Adjoint Aerodynamic Shape Optimisation of Transonic Airfoils

This coursework aims to give you hands-on experience in aerodynamic shape optimisation using Computational Fluid Dynamics (CFD) tools. You will also get familiar with the Class Shape Transformation (CST) technique for airfoil shape parameterisation and the adjoint method for sensitivity analysis. The assignment has two parts. In the first part, you need to fit a CST curve to a given airfoil coordinates. In the second part, you will use that airfoil (CST curves) as the starting point of adjoint shape optimisation for minimising airfoil drag subject to constraints on the lift, pitching moment and airfoil thickness.

You need to do the assignment in pairs. Please find your teammate yourself. In the first step, you need to identify the inputs for your assignment. The inputs are assigned to each group randomly based on your student numbers, as follows.

- Start from RIGHT to LEFT in the student number of the first student. Identify the first digit from 1 to 5 → Digit 1
- Start from RIGHT to LEFT in the student number of the second student. Identify the first digit from 1 to 5 → Digit 2
- Add the two student numbers together! Start from Right to LEFT and identify the first digit from 1 to 5 → Digit 3

For example, if the student number of the first student is 30104567, and the student number of the second student is 30236721, then Digit 1 is 5 (30104567), Digit 2 is 1 (30236721), and Digit 3 is 2 (sum of the two student numbers is 60341288). Then please pick up the number of CST modes for each surface of your airfoil using Digit 1, the design lift coefficient (Cl) using Digit 2 and the Mach number using Digit 3 from the Table below.

Digit	1	2	3	4	5
CST modes	6	7	8	9	10
Cl	0.71	0.72	0.73	0.74	0.75
M	0.730	0.735	0.740	0.745	0.750

For the example above, the number of CST modes for each surface is 10, Cl is 0.71, and the Mach number is 0.735. Please note that the number of CST modes given in the table above is for each side of the airfoil, so the total number of your design variables is twice this number.

Part 1:
You are given the coordinates of the RAE2822 airfoil. Please fit a CST curve to each side of this airfoil using the number of modes assigned to you. You need to use the codes given to you to create coordinates using CST modes and combine them with an unconstrained optimisation to minimise the error between the actual airfoil coordinates and the coordinates from your CST curves.

Part 2:
Please use the OpenFEMflow code to run a shape optimisation starting using the CST curves you obtained in part 1. You need to fix the Mach number using the inputs of your assignment and then minimise drag subject to a constraint on lift coefficient using the inputs of your assignment. You also need a constraint on the pitching moment. Please use the value of -0.15 as your airfoil's minimum allowed pitching moment coefficient. Also, you need two more constraints on the thickness of the airfoil. Please ensure the airfoil's thickness is not lower than the thickness of the initial airfoil at 20% and 60% of the chord.

Please use this template to write your report. Good luck! Ali Elham

Part 1: Fit CST curves using unconstrained optimisation (40 marks)	
In the first step, you need to fit a CST curve to the airfoil coordinates given to you, using the same number of CST coefficients identified in the table. You need to report the following: - The mathematical formulation of the optimisation you used for this curve fitting, i.e. objective function, design variables, constraints and bounds (if they exist). - The starting and final values of the CST coefficient for each airfoil surface. - The final value of the objective function. - The history of optimisation, i.e. the convergence plots of objective function and first order optimality (for each surface separately). - A figure showing the airfoil coordinates and the fitted CST curves. Please add proper labels and legends to your plots.	
Indicative marks grid:	
Only mathematical formulation, with no results.	0 - 5
Optimisations are performed but the outputs are not correct.	6 - 14
Optimisations are performed, the outputs are correct, but not all the requested information are presented.	15 – 30
Optimisations are performed, correct results are obtained, and all the requested information are presented.	31 - 40
Page limit: do not exceed the rest of the page plus one other.	

The mathematical formulation to optimize the airfoil is shown in Eqn.1. This equation is optimized using the "fmincunc" function available in Matlab. The "fmincunc" describes a method applying the Quasi-Newton method, updating numerically, an approximation to the inverse Hessian matrix, similar to the steepest descent algorithm, however with an improved directional term. The directional information is contained in B_i, as depicted in Eqn.2, with common, optimal method in calculating it concludes to the BFGS method.

$$\min(\text{Error}) = min[\sum_{i=1}^{n}(y_{cst_i} - y_i)^2] \qquad \textit{Eqn. 1}$$

$$[B_{i+1}] = [B_i] + \frac{d_i d_i^T}{d_i^T g_i}\left(1 + \frac{g_i^T [B_i] g_i}{d_i^T g_i}\right) - \frac{[B_i] g_i d_i^T}{d_i^T g_i} - \frac{d_i g_i^T [B_i]}{d_i^T g^i} \qquad \textit{Eqn. 2}$$

	Suction (Upper) Surface						
Initial	1	1	1	1	1	1	1
Optimised	0.126275	0.139415	0.157457	0.171029	0.213332	0.176664	0.211278
	Pressure (Lower) Surface						
Initial	1	1	1	1	1	1	1
Optimised	-0.12889	-0.14279	-0.13067	-0.26221	-0.02445	-0.11506	0.07298

Table 1: Comparison of initial and optimised CST coefficient.

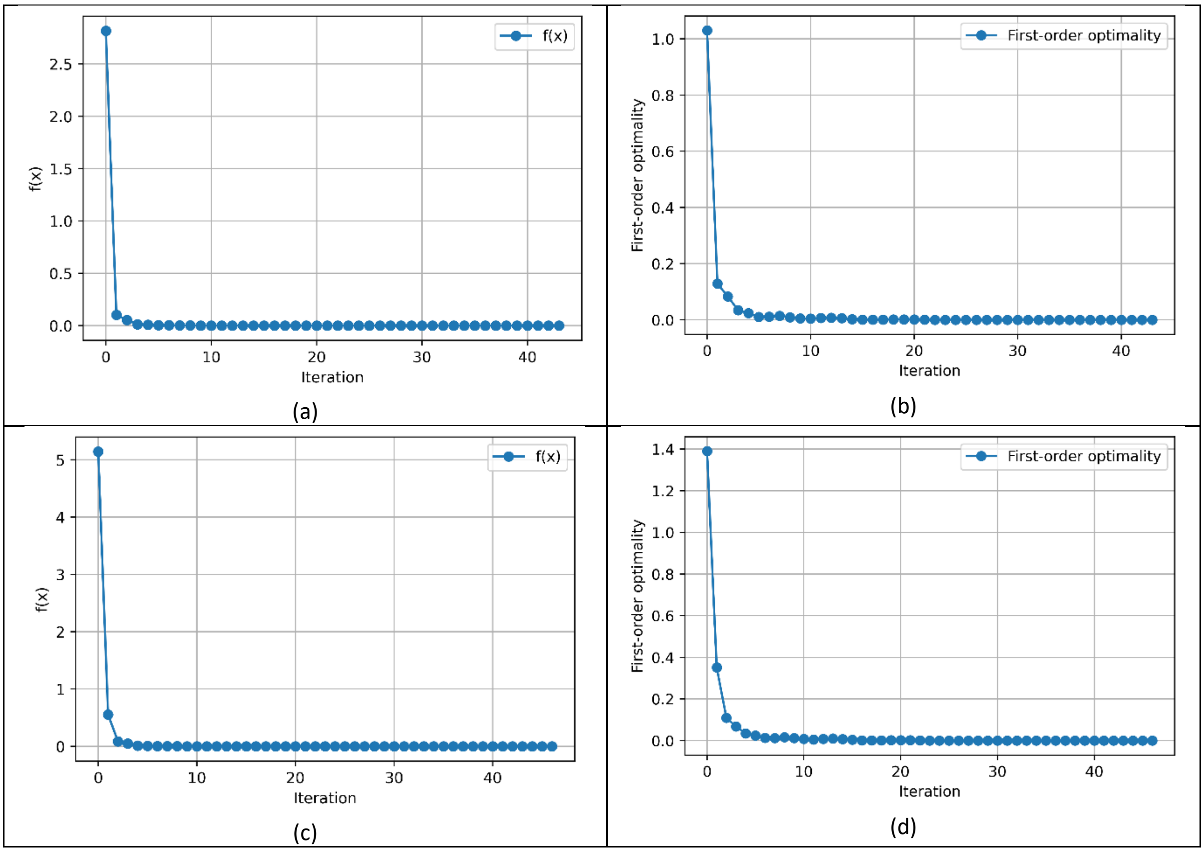

Figure 1: Convergence plots of objective function (left) and First-order optimality (right) for the upper surface in (a) and (b), and for the lower surface in (c) and (d).

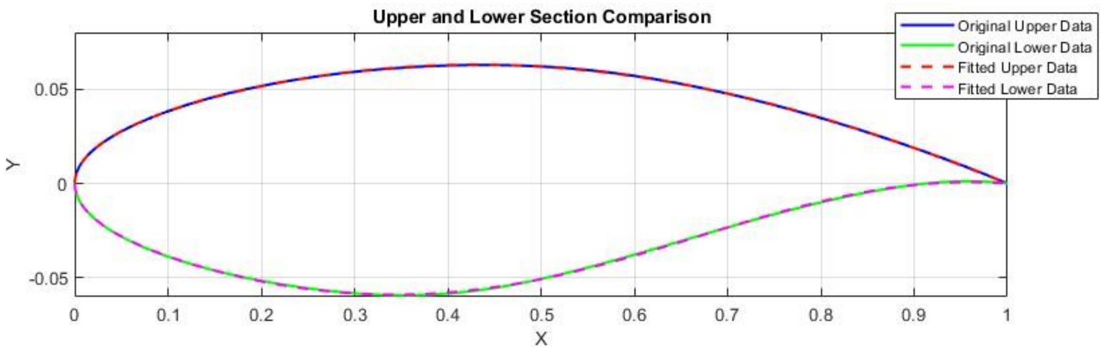

Figure 2: Comparison of the RAE2822 airfoil coordinate to the CST curves.

Figure 1 shows the convergence plots for both surfaces, . Figure 2 demonstrated that the CST curves are exactly identical to the RAE2822 airfoil. This depicted the correct objective function, and method applied for the optimisation process.

Part 2: Airfoil shape optimisation (60 marks)	
In the second step, you need to use the CST curves obtained in part 1 as the starting point of an airfoil shape optimisation, as described on the first page. You need to report the following: - The mathematical formulation of the optimisation you used in this part, i.e. objective function, design variables, constraints and bounds. - The starting and final values of the CST coefficient for each airfoil surface. - The starting and final values of the airfoil lift, drag and pitching moment coefficients and the thickness at the given locations. - The history of optimisation, i.e., the convergence plots of the objective function, constraint violation and first-order optimality. - A figure showing the initial and final airfoil shapes. - A figure showing the pressure distributions on the initial and the optimised airfoils. Please add proper labels and legends to your plots.	
Indicative marks grid:	
Only mathematical formulation, with no results.	0 - 5
Optimisations are performed but the outputs are not correct.	6 - 20
Optimisations are performed, the outputs are correct, but not all the requested information are presented.	21 – 40
Optimisations are performed, correct results are obtained, and all the requested information are presented.	41 - 60
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The optimizer operates under the Eqn. 3, as illustrated below. While the objectives are to minimize the drag coefficient, the initial CST curve, as well as the angle of attack act as design variables. The constrains in this objectives function is defined by the target lift coefficient, the thickness of the airfoil and the moment coefficient. The optimizer is also allowed to alter the angle of attack until the constrains are satisfied.

$$
\begin{aligned}
min \quad & C_d(X) \\
& X = [\mathrm{CST}_i, \alpha] i = 1..n \\
\text{s.t.} \quad & C_l - C_{l_t} = 0 \\
& C_{m_t} - C_m \leq 0 \\
& t_{max_{init}} - t_{max} \leq 0
\end{aligned}
\qquad \textit{Eqn. 3}
$$

Where $t = [t_{c20,} t_{c60,}]$

$$
\begin{aligned}
& t_{c20_{init}} - t_{c20} \leq 0 \\
& t_{c60_{init}} - t_{c60} \leq 0
\end{aligned}
$$

	Suction (Upper) Surface						
Original	0.126275	0.139415	0.157457	0.171029	0.213332	0.176664	0.211278
Optimised	0.127198	0.139002	0.13588	0.156244	0.187489	0.231575	0.304441
	Pressure (Lower) Surface						
Original	-0.12889	-0.14279	-0.13067	-0.26221	-0.02445	-0.11506	0.07298
Optimised	-0.04296	-0.26619	-0.08814	-0.15045	-0.03137	-0.27267	0.039963

Table 2: Original and Optimised CST coefficient.

	Cl	**Cd**	**Cm**	**t @ c =0.2**	**t @ c =0.6**
Original	0.128896	0.093841	-0.15	0.1031	0.0948
Optimised	0.739988	0.016228	-0.06507	0.1031	0.0948

Table 3: Original and Optimized performance coefficients and thickness at given location.

Table 2 shows the initial and optimised CST coefficients for the airfoil. As Table 3 shows, the optimized airfoil reaches the target lift coefficient of 0.74, while successfully reduced the drag coefficient. Both constrains, the moment of coefficient and the thickness at 20 and 60 percent chord is not breach, suggesting the correct implementation of the mathematical equations. Figure 3: History of Optimization.demonstrated the optimization history, with the objective function value and first-order optimality converging under numerically minimal of the maximum constrain violation, the optimizer is able to provide an optimal solution.

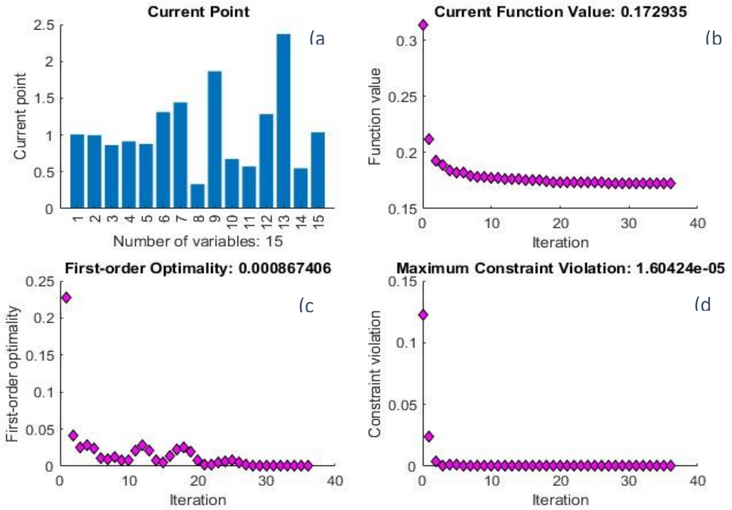

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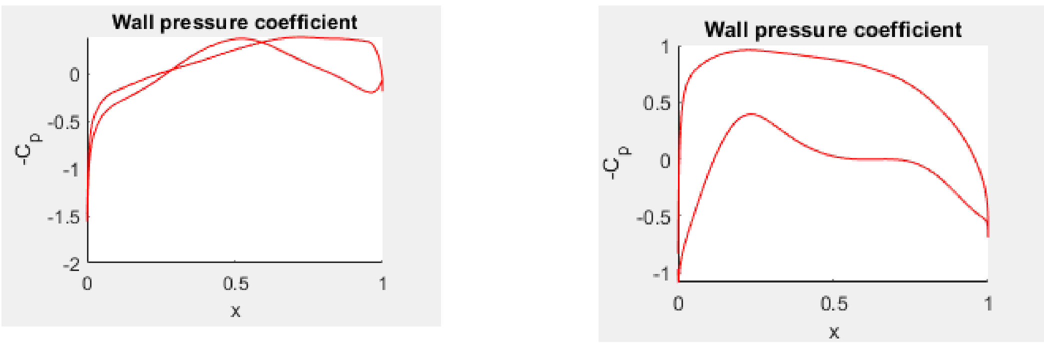
Figure 3: History of Optimization.



(a) *(b)*

Figure 4: (a) Original Airfoil Pressure Distribution. (b) Optimized Airfoil Pressure Distribution.

Figure 4 illustrated the pressure distribution of the airfoil. As shown, the optimized airfoil has greater area under the curve, hence indicating a higher total lift generation. Figure 4: (a) Original Airfoil Pressure Distribution. (b) Optimized Airfoil Pressure Distribution.

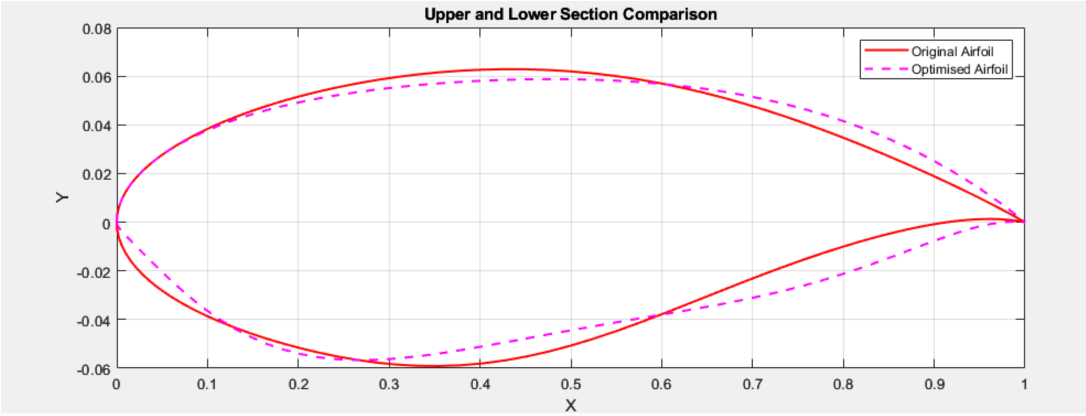


Figure 5: Original and Optimized Airfoil Shape.