

# Cartesian constrained time-optimal point-to-point motion planning for robots: the waiter problem

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## 1 Introduction

Time-optimal point-to-point motion is of significant importance for maximizing the productivity of robot systems. This type of motion planning for robots is however a complex problem and is therefore often solved in two phases. First, a high level planner determines a geometric path ignoring the system dynamics but taking into account geometric path constraints. Second, an optimal trajectory along the geometric path is determined taking into account system dynamics and limitations. Since the dynamics along a geometric path can be described in terms of a scalar path coordinate  $s$  and its time derivatives [1], the decoupled approach simplifies the motion planning problem to great extent. In recent work it was shown for a simplified robotic manipulator that the path following problem with Cartesian acceleration constraints can be cast as a convex optimization problem [2], which allows for the highly efficient computation of the global optimum.

In this work we explore ways to solve the motion planning problem in application to the waiter problem. The waiter problem considers moving a non-fixed object time-optimally from an initial pose to a final pose while preventing the object to slide, lift or tip over. This problem is akin to a robot that transports, e.g., a pallet with a loosely stacked payload. In [2] the convex formulation of the path following problem is implemented to solve a simplified version of the waiter problem. Assuming a fixed geometric path, the optimal timing along this path  $s(t)$  is determined subject to the earlier outlined criteria. This convex problem can be solved efficiently, but is still conservative. Namely, appropriate tilting of the tray allows reduce the overall motion time. We present attempts to include the shape of the path in the optimization and to implement an efficient solution technique.

## 2 Approach

We compare two methods to tackle the overall time-optimal trajectory planning problem. For both methods the path is parameterized as a convex combination between lower- and upper-bound paths in joint space to achieve freedom in the Cartesian trajectory [3]. In the first approach, a high-level gradient descent optimization is applied to the parametric sensitivity of the solution for each convex path following subproblem towards the shape of the path. This algorithm



Figure 1: Experimental setup, the ABB IRB120 manipulator.

can be very slow due to the many convex problem solutions that are needed to compute the gradients. The second alternative is to formulate one single non-linear program for the entire time-optimal motion planning problem. And to subsequently use a state-of-art non-linear programming solver to obtain the optimal trajectory. The goal of this case study is to assess the potential performance of both techniques in a real-time solution scheme for nonlinear model predictive control. The presentation is primarily focused towards the intermediate findings for the first described gradient-descent method.

## 3 Example

To illustrate the results and show the experimental validity, we apply the solution methods for the waiter problem to an industrial robot. Considered here is a six-DOF ABB IRB120 serial link manipulator with an open controller for tracking the joint trajectories, see Figure 1. The time-optimal trajectory is generated off-line and applied in open-loop for the validation of the numerical results.

## References

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