Precalculus

Me. I am Him.

11/28/2022

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3.1 Properties of functions and Complex Zeros

3.1.1 3.1 - Completing the square

Definition 3.1.1: Completing the square

In form $ax^2 + bx + c$, you half b, and then square.

Note:-

Forms:

Standard: $ax^2 + bx + c$; $a \neq 0$

Vertex: $a(x-h)^2 + k$; $a \neq 0$

Note:-

Verticies:

Standard: (-b/2a , f(-b/2a))

Vertex: (h , k)

Note:-

Axis of Symmetry:

Standard: x=-b/2a

Vertex: x=h

Note:-

y-Intercept

Standard: (0 , c)

Vertex: *Plug in 0 for x and solve*

Note; There is always one.

Note:-

x-Intercept

Standard: $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, otherwise known as the quadratic formula.

If the inside of the quadratic is < 0, there are no x-intercepts.

If the inside of the quadratic is $= 0$, there is exactly one x-intercept.

If the inside of the quadratic is > 0, there are exactly two x-intercepts.

Vertex: *Plug in 0 for y and solve*

Chapter 4

4

4.1 4: Composite Functions

4.1.1

Note:-

11/28/2022 - Didn't really do much. Just reviewed what $f \cdot g$ or f(g(x)) was.

4.1.2

4.1.3 Exponential Functions

Are You Prepared? 4.1.1

- $4^3 = 8$
- $8^{\frac{2}{3}} = 4$
- $3^{-2} = \frac{1}{9}$

Note:-

In a^n, a is known as the base whereas n is known as the exponent, index, or power.

Note:-

Law of Exponents:

(1) $a^m \cdot a^n = a^{m+n}$ Example: $3^2 \cdot 3^5 = 3^{2+5} = 3^7 = 2187$

(2) $(a^m)^n = a^{mn}$ Example: $(2^3)^2 = 2^{3\cdot 2} = 2^6 = 64$

(3) $(ab)^m = a^m b^m$ Example: $(5x)^3$

(4) $1^n = 1$ Example: $1^{1001} = 1$

(5) $a^{-n} = \frac{1}{a^n}$ Example: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

(6) $a^0 = 1$ Example: $7^0 = 1$

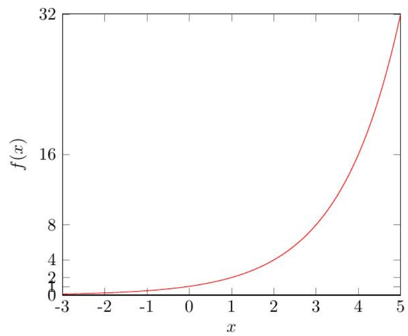
(7) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ Example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$

Definition 4.1.1: Exponential Function

A function of the form f(x) = a^x where x is a positive real number (a>0) and a $\neq$ 1. The domain of f is $\mathbb{R}$

Example 4.1.1 (2: Graph the exponential function: $f(x) = 2^x$)

Figure 4.1: $f(x) = 2^x$



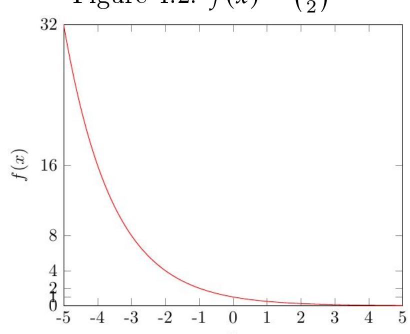
Note:-

Properties of the Exponential Function: $f(x) = a^x$, where $a > 1$

1. The *domain* is the set of all real numbers. The *range* is the set of all positive real numbers.
2. There are no *x-intercepts*. The *y-intercept* is 1.
3. The *x-axis* (y=0) is a horizontal asymptote as $x \longrightarrow -\infty$
4. The function is an increasing function and is one-to-one.
5. The graph of f contains the points (0,1),(1,a), and (-1,1/a).
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.2 (3: Graph the exponential function: $f(x) = \left(\frac{1}{2}\right)^2$)

Figure 4.2: $f(x) = \left(\frac{1}{2}\right)^2$



Note:-

Properties of Exponential Function: $f(x) = a^x$, where $0 < x < 1$.

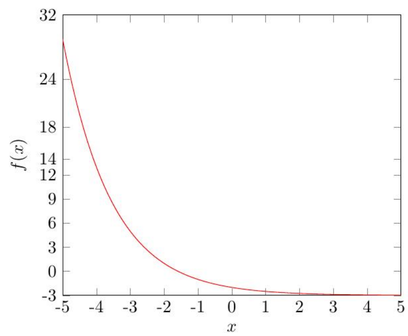
1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis ($y = 0$) is a horizontal asymptote as $x \longrightarrow \infty$.

4. The function is a decreasing function and is one-to-one.
5. The graph of f contains the points (0, 1), (1, a), and (-1, 1/a).
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.3 (Graph $f(x) = 2^{-x} - 3$ and determine the domain, range, and horizontal asymptote of f.)

- Domain: $x|x \in \mathbb{R}$ or $[-\infty, \infty]$
- Range: $y|y > -3$ or $[-3, \infty]$
- Horizontal Asymptote: $y = -3$

Figure 4.3: $f(x) = 2^{-x} - 3$



Example 4.1.4 (Explain the transformation of g(x) from $f(x) = e^x$)

- $g(x) = -e^{x-3}$
- $g(x) = 3e^{-x} - 5$

Example 4.1.5 (6: Solve $3^{x+1} = 81$)

- $3^{x+1} = 3^4$
- $x + 1 = 4$
- $x = 3$

Example 4.1.6 (7: Solve $e^{-x^2} = \left(e^{x^2} \cdot \frac{1}{e^3}\right)$)

- $e^{-x^2} = e^{2x} \cdot e^{-3}$
- $e^{-x^2} = e^{2x-3}$
- $-x^2 = 2x - 3$
- $x^2 + 2x - 3$
- $(x + 3)(x - 1) = 0$
- $x = -3, 1$

Example 4.1.7 (8: Between 9 AM and 10 PM cars arrive at burger king's drive-thru at the rate of 12 cars per hour (0.2 cars per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 9 PM)

$F(t) = 1 - e^{-2t}$

(a) 63%

(b) 99.7%

(c) graph

(d) other thing

4.2

4.3

4.4 Logarithmic Functions

Definition 4.4.1: Logarithmic Function:

The opposite to an exponential function. The logarithmic function to the base a, where $a > 0$ and $a \neq 0$, is denoted and defined by $y = \log_x x$ if and only if $x = a^y$

Note:-

You can remember the format by thinking log-base-answer-exponent.

Example 4.4.1 (2: Change each exponential expression to an equivalent expression involving a logarithm.)

1. $1.2^3 \longrightarrow$

Example 4.4.2 (3: Change each logarithmic expression to an equivalent expression involving an exponent.)

1. $\log_a 4 = 5 \longrightarrow a^5 = 4$
2. $\log_b e = -3 \longrightarrow b^{-3} = e$
3. $\log_3 5 = c \longrightarrow 3^c = 5$

Theorem 4.4.1

Get that exponential theorem from slides

Example 4.4.3 (4: Find he exact value of:)

1. $\log_2 16 = x \longrightarrow x = 4$
2. $\log_3 \frac{1}{27} = x \longrightarrow x = -3$ Convert to exponential then use the rules of exponents.
3. $\log_4 2 = x \longrightarrow x = \frac{1}{2}$

Theorem 4.4.2 Determine the Domain of a logarithmic function:

- Domain of the logarithmic function = range of the exponential function = $(0, \infty)$

- Range of the logarithmic function = domain of the exponential function = $(-\infty, \infty)$

Example 4.4.4 (5: Find the domain of each logarithmic function:)

1. $f(x) = \log_2(x+3) \to x+3 > 0$
   - $x > -3$ or $(-3, \infty)$
2. $g(x) = \log_b\left(\frac{1+x}{1-x}\right) \to \frac{1+x}{1-x} > 0$
   - $x \neq 1, -1$. Now use a number line to find out where it applies. In this case it is $-1 < x < 1$ or $(-1, 1)$ or $x|x \neq 1, -1$
3. $h(x) = \log_{\frac{1}{2}}|x| \to |x| > 0$
   - **Domain** = $\mathbb{R}$ where $x \neq 0$, or All Real Numbers where $x \neq 0$, or $x|x \neq 0$

4.4.1 Natural Logarithm

Theorem 4.4.3 If the base of a logarithmic function is the number e, then we have the natural logarithm function. That is,

1. y = ln x if and only if x = ey
2. y = ln x and y = ex are inverse functions

Theorem 4.4.4 Common Logarithm Function

If the base of a logarithmic function is the number 10, then we have the common logarithm function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is,

- $y = log_x$ if and only if $x = 10^y$

Example 4.4.5 (6 & 7: Determine the domain, range, and vertical asymptote of each logarithmic function. List any transformations.)

a. $f(x) = \ln(x) \to g(x) = -\ln(x+2)$
   - Domain: $x > -2$
   - Range: $(-\infty, \infty)$
   - Vertical Asymptote: $x \neq -2$

   Note:-

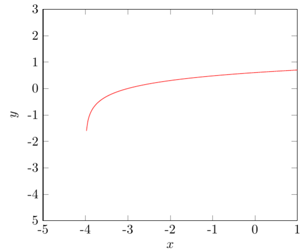
   The negtive applied to the natural log, seen in the equation $-\ln(x+2)$, is causing it to reflect over the x-axis.

b. $f(x) = \log(x) \to g(x) = 3\log(-x) - 1$

Theorem 4.4.5 Equations that contain logarithms are called logarithmic equations. Be sure to check each solution in the original equation and discard any extraneous solutions. Remember in logaM, a and M are positive and $a \neq 1$.

1. Change the logarithmic equation to an exponential equation and solve for x

Figure 4.4: $\log_{10} x$



2. If the exponential equation has base e, change it to the natural logarithm function
3. If the exponential equation has base 10, change it to the common logarithm function

Example 4.4.6 (8: Solve for x)

1. $\log_3(4x - 7) = 2$
   - $3^2 = 4x - 7$
   - $9 = 4x - 7$
   - $x = 4$
2. $\log_x(64) = 2$
   - $x^2 = 64$
   - $x = \sqrt[2]{64}$
   - $x = 8$ Note: -8 does not work as a solution as base values for a logarithm must be greater than 1.

Example 4.4.7 (8.5: Solve for x. Give the exact solution then use your calculator to give the approximate solution.)

1. $e^{2x} = 5$
   - $\log_e 5 = 2x$
   - $\ln 5 = 2x$
   - $\frac{\ln 5}{2} = x$

Example 4.4.8 (Additional Example:)

1. $10^{x^2+2x+1} = 50$
   - $\log(50) = x^2 + 2x + 1$
   - $\pm\sqrt{\log(50)} = \sqrt{(x+1)^2}$
   - $\pm\sqrt{\log(50)} = x + 1$
   - $x = \pm\sqrt{\log(50)} + 1$

Example 4.4.9 (10: The concentration of alcohol in a person's blood is measurable. Recent medical research suggests that the risk R (given as a percent) of having an accident while driving a car can be modeled by the equation $6e^{kx}$ where x is the variable concentration of alcohol n the blood and k is a constant.

1. Suppose that a concentration in the blood of 0.04 results in a 10% risk (R=10) of an accident. Find the constant k in the equation. Graph $R = 6e^{kx}$ using the k value.
   - do stuff so that k=20.62. She literally used her calc
2. Using the value of k, what is the risk if the concentration is 0.17?
   - uhhhh she didn't do this.
3. Using the same value of k, what concentration of alcohol corresponds to a risk of 100%?
   - didn't do this one either. apparently D is the most important.
4. If the law asserts that anyone with a risk of having an accident of 20% or more should not have driving privileges, at what concentration of alcohol in the blood should a driver be arrested and charged with a DUI?
   - $20 = 6e^{12.77x}$
   - $\frac{10}{3} = e^{12.77x}$
   - $\ln\left(\frac{10}{3}\right) = 12.77x$
   - $x = 0.94$

)

4.5 Properties of Logarithms

Note:-

Properties of logarithms:

1. Identity $\longrightarrow \log_a 1 = 0$ or $\log_a a = 1$
2. Inverse $\longrightarrow \log_b b^x = x$ or $b^{\log_b(x)}x$
3. Product $\longrightarrow \log_a xy = \log = \log_a x + \log_a y$
4. Quotient $\longrightarrow \log_a \frac{x}{y} = \log_a - \log_a y$
5. Equality $\longrightarrow \log_b a = \log_b c \Rrightarrow a = c$
6. Change of Base Formula $\longrightarrow \log_a b = \frac{\log_c b}{\log_c b}$

Example 4.5.1 (1: Use properties of logarithms to find the exact value of each expression. Do not use a calculator.)

1. $\ln e^{\sqrt{2}}$
   - $\log_e e^{\sqrt{2}}$
   - $\sqrt{2} \times \log_e e$
   - $\sqrt{2} \times \ln e$
2. $\log_8 16 - \log_8 2$

- $\log_8 \frac{16}{2}$
- $\log_8 8$
- 1

Example 4.5.2 (3:Write the expression as a sum of logarithms. Express all powers as factors.)

1. $\log_a \left(x\sqrt{x^2+1}\right)$
   - $\log_a x + \log_a \sqrt{x^2+1}$
   - $\log_a x + \log_a (x^2+1)^{\frac{1}{2}}$
   - $\log_a x + \frac{1}{2}\log_a (x^2+1)$

Example 4.5.3 (4: Write the expression as a difference in logarithms. Express all powrs as factors.)

1. $\ln\left(\frac{x^2}{(x-1)^3}\right)$
   - $\ln(x^2) - \ln(x-1)^3$
   - $2\ln(x) - 3\ln(x-1)$

Example 4.5.4 (6: Write each of the following as a single logarithm.)

1. $\log_a 7 + 4\log_a 3$
   - $\log_a 7 + \log_a 3^4$
   - $\log_a (7 \times 3^4)$
   - $\log_a 567$
2. $\frac{2}{3}\ln 8 - \ln(3^4 - 8)$
   - $\ln 8^{\frac{2}{3}} - \ln(3^4 - 8)$
   - $\ln\left(\frac{8^{\frac{2}{3}}}{3^4-8}\right)$
   - $\ln\left(\frac{4}{7^3}\right)$
3. $\log_a x + \log_a 9 + \log_a \left(x^2+1\right) - \log_a 5$
   - blah

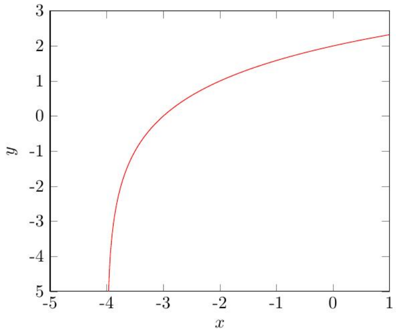
Example 4.5.5 (7: Approximate the following. Round answers to four decimal places.)

1. $\log_2 27$
   - $\frac{log_{10}27}{\log_{10} 2}$ Note: 10 is the common base, thus it can be omitted.
   - Another answer could be, $\frac{\ln 27}{\ln 2}$

Example 4.5.6 (9: Use a graphing utility to graph the following)

1. $y = \log_2 x$

Figure 4.5: $\log_2 x$



4.6

Example 4.6.1 (1: Solve.)

1. $2\log_5 x = \log_5 9$
   - $x^2 = 9$
   - Ergo, $x = \pm 3$

Example 4.6.2 (2: Solve.)

1. $\log_4(x+3) + \log_4(2-x) = 1$
   - $\log_4(x+3)(2-x) = 1$
   - $4^1 = (x+3)(2-x)$
   - $4 = x^2 - x + 6$
   - $x^2 + x - 2$
   - $(x+2)(x-1)$
   - $x = -2, 1$

4.7 Interest

Definition 4.7.1: Interest Formulas

Note:-

I = amount of interest, A = final amount, P = principal, r = interest rate (as a decimal), t = time (in years), n = number of times compounded per year)

Formulas:

1. Simple Interest:
$$I = Prt$$
or
$$A = P + I = P + Prt = P(1 + rt)$$

2. Compound Interest:
$$A = P(1 + \frac{r}{n})^{nt}$$

3. Continuous Compounding:
$$A = Pe^{rt}$$

Example 4.7.1 (1: A credit union pays interest of 8% per annum compounded quarterly on a certain savings plan. If $100 is deposited in such a plan and the interest is left to accumulate, how much will be in the account after a year?)

1. A = P$(1+\frac{r}{n})^{nt}$
2. A = 100$(1+\frac{0.08}{4})^{4*1}$
3. A = $108.24

Example 4.7.2 (2 & 3 : Determine the final amount you invest $1000 at an annual rate of 10% for 5 years, compounding at the amounts shown below.)

1. A = P$(1+\frac{r}{n})^{nt}$
2. P = $1000
3. r = 0.1
4. t = 5
    - Annually:
        - $1610.51
    - Semiannually
        - $1628.89
    - Quarterly
        - $1638.12
    - Monthly
        - $1645.31
    - Daily
        - $1648.61
    - Continuously
        - New Formula: Pe^{rt}
        - $1648.72

Note:-

Calculate Effective rates of return:
The effective rate of interest is the equivalent annual simple rate of interest that would yield the same amount of compounding after 1 year.
To find the effective rate of interest:

1. Using the appropriate formula, find the final amount A.
2. Subtract the principal P from the final amount A to get the interest earned I.
3. Using the simple interest formula, $I = Prt$, plug in values and solve for r.

Example 4.7.3 (4: On January 2, 2004, $2000 is placed in an Individual Retirement Account (IRA) that will pay interest of 7% per annum compounded continuously.)

a. What will the IRA be worth on January 1, 2024?:

$$Pe =^{rt} = 2000e^{0.07 \cdot 20} = \$8110.40$$

b. Effective Rate of Interest?: (use the last note for guidance)

$$A = Pe^{rt} = 2000e^{0.07 \cdot 1} = 2145.02$$

$$2145.02 - 2000 = 145.02 = I$$

$$I = Prt = 145.02 = 2000r \longrightarrow r = 0.07251 or 7.25\%$$

Definition 4.7.2: Determine the Present Value of a Lump Sum of Money

Present Value Formulas are the compound interest and continuous compounding formulas solved for the principal P. So,

$$P = A(1 + \frac{r}{n})^{-nt}$$

or

$$Ae^{-rt}$$

Example 4.7.4 (5: A zero-coupon (non-interest bearing) bond can be redeemed in 10 years for $1000. How much should you be willing to pay for it now if you want a return of:)

1. 8% compounded monthly?
   - $$P = 1000(1 + \frac{0.08}{12})^{-12 \cdot 10}$$
   - $$\$450.52$$
2. 7% compounded continuously?
   - $$P = 1000e^{-0.07 \cdot 10}$$
   - blah, its higher.

Note:-

To determine the time required to double or triple lump sums of money, if we want to double P, A will be equal to 2p. Triple is the same thing.

Example 4.7.5 (6: What rate of interest compounded annually should you seek if you want to double your investment in 5 years?)

$$A = P(1 + \frac{r}{n})^{nt}$$

$$2p = P(1 + \frac{r}{1})^{1 \cdot 5}$$

Divide by P

$$2 = (1 + \frac{r}{1})^{1 \cdot 5}$$

5th Root

$$\sqrt[5]{2} = 1 + r$$

ANS:

$$r = \sqrt[5]{2} - 1$$

Or:

$$r = 14.9\%$$

Example 4.7.6 (7:)

1. How long will it take for an investment to double in value if it earns 5% interest compounded continuously?

$$A = Pe^{rt}$$
$$2P = Pe^{0.05t}$$
$$2 = e^{0.05t}$$
$$\ln(2) = 0.05t$$
$$t = \frac{\ln(2)}{0.05}$$
$$t = 13.86$$

2. How long will it take to triple in value?

$$3P = Pe^{0.05t}$$
$$t = 21.97$$

4.8 Exponential Growth and Decay

Definition 4.8.1: Exponential Growth and Decay follow one formula

Many natural phenomena have been found to follow the law that an amount A varies with the time t according to

$$A(t) = A_0 e^{kt}$$

Where A_0 is the initial amount at $t = 0$and k is a constant not equal to zero.
If $k > 0$, then A increases over time (growth).
If $k < 0$, then A decreases over time (decay).

Example 4.8.1 (1: The population of a midwestern city follows the exponential law. The population decreased from 900,000 to 800,000 from 2003 to 2005.)

k: Figure out k first.

(a)
$$t = 2 \longrightarrow A(t) = A_0 e^{kt}$$

(b)
$$800,000 = 900,000 e^{2k}$$

(c) Divide by 900,000.
$$\frac{8}{9} = e^{2k}$$

(d) Convert to a log.
$$ln \frac{9}{8} = 2k$$

(e) Divide by two.
$$\frac{\ln \frac{8}{9}}{2} = k$$

(f)
$$k = \frac{\ln \frac{8}{9}}{2}$$

(g)
$$k = 0.05 \dots$$

a. What will the population be in 2007?

(a)
$$A(4) = 900,000 e^{\frac{\ln \frac{8}{9}}{2} \cdot 4}$$

(b)
$$711,111$$

b. When will the population be half its original amount?

(a)
$$450,000 = 900,000 e^{\frac{\ln \frac{8}{9}}{2} \cdot\ t}$$

(b)
$$\frac{1}{2} = e^{\frac{\ln \frac{8}{9}}{2} \cdot\ t}$$

(c)

$$\ln\frac{1}{2}=\frac{\ln\frac{8}{9}}{2}\cdot\ t$$

(d)

$$t=\frac{\ln\frac{1}{2}}{\frac{\ln\frac{8}{9}}{2}}$$

c. When will the population reach 300,000?

(a)

$$\frac{1}{3}=e^{\frac{\ln\frac{8}{9}}{2}\cdot\ t}$$

(b)

$$t=18.65or19years$$

Example 4.8.2 (2: A colony of bacteria grows according to the law of unhibited growth according to the function $N(t)=100e^{0.045t}$, where N is measured in grams and t is in days.)

1. Determine the initial amount of bacteria.

   (a)

   $$N(0)=100e^{0.045(0)}$$

2. What is the growth rate of the bacteria?

   (a)

   $$4.5\%$$

3. Graph the function using your calculator

   (a) Insert Image here later

4. What is the population after 5 days?

   (a)

   $$N(5)=100e^{0.045(5)}$$

   (b)

   $$N(5)=125.2$$

5. How long will it take for the population to reach 140 grams?

   (a)

   $$N(t)=140\longrightarrow 140=100e^{0.045(t)}$$

   (b)

   $$t=\frac{\ln\frac{7}{5}}{0.045}=7.5$$

6. What is the doubling time for the population?

   (a)

   $$N(t)=200\longrightarrow 200=100e^{0.045(t)}$$

   (b)

   $$t=\frac{\ln 2}{0.045}=15.4$$

Thanks for reading