

# Precalculus

Me. I am Him.

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# Chapter 1

Are You Prepared? 1.0.1: Yeet2

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## Chapter 2

# Chapter 3

## 3.1 Properties of functions and Complex Zeros

### 3.1.1 3.1 - Completing the square

#### Definition 3.1.1: Completing the square

In form  $ax^2 + bx + c$ , you half b, and then square.

#### Note:-

Forms:

Standard:  $ax^2 + bx + c ; a \neq 0$

Vertex:  $a(x - h)^2 + k ; a \neq 0$

#### Note:-

Vertices:

Standard:  $(-b/2a, f(-b/2a))$

Vertex:  $(h, k)$

#### Note:-

Axis of Symmetry:

Standard:  $x = -b/2a$

Vertex:  $x = h$

#### Note:-

y-Intercept

Standard:  $(0, c)$

Vertex: *Plug in 0 for x and solve*

Note; There is always one.

#### Note:-

x-Intercept

Standard:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ , otherwise known as the quadratic formula.

If the inside of the quadratic is  $< 0$ , there are no x-intercepts.

If the inside of the quadratic is  $= 0$ , there is exactly one x-intercept.

If the inside of the quadratic is  $> 0$ , there are exactly two x-intercepts.

Vertex: *Plug in 0 for y and solve*

# Chapter 4

## 4

### 4.1 4: Composite Functions

#### 4.1.1

**Note:-**

11/28/2022 - Didn't really do much. Just reviewed what  $f \cdot g$  or  $f(g(x))$  was.

#### 4.1.2

#### 4.1.3 Exponential Functions

**Are You Prepared? 4.1.1**

- $4^3 = 8$
- $8^{\frac{2}{3}} = 4$
- $3^{-2} = \frac{1}{9}$

**Note:-**

In  $a^n$ ,  $a$  is known as the base whereas  $n$  is known as the exponent, index, or power.

**Note:-**

Law of Exponents:

- (1)  $a^m \cdot a^n = a^{m+n}$  Example:  $3^2 \cdot 3^5 = 3^{2+5} = 3^7 = 2187$
- (2)  $(a^m)^n = a^{mn}$  Example:  $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$
- (3)  $(ab)^m = a^m b^m$  Example:  $(5x)^3$
- (4)  $1^n = 1$  Example:  $1^{1001} = 1$
- (5)  $a^{-n} = \frac{1}{a^n}$  Example:  $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
- (6)  $a^0 = 1$  Example:  $7^0 = 1$
- (7)  $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$  Example:  $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

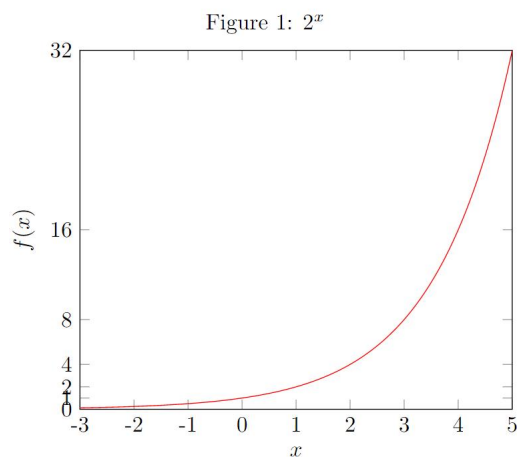
**Definition 4.1.1: Exponential Function**

A function of the form  $f(x) = a^x$  where  $x$  is a positive real number ( $a > 0$ ) and  $a \neq 1$ . The domain of  $f$  is  $\mathbb{R}$



**Example 4.1.1** (2: Graph the exponential function:  $f(x) = 2^x$ )

Figure 4.1:  $f(x) = 2^x$



**Note:-**

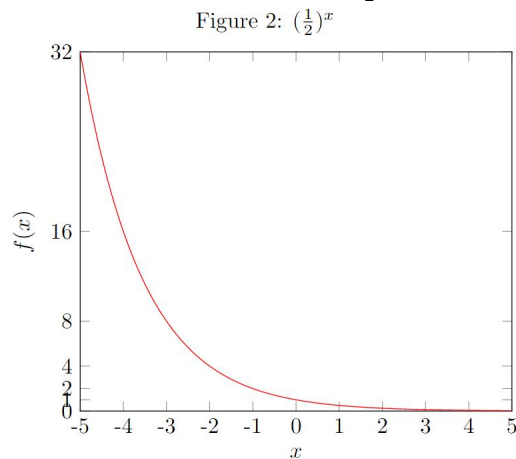
Properties of the Exponential Function:  $f(x) = a^x$ , where  $a > 1$

1. The *domain* is the set of all real numbers. The *range* is the set of all positive real numbers.
2. There are no *x-intercepts*. The *y-intercept* is 1.
3. The *x-axis* ( $y=0$ ) is a horizontal asymptote as  $x \rightarrow -\infty$
4. The function is an increasing function and is one-to-one.
5. The graph of  $f$  contains the points  $(0,1)$ ,  $(1,a)$ , and  $(-1,1/a)$ .
6. The graph of  $f$  is smooth and continuous, with no corners or gaps.

**Example 4.1.2** (3: Graph the exponential function:  $f(x) = (\frac{1}{2})^x$ )

s

Figure 4.2:  $f(x) = (\frac{1}{2})^x$



**Note:-**

Properties of Exponential Function:  $f(x) = a^x$ , where  $0 < a < 1$ .

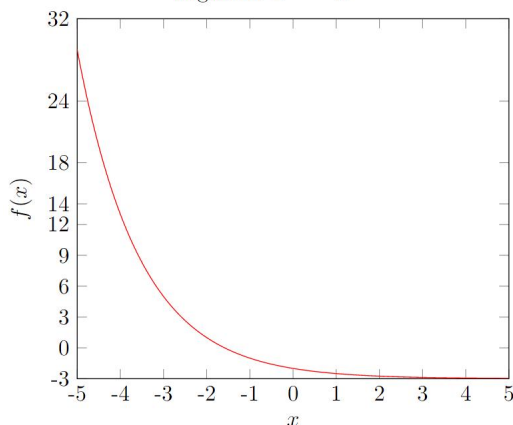
1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis ( $y = 0$ ) is a horizontal asymptote as  $x \rightarrow \infty$ .
4. The function is an decreasing function and is one-to-one.
5. The graph of  $f$  contains the points  $(0, 1)$ ,  $(1, a)$ , and  $(-1, 1/a)$ .
6. The graph of  $f$  is smooth and continuous, with no corners or gaps.

**Example 4.1.3** (Graph  $f(x) = 2^{-x} - 3$  and determine the domain, range, and horizontal asymptote of  $f$ .)

- Domain:  $x|x \in \mathbb{R}$  or  $[-\infty, \infty]$
- Range:  $y|y > -3$  or  $[-3, \infty]$
- Horizontal Asymptote:  $y = -3$

Figure 4.3:  $f(x) = 2^{-x} - 3$

Figure 3:  $2^{-x} - 3$



**Example 4.1.4** (Explain the transformation of  $g(x)$  from  $f(x) = e^x$ )

- $g(x) = -e^{x-3}$
- $g(x) = 3e^{-x} - 5$

**Example 4.1.5** (6: Solve  $3^{x+1} = 81$ )

- $3^{x+1} = 3^4$
- $x + 1 = 4$
- $x = 3$

**Example 4.1.6** (7: Solve  $e^{-x^2} = (e^{x^2} \cdot \frac{1}{e^3})$ )

- $e^{-x^2} = e^{2x} \cdot e^{-3}$
- $e^{-x^2} = e^{2x-3}$
- $-x^2 = 2x - 3$
- $x^2 + 2x - 3$
- $(x + 3)(x - 1) = 0$
- $x = -3, 1$

**Example 4.1.7** (8: Between 9 AM and 10 PM cars arrive at burger king's drive-thru at the rate of 12 cars per hour (0.2 cars per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 9 PM)

$$F(t) = 1 - e^{-2t}$$

- (a) 63%
- (b) 99.7%
- (c) graph
- (d) other thing

## 4.2

## 4.3

## 4.4 Logarithmic Functions

### Definition 4.4.1: Logarithmic Function:

The opposite to an exponential function. The logarithmic function to the base a, where  $a > 0$  and  $a \neq 1$ , is denoted and defined by  $y = \log_a x$  if and only if  $x = a^y$

#### Note:-

You can remember the format by thinking log-base-answer-exponent.

**Example 4.4.1** (2: Change each exponential expression to an equivalent expression involving a logarithm.)

1.  $1.2^3 \rightarrow$

**Example 4.4.2** (3: Change each logarithmic expression to an equivalent expression involving an exponent.)

1.  $\log_a 4 = 5 \rightarrow a^5 = 4$

2.  $\log_b e = -3 \rightarrow b^{-3} = e$

3.  $\log_3 5 = c \rightarrow 3^c = 5$

### Theorem 4.4.1

Get that exponential theorem from slides

**Example 4.4.3** (4: Find the exact value of:)

1.  $\log_2 16 = x \rightarrow x = 4$
2.  $\log_3 \frac{1}{27} = x \rightarrow x = -3$  Convert to exponential then use the rules of exponents.
3.  $\log_4 2 = x \rightarrow x = \frac{1}{2}$

**Theorem 4.4.2** Determine the Domain of a logarithmic function:

- Domain of the logarithmic function = range of the exponential function =  $(0, \infty)$
- Range of the logarithmic function = domain of the exponential function =  $(-\infty, \infty)$

**Example 4.4.4** (5: Find the domain of each logarithmic function:)

1.  $f(x) = \log_2(x+3) \rightarrow x+3 > 0$ 
  - $x > -3$  or  $(-3, \infty)$
2.  $g(x) = \log_b\left(\frac{1+x}{1-x}\right) \rightarrow \frac{1+x}{1-x} > 0$ 
  - $x \neq 1, -1$ . Now use a number line to find out where it applies. In this case it is  $-1 < x < 1$  or  $(-1, 1)$  or  $x|x \neq 1, -1$
3.  $h(x) = \log_{\frac{1}{2}}|x| \rightarrow |x| > 0$ 
  - **Domain** =  $\mathbb{R}$  where  $x \neq 0$ , or All Real Numbers where  $x \neq 0$ , or  $x|x \neq 0$

#### 4.4.1 Natural Logarithm

**Theorem 4.4.3** If the base of a logarithmic function is the number e, then we have the natural logarithm function. That is,

1.  $y = \ln x$  if and only if  $x = e^y$
2.  $y = \ln x$  and  $y = e^x$  are inverse functions

**Theorem 4.4.4** Common Logarithm Function

If the base of a logarithmic function is the number 10, then we have the common logarithm function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is,

- $y = \log_{10} x$  if and only if  $x = 10^y$

**Example 4.4.5** (6 & 7: Determine the domain, range, and vertical asymptote of each logarithmic function. List any transformations.)

- a.  $f(x) = \ln(x) \rightarrow g(x) = -\ln(x+2)$ 
  - Domain:  $x > -2$
  - Range:  $(-\infty, \infty)$
  - Vertical Asymptote:  $x \neq -2$

**Note:-**

The negative applied to the natural log, seen in the equation  $-\ln(x + 2)$ , is causing it to reflect over the x-axis.

b.  $f(x) = \log(x) \rightarrow g(x) = 3 \log(-x) - 1$

**Theorem 4.4.5** Equations that contain logarithms are called logarithmic equations. Be sure to check each solution in the original equation and discard any extraneous solutions. Remember in  $\log_a M$ ,  $a$  and  $M$  are positive and  $a \neq 1$ .

1. Change the logarithmic equation to an exponential equation and solve for  $x$
2. If the exponential equation has base  $e$ , change it to the natural logarithm function
3. If the exponential equation has base 10, change it to the common logarithm function

**Example 4.4.6** (8: Solve for  $x$ )

1.  $\log_3(4x - 7) = 2$

- $3^2 = 4x - 7$
- $9 = 4x - 7$
- $x = 4$

2.  $\log_x(64) = 2$

- $x^2 = 64$
- $x = \sqrt[2]{64}$
- $x = 8$  Note: -8 does not work as a solution as base values for a logarithm must be greater than 1.

**Example 4.4.7** (8.5: Solve for  $x$ . Give the exact solution then use your calculator to give the approximate solution.)

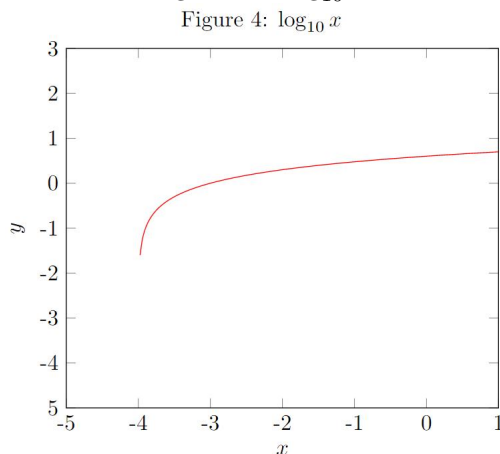
1.  $e^{2x} = 5$

- $\log_e 5 = 2x$
- $\ln 5 = 2x$
- $\frac{\ln 5}{2} = x$

**Example 4.4.8** (Additional Example:)

1.  $10^{x^2+2x+1} = 50$

- $\log(50) = x^2 + 2x + 1$
- $\pm\sqrt{\log(50)} = \sqrt{(x+1)^2}$
- $\pm\sqrt{\log(50)} = x + 1$
- $x = \pm\sqrt{\log(50)} + 1$

Figure 4.4:  $\log_{10} x$ 

**Example 4.4.9** (10: The concentration of alcohol in a person's blood is measurable. Recent medical research suggests that the risk  $R$  (given as a percent) of having an accident while driving a car can be modeled by the equation  $6e^{kx}$  where  $x$  is the variable concentration of alcohol in the blood and  $k$  is a constant.

- Suppose that a concentration in the blood of 0.04 results in a 10% risk ( $R=10$ ) of an accident. Find the constant  $k$  in the equation. Graph  $R = 6e^{kx}$  using the  $k$  value.
  - do stuff so that  $k=20.62$ . She literally used her calc
- Using the value of  $k$ , what is the risk if the concentration is 0.17?
  - uhhhh she didn't do this.
- Using the same value of  $k$ , what concentration of alcohol corresponds to a risk of 100%?
  - didn't do this one either. apparently D is the most important.
- If the law asserts that anyone with a risk of having an accident of 20% or more should not have driving privileges, at what concentration of alcohol in the blood should a driver be arrested and charged with a DUI?
  - $20 = 6e^{12.77x}$
  - $\frac{10}{3} = e^{12.77x}$
  - $\ln\left(\frac{10}{3}\right) = 12.77x$
  - $x = 0.94$

)

## 4.5: Properties of Logarithms

### Note:-

Properties of logarithms:

- Identity  $\rightarrow \log_a 1 = 0$  or  $\log_a a = 1$
- Inverse  $\rightarrow \log_b b^x = x$  or  $b^{\log_b(x)} = x$
- Product  $\rightarrow \log_a xy = \log_a x + \log_a y$

4. Quotient  $\rightarrow \log_a \frac{x}{y} = \log_a x - \log_a y$
5. Equality  $\rightarrow \log_b a = \log_b c \Rightarrow a = c$
6. Change of Base Formula  $\rightarrow \log_a b = \frac{\log_c b}{\log_c a}$

**Example 4.5.1** (1: Use properties of logarithms to find the exact value of each expression. Do not use a calculator.)

1.  $\ln e^{\sqrt{2}}$ 
  - $\log_e e^{\sqrt{2}}$
  - $\sqrt{2} \times \log_e e$
  - $\sqrt{2} \times \ln e$
2.  $\log_8 16 - \log_8 2$ 
  - $\log_8 \frac{16}{2}$
  - $\log_8 8$
  - 1

**Example 4.5.2** (3: Write the expression as a sum of logarithms. Express all powers as factors.)

1.  $\log_a(x\sqrt{x^2+1})$ 
  - $\log_a x + \log_a \sqrt{x^2+1}$
  - $\log_a x + \log_a (x^2+1)^{\frac{1}{2}}$
  - $\log_a x + \frac{1}{2} \log_a (x^2+1)$

**Example 4.5.3** (4: Write the expression as a difference in logarithms. Express all powers as factors.)

1.  $\ln\left(\frac{x^2}{(x-1)^3}\right)$ 
  - $\ln(x^2) - \ln(x-1)^3$
  - $2\ln(x) - 3\ln(x-1)$

**Example 4.5.4** (6: Write each of the following as a single logarithm.)

1.  $\log_a 7 + 4\log_a 3$ 
  - $\log_a 7 + \log_a 3^4$
  - $\log_a (7 \times 3^4)$
  - $\log_a 567$
2.  $\frac{2}{3}\ln 8 - \ln(3^4 - 8)$ 
  - $\ln 8^{\frac{2}{3}} - \ln(3^4 - 8)$
  - $\ln\left(\frac{8^{\frac{2}{3}}}{3^4 - 8}\right)$
  - $\ln\left(\frac{4}{7^3}\right)$





Thanks for reading

