# 概率统计 18-19-2(A)参考答案

## 一、选择题

1) C,　2) C,　3) C,　4) D,　5) D

## 二、填空题

1) 2/7=0.2857;

2) 15/16

3) $2e^{-2}=0.271$

4) 0

5)

| X+Y | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| p | 0.3 | 0.2 | 0.1 | 0.4 |

6) -1.4

7) 50

8) $\chi^2(9)$

9) $$F(x)=\begin{cases}0 & x<-10\\ 0.3 & -10\le x<10\\ 1 & x\ge 10\end{cases}$$

10) $$f_Y(y)=\begin{cases}\dfrac{1-y}{2} & -1<y<1\\ 0 & \text{其它}\end{cases}$$

11) 1/20=0.05

12) 1.65

13) 2/3=0.667.

三、(1) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy=1;$

$$a\int_{-1}^{0}\int_{0}^{y+1} xy^2dxdy=1;.$$

$$a=60$$

(2) $f_Y(y)=\int_{-\infty}^{\infty} f(x,y)dx$

当-1<y<0时 $f_Y(y)=\int_0^{1+y} axy^2dx=30y^2(y+1)^2$

当y ≤ -1，或 y ≥ 0时 $f_Y(y)=0$

(3) $f_{X|Y}(x|y)=\dfrac{f(x,y)}{f_Y(y)}=\begin{cases}\dfrac{2x}{(y+1)^2} & 0<x<y+1 \\ 0 & \text{其它}\end{cases} \quad (-1<y<0)$

$$f_{X|Y}(x|-0.6)=\begin{cases}\dfrac{2x}{(-0.6+1)^2} & 0<x<0.4 \\ 0 & \text{其它}\end{cases}=\begin{cases}12.5x & 0<x<0.4 \\ 0 & \text{其它}\end{cases}$$

$$P(X<0.5|Y=-0.6)=\int_{-\infty}^{0.5} f_{X|Y}(x|-0.6)dx=\int_0^{0.4}12.5xdx=1$$

四、A1,A2,A3 分别表示所选到地区的报名考生数量为 10 名，15 名和 20 名；
B1 表示第一次抽到的是女生报名表；B2 表示第二次抽到的是女生报名表；
则

$$P(A_1)=P(A_2)=P(A_3)=1/3;$$

$$P(B_1|A_1)=\frac{3}{10};P(B_1|A_2)=\frac{5}{15};P(B_1|A_3)=\frac{10}{20};$$

$$(1)P(B_1)=P(A_1)P(B_1|A_1)+P(A_2)P(B_1|A_2)+P(A_3)P(B_1|A_3)$$

$$=\frac{1}{3}(\frac{3}{10}+\frac{5}{15}+\frac{10}{20})=\frac{34}{90}=0.378$$

(2)

$$P(B_2)=P(B_1)=P(A_1)P(B_2|A_1)+P(A_2)P(B_2|A_2)+P(A_3)P(B_2|A_3)=\frac{34}{90}$$

$$P(B_1|B_2)=\frac{P(B_1B_2)}{P(B_2)}=\frac{1}{P(B_2)}(\sum_{i=1}^{3}P(A_i)P(B_1B_2|A_i))$$

$$=\frac{\frac{1}{3}\times(\frac{C_3^2}{C_{10}^2}+\frac{C_5^2}{C_{15}^2}+\frac{C_{10}^2}{C_{20}^2})}{\frac{34}{90}}=\frac{\frac{1}{3}\times\frac{1591}{3990}}{\frac{34}{90}}\approx\frac{0.133}{0.378}\approx 0.352$$

五、 $X$和$Y$的概率密度为：

$$f_X(x)=\begin{cases}1/2 & 0<x<2\\ 0 & 其它\end{cases},\ f_Y(y)=\begin{cases}1/2 & 0<y<2\\ 0 & 其它\end{cases}$$

$X$和$Y$的联合密度为：

$$f(x,y)=\begin{cases}1/4 & 0<x<2,0<y<2\\ 0 & 其它\end{cases}$$

Z的分布函数$F_Z(z)=P(Z\le z)=P(X+2Y\le z)$

当 $z<0$时，$F_Z(z)=0$;

当 $z>6$时，$F_Z(z)=1$;

当$0<z<2, F_Z(z)=\int_0^z\int_0^{\frac{z-x}{2}}\frac{1}{4}dydx=\frac{1}{16}z^2$

当$2<z<4, F_Z(z)=\int_0^2\int_0^{\frac{z-x}{2}}\frac{1}{4}dydx=\frac{1}{4}(z-1)$

当$4<z<6, F_Z(z)=1-\int_{z-4}^2\int_{\frac{z-x}{2}}^2\frac{1}{4}dydx=1-\frac{1}{16}(6-z)^2$

$Z$的概率密度为

$$f_Z(z)=[F_Z(z)]'=\begin{cases}z/8 & 0<z<2\\ 1/4 & 2\le z<4\\ (6-z)/8 & 4\le z<6\\ 0 & 其它\end{cases}$$

或者：

令 $T=2Y$

$$f_T(t)=\frac{1}{2}f_Y(\frac{t}{2})=\begin{cases}1/4,0<t<4\\ 0,其他\end{cases}$$

$$\therefore f_Z(z)=\int_{-\infty}^{+\infty}f_X(x)f_T(z-x)dx$$

$$=\int_{z-4}^{z}f_X(x)\times\frac{1}{4}dx$$

$$=\begin{cases}0,\ z<0\\ \int_0^z\frac{1}{8}dx=\frac{z}{8},\ 0\le z<2\\ \int_0^2\frac{1}{8}dx=\frac{1}{4},\ 2\le z<4\\ \int_{z-4}^2\frac{1}{8}dx=\frac{6-z}{8},\ 4\le z<6\\ 0, z\ge 6\end{cases}$$

六、设第i页有错误$X_i$个，则$X_i \sim P(2); \mu = EX_i = 2; \sigma^2 = DX_i = 2; n = 200$

所求概率为：

$$P(\sum_{i=1}^{200} X_i > 420) \approx 1 - \Phi(\frac{420 - n\mu}{\sqrt{n}\sigma})$$

$$= 1 - \Phi(\frac{420 - 400}{\sqrt{200} \times \sqrt{2}}) = 1 - \Phi(1) = 0.1587$$

七、(1)似然函数为：$L(\theta) = \prod_{i=1}^{n} f(X_i, \theta) = \prod_{i=1}^{n} \theta X_i^{-(\theta+1)} = \theta^n (\prod_{i=1}^{n} X_i)^{-(\theta+1)}$

对数似然函数：$l(\theta) = \ln L(\theta) = n \ln \theta - (\theta+1) \ln \prod_{i=1}^{n} X_i$

对$\theta$求导数： $[l(\theta)]' = \frac{n}{\theta} - \ln \prod_{i=1}^{n} X_i,$

令$[l(\theta)]'$=0，解得$\theta$的最大似然估计：$\hat{\theta} = \frac{n}{\ln \prod_{i=1}^{n} X_i}$

所有$\eta$的最大似然估计为：$\hat{\eta} = \frac{1}{\hat{\theta}} = \frac{\ln \prod_{i=1}^{n} X_i}{n} = \frac{1}{n} \sum_{i=1}^{n} \ln X_i$

$$(2) E\hat{\eta} = E \frac{1}{n} \sum_{i=1}^{n} \ln X_i = E \ln X$$

$$= \int_1^{\infty} \ln x \theta x^{-(\theta+1)} dx = -x^{-\theta} \ln x \Big|_1^{\infty} + \int_1^{\infty} x^{-(\theta+1)} dx$$

$= \frac{1}{\theta} = \eta;$所以，$\hat{\eta}$是$\eta$的无偏估计量。

八、(1)$n = 25, \alpha = 0.05,$

检验统计量 $T = \frac{\bar{X} + 6}{S_n} \sqrt{n} \mid H_0 \sim t(n-1)$

拒绝域：$D = \{T > t_\alpha(n-1)\} = \{T > 1.711\}$

$\bar{x} = -5, s_n = 2$

$T$的观测值：$T = \frac{-5+6}{2} \sqrt{5} = 2.5 > 1.711,$

所以，拒绝原假设。

(2)$\sigma^2$的置信度为95%的置信区间为：$[\frac{(n-1) S_n^2}{\chi^2_{0.025}(24)}, \frac{(n-1) S_n^2}{\chi^2_{0.975}(24)}]$

$$= [\frac{24 \times 4}{39.36}, \frac{24 \times 4}{12.4}] = [2.44, 7.74]$$