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Near-zone fields behind circular apertures in thick, perfectly conducting screens

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A rigorous electromagnetic theory describing diffraction by a circular aperture in a perfectly conducting screen is used to model the flux and field intensity profiles in the near field behind the screen. In particular, the case of an aperture having a diameter one-fifth that of the wavelength of the radiation being used is considered. The degree of collimation of both the Poynting vector and the electric field intensity profiles of the transmitted beam is observed and its decrease with increasing distance from the screen is investigated. The effect of increasing the thickness of the screen is observed. These results are relevant to the study of near-field microscopy where the sample under investigation is either imaged with a collimated beam or light transmitted through the object is sampled using a probe having an electrically small diameter.

I. INTRODUCTION

Passing a plane wave through an aperture produces a well-defined, locally collimated beam immediately behind the screen, with a cross section given by the shape of the aperture. As the distance from the screen increases, however, the field profile spreads For wavelengths λ a, this phenomenon has been incorporated into a technique known as "near-field microscopy," where the resolution is determined by the size of the aperture rather than the wavelength. A scanning microwave microscope with a resolution of $\lambda/60$ was investigated by Ash and Nicholls[1] in 1971 and subsequently Massey *et al.*[2,3] demonstrated resolution in the far infrared of 30 μm using a laser with a wavelength of 118.8 μm (i. e., $\lambda/4$). More recently Lewis and Isaacson have obtained a resolution of 30 nm at optical wavelengths.[4] In all these experiments the object being imaged was illuminated through an electrically small aperture. The physical dimensions of the aperture were much smaller than the wavelength and the resolution was determined by the spot size rather than λ. Current attempts at producing a near-field optical microscope could, if successful, be used, for example, to nondestructively image biological surfaces and to extend the limits of microlithography. It may also enable a high-resolution investigation of the time evolution of large molecules in living cells. Initial studies[5–8] involving the illumination of the sample under investigation through a small aperture and the collection of the radiation transmitted through the sample with a conventional microscope suggest that resolution better than $\lambda/10$ is possible. Recently, however, it has been found that using the small aperture in collection mode may yield higher resolution. Fischer[9] used the aperture to restrict the area of observation of a sample illuminated by light which has been totally internally reflected along a glass slide lying between the aperture and the sample. Betzig, Isaacson, and Lewis[10] used a probe containing an electrically small aperture to sample light transmitted through the object under investigation.

The main limitation of near-field microscopy is the rapid decrease in the collimation of the beam with increasing distance from the aperture, which is accompanied by a decrease in the peak intensity of the transmitted radiation. Although the actual experimental arrangements are generally quite complex, it is possible to use models describing diffraction by simple structures to gain an estimate of the distance over which the transmitted beam has the required degree of collimation. Leviatan[11] used the small hole theory of Bethe[12] to investigate the near-zone fields behind an aperture having a radius $\lambda/150$ in a screen of zero thickness. Betzig and co-workers[13] pointed out that at optical wavelengths, a metallic screen must have a non-negligible thickness to be sufficiently opaque. In order to investigate the effect of a finite screen thickness on near-zone fields, they used a rigorous Green's function formulation[14] for a slit in a thick, perfectly conducting screen. As they conceded in their conclusion, however, there are some fundamental differences between this problem and that of a circular aperture which is necessary for two-dimensional imaging. First, the transmission through a circular aperture is generally lower than that through a slit having roughly the same dimensions. As the thickness of the screen is increased, transmission through a small circular aperture decreases exponentially. If the ratio of radius to wavelength is less than 0.3 all modes are evanescent. Since a slit can always support a propagating mode the transmission coefficient varies sinusoidally with screen thickness and in Ref. 4 Betzig and co-workers selected a screen thickness which produced a transmission maximum. Hence, the effect on transimission through a circular aperture of finite screen thickness is more significant than on that of the corresponding slit

II. FORMULATION

Here, a modal theory for a circular aperture in a thick, perfectly conducting screen[15] is used to model the fields behind an aperture of radius $\lambda/10$. This model involves separating the space around the aperture into three distinct regions. The plane passing through the center of the screen is

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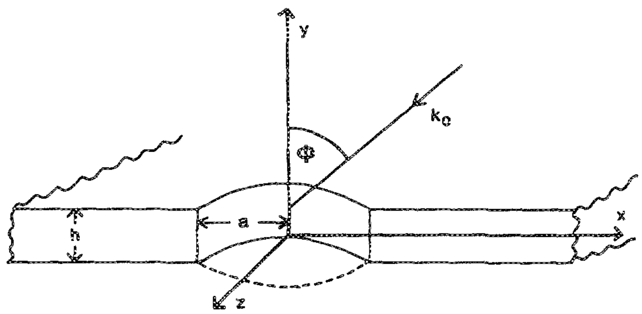

FIG. 1. Parameters used in the description of the aperture, screen, and the incident radiation.

taken to be the x-z plane and the y axis is normal to the screen surface. The screen occupies the region of space between $y = +h/2$ and $y = -h/2$ and the aperture radius is designated by a. (See Fig. 1.) The wave number of the incident radiation (assumed to be a normally incident plane wave) is $k_0 = 2\pi/\lambda$. In the free-space regions above and below the screen the electric and magnetic fields are written as a Fourier integral over plane waves, while within the aperture itself the fields are written as a discrete sum over an infinite set of circular waveguide modes. The plane-wave amplitudes and modal coefficients are determined by applying the continuity conditions at the upper and lower screen interfaces. This produces an infinite set of linear equations in the mode coefficients. The set of modes is truncated and the resulting set of equations is solved using standard matrix inversion techniques. It is also necessary to calculate some of the integrals appearing in the matrix numerically. Once the modal coefficients have been determined, the Fourier integrals which give the electric and magnetic fields below the screen are performed numerically. Although no approximations were necessary in the development of the theory it is important to minimize inaccuracies which appear when using the numerical implementation of the model. Once the modal coefficients have been determined, typical run times on a CDC CYBER 830 are of the order of 70 s when 200 integration points and 10 modes are used in the determination of the electric and magnetic field at a single point below the aperture. For distances less than one-fifth of an aperture radius from the screen more than 1000 points are required to obtain convergence to within 5%.

Since many detectors, including photographic plates and dipoles, actually respond to the electric field strength rather than the Poynting vector, the corresponding electric field intensity plots are also given. In order to investigate the variation in the collimation of the "beam" behind an electrically small aperture, the specific case of an aperture with a radius of $\lambda/10$ was considered. A plane wave with its electric field vector parallel to the z axis was normally incident upon the aperture. Although it is difficult to model to fields near the edges of the aperture using the modal theory, it has been seen that the formulation can accurately predict field quantities away from the aperture itself. It has been suggested by other authors[10,11] that the quantity of interest in near-field microscopy is the component of the Poynting vector normal to the screen surface, that is the component of the energy density propagating away from the screen.

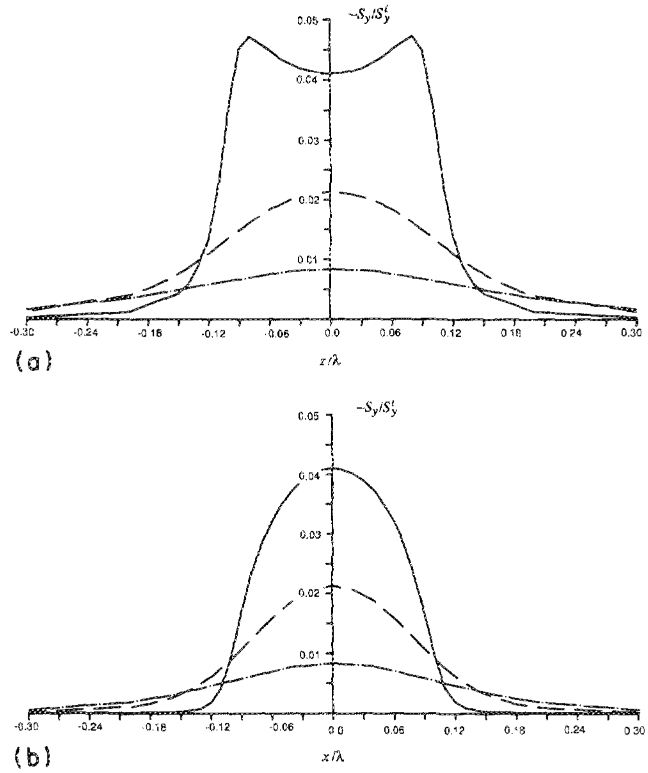

FIG. 2. The component of the Poynting vector normal to the screen divided by that of the incident radiation, in (a) the E plane and (b) the H plane at distances below the screen of $0.2a$ (solid curve), a (dashed line), and $2a$ (broken line). The radius of the aperture is $a = \lambda/10$ and the screen has zero thickness.

III. NUMERICAL RESULTS

In Fig. 2 the variation of S_y in planes parallel to a screen of zero thickness along planes parallel to the electric field of the incident beam (E plane) and the orthogonal (H) plane is shown. The planes are distant $a/5$, a, and $2a$ from the screen. As expected, the collimation of the beam deteriorates with increasing distance from the screen while the peak intensity of the "beam" decreases. In Fig. 3, the thickness of the screen has been increased to $a/4$, which corresponds to a thickness of 12.5 nm when $\lambda = 500$ nm. The most noticeable effect is the drop in intensity, although the rate of spreading of the transmitted field does not appear to be significantly affected. At this wavelength aluminum has a skin depth δ of 6.5 nm. Thus, an undiffracted plane wave passing through the screen emerges with a normalized intensity of $\exp(-2h/\delta)$ which, in this case, is numerically equal to 2%. Figure 3, however, shows that using a perfectly conducting screen with this thickness produces a drop in the normalized Poynting vector from approximately 4% to less than 2% at a distance $Y = a/5$ below the screen. The small ratio of the transmitted flux intensity to that incident upon the aperture suggests that the contrast between the radiation passing through the aperture and that passing undiffracted through the screen is not high enough for microscopy purposes. Many detectors, however, including photographic plates and dipoles, actually respond to the electric field strength rather than the Poynting vector. The normalized intensity of the electric field at zero screen thickness is approximately 27%, dropping to around 11%

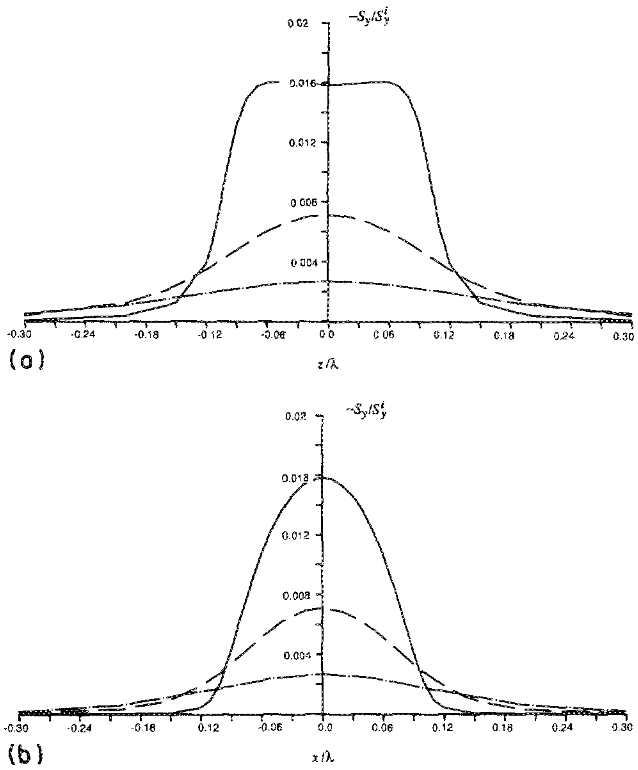

FIG. 3. Same as for Fig. 2, although the screen thickness is now $\lambda/4$.

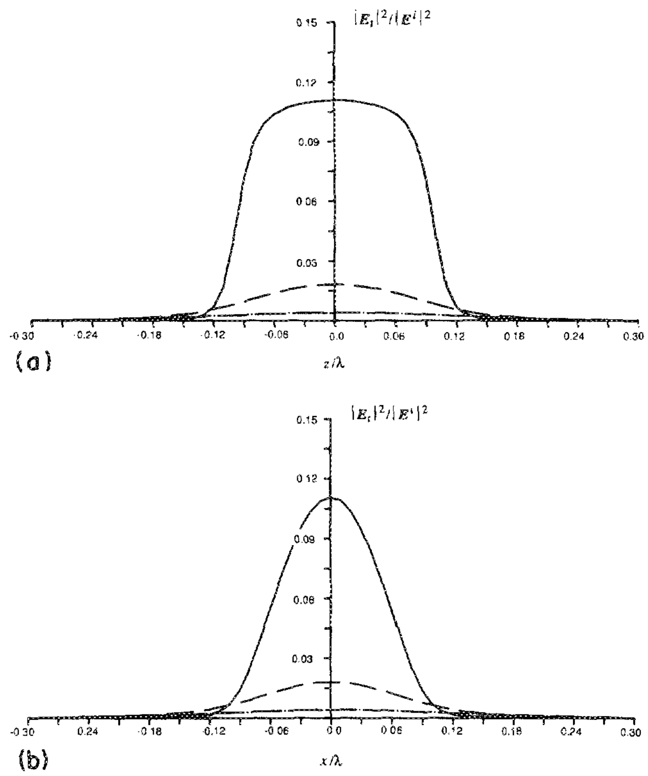

FIG. 5. Same as for Fig. 4, except the screen thickness is now $\lambda/4$.

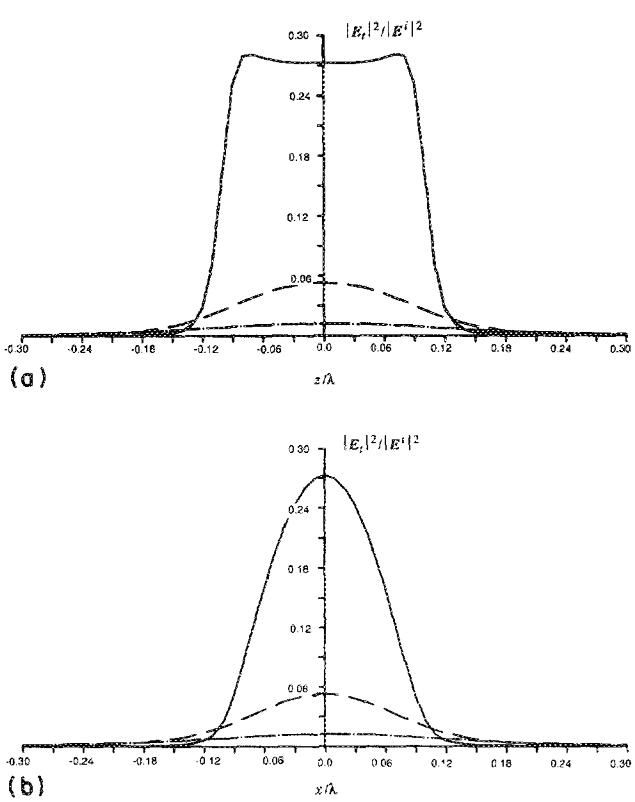

FIG. 4. The normalized electric field profile in planes distant $0.2a$ (solid line), a (dashed line), and $2a$ (broken line) below an aperture of radius $a = \lambda/10$. The screen has zero thickness and graph (a) shows the E plane variation, while (b) shows the H plane profile.

when $h = a/4$. (See Figs. 4 and 5.) As the Poynting vector and the electric field intensity are attenuated in the same manner when passing through a uniform, thick screen, the contrast in the electric field strength should be sufficient for microscopy purposes. Note, however, that when the electric field is regarded as the quantity of interest, the intensity and degree of collimation of the beam drop more rapidly with distance from the screen than those of the Poynting vector.

Bethe[12] pointed out that the fields produced below a small aperture are to first order equivalent to those produced by a magnetic dipole lying in the plane of the aperture and an electric dipole directed perpendicular to the plane of the screen. The behavior of the electric field and Poynting vector exhibited in Figs. 2–5 is similar to that of an electric dipole in the quasi-static limit, where at small distances from the dipole, the dominant term in the experessions for the electric and magnetic fields is the static term. Since, to lowest order in k_0a, the magnetic field is 90° out of phase with the electric field, the Poynting vector is of higher order in k_0a than either the electric or magnetic fields. Another result which is analogous to the behavior of a dipole in the quasi-static limit is the decrease in the field intensity with increasing distance from the aperture. Close to an electric dipole the square of the electric field decreases as r^{-6} with distance r from the dipole, while the Poynting vector varies as r^{-2}. At larger distances from the source of the radiation, the electric field intensity also falls off as r^{-2}

IV. CONCLUSION

Although the assumption regarding the perfect conductivity of the screen material is highly inaccurate, it is possible using the modal theory to gain some insight into the behavior of fields transmitted through a circular aperture when a screen with nonzero thickness is used. The electric field intensity plots presented here confirm that resolution significantly better than one-fifth of a wavelength should be obtainable provided measurements can be made within a distance equal to one aperture radius from the screen.

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