Precalculus

Me. I am Him.

11/28/2022

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3.1 Properties of functions and Complex Zeros

3.1.1 3.1 - Completing the square

Definition 3.1.1: Completing the square

In form $ax^2 + bx + c$, you half b, and then square.

Note:-

Forms:

Standard: $ax^2 + bx + c$; $a \neq 0$

Vertex: $a(x - h)^2 + k$; $a \neq 0$

Note:-

Verticies:

Standard: (-b/2a , f(-b/2a))

Vertex: (h , k)

Note:-

Axis of Symmetry:

Standard: x=-b/2a

Vertex: x=h

Note:-

y-Intercept

Standard: (0 , c)

Vertex: *Plug in 0 for x and solve*

Note; There is always one.

Note:-

x-Intercept

Standard: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, otherwise known as the quadratic formula.

If the inside of the quadratic is < 0, there are no x-intercepts.

If the inside of the quadratic is $= 0$, there is exactly one x-intercept.

If the inside of the quadratic is > 0, there are exactly two x-intercepts.

Vertex: *Plug in 0 for y and solve*

Chapter 4

4.1 4: Composite Functions

4.1.1

Note:-

11/28/2022 - Didn't really do much. Just reviewed what $f \cdot g$ or f(g(x)) was.

4.1.2

4.1.3 Exponential Functions

Definition 4.1.1: Are you prepared? MAKE NEW COMMAND

- $4^3 = 8$
- $8^{\frac{2}{3}} = 4$
- $3^{-2} = \frac{1}{9}$

Note:-

In a^n, a is known as the base whereas n is known as the exponent, index, or power.

Note:-

Law of Exponents:

(1) $a^m \cdot a^n = a^{m+n}$ Example: $3^2 \cdot 3^5 = 3^{2+5} = 3^7 = 2187$

(2) $(a^m)^n = a^{mn}$ Example: $(2^3)^2 = 2^{3\cdot 2} = 2^6 = 64$

(3) $(ab)^m = a^m b^m$ Example: $(5x)^3$

(4) $1^n = 1$ Exaple: $11001 = 1$

(5) $a^{-n} = \frac{1}{a^n}$ Example: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

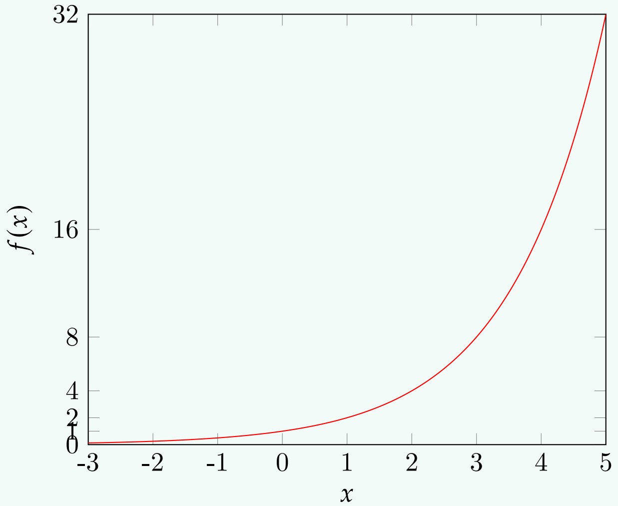
(6) $a^0 = 1$ Example: $7^0 = 1$

(7) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ Example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

Definition 4.1.2: Exponential Function

A function of the form f(x) = a^x where x is a positive real number (a>0) and a $\neq$ 1. The domain of f is $\mathbb{R}$

Example 4.1.1 (2: Graph the exponential function: $f(x) = 2^x$)

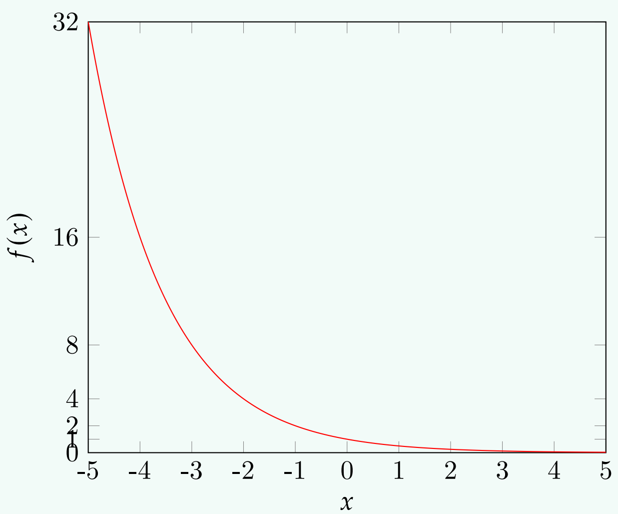


Note:-

Properties of the Exponential Function: $f(x) = a^x$, where $a > 1$

1. The *domain* is the set of all real numbers. The *range* is the set of all positive real numbers.
2. There are no *x-intercepts*. The *y-intercept* is 1.
3. The *x-axis* (y=0) is a horizontal asymptote as $x \to -\infty$
4. The function is an increasing function and is one-to-one.
5. The graph of f contains the points (0,1),(1,a), and (-1,1/a).
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.2 (3: Graph the exponential function: $f(x) = (\frac{1}{2})^2$)



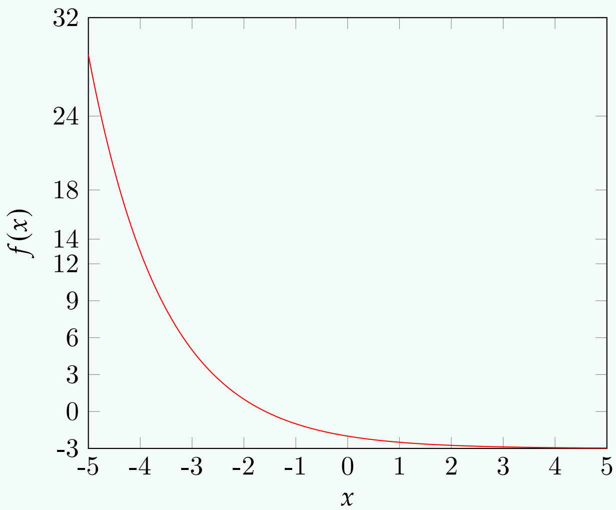
Note:-

Properties of Exponential Function: $f(x) = a^x$, where $0 < x < 1$.

1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no x-intercepts; the y-intercept is 1.

3. The x-axis (y = 0) is a horizontal asymptote as $x \longrightarrow \infty$.
4. The function is an decreasing function and is one-to-one.
5. The graph of f contains the points (0, 1), (1, a), and (-1, 1/a).
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.3 (Graph $f(x) = 2^{-x} - 3$ and determine the domain, range, and horizontal asymptote of f.)



- Domain: $x|x \in \mathbb{R}$ or $[-\infty, \infty]$
- Range: $y|y > -3$ or $[-3, \infty]$
- Horizontal Asymptote: $y = -3$

Example 4.1.4 (Explain the transformation of g(x) from $f(x) = e^x$)

- $g(x) = -e^{x-3}$
- $g(x) = 3e^{-x} - 5$

Example 4.1.5 (6: Solve $3^{x+1} = 81$)

- $3^{x+1} = 3^4$
- $x + 1 = 4$
- $x = 3$

Example 4.1.6 (7: Solve $e^{-x^2} = (e^{x^2} \cdot \frac{1}{e^3})$)

- $e^{-x^2} = e^{2x} \cdot e^{-3}$
- $e^{-x^2} = e^{2x-3}$
- $-x^2 = 2x - 3$
- $x^2 + 2x - 3$
- $(x + 3)(x - 1) = 0$
- $x = -3, 1$

Example 4.1.7 (8: Between 9 AM and 10 PM cars arrive at burger king's drive-thru at the rate of 12 cars per hour (0.2 cars per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 9 PM)

$F(t) = 1 - e^{-2t}$

(a) 63%

(b) 99.7%

(c) graph

(d) other thing

4.2 Logarithmic Functions

Definition 4.2.1: Logarithmic Function:

The opposite to an exponential function. The logarithmic function to the base a, where $a > 0$ and $a \neq 0$, is denoted and defined by $y = \log_x x$ if and only if $x = a^y$

Note:-

You can remember the format by thinking log-base-answer-exponent.

Example 4.2.1 (2: Change each exponential expression to an equivalent expression involving a logarithm.)

1. $1.2^3 \rightarrow$

Example 4.2.2 (3: Change each logarithmic expression to an equivalent expression involving an exponent.)

1. $\log_a 4 = 5 \rightarrow a^5 = 4$
2. $\log_b e = -3 \rightarrow b^{-3} = e$
3. $\log_3 5 = c \rightarrow 3^c = 5$

Theorem 4.1 G

t that exponential theorem from slides

Example 4.2.3 (4: Find he exact value of:)

1. $\log_2 16 = x \rightarrow x = 4$
2. $\log_3 \frac{1}{27} = x \rightarrow x = -3$ Convert to exponential then use the rules of exponents.
3. $\log_4 2 = x \rightarrow x = \frac{1}{2}$

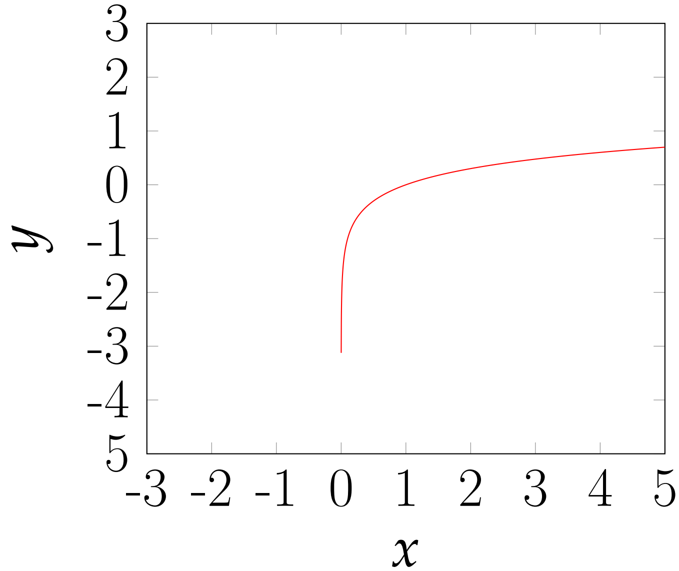
Theorem 4.2.1 Determine the Domain of a logarithmic function:

- Domain of the logarithmic function = range of the exponential function = $(0, \infty)$

- Range of the logarithmic function = domain of the exponential function = $(-\infty, \infty)$

Example 4.2.4 (5: Find the domain of each logarithmic function:)

1. $f(x) = \log_2(x+3) \rightarrow x+3 > 0 \rightarrow x > -3 \rightarrow (-3, \infty)$
2. $g(x) = \log_b(\frac{1+x}{1-x}) \rightarrow \frac{1+x}{1-x} > 0 \rightarrow x \neq 1, -1$. Now use a number line to find out where it applies. In this case it is $-1 < x < 1$ or $(-1, 1)$ or $x | x \neq 1, -1$
3. $h(x) = \log_{\frac{1}{2}} |x| \rightarrow |x| > 0 \rightarrow$ **Domain** = $\mathbb{R}$ where $x \neq 0$, or All Real Numbers where $x \neq 0$, or $x | x \neq 0$



Graphs

Thanks for reading