

# Global Path Planning Using Artificial Potential Fields

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## Abstract

This paper describes a path planning technique for robotic manipulators and mobile robots in the presence of stationary obstacles. The planning consists of applying potential fields around *C-Space* obstacles and using this field to select a safe path for the robot to follow. The advantage of using potential fields in path planning is that they offer a relatively fast and effective way to solve for safe paths around obstacles. In the method used to accomplish path planning presented here, a trial path is chosen and then modified under the influence of the potential field until an appropriate path is found. By considering the *entire path*, the problem of being trapped in a local minimum is greatly reduced, allowing the method to be used for global planning. The algorithm was tried with success on many different realistic planning problems. The examples in this paper illustrate the algorithm applied to a two dimensional revolute manipulator, a mobile robot capable of translation only, and a mobile robot capable of both translation and rotation.

## 1 Introduction

This work is an examination of a technique for planning a collision free path of both fixed base revolute robots and mobile robots in obstacle cluttered workspaces. Artificial potential fields placed around the obstacles are used to influence the path of the robot so that a safe path is selected.

The first step in the planning process is to map the workspace into the configuration space, (*C-Space*), of the robot[1,2]. Configuration space is a multi-dimensional space spanned by a set of generalized coordinates which describe the position and orientation of a rigid body. The advantage of using *C-Space* is that the position and orientation of a moving body is represented as a point and the obstacles are represented by forbidden regions. The planning problem is then reduced to moving the point representing the robot configuration from the initial to the goal position without entering any of the forbidden regions. The major goal in the research presented here is path planning in configuration space. It is assumed here that we have knowledge of the workspace and that the configuration space has already been produced.

In the technique to be described here, once the *C-Space* of the robot has been created, artificial potential fields are placed around the obstacles and the resulting field is used to choose an appropriate path. To accomplish this, a trial path is chosen and then modified under the influence of the potential field. By

considering the entire path, the problem of becoming trapped by a local minimum is greatly reduced allowing the method to be used for global planning.

### 1.1 Background

The path planning problem has attracted quite a bit of attention in recent years. Most of the approaches to the problem can be classified into a few areas. The two main areas are graph searching and potential field methods. The graph searching techniques are so named because a chart or graph is produced showing free spaces where no collision will occur and forbidden spaces where a collision will occur. Based on this graph, a path is then selected by piecing together the free spaces or by tracing around the forbidden spaces.

A variety of potential field methods have been presented in the past few years, but most have a common theme. These methods use artificial potential fields applied to the obstacles and goal positions and use the resulting field to influence the path of the robot which is subject to this potential. Although not as thorough as the graph searching techniques, the speed of the algorithms and the easy extension to higher dimensions make them an excellent alternative to the graph searching techniques.

An early researcher who used artificial potential fields is Khatib [3]. His strategy involves the use of potential fields in real space as opposed to *C-Space*. Positive (repulsive) potential fields are placed around the obstacles and points subject to this potential (PSP) are placed on the manipulator links. Also, negative (attractive) potential is placed at the goal position which acts on a PSP located at the manipulator's end effector. The result is that the position to be reached is an attractive pole for the end effector and the obstacles are repulsive surfaces for the manipulator parts. The sum of the potentials is then placed into a Lagrangian formulation along with the manipulator kinetic energy to determine the end effector equations of motion. These can be solved to determine a free path to the goal position. Since the mapping from real space to configuration space is not performed, the total solution time for the algorithm is greatly reduced. Also, the dimensionality of the problem is no greater than three dimensions, since it uses real space instead of *C-Space*. This algorithm is very susceptible to local minima in the potential field, however, which limits its usefulness.

Other early researchers in the application of potential fields to path planning include Andrews and Hogan [4], Hogan [5], and Newman and Hogan [6]. Andrews' and Hogan's work and Hogan's later works were in impedance control, which is a control scheme that considers the interaction of the manipulator and its

environment, including external and inertial forces when planning trajectories. The artificial potential fields were used to impede the movement of the manipulator in the direction of obstacles. This work differs from most others in that it is a complete manipulator controller and not just a path planner. Newman and Hogan [6] extended this to include revolute manipulators by using the *C-Space* of the manipulator.

Again, the major problem with these potential field methods is that they are subject to local minima. Since the planner tends toward lower potential areas, it can reach a state of equilibrium, or a potential basin, and becomes trapped. Okutomi and Mori [7] attempted to overcome the problem of local minima in the potential field when path planning. The potential fields selected were different from those previously mentioned in that they were oval in nature and not as susceptible to local minima. Khosla and Volpe, [8], used a similar approach with the use of superquadratic potential fields. The problem, however, was not eliminated.

Koditschek [9] presents a rigorous description of the topological considerations of the potential fields. He also introduces potential fields that have only one global minimum and no local minima, but only for very simple geometric shapes in two dimensions. This was extended [10] to include robots moving amidst spherically bounded obstacles in three dimensions.

Two attempts at using the best features of both graph searching and potential fields were presented by Krogh and Thorpe [11] and by Tournassoud [12]. They used the geometrical solutions for global planning and potential fields for local planning. It has been accepted that the past potential field methods are only reliable in local planning situations. These benefit from the global planning ability of the graph searching methods, but suffer the same shortcomings as well.

A recent paper by Kanayama [13] presents the idea of modifying the entire path under the influence of "costs". A combination of Voronoi diagrams in configuration space [14] and calculus of variation are used to select safe paths that balance the length of the path and the safety of the path.

The research to be presented in this paper is based on the assumption that the obstacles have been identified and there is no uncertainty about their positions. There have, however, been several papers that addressed the question of obstacle uncertainty [15],[16], [17],[18],[19], [20].

## 2 Path Planning Algorithm

Most previously reported potential function schemes have concentrated on the potential fields affecting a point which represents the manipulator configuration. The high potentials on the obstacles force the point into the low potential area around the goal and a path is traced out by the moving point. An alternative planning scheme has been developed in which an arbitrary path is constructed through the potential field with all points along the path being subject to the potential simultaneously. The difference to note is that the potential affects the *entire path* simultaneously and not just one configuration. This will be shown to be much more effective at global path planning. Using optimization techniques, the path is altered so that it passes through minimum potential and, therefore, avoids the obstacles. Although not entirely eliminated, with careful selection of the potential fields, the problem of local minima has been

greatly reduced. This idea can be readily extended into any arbitrary *C-Space* of higher dimension, but the development shown here will be in two and three-dimensional *C-Space*.

Since an arbitrary initial path laid through the *C-Space* will probably pass through the *C-Space* obstacles, a potential function must be selected so that the path is driven away from the obstacle. The total potential field will be broken down into two parts, points inside a *C-Space* obstacle and points outside. The potential field selected for points inside an obstacle,  $U_{in}$ , is one such that the maximum value is inside of the *C-Space* obstacle and the potential decreases linearly as the distance away from the centroid is increased. In general, the best expression for potential inside an obstacle [21] may be written as:

$$U_{in} = U_{max} \left( 1 - \frac{R_{in}}{R_{max}} \right) + U_{offset} \quad (1)$$

where  $U_{max}$  is the maximum potential allowed,  $R_{in}$  is the distance from the point in question to the centroid,  $R_{max}$  is the distance from the centroid to the farthest point on the boundary of the obstacle, and  $U_{offset}$  is an additional potential applied to all points on the obstacle used to produce more of a penalty for crossing an obstacle. For the two dimensional case, this can be envisioned as a cone placed over the obstacle with the vertex of the cone directly above the centroid of the obstacle. The nice feature of using this type of potential field is that the path is driven away from the centroid of the *C-Space* obstacle. In most cases involving revolute manipulators, the resulting path will be the shortest joint space path around the obstacle.

Points outside of the *C-Space* obstacles will be subject to a lower potential. This is to insure that driving the path off the obstacle will be given top priority. This potential,  $U_{out}$ , may be based on the inverse of the distance to the closest boundary:

$$U_{out} = .5 U_{offset} \left( \frac{1}{1 + R_{out}} \right) \quad (2)$$

where  $R_{out}$  in this case is the distance to the closest boundary of the obstacle. This limits the influence of the potential outside of the obstacle to the immediate vicinity of the obstacle. The purpose of the potential outside of the obstacle is to provide a margin of safety. Figure 1 illustrates the potential field associated with a two dimensional square *C-Space* obstacle.

The *C-Space* path is approximated by a set of connected straight line segments. The path selection technique is based on adjusting the endpoints of the line segments. One of the endpoints of the first line segment is fixed at the starting configuration and one of the endpoints of the last line segment is fixed at the goal configuration. The remaining  $n$  endpoints of the  $n + 1$  line segments will be called nodes. The "design variables" used by the minimization routine are the coordinates of the nodes. There will be one design variable assigned to each coordinate of each node. For an  $m$  dimensional *C-Space*, each node will have  $m$  coordinates. Since there are  $n$  nodes, there will be  $m \times n$  separate coordinates that need to be adjusted so that the resulting path passes around the obstacles. In the case of the 2-D joint space, the variables are the  $\theta_1$  and  $\theta_2$  coordinates of each node. More nodes are required if there is more direction changing in the path, resulting in slower convergence. Although rules of thumb have been used to find the number of nodes, an exact number cannot be found for an arbitrary workspace without a priori knowledge of the correct path. Also, the number of

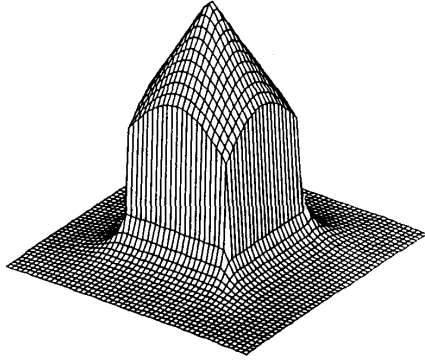


Figure 1: Potential Field

nodes can be updated dynamically during optimization, but this is also be heuristic.

The penalty function used by the minimization routine consists of the potential through which individual line segments pass,  $U_{APF}$ , and the length of the line segments. The penalty due to the segment length tends to minimize the resulting path length. A simplistic 2-D analogy is an elastic cord that has weight. Since there are "peaks and valleys" associated with the potential field, the two ends of the elastic cord can be held on the initial and goal positions of the manipulator and the cord will naturally slip down between the peaks into the valleys. The valleys are the free spaces between the obstacles.

The potential function formulated for the length of each line segment,  $U_{length_i}$ , is based on the potential energy stored in an elastic cord.

$$U_{length_i} = (length_i - limit_i)^2 \quad (3)$$

where  $length_i$  is the length of the  $i^{th}$  line segment and  $limit_i$  is the initial length of the line segment. The difference is the "stretch" in the line segment.

The total penalty function can now be written as:

$$I.P. = \sum_{i=1}^{n+1} w [max(U_{APF_i})] + U_{length_i} \quad (4)$$

where  $w$  is a weighting constant for the artificial potential field, ( $APF$ ),  $max(U_{APF_i})$  is the maximum potential experienced by the  $i^{th}$  line segment due to the artificial potential field, and  $n+1$  is the number of line segments. The design parameter,  $w$ , influences the ability to reach a safe path and must be chosen carefully. It is included to regulate the trade off between obstacle avoidance and path length minimization. Situations can arise where equilibrium positions are reached which do not represent a suitable path due to a poor choice of this constant.

Although the algorithm for planning the path of a manipulator is very similar to that of planning the path of a mobile robot, there are slight differences. Examples of the use of the algorithm on each of these will be presented separately.

## 2.1 Revolute Manipulators

The implementation of the path planning algorithm used in this research discretizes the  $C$ -Space of the robot into a  $101 \times 101$

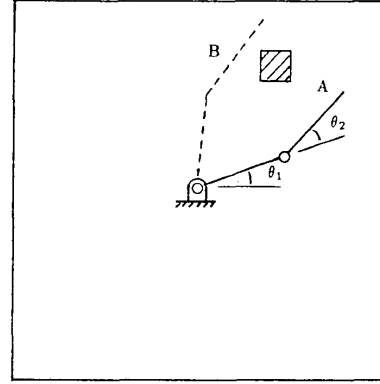


Figure 2: Simple Path Planning Problem

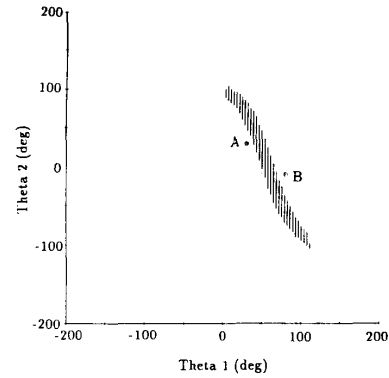


Figure 3: Joint Space

grid for a two degree of freedom revolute robot. The artificial potential field based on the  $C$ -Space was also discrete. This was done to speed the planning algorithm once the potential field was applied. The path segments were laid through the discretized potential field using a line drawing algorithm used in raster scan computer graphics. The discretization process used is conservative. That is, the discretized  $C$ -Space obstacles will be larger than the actual  $C$ -Space obstacles.

As an example of the path planning algorithm, consider the simple problem shown in Figure 2. It is desired to travel from configuration A to configuration B without colliding with the obstacle shown. The joint space map for the manipulator is shown in Figure 3, with the dark area representing the forbidden configurations that would result in a collision between the manipulator and the obstacle. Configurations A and B are shown on the joint space map.

Figure 4 shows the joint space obstacle with the artificial potential field applied. The center of the potential field was applied at the centroid of the obstacle and the potential field varied linearly away from this point. The design variables that the optimization routine uses are the coordinates of the node points. The figure also shows the initial straight path before the optimization

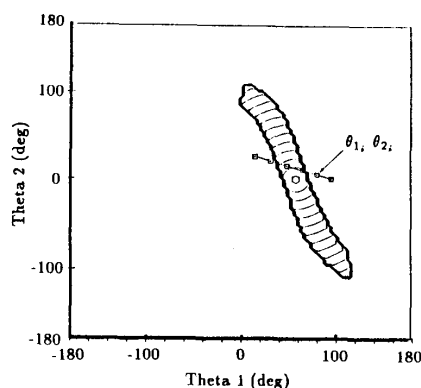


Figure 4: Contour Plot with Initial Path

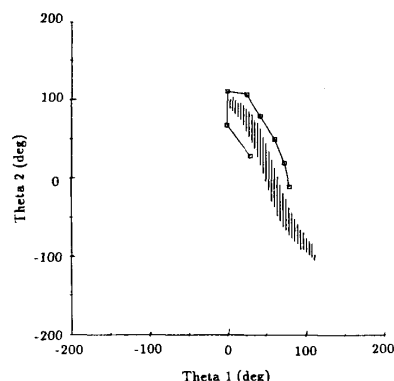


Figure 5: Safe Path Selected

routine begins iterating. The coordinates of the nodes are shown as  $\theta_1$ , and  $\theta_2$ .

The minimization routine selected was a simplex method routine proposed by Nelder and Mead [22]. Most gradient based searching algorithms will work. A Quasi-Newton method and a Conjugate Gradient routine were also tried with success.

The planning algorithm was run and the resulting safe path is shown in Figure 5. The sequence of manipulator moves based on this is shown in Figure 6.

For multiple obstacles in the workspace of the robot, the potential fields of the individual *C-Space* obstacles are superimposed to yield a total potential field. There are some problems that arise that must be dealt with before the algorithm can be applied effectively.

It is possible for a manipulator to collide with two or more obstacles while it is in one configuration. If this happens, then two or more *C-Space* obstacles will overlap, even though the Cartesian space obstacles could be relatively far apart. If the individual potential fields are produced without regard to the overlap, "saddles" will result when the fields are superimposed. These saddles will trap the path that is subject to potential, resulting in no safe path being found. Koditschek [9] addresses

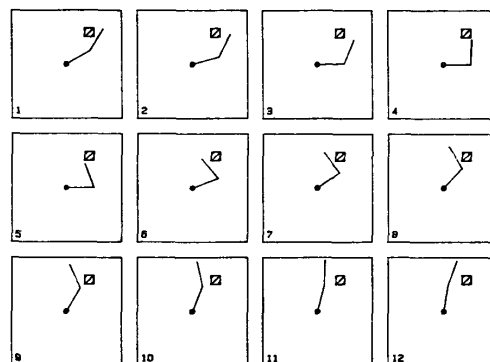


Figure 6: Sequence of Manipulator Moves

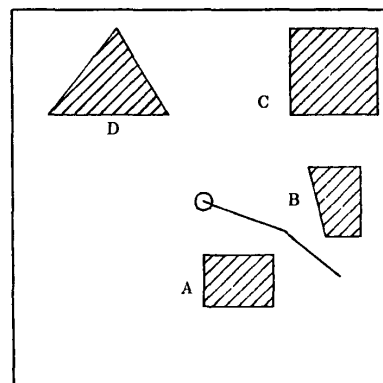


Figure 7: Real Space

the problem of saddles in a point navigation scheme.

The saddle problem may be avoided in a straight forward manner. If two or more *C-Space* obstacles overlap, they will be treated as one obstacle when applying the potential fields. A new total centroid of the two obstacles is then used to compute the potential. Finally, all of the potentials for each separate obstacle will be superimposed as before. Since there is no longer any overlap, there will be no saddle points.

As an example of multiple obstacle avoidance, an example from Red an Truong-Cao [23] is used. The real space of the robot is shown in Figure 7. The configuration space of the robot and the safe path found are shown in Figure 8. The sequence of manipulator moves from the start to the goal position is shown in Figure 9. This technique has also been applied to three dimensional revolute and redundant planner robot planning problems [21].

## 2.2 Mobile Robots

For the most part, the path planning algorithm for mobile robots is the same as that for revolute manipulators. The configuration space of the robot is created, the potential fields are applied, and the path is planned.

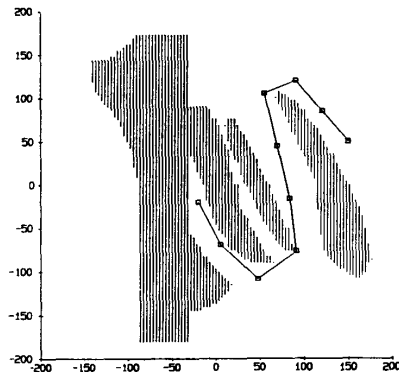


Figure 8: Safe Path Found (Joint Space)

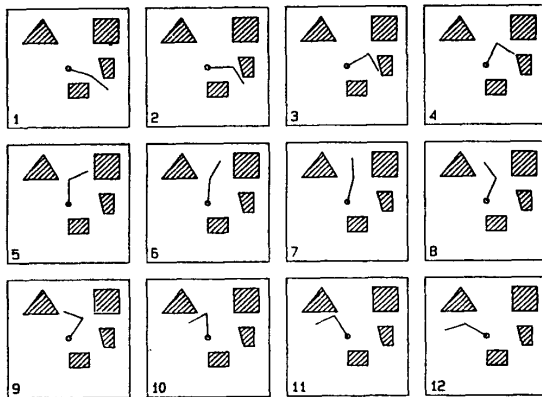


Figure 9: Incremental Sequence of Robot Motion

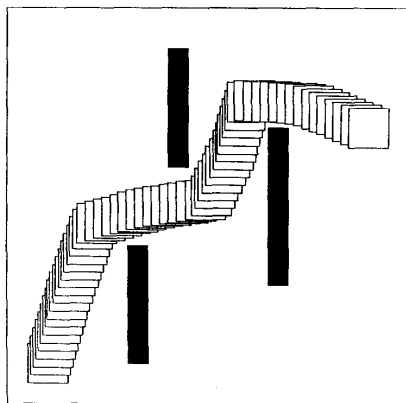


Figure 10: Safe Path for Mobile Robot (Translation)

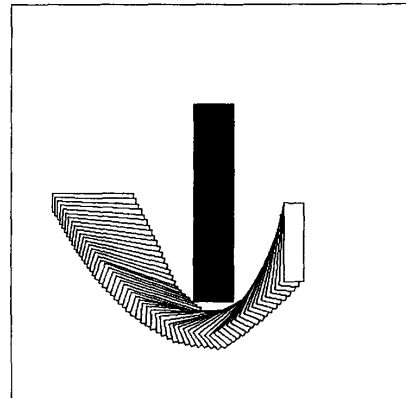


Figure 11: Safe Path (Translation and Rotation)

As an example, a robot capable of pure translation is placed in an obstacle cluttered workspace. The solution to this problem is identical the previous examples shown for revolute manipulators. Figure 10 shows the path planned for the robot from the start to the goal position.

As a second example, a robot capable of both rotation and translation is to move from one side of a obstacle to another. To describe translation in a plane and rotation about one axis, three generalized coordinates are required. Therefore, the configuration space of the robot will be three dimensional. The path generated by the planning algorithm is shown in Figure 11. A multiple obstacle version of this algorithm is being coded.

### 3 Conclusions

The primary goal of this research was to develop an artificial potential field technique for planning the path of a robot that was less susceptible to local minima than other potential field methods. This was accomplished by establishing a trial path and then modifying the *entire path* under the influence of the potential fields. This method was very successful in a large number of revolute robot examples. One drawback to this method is that the global workspace must be known at the time of the planning, whereas only local information is required for the potential methods of the past. However, the benefits of global planning outweigh this in problems with no uncertainty.

For mobile robots, the algorithm was effective in problems involving sets of convex obstacles, or simple concave ones. The nature of the application of the potential fields shown here, however, makes successful application of this to "maze" type problems unlikely.

This method could be used to solve planning problems with cylindrical, spherical, and cartesian geometries and moving obstacle problems. Extension of this technique to higher dimensional problems is straightforward. It is limited only by the computational power of the robot's planner. For most fixed base and simple mobile robot planning problems, this technique offers a fast, effective solution to global planning problems.

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