Precalculus

Me. I am Him.

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3.1 Properties of functions and Complex Zeros

3.1.1 3.1 - Completing the square

Definition 3.1.1: Completing the square

In form $ax^2 + bx + c$, you half b, and then square.

Note:-

Forms:

Standard: $ax^2 + bx + c$; $a \neq 0$

Vertex: $a(x - h)^2 + k$; $a \neq 0$

Note:-

Verticies:

Standard: (-b/2a , f(-b/2a))

Vertex: (h , k)

Note:-

Axis of Symmetry:

Standard: x=-b/2a

Vertex: x=h

Note:-

y-Intercept

Standard: (0 , c)

Vertex: *Plug in 0 for x and solve*

Note; There is always one.

Note:-

x-Intercept

Standard: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, otherwise known as the quadratic formula.

If the inside of the quadratic is < 0, there are no x-intercepts.

If the inside of the quadratic is $= 0$, there is exactly one x-intercept.

If the inside of the quadratic is > 0, there are exactly two x-intercepts.

Vertex: *Plug in 0 for y and solve*

Chapter 4

4.1 4: Composite Functions

4.1.1

Note:-

11/28/2022 - Didn't really do much. Just reviewed what $f \cdot g$ or f(g(x)) was.

4.1.2

4.1.3 Exponential Functions

Definition 4.1.1: Are you prepared? MAKE NEW COMMAND

- $4^3 = 8$
- $8^{\frac{2}{3}} = 4$
- $3^{-2} = \frac{1}{9}$

Note:-

In a^n, a is known as the base whereas n is known as the exponent, index, or power.

Note:-

Law of Exponents:

(1) $a^m \cdot a^n = a^{m+n}$ Example: $3^2 \cdot 3^5 = 3^{2+5} = 3^7 = 2187$

(2) $(a^m)^n = a^{mn}$ Example: $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$

(3) $(ab)^m = a^m b^m$ Example: $(5x)^3$

(4) $1^n = 1$ Example: $1^{1001} = 1$

(5) $a^{-n} = \frac{1}{a^n}$ Example: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

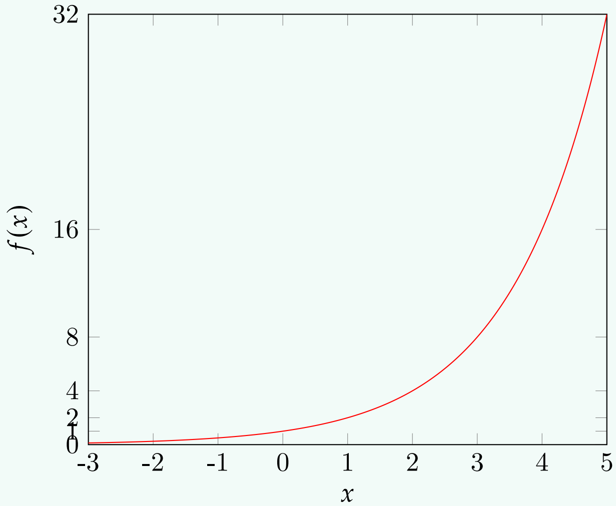
(6) $a^0 = 1$ Example: $7^0 = 1$

(7) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ Example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

Definition 4.1.2: Exponential Function

A function of the form f(x) = a^x where x is a positive real number (a>0) and a $\neq$ 1. The domain of f is $\mathbb{R}$

Example 4.1.1 (2: Graph the exponential function: $f(x) = 2^x$)

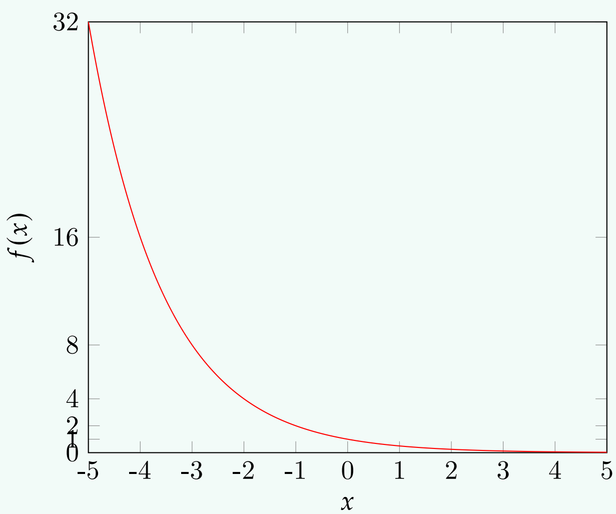


Note:-

Properties of the Exponential Function: $f(x) = a^x$, where $a > 1$

1. The *domain* is the set of all real numbers. The *range* is the set of all positive real numbers.
2. There are no *x-intercepts*. The *y-intercept* is 1.
3. The *x-axis* (y=0) is a horizontal asymptote as $x \to -\infty$
4. The function is an increasing function and is one-to-one.
5. The graph of f contains the points (0,1),(1,a), and (-1,1/a).
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.2 (3: Graph the exponential function: $f(x) = (\frac{1}{2})^2$)



Note:-

Properties of Exponential Function: $f(x) = a^x$, where $0 < x < 1$.

1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no x-intercepts; the y-intercept is 1.

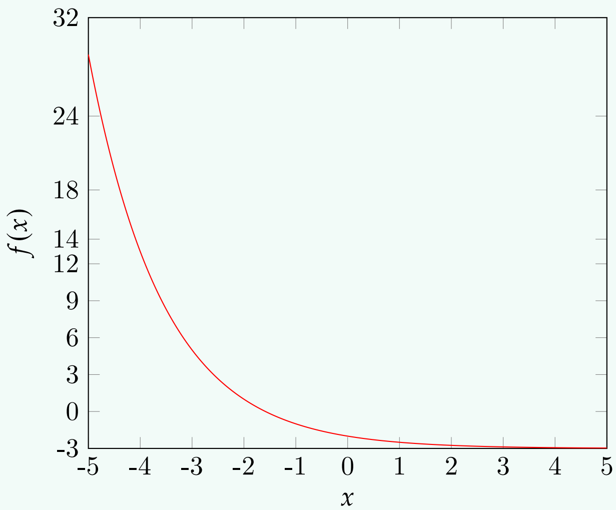
3. The x-axis (y = 0) is a horizontal asymptote as $x \longrightarrow \infty$.

4. The function is an decreasing function and is one-to-one.

5. The graph of f contains the points (0, 1), (1, a), and (-1, 1/a).

6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.3 (Graph $f(x) = 2^{-x} - 3$ and determine the domain, range, and horizontal asymptote of f.)



- Domain: $x|x \in \mathbb{R}$ or $[-\infty, \infty]$

- Range: $y|y > -3$ or $[-3, \infty]$

- Horizontal Asymptote: $y = -3$

Example 4.1.4 (Explain the transformation of g(x) from $f(x) = e^x$)

- $g(x) = -e^{x-3}$
- $g(x) = 3e^{-x} - 5$

Example 4.1.5 (6: Solve $3^{x+1} = 81$)

- $3^{x+1} = 3^4$
- $x + 1 = 4$
- $x = 3$

Example 4.1.6 (7: Solve $e^{-x^2} = (e^{x^2} \cdot \frac{1}{e^3})$)

- $e^{-x^2} = e^{2x} \cdot e^{-3}$
- $e^{-x^2} = e^{2x-3}$
- $-x^2 = 2x - 3$
- $x^2 + 2x - 3$
- $(x + 3)(x - 1) = 0$
- $x = -3, 1$

Example 4.1.7 (8: Between 9 AM and 10 PM cars arrive at burger king's drive-thru at the rate of 12 cars per hour (0.2 cars per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 9 PM)

$F(t) = 1 - e^{-2t}$

(a) 63%

(b) 99.7%

(c) graph

(d) other thing

4.2

4.3

4.4 Logarithmic Functions

Definition 4.4.1: Logarithmic Function:

The opposite to an exponential function. The logarithmic function to the base a, where $a > 0$ and $a \neq 0$, is denoted and defined by $y = \log_x x$ if and only if $x = a^y$

Note:-

You can remember the format by thinking log-base-answer-exponent.

Example 4.4.1 (2: Change each exponential expression to an equivalent expression involving a logarithm.)

1. $1.2^3 \longrightarrow$

Example 4.4.2 (3: Change each logarithmic expression to an equivalent expression involving an exponent.)

1. $\log_a 4 = 5 \longrightarrow a^5 = 4$
2. $\log_b e = -3 \longrightarrow b^{-3} = e$
3. $\log_3 5 = c \longrightarrow 3^c = 5$

Theorem 4.4.1

Get that exponential theorem from slides

Example 4.4.3 (4: Find he exact value of:)

1. $\log_2 16 = x \longrightarrow x = 4$
2. $\log_3 \frac{1}{27} = x \longrightarrow x = -3$ Convert to exponential then use the rules of exponents.
3. $\log_4 2 = x \longrightarrow x = \frac{1}{2}$

Theorem 4.4.2 Determine the Domain of a logarithmic function:

- Domain of the logarithmic function = range of the exponential function = $(0, \infty)$
- Range of the logarithmic function = domain of the exponential function = $(-\infty, \infty)$

Example 4.4.4 (5: Find the domain of each logarithmic function:)

1. $f(x) = \log_2(x+3) \longrightarrow x+3 > 0$
   - $x > -3$ or $(-3, \infty)$
2. $g(x) = \log_b(\frac{1+x}{1-x}) \longrightarrow \frac{1+x}{1-x} > 0$
   - $x \neq 1, -1$. Now use a number line to find out where it applies. In this case it is $-1 < x < 1$ or $(-1, 1)$ or $x|x \neq 1, -1$
3. $h(x) = \log_{\frac{1}{2}}|x| \longrightarrow |x| > 0$
   - **Domain** = $\mathbb{R}$ where $x \neq 0$, or All Real Numbers where $x \neq 0$, or $x|x \neq 0$

4.4.1 Natural Logarithm

Theorem 4.4.3 If the base of a logarithmic function is the number e, then we have the natural logarithm function. That is,

1. y = ln x if and only if x = ey
2. y = ln x and y = ex are inverse functions

Theorem 4.4.4 Common Logarithm Function

If the base of a logarithmic function is the number 10, then we have the common logarithm function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is,

- $y = log_x$ if and only if $x = 10^y$

Example 4.4.5 (6 & 7: Determine the domain, range, and vertical asymptote of each logarithmic function. List any transformations.)

a. $f(x) = \ln(x) \longrightarrow g(x) = -\ln(x+2)$
   - Domain: $x > -2$
   - Range: $(-\infty, \infty)$
   - Vertical Asymptote: $x \neq -2$

   Note:-

   The negtive applied to the natural log, seen in the equation $-\ln(x+2)$, is causing it to reflect over the x-axis.

b. $f(x) = \log(x) \longrightarrow g(x) = 3\log(-x) - 1$

Theorem 4.4.5 Equations that contain logarithms are called logarithmic equations. Be sure to check each solution in the original equation and discard any extraneous solutions. Remember in logaM, a and M are positive and $a \neq 1$.

1. Change the logarithmic equation to an exponential equation and solve for x
2. If the exponential equation has base e, change it to the natural logarithm function
3. If the exponential equation has base 10, change it to the common logarithm function

Example 4.4.6 (8: Solve for x)

1. $\log_3(4x - 7) = 2$
   - $3^2 = 4x - 7$
   - $9 = 4x - 7$
   - $x = 4$
2. $\log_x(64) = 2$
   - $x^2 = 64$
   - $x = \sqrt[2]{64}$
   - $x = 8$ Note: -8 does not work as a solution as base values for a logarithm must be greater than 1.

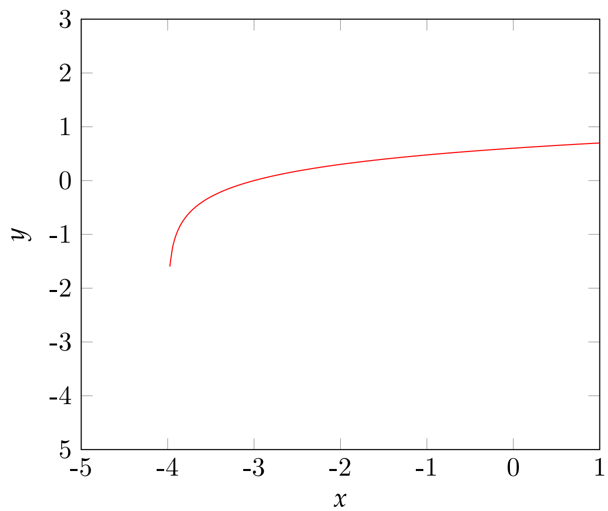
Example 4.4.7 (8.5: Solve for x. Give the exact solution then use your calculator to give the approximate solution.)

1. $e^{2x} = 5$
   - $\log_e 5 = 2x$
   - $\ln 5 = 2x$
   - $\frac{\ln 5}{2} = x$

Example 4.4.8 (Additional Example:)

1. $10^{x^2+2x+1} = 50$
   - $\log(50) = x^2 + 2x + 1$
   - $\pm\sqrt{\log(50)} = \sqrt{(x+1)^2}$
   - $\pm\sqrt{\log(50)} = x + 1$
   - $x = \pm\sqrt{\log(50)} + 1$

Figure 4.1: $\log_{10} x$



Thanks for reading