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# Kuka youBot Arm Shortest Path Planning Based on Geodesics

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***Abstract*— The state-of-the-art Kuka youBot is an open-source robot platform. In order to improve youBot arm manipulation performance, a novel robotic trajectory planning method based on geodesics is used for Kuka youBot arm shortest path trajectory planning in this paper. Geodesic is the necessary condition of the shortest length between two points on the Riemannian surface in which the covariant derivative of the geodesic's tangent vector is zero. The Riemannian metric is constructed according to the distance metric by arc length of the youBot arm trajectory to achieve shortest path. Once the Riemannian metric is obtained, the corresponding Riemannian surface is solely determined. Then the geodesic equations on this surface can be determined and calculated. For the given initial conditions of the trajectory, the geodesic equations can be solved and the results are the optimal trajectory of the youBot arm in the joint space for the given metric. The planned trajectories in the joint space can also be mapped into the workspace. A simple trajectory planning example on Kuka youBot arm from camera pose ready point to object grasping point is given to demonstrate the feasibility of the proposed approach.**

## I. INTRODUCTION

Kuka youBot [1] is a state-of-art robot for mobile manipulation research and education. It is also the official platform for RoboCup@work competition. Kuka youBot system is made up of a omni-directional mobile platform and a 5-degree-of-freedom arm with a two-finger gripper. Kuka youBot is fully open source control and this gives us an opportunity to make an optimal trajectory planning to improve its performance. During the RoboCup@work competition and the research works on this platform, we find that the existing kinematics and trajectory planning approach is not satisfied. For example, the moving trajectory of youBot arm from the initial position to the camera ready for detecting position is long and more time is needed. So a novel shortest path planning method based on geodesics is introduced in this paper to try to improve the working efficiency of the youBot arm.

The traditional trajectory planning method [2] for manipulator is the polynomial interpolating method, and the interpolating is performed either in joint space or Cartesian space. This method is simple and quick and it is still popular used nowadays [3]. But the joints of the manipulator are regarded as linearly independent in this simple method, and the potential performance of the manipulator has not been developed sufficiently. For this reason, many research works on optimal trajectory planning have been carried out. On kinematic optimal trajectory planning aspect, Chiu [4] developed Asada's inertia ellipsoid and Yoshikawa's manipulability ellipsoid, and established a combined performance index of velocity and static force. Then a task compatibility planning method is introduced. Zha [5][6] researched kinematic trajectory planning in Cartesian workspace by genetic algorithm. He regarded the manipulator's trajectory as a ruled surface and interpolated between two pose by Bezier curves. Eldershaw and Cameron [7] and Tian et al [8] also used genetic algorithm to study the trajectory planning problem in joint space, and these methods were based on the polynomial interpolation. Bobrow [9] studied the minimum time trajectory planning and the control method of his planning result was also studied [10]. In order to perform a good trajectory planning, the dynamics of the manipulator also need to be taken into account. Generally speaking, all the methods mentioned above are all based on polynomial interpolation. When to implement trajectory planning, it is better that the trajectory is unrelated to the world coordinates besides it can optimize certain index. Milos Zefran et al [11] investigated the rigid body motion planning in three-dimensional space based on Lie group and covariant derivative. Guy Rodnay and Elon Rimon[12] visualized the dynamics properties of 2-dof robot in three-dimensional Euclidian space. They introduced geodesics on dynamic surface to represent the trajectory when robot system makes free motion. But there is no method of how to identify the geodesics under arbitrary initial conditions and how to plan robot trajectories based on these geodesics. Park [13] used the intrinsic concept of Riemannian metric in the optimal design of open loop mechanism.

In this paper, we introduce a geodesic method of trajectory planning for Kuka youBot arm. It is quite different from the traditional polynomial method. It is an precise optimal trajectory solution but not an approximate one as polynomial interpolation between two points. We regard arc length of the arm trajectory as Riemannian metric and get the shortest path optimal trajectory by our geodesic method for Kuka youBot arm.

## II. KUKA YOUBOT

The KUKA youBot is a mobile manipulator that was primarily developed for education and research in mobile manipulation. KUKA youBot (shown in Fig. 1) comes with fully open interfaces and allows the developers to access the system on nearly all levels of hardware control. It further

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comes with an application programming interface (KUKA youBot API), with interfaces and wrappers for recent robotic frameworks, with an open source simulation in Gazebo [1].

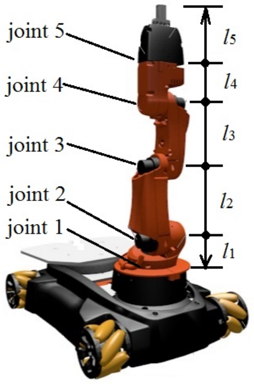


Fig. 1. Kuka youBot arm.

## III. GEODESIC TRAJECTORY PLANNING

### A. The Concept Of Geodesic

Geodesic is the shortest curve between two points on the Riemannian surface and the velocity along geodesic curve remains invariant [14]. It is noticed that the meanings of velocities are different according to different Riemannian surfaces determined by different Riemannian metrics. When arc length is regarded as the metric, the velocity along geodesic refers to the normal velocity that we all know well. If kinetics of the robotic system is defined as the metric, the velocity along geodesic will be the kinetic invariant vector and the system kinetic energy will be minimal along geodesics. For each point in the Riemannian surface, there is only one geodesic according to the given direction. That is, geodesic is determined by the initial conditions. The geodesic equation is

$$\frac{d^2\theta_i}{ds^2} + \Gamma^i_{kj}\frac{d\theta_k}{ds}\frac{d\theta_j}{ds} = 0 \tag{1}$$

$\theta_i$ is the curve coordinate, $s$ is the curve length, $ds$ represents the derivative of the arc length, $\Gamma^i_{kj}$ is the Christoffel symbols,

$$\Gamma^i_{kj} = \frac{1}{2}g^{mi}\left(\frac{\partial g_{km}}{\partial\theta_j} + \frac{\partial g_{jm}}{\partial\theta_k} - \frac{\partial g_{kj}}{\partial\theta_m}\right) \tag{2}$$

where $g_{km}$ is the element of the Riemannian metric matrix, $g^{mi}$ is the element of the inverse matrix of Riemannian metric matrix. $i, j, k, m = 1, \cdots, n$, $n$ is the dimension of the Riemannian space which corresponds to the degree of freedoms when the geodesic is used in robotics.

### B. The Steps of Shortest Path Geodesic Trajectory Planning

According to the definition of geodesics above, the steps of robotic shortest path trajectory planning based on geodesics are as followings:

- To calculate the kinematics (arc length of robot trajectory) of the robot;
- To define the Riemannian metrics according to the kinematics (square of arc length);
- To solve the geodesic equations according to the Riemannian metrics and obtain the kinematic optimal trajectory planning (the shortest path) result.

## IV. KUKA YOUBOT SHORTEST PATH PLANNING BASED ON GEODESIC

In order to make a shortest path planning using geodesic, the Riemannian metric on the square of arc length must be calculated first. This Riemannian metric will determine a geodesic which has the kinematic intrinsics. Solving this equation will obtain the shortest path trajectory of the robot.

### A. The Kinematics Of Kuka youBot

Some of the geometric and physical parameters used in our geodesic trajectory planning are listed in Table I. The geometric model of Kuka youBot arm is shown in Fig. 2, where $x_S$ and $y_S$ represent the spatial coordinate, $x_T$ and $y_T$ represent the tool (gripper) coordinate, $\xi_i (i = 1, \ldots, 5)$ is the twist of the revolute joint.

The Lie group and screw theory [15] are used to calculate the kinematics and dynamics of the Kuka youBot arm. The spatial base frame $x_S, y_S, z_S$ is attached to the base of the manipulator and tool frame $x_T, y_T, z_T$ is attached to the gripper as shown in Fig. 2.The reference configuration is fully extended $\theta_i = 0$ and let $g_{st}(0)$ represent the rigid body transformation between frames $T$ and $S$ when the Kuka youBot arm is in the reference configuration. The twist $\xi_i$ corresponds to the screw motion for the $ith$ joint $\theta_i$ with all other joint angles being remained fixed at $\theta_j = 0$. For a revolute joint,

TABLE I

PARAMETERS OF KUKA YOUBOT ARM

| Description | Notation | Value | Units |
|---|---|---|---|
| Length of link 1 | $l_1$ | 0.075 | m |
| Length of link 2 | $l_2$ | 0.155 | m |
| Length of link 3 | $l_3$ | 0.135 | m |
| Length of link 4 | $l_4$ | 0.081 | m |
| Length of link 5 | $l_5$ | 0.137 | m |
| Center of mass of $l_1$ | $l_{c_1}$ | 0.038 | m |
| Center of mass of $l_2$ | $l_{c_2}$ | 0.078 | m |
| Center of mass of $l_3$ | $l_{c_3}$ | 0.068 | m |
| Center of mass of $l_4$ | $l_{c_4}$ | 0.041 | m |
| Center of mass of $l_5$ | $l_{c_5}$ | 0.069 | m |
| Mass of link 1 | $m_1$ | 1.39 | kg |
| Mass of link 2 | $m_2$ | 1.32 | kg |
| Mass of link 3 | $m_3$ | 0.82 | kg |
| Mass of link 4 | $m_4$ | 0.77 | kg |
| Mass of link 5 | $m_5$ | 0.69 | kg |
| Joint limits of link 1 | $\theta_1$ | -2.95 +2.95 | rad |
| Joint limits of link 2 | $\theta_2$ | -1.13 +1.57 | rad |
| Joint limits of link 3 | $\theta_3$ | -2.64 +2.55 | rad |
| Joint limits of link 4 | $\theta_4$ | -1.79 +1.79 | rad |
| Joint limits of link 5 | $\theta_5$ | -2.92 +2.92 | rad |

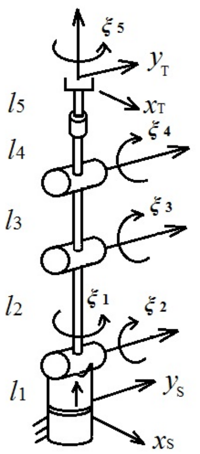


Fig. 2. The geometric model of Kuka youBot arm.

$$\xi_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} -\omega_i \times q_i \\ \omega_i \end{bmatrix} \quad (3)$$

where $\omega_i \in R^3$ is a unit vector in the direction of the twist axis and $q_i \in R^3$ is any point on the axis. Combining the individual joint motions, the forward kinematics map from joint space $Q$ to state space $SE(3)$ (3-dimensional Special Euclidean group), $g_{st} : Q \to SE(3)$, is given by

$$g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} e^{\hat{\xi}_2\theta_2} \ldots e^{\hat{\xi}_5\theta_5} g_{st}(0) \quad (4)$$

Equation (4) is called the product of exponentials formula for the youBot arm forward kinematics. The screw motion $e^{\hat{\xi}_i\theta_i}, i = 1, \ldots, 5$, in (4) is calculated by

$$e^{\hat{\xi}_i\theta_i} = \begin{bmatrix} e^{\hat{\omega}_i\theta_i} & (I - e^{\hat{\omega}_i\theta_i})(\omega_i \times v_i) + \omega_i\omega_i^T v_i\theta_i \\ 0 & 1 \end{bmatrix} \quad (5)$$

which is an element of $SE(3)$. $I$ is the unit matrix and the $e^{\hat{\omega}_i\theta_i}$ is expressed as

$$e^{\hat{\omega}_i\theta_i} = I + \hat{\omega}_i \sin\theta_i + \hat{\omega}_i^2(1 - \cos\theta_i) \quad (6)$$

$\hat{\omega}_i$ in (6) is defined as

$$\hat{\omega}_i = \begin{bmatrix} 0 & -\omega_{i3} & \omega_{i2} \\ \omega_{i3} & 0 & -\omega_{i1} \\ -\omega_{i2} & \omega_{i1} & 0 \end{bmatrix} \quad (7)$$

where $\omega_i = [\omega_{i1} \ \ \omega_{i2} \ \ \omega_{i3}]^T$ is the unit vector of angular velocity of the joint $i$.

The transformation between base and tool frames at reference configuration ($\theta_i = 0$) in Fig. 2 is given by

$$g_{st}(0) = \begin{bmatrix} I & \begin{pmatrix} 0 \\ 0 \\ l_1 + l_2 + l_3 + l_4 + l_5 \end{pmatrix} \\ 0 & 1 \end{bmatrix} \quad (8)$$

The twist of joint 1 is

$$\xi_1 = \begin{bmatrix} -\omega_1 \times q_1 \\ \omega_1 \end{bmatrix} = \begin{bmatrix} -\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (9)$$

Since $q_1$ can be any point on the axis of joint 1 as mentioned in (3), here we select $(0 \ \ 0 \ \ 0)^T$ as $q_1$. The twist of joint 2 is

$$\xi_2 = \begin{bmatrix} -\omega_2 \times q_2 \\ \omega_2 \end{bmatrix} = \begin{bmatrix} -\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ l_1 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \end{bmatrix} = \begin{bmatrix} -l_1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (10)$$

In (10), the point $(0 \ \ 0 \ \ 0)^T$ on the joint axis 2 is chosen as $q_2$. The unit vector of angular velocity of joint 2, $\omega_2$, is $(0 \ \ 1 \ \ 0)^T$. The other twists are calculated in the similar manner as followings,

$$\xi_3 = \begin{bmatrix} -l_1 - l_2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \xi_4 = \begin{bmatrix} -l_1 - l_2 - l_3 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \xi_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (11)$$

The final forward kinematics of the Kuka youBot arm is

$$g_{st}(\theta) = e^{\hat{\xi}_1\theta_1} \ldots e^{\hat{\xi}_5\theta_5} g_{st}(0) = \begin{bmatrix} R(\theta) & p(\theta) \\ 0 & 1 \end{bmatrix} \quad (12)$$

where

$$R(\theta) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad (13)$$

$$p(\theta) = \begin{bmatrix} c_1[l_2 s_2 + l_3 s_{23} + (l_4 + l_5)s_{234}] \\ s_1[l_2 s_2 + l_3 s_{23} + (l_4 + l_5)s_{234}] \\ l_1 + l_2 c_2 + l_3 c_{23} + (l_4 + l_5)c_{234} \end{bmatrix} \quad (14)$$

The detail results of $r_{ij}$ will not be listed here in order to save space. $s_1$, $s_{23}$ and $s_{234}$ represent $\sin\theta_1$, $\sin(\theta_2 + \theta_3)$ and $\sin(\theta_2 + \theta_3 + \theta_4)$ respectively.

### *B. Riemannian metric of the Kuka youBot arm*

Riemannian metric is the square of arc length differentiation of the end effector and is obtained from the forward kinematics,

$$\begin{aligned} ds^2 &= dp(\theta)^2 \\ &= \begin{bmatrix} d\theta_1 & d\theta_2 & d\theta_3 & d\theta_4 \end{bmatrix} G \begin{bmatrix} d\theta_1 \\ d\theta_2 \\ d\theta_3 \\ d\theta_4 \end{bmatrix} \end{aligned} \quad (15)$$

where

$$
G = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix}
$$

$$
= \begin{bmatrix} H^2 & 0 & 0 & 0 \\ 0 & H^2 + {H_2}^2 & H_2H_3 + H^2 - Hl_2s_2 & H_2H_4 + H(l_4+l_5)s_{234} \\ 0 & H_2H_3 + H^2 - Hl_2s_2 & {H_3}^2 + (H - l_2s_2)^2 & H_3H_4 + (H - l_2s_2)(l_4+l_5)s_{234} \\ 0 & H_2H_4 + H(l_4+l_5)s_{234} & H_3H_4 + (H - l_2s_2)(l_4+l_5)s_{234} & {H_4}^2 + (l_4+l_5)^2{s_{234}}^2 \end{bmatrix} \qquad (16)
$$

The parameters $H, H_2, H_3, H_4$ in the above formula are used to simply the expression. The exact values are,

$$
\begin{gathered} H = l_2s_2 + l_3s_{23} + (l_4 + l_5)s_{234} \\ H_2 = l_2c_2 + l_3c_{23} + (l_4 + l_5)c_{234} \\ H_3 = l_3c_{23} + (l_4 + l_5)c_{234} \\ H_4 = (l_4 + l_5)c_{234} \end{gathered}
$$

Please notice that joint angle $\theta_5$ does not shown in the Riemannian metric because we define arc length of the trajectory as the metric, the joint $\theta_5$ has no effect of the arc length, its joint trajectory can use classical polynomial interpolation. There are four angles in (1) and (2). In (2), the index $m$ is always from 1 to 4, $m = 1, \ldots, 4$ for each $i, j, k = 1, \ldots, 4$, this is the Einstein summation convention. For example, when $i = j = k = 1$, $\Gamma^i_{kj}$ is calculated as following,

$$
\begin{aligned} \Gamma^1_{11} &= \frac{1}{2}g^{11}\left(\frac{\partial g_{11}}{\partial \theta_1} + \frac{\partial g_{11}}{\partial \theta_1} - \frac{\partial g_{11}}{\partial \theta_1}\right) \\ &+ \frac{1}{2}g^{21}\left(\frac{\partial g_{12}}{\partial \theta_1} + \frac{\partial g_{12}}{\partial \theta_1} - \frac{\partial g_{11}}{\partial \theta_2}\right) \\ &+ \frac{1}{2}g^{31}\left(\frac{\partial g_{13}}{\partial \theta_1} + \frac{\partial g_{13}}{\partial \theta_1} - \frac{\partial g_{11}}{\partial \theta_3}\right) \\ &+ \frac{1}{2}g^{41}\left(\frac{\partial g_{14}}{\partial \theta_1} + \frac{\partial g_{14}}{\partial \theta_1} - \frac{\partial g_{11}}{\partial \theta_4}\right) \end{aligned}
$$

The other Christoffel symbols $\Gamma_{kj}i$ can be calculated in the same way. Please notice that the determinant of $G$ in (16) is 0, so this Riemannian metric matrix with respect to arc length is singular matrix. The reason is because the three joint axes are parallel. This redundant can be used for obstacle avoidance. Here we simply set the angle of joint 4 as $\theta_4 = \frac{\pi}{2}$ because the youBot arm is working in this position in most of the time, then we can obtain a non-singular Riemannian metric matrix as (17) and make a short path trajectory planning based on geodesics.

$$
G = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix}
$$

$$
= \begin{bmatrix} (l_2s_2 + l_3s_{23} + l_{45}c_{23})^2 & 0 & 0 \\ 0 & l_2^2 + l_3^2 + l_{45}^2 + 2l_2l_3c_3 - 2l_2l_{45}s_3 & l_{45}^2 + l_2l_3c_3 + l_3^2 - l_2l_{45}s_3 \\ 0 & l_{45}^2 + l_2l_3c_3 + l_3^2 - l_2l_{45}s_3 & l_3^2 + l_{45}^2 \end{bmatrix} \qquad (17)
$$

Once the Riemannian metric is set, the Christoffel symbols $\Gamma^i_{kj}$ can be calculated, then the geodesic equation (1) can be solved. For example, let $i = 1, j = 3, k = 1, m = 1, 2, 3$ in (2), then the Chirstoffel symbol $\Gamma^1_{13} = 1/(s_2l_2 + l_3s_{23} + l_{45}c_{23})(l_3c_{23} - l_{45}s_{23})$, the other Christoffel symbols can be calculated in the same method. In order to solve the geodesic equations, they are expressed in standard state equations as

$$
\begin{aligned} \frac{d\theta_1}{ds} &= \dot{\theta}_1 \\ \frac{d\dot{\theta}_1}{ds} &= -\Gamma^1_{11}\dot{\theta}_1^2 - 2\Gamma^1_{12}\dot{\theta}_1\dot{\theta}_2 - 2\Gamma^1_{13}\dot{\theta}_1\dot{\theta}_3 - 2\Gamma^1_{23}\dot{\theta}_2\dot{\theta}_3 \\ &\quad -\Gamma^1_{22}\dot{\theta}_2^2 - \Gamma^1_{33}\dot{\theta}_3^2 \\ \frac{d\theta_2}{ds} &= \dot{\theta}_2 \\ \frac{d\dot{\theta}_2}{ds} &= -\Gamma^2_{11}\dot{\theta}_1^2 - 2\Gamma^2_{12}\dot{\theta}_1\dot{\theta}_2 - 2\Gamma^2_{13}\dot{\theta}_1\dot{\theta}_3 - 2\Gamma^2_{23}\dot{\theta}_2\dot{\theta}_3 \\ &\quad -\Gamma^2_{22}\dot{\theta}_2^2 - \Gamma^2_{33}\dot{\theta}_3^2 \\ \frac{d\theta_3}{ds} &= \dot{\theta}_3 \\ \frac{d\dot{\theta}_3}{ds} &= -\Gamma^3_{11}\dot{\theta}_1^2 - 2\Gamma^3_{12}\dot{\theta}_1\dot{\theta}_2 - 2\Gamma^3_{13}\dot{\theta}_1\dot{\theta}_3 - 2\Gamma^3_{23}\dot{\theta}_2\dot{\theta}_3 \\ &\quad -\Gamma^3_{22}\dot{\theta}_2^2 - \Gamma^3_{33}\dot{\theta}_3^2 \end{aligned}
$$

To assume $y(1) = \theta_1, y(2) = \dot{\theta}_1, y(3) = \theta_2, y(4) = \dot{\theta}_2, y(5) = \theta_3, y(6) = \dot{\theta}_3$, the above equations set of geodesics can be further written in the M-file format so that it can be solved by ODE45 in the Matlab environment. We select the start point as $\theta_1 = 0, \theta_2 = \pi/6, \theta_3 = \pi/3$, this is the position which Kuka youBot arm searching objects by its camera installed in the end of the joint 5. The end point is $\theta_1 = \pi/18, \theta_2 = \pi/12, \theta_3 = \pi/18$, this is one of the position for Kuka youBot arm to pick the object. $\theta_4$ is set a fixed value as $\pi/2$ in this example. The trajectory planning results in joint space are shown in Fig. 3 and end-effector trajectory simulation in Cartesian space is shown in Fig. 4. This simulation shows that the geodesic trajectory is a straight line in the Cartesian space which is the shortest path between the given start point ($\theta_1 = 0, \theta_2 = \pi/6, \theta_3 =

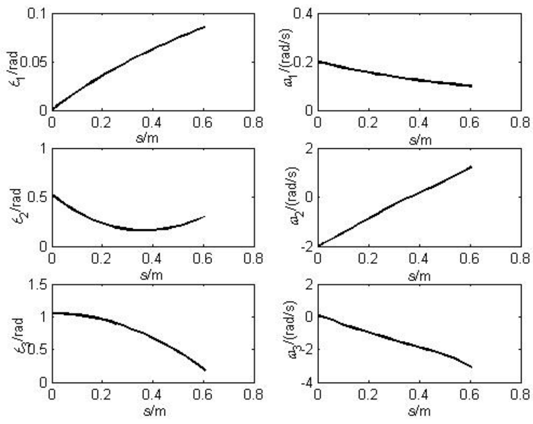


Fig. 3. Trajectory planning results

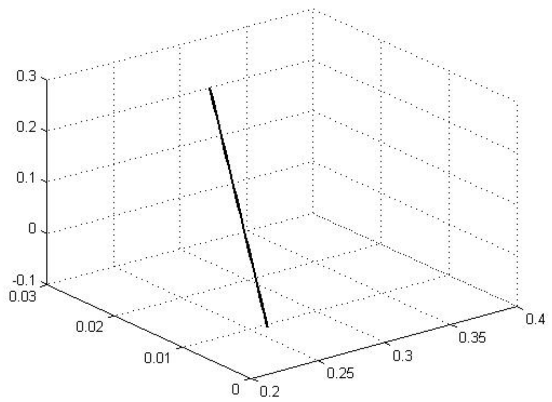

Fig. 4. End effector simulation

$\pi/3, \theta_4 = \pi/2)$ and end point ($\theta_1 = \pi/18, \theta_2 = \pi/12, \theta_3 = \pi/18, \theta_4 = \pi/2$).

The simulation result shows that the the trajectory is the shortest path and is obtained directly by solving the differential equations of geodesics. The traditional polynomial interpolation trajectory planning method is light computing payload but needs to set up several knots on the desired trajectory. The precision of the trajectory depends on how many knots are selected and the knots are difficult to be deployed if the optimal trajectory is not easy to predict.

## V. CONCLUSIONS AND FUTURE WORKS

### A. Conclusions

In this paper, a novel geodesic trajectory planning method is introduced and applied on Kuka youBot arm's shortest path trajectory planning. The kinematics of youBot arm is obtained using Lie group and screw theory. Then the square of arc length is defined as the Riemannian metric and the corresponding geodesic equations can be determined and calculated according to the given initial conditions of the trajectory. A trajectory planning example is given to verify the proposed approach. Comparing with the traditional polynomial interpolation method, geodesic-based trajectory planning method can obtain the optimal path directly. The proposed geodesic trajectory planning method can also be used in other robotic arms.

### B. Future Works

The next step of our research work on Kuka youBot arm geodesic trajectory planing will be to create a new Riemannian metric which includes all the five joints, so full trajectory planning for all the five joints can be obtained. We will also implement the geodesic trajectory planning results on the real youBot arm to verify our method used in this paper. By measuring the time, stability and currents in each joints, to compare our geodesic trajectory planning method with the existing open source trajectory planning algorithm. Furthermore, the dynamics of the youBot arm will also be considered in the geodesics trajectory planning in the future.

## VI. ACKNOWLEDGMENTS


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