Stochastic Modeling of Light-weight Floating Objects

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(a) Passive advection (b) Random noise (c) Our method

Figure 1: *Flying leaves past a house.*

1 Introduction

Light-weight objects floating inside a flow play a significant role in the liveliness of our world (e.g., leaves, dust, snowflakes, bubbles). They follow the flow and show complex and chaotic motion. First, animators usually add simple random noise to the streaming path. However, this method yields low-quality floating behavior since the random noise does not take into account the spatial and temporal distribution of underlying flow turbulence. For example, obstacle-induced oscillation cannot be easily created as a major source of the unique motion. Second, floating objects can be passively advected by flow velocities from physically-based simulation. A critical challenge is that modeling the important jiggling motion requires turbulent flow field, which is hard to achieve by direct numerical simulation (DNS) due to limited computational resources and numerical dissipation. This situation deteriorates severely when realtime performance and interactivity are demanded in a 3D gaming environment. Moreover, the floating motion has intrinsic stochastic nature, i.e., the repeated executions result in nonidentical dynamics, which is not achievable with deterministic simulation. Third, adding noise to fluid solvers can introduce chaotic flow velocities (e.g. [Pfaff et al. 2010]). In an approach, special Langevin particles affect fluid solver with forces [Chen et al. 2011]. However, such methods rely on DNS and apply chaotic addition to the whole fluid domain, which is inefficient in handling a group of floating objects inside.

In this work, we model the movement of light-weight floating objects as a random process. A Langevin stochastic differential equation (SDE) is employed to compute the momentum change of objects, which adaptively show placid motion along a quiet flow, while behave in a chaotic manner in unsteady regions. In particular, a pre-computed or designed base flow defines the main stream. The location and intensity of fluctuation are determined by $k - \varepsilon$ turbulence equations. These equations are explicitly solved with finite difference schemes for each object, achieving very fast computation, minimal programming effort and convenient control.

2 Stochastic Object Motion

The generalized Langevin SDE [Pope 2000] describes the random motion of a suspended particle inside a fluid. We utilize and modify it for modeling light-weight floating objects moving inside a turbulent flow as:

$$m\frac{d\mathbf{v}_t}{dt} = -C_{l1}\frac{\varepsilon_t}{k_t}(\mathbf{v}_t - \overline{\mathbf{U}}_t) + C_{l2}k_t\psi_t + \mathbf{F}, \qquad (1)$$

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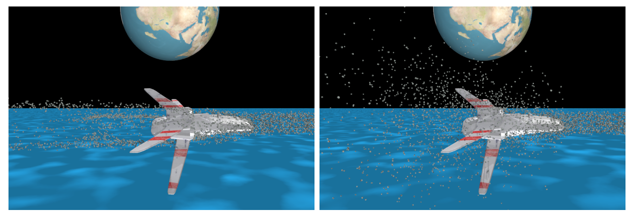

(a) Passive advection (b) Our method

Figure 2: *Space fighter interacting with cosmic dust.*

where $\mathbf{v}_t$ is the velocity of an object with mass m, and $\mathbf{F}$ is the body force. The equation includes a relaxation of the agitated velocity towards the base flow $\overline{\mathbf{U}}_t$, and a random agitation ψ_t, which is considered as the *diffusion process*, a statistical Markov process, implemented as a standardized Gaussian random variable in vector format. C_{l1} and C_{l2} are control parameters. k (turbulence energy) and ε (turbulence dissipation) are the measurement of estimated turbulence computed from:

$$\frac{\partial k}{\partial t} = \mathsf{P} - \varepsilon; \quad \frac{\partial \varepsilon}{\partial t} = \frac{\varepsilon}{k}(C_{\varepsilon 1}\mathsf{P} - C_{\varepsilon 2}\varepsilon). \qquad (2)$$

$C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are control parameters. P is the production of the turbulent energy $\mathsf{P} = 2\nu_t\overline{S_{ij}}^2$, where $\overline{S_{ij}}^2 = \sum_{i,j}(\frac{1}{2}(\frac{\partial \overline{\mathbf{U}}_i}{\partial \mathbf{x}_j} + \frac{\partial \overline{\mathbf{U}}_j}{\partial \mathbf{x}_i}))^2$. Here ν_t is the turbulent viscosity $\nu_t = C_\mu k^2/\varepsilon$ with a constant $C_\mu = 0.09$.

Fig. 1a displays leaves flowing along the wind with steady motion. Fig. 1b uses random noise for unsteady dynamics, however, with an unpredicted dispersive propagation. Our method, Fig. 1c, models leaves flowing past the roof, climbing upwards, and oscillating behind the house according to wall-induced turbulence. Fig. 2 shows another example of a space fighter interacting with cosmic dust. On an ordinary PC, the examples run with an average speed less than 1 millisecond per step given a steady base flow.

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