第二次作业部分答案：

2.（1）1 ,（2）$\max\{a_1, a_2, \cdots, a_k\}$,(3)1.

3.(2) **证明**: (S1) 若有极限，则 $\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} x_{n+1} = a$. 对$x_{n+1} = \sqrt{2x_n}$，令$n \to \infty$有$a = \sqrt{2a}$, $\therefore a = 0$或2，$\because x_n > 0, \therefore a = 0$(舍去)$a = 2$，

(S2) 当$n = 1$时，$x_1 = \sqrt{2} < 2$，当$n = 2$时，$x_2 = \sqrt{x_1} = \sqrt{2\sqrt{2}} < 2$，$\therefore x_1 < x_2 < 2$，设$n = k$时，有$x_k < x_{k+1}$，且$x_k < 2 (n \in N^+)$，$n = k+1$时，$x_{k+1} = \sqrt{2x_k} < \sqrt{2 \times 2} = 2$，$\therefore n = k+1$时，$x_n < x_{n+1} < 2$成立，

综上，x_n单调递增且有上界，所以它收敛，且极限为2. **证毕**.

(3)**证明**: (S1) 若有极限，则 $\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} x_{n+1} = a$，对$x_{n+1} = 1 + \frac{x_n}{1+x_n}$，令$n \to \infty, a = 1 + \frac{a}{a+1}$，$\therefore a = \frac{1\pm\sqrt{5}}{2}$，$\because x_n > 1$，$\therefore a = \frac{1+\sqrt{5}}{2}$。

(S2) $\because x_n > 1$，所以$x_{n+1} = 1 + \frac{x_n}{1+x_n} \leq 2$, 即$x_n$有界，又$\because x_{n+1} - x_n = 1 + \frac{x_n}{1+x_n} - (1 + \frac{x_{n-1}}{1+x_{n-1}}) = \frac{x_n - x_{n-1}}{(1+x_n)(1+x_{n-1})} > 0 \therefore x_n$单调.

$\therefore x_n$收敛并且有极限为$\frac{1+\sqrt{5}}{2}$. **证毕**.

(5) **证明**: (S1) 若有极限$\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} x_{n+1} = m$，对$x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n})$，令$n \to \infty$，$m = \frac{1}{2}(m + \frac{a}{m})$. $\therefore m = \pm\sqrt{a}, \because m > 0, \therefore m = \sqrt{a}$

(S2) $\therefore x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}) \geq \sqrt{a}$, $(n \geq 1)$; $\therefore x_{n+1} - x_n = \frac{1}{2}(\frac{a}{x_n} - x_n) \leq 0$，$\therefore x_n$单调递减有下界.

综上, $\lim_{n\to\infty} x_n = \sqrt{a}$. **证毕**.

4.略

5. （1）**证明：**由$\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_n} = r < 1$，可知$\exists N$, 当$n > N$ 时, 有$a_{n+1} < a_n$. 即a_n单调递减$(n > N)$. 又$\because a_n > 0$, $a_n > 0$, $\therefore$根据单调有界原理知a_n收敛.

令$\lim\limits_{n\to\infty} a_n = A, \therefore A = rA$, 又$\because 0 < r < 1, \therefore A = 0, \therefore \lim\limits_{n\to\infty} a_n = 0$. **证毕**.

(2) **证明：**$\because \lim\limits_{n\to\infty}(b_n - a_n) = 0$，所以$a_n - b_n$有界; 即, 存在$M > 0$, s.t. $-M \leq a_n - b_n \leq M$. 注意到b_n 单调递减, 则可得$a_n \leq M + b_n \leq M + b_1, \forall n$. 从而$a_n$ 有上界. 再由于，a_n单调递增, 由单调有界定理, $\lim_{n\to\infty} a_n$ 存在, 并由极限的四则运算性质, $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} b_n$. **证毕**.

2. (4)略(5)略

(6)**证明：**由

$$\left|\frac{x^2-4}{2x^2+x}-\frac{1}{2}\right| = \left|\frac{x-8}{4x^2+2x}\right| = \frac{1}{|x|}\left|\frac{1-8/x}{4+2/x}\right|,$$

注意到当$|x|>10$ 时, 有$1/|x|\le 10^{-1}$, 从而

$$\left|\frac{1-8/x}{4+2/x}\right| \le \frac{1+8/|x|}{4-2/|x|} \le \frac{1+8/10}{4-2/10} \le 1$$

由上, $\forall \epsilon>0, \exists L=\max\{\frac{1}{\epsilon},10\}$，使得当$|x|>L$ 时, 有

$$\left|\frac{x^2-4}{2x^2+x}-\frac{1}{2}\right| \le \frac{1}{|x|} < \epsilon.$$

证毕.

7(1)存在;

(2)左右极限不相等，所以不存在;

(3)左右极限不相等，所以不存在.