

题 1

- **Amortized costs: more cases**
 - **insert, delete**
 - **$\alpha \geq 1/2$, $\alpha < 1/2$ (use α_i , since α can vary a lot)**
 - **size does/doesn't change**
- **Exercise**

$\alpha \geq 1/2$ 时, $F(T) = 2\text{num}[T] - \text{size}[T]$

$\alpha < 1/2$ 时, $F(T) = \text{size}[T]/2 - \text{num}[T]$

Case1: insert, $\alpha \geq 1/2$, size doesn't change

$\text{size}(i) = \text{size}(i-1)$
 $\text{num}(i) = \text{num}(i-1) + 1$
 $c(i) = 1$;
Amortized cost
 $= c(i) + \Delta F = c(i) + F(i) - F(i-1)$
 $= 1 + (2 * \text{num}(i) - \text{size}(i)) - (2 * \text{num}(i-1) - \text{size}(i-1))$
 $= 3$

Case2: insert, $\alpha \geq 1/2$, size change

$\text{size}(i) = 2 * \text{size}(i-1)$
 $\text{size}(i-1) = \text{num}(i-1) = \text{num}(i) - 1$
 $c(i) = \text{num}(i-1) + 1 = \text{num}(i)$
Amortized cost
 $= c(i) + \Delta F = c(i) + F(i) - F(i-1)$
 $= \text{num}(i) + (2 * \text{num}(i) - \text{size}(i)) - (2 * \text{num}(i-1) - \text{size}(i-1))$
 $= 3$

Case3: insert, $\alpha < 1/2$, size doesn't change

$\text{size}(i) = \text{size}(i-1)$
 $\text{num}(i) = \text{num}(i-1) + 1$
 $c(i) = 1$
Amortized cost

$$\begin{aligned}
&=c(i)+\Delta F=c(i)+F(i)-F(i-1) \\
&=1+(size(i)/2-num(i))-(size(i-1)/2-num(i-1)) \\
&=0
\end{aligned}$$

Case4: insert, $\alpha < 1/2$, size change

不存在这种情况;

Case5: delete, $\alpha \geq 1/2$, size doesn't change

$$\begin{aligned}
size(i) &= size(i-1) \\
num(i) &= num(i-1)-1 \\
c(i) &= 1 \\
\text{Amortized cost} \\
&=c(i)+\Delta F=c(i)+F(i)-F(i-1) \\
&=1+(2*num(i)-size(i))-(2*num(i-1)-size(i-1)) \\
&=-1
\end{aligned}$$

Case6: delete, $\alpha \geq 1/2$, size change

$\alpha \geq 1/2 > 1/4$, 所以不存在该情况;

Case7: delete, $\alpha < 1/2$, size doesn't change

$$\begin{aligned}
size(i) &= size(i-1) \\
num(i) &= num(i-1)-1 \\
c(i) &= 1 \\
\text{Amortized cost} \\
&=c(i)+\Delta F=c(i)+F(i)-F(i-1) \\
&=1+(size(i)/2-num(i))-(size(i-1)/2-num(i-1)) \\
&=2
\end{aligned}$$

Case8: delete, $\alpha < 1/2$, size change

$$\begin{aligned}
size(i) &= size(i-1)/2 \\
size(i-1) &= 2*num(i-1)=2*(num(i)+1) \\
num(i) &= num(i-1)-1 \\
c(i) &= num(i-1) \\
\text{Amortized cost} \\
&=c(i)+\Delta F=c(i)+F(i)-F(i-1) \\
&=num(i-1)+(size(i)/2-num(i))-(size(i-1)/2-num(i-1)) \\
&=1
\end{aligned}$$

题 2

Exercise

- $A = \langle 1\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0 \rangle$
- Find $A[6]$ ' Children with Rank/Selections
- Find $A[13]$ ' Parent with Rank/Selections
- $A[?]$

① 过程: $A[\text{rank}(6)*2]$ $A[\text{rank}(6)*2+1]$

结果: $A[10]$ $A[11]$

② 过程: $A[\text{selections}([13/2])]$

结果: $A[7]$