{em-1} +,\ldots,+ a_1 x^{e1}$$

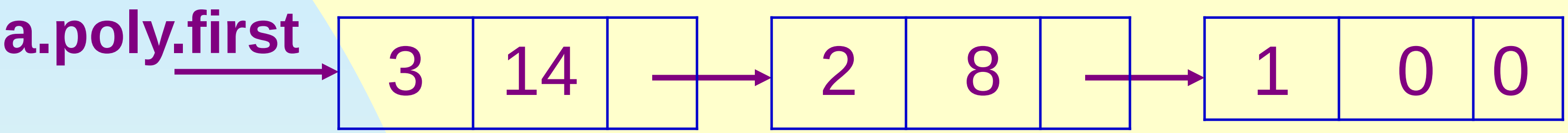
$$\text{Where } a_i \neq 0, \quad e_m > e_{m-1} >,\ldots, e_1 \geq 0$$

Since a polynomial is to be represented by a list, we say Polynomial is IS-IMPLEMENTED-BY List.

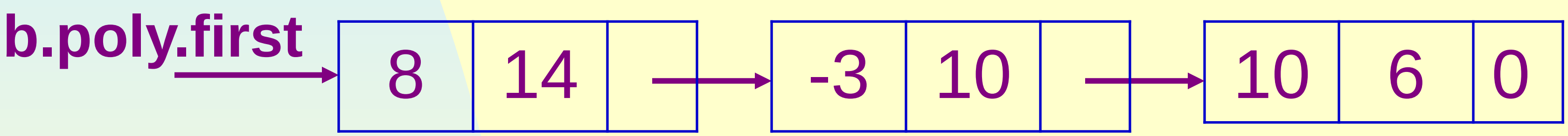
Definition: **a data object of Type A IS-IMPLEMENTED-IN-TERMS-OF a data object of Type B if the Type B object is central to the implementation of Type A object. ---Usually by declaring the Type B object as a data member of the Type A object.**

- **Make the chain poly a data member of Polynomial.**
- **Each ChainNode will represent a term. The template T is instantiated to struct Term:**

```
struct Term
{ // all members of Term are public by default
     int coef;
     int exp;
     Term Set(int c, int e) { coef=c; exp=e; return *this;};
};
class Polynomial {
public:
   // public functions defined here
private:
   Chain<Term> poly;
};
```



$$a = 3x^{14} + 2x^{8} + 1$$



$$b = 8x^{14} - 3x^{10} + 10x^{6}$$

4.7.2 Adding Polynomials

To add two polynomials a and b, use the chain iterators ai and bi to move along the terms of a and b.

```
Polynomia Polynomial::operaor+ (const Polynomial& b) const
{ // *this (a) and b are added and the sum returned
  Term temp;
  Chain<Term>::ChainIterator ai = poly.begin(),
                             bi = b.poly.begin();
  Polynomial c;
```

```
while (ai != poly.end() && bi != b.poly.end()) { //not null
    if (ai→exp == bi→exp) {
        int sum = ai→coef + bi→coef;
        if (sum) c.poly.InsertBack(temp.Set(sum, bi→exp);
          ai++; bi++; // to next term
    }
    else if (ai→exp < bi→exp) {
            c.poly.InsertBack(temp.Set(bi→coef, bi→exp));
            bi++;  // next term of b
    }
    else {
            c.poly.InsertBack(temp.Set(ai→coef, ai→exp));
            ai++;  // next term of a
     }
  }
```

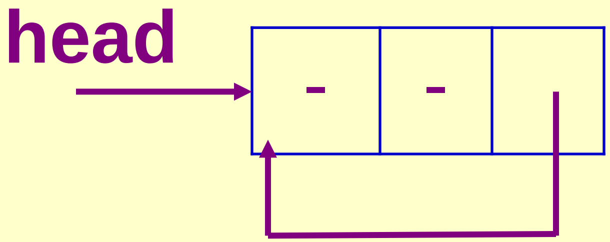
```
  while (ai != poly.end()) { // copy rest of a
      c.poly.InsertBack(temp.Set(ai→coef, ai→exp));
      ai++;
  }
  while (bi != b.poly.end()) { // copy rest of b
      c.poly.InsertBack(temp.Set(bi→coef, bi→exp));
      bi++;
  }
  return c;
}
```

Analysis:

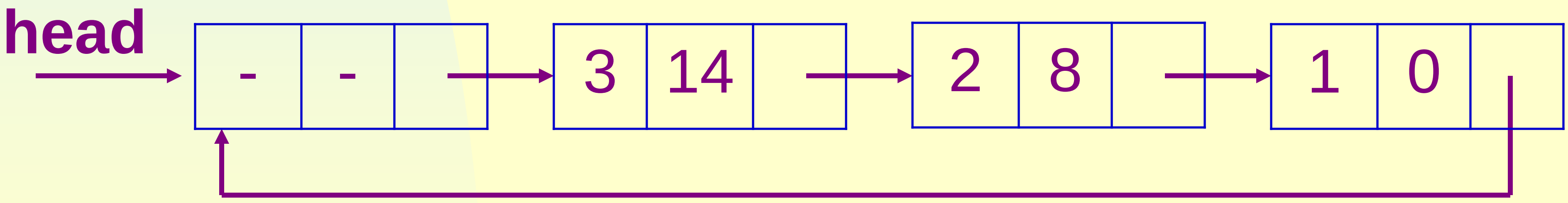
Assume a has m terms, b has n terms. The computing time is O(m+n).

4.7.3 Circular List Representation of Polynomials

Polynomials represented by circular lists with head node are as in the next slide:



(a) Zero polynomial



(b) $3x^{14} + 2x^{8} + 1$

Adding circularly represented polynomials

- **The exp of the head node is set to –1 to push the rest of a or b to the result.**

- **Assume the begin() function for class CircularListWithHead returns an iterator that points to the node head→link.**

```
Polynomial Polynomial::operaor+(const Polynomial& b) const
{ // *this (a) and b are added and the sum returned
  Term temp;
  CircularListWithHead<Term>::Iterator ai = poly.begin(),
                                       bi = b.poly.begin();
  Polynomial c; //assume constructor sets head→exp = -1
  while (1) {
     if (ai→exp == bi→exp) {
       if (ai→exp == -1) return c;
       int sum = ai→coef + bi→coef;
       if (sum) c.poly.InsertBack(temp.Set(sum, ai→exp);
         ai++; bi++; // to next term
     }
```

```
        else if (ai→exp < bi→exp) {
              c.poly.InsertBack(temp.Set(bi→coef, bi→exp));
              bi++;  // next term of b
        }
        else {
              c.poly.InsertBack(temp.Set(ai→coef, ai→exp));
              ai++;  // next term of a
        }
    }
}
```

Experiment: P209-5

4.10 Doubly Linked Lists

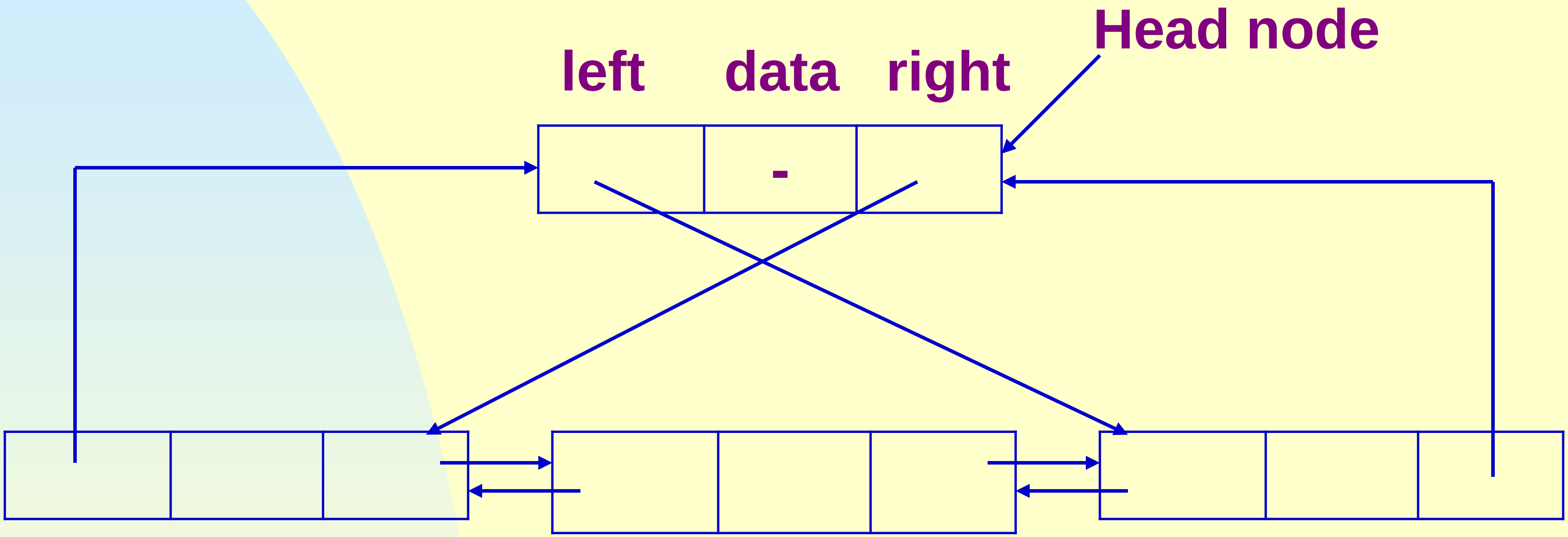
Difficulties with singly linked list:

- **can easily move only in one the direction of the link**
  - **not easy to delete an arbitrary node**
    - **requires knowing the preceding node**

A node in doubly linked list has at least 3 field: data, left and right, this makes moving in both directions easy.

left	data	right

A doubly linked list may be circular. The following is a doubly linked circular list with head node:



Suppose p points to any node, then

$p == p \rightarrow left \rightarrow right == p \rightarrow right \rightarrow left$

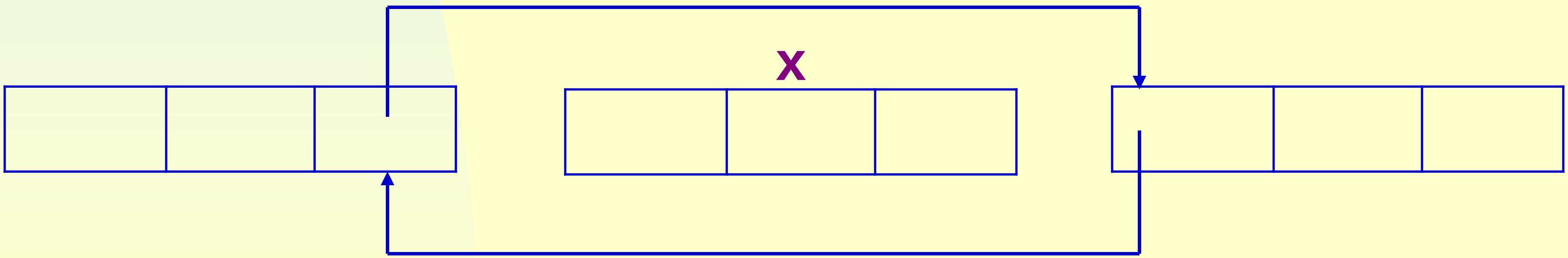
```
class DblList;

class DblListNode {
friend class DblList;
private:
    int data;
    DblListNode *left, *right;
};

class DblList {
public:
    // List manipulation operations
    …
private:
    DblListNode *first; // points to head node
};
```

Delete

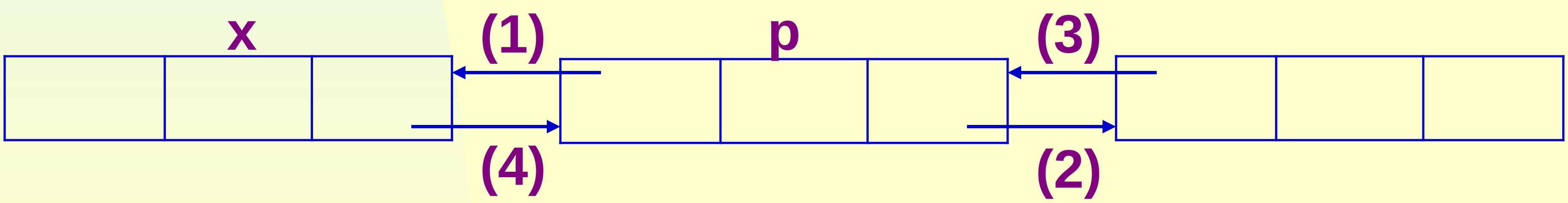
```
void DblList::Delete(DblListNode *x )
{
   if(x == first) throw “Deletion of head node not permitted”;
   else  {
      x→left→right = x→right;
      x→right→left = x→left;
      delete x;
   }
}
```



Insert

```
void DblList::Insert(DblListNode *p, DblListNode *x )
{ // insert node p to the right of node x

      p→left = x;                // (1)
      p→right = x→right;  // (2)
      x→right→left = p;     // (3)
      x→right = p;              // (4)
}
```



Exercises: P225-2

1. Write an algorithm to construct a Chain from an Array.

2.Given a sorted single linked list L = <a_1, …., a_n>, where a_i.data<= a_j.data (i < j).
Try to write an algorithm of inserting a new data element X to L, and analysis its complexities.

3.Given a linear list L = <a_1, …., a_n>, implemented by a single linked list.
Delete data a_i with Time Complexity O(1). We have a pointer to node(a_i).