

Precalculus

Me. I am Him.

11/28/2022

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Chapter 1

Chapter 2

Chapter 3

3.1 Properties of functions and Complex Zeros

3.1.1 3.1 - Completing the square

Definition 3.1.1: Completing the square

In form $ax^2 + bx + c$, you half b, and then square.

Note:-

Forms:

Standard: $ax^2 + bx + c ; a \neq 0$

Vertex: $a(x - h)^2 + k ; a \neq 0$

Note:-

Vertices:

Standard: $(-b/2a , f(-b/2a))$

Vertex: (h , k)

Note:-

Axis of Symmetry:

Standard: $x = -b/2a$

Vertex: $x = h$

Note:-

y-Intercept

Standard: $(0 , c)$

Vertex: *Plug in 0 for x and solve*

Note; There is always one.

Note:-

x-Intercept

Standard: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, otherwise known as the quadratic formula.

If the inside of the quadratic is < 0 , there are no x-intercepts.

If the inside of the quadratic is $= 0$, there is exactly one x-intercept.

If the inside of the quadratic is > 0 , there are exactly two x-intercepts.

Vertex: *Plug in 0 for y and solve*

Chapter 4

4

4.1 4: Composite Functions

4.1.1

Note:-

11/28/2022 - Didn't really do much. Just reviewed what $f \cdot g$ or $f(g(x))$ was.

4.1.2

4.1.3 Exponential Functions

Are You Prepared? 4.1.1

- $4^3 = 8$
- $8^{\frac{2}{3}} = 4$
- $3^{-2} = \frac{1}{9}$

Note:-

In a^n , a is known as the base whereas n is known as the exponent, index, or power.

Note:-

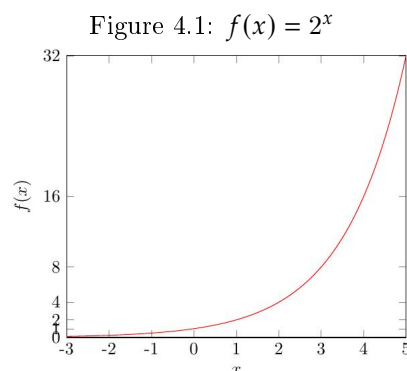
Law of Exponents:

- (1) $a^m \cdot a^n = a^{m+n}$ Example: $3^2 \cdot 3^5 = 3^{2+5} = 3^7 = 2187$
- (2) $(a^m)^n = a^{mn}$ Example: $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$
- (3) $(ab)^m = a^m b^m$ Example: $(5x)^3$
- (4) $1^n = 1$ Example: $1^{1001} = 1$
- (5) $a^{-n} = \frac{1}{a^n}$ Example: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
- (6) $a^0 = 1$ Example: $7^0 = 1$
- (7) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ Example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

Definition 4.1.1: Exponential Function

A function of the form $f(x) = a^x$ where x is a positive real number ($a > 0$) and $a \neq 1$. The domain of f is \mathbb{R}

Example 4.1.1 (2: Graph the exponential function: $f(x) = 2^x$)

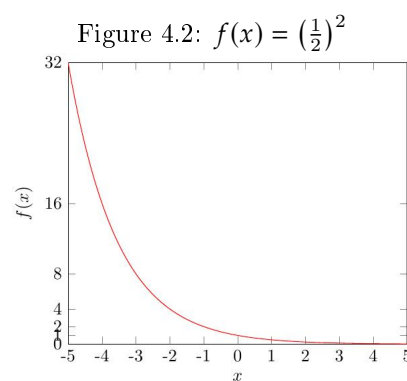


Note:-

Properties of the Exponential Function: $f(x) = a^x$, where $a > 1$

1. The *domain* is the set of all real numbers. The *range* is the set of all positive real numbers.
2. There are no *x-intercepts*. The *y-intercept* is 1.
3. The *x-axis* ($y=0$) is a horizontal asymptote as $x \rightarrow -\infty$
4. The function is an increasing function and is one-to-one.
5. The graph of f contains the points $(0,1)$, $(1,a)$, and $(-1,1/a)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.2 (3: Graph the exponential function: $f(x) = \left(\frac{1}{2}\right)^x$)



Note:-

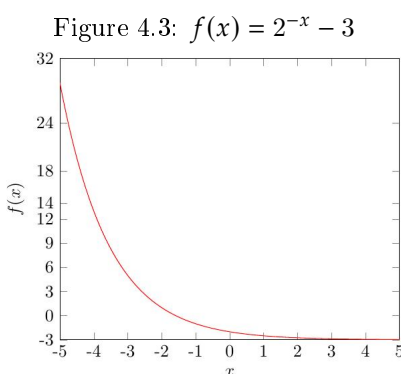
Properties of Exponential Function: $f(x) = a^x$, where $0 < a < 1$.

1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no *x-intercepts*; the *y-intercept* is 1.
3. The *x-axis* ($y = 0$) is a horizontal asymptote as $x \rightarrow \infty$.

4. The function is a decreasing function and is one-to-one.
5. The graph of f contains the points $(0, 1)$, $(1, a)$, and $(-1, 1/a)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.3 (Graph $f(x) = 2^{-x} - 3$ and determine the domain, range, and horizontal asymptote of f .)

- Domain: $x|x \in \mathbb{R}$ or $[-\infty, \infty]$
- Range: $y|y > -3$ or $[-3, \infty]$
- Horizontal Asymptote: $y = -3$



Example 4.1.4 (Explain the transformation of $g(x)$ from $f(x) = e^x$)

- $g(x) = -e^{x-3}$
- $g(x) = 3e^{-x} - 5$

Example 4.1.5 (6: Solve $3^{x+1} = 81$)

- $3^{x+1} = 3^4$
- $x + 1 = 4$
- $x = 3$

Example 4.1.6 (7: Solve $e^{-x^2} = \left(e^{x^2} \cdot \frac{1}{e^3}\right)$)

- $e^{-x^2} = e^{2x} \cdot e^{-3}$
- $e^{-x^2} = e^{2x-3}$
- $-x^2 = 2x - 3$
- $x^2 + 2x - 3$
- $(x + 3)(x - 1) = 0$
- $x = -3, 1$

Example 4.1.7 (8: Between 9 AM and 10 PM cars arrive at burger king's drive-thru at the rate of 12 cars per hour (0.2 cars per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 9 PM)

$$F(t) = 1 - e^{-2t}$$

- (a) 63%
- (b) 99.7%
- (c) graph
- (d) other thing

4.2

4.3

4.4 Logarithmic Functions

Definition 4.4.1: Logarithmic Function:

The opposite to an exponential function. The logarithmic function to the base a , where $a > 0$ and $a \neq 1$, is denoted and defined by $y = \log_a x$ if and only if $x = a^y$

Note:-

You can remember the format by thinking log-base-answer-exponent.

Example 4.4.1 (2: Change each exponential expression to an equivalent expression involving a logarithm.)

1. $1.2^3 \rightarrow$

Example 4.4.2 (3: Change each logarithmic expression to an equivalent expression involving an exponent.)

1. $\log_a 4 = 5 \rightarrow a^5 = 4$

2. $\log_b e = -3 \rightarrow b^{-3} = e$

3. $\log_3 5 = c \rightarrow 3^c = 5$

Theorem 4.4.1

Get that exponential theorem from slides

Example 4.4.3 (4: Find the exact value of:)

1. $\log_2 16 = x \rightarrow x = 4$

2. $\log_3 \frac{1}{27} = x \rightarrow x = -3$ Convert to exponential then use the rules of exponents.

3. $\log_4 2 = x \rightarrow x = \frac{1}{2}$

Theorem 4.4.2 Determine the Domain of a logarithmic function:

- Domain of the logarithmic function = range of the exponential function = $(0, \infty)$

- Range of the logarithmic function = domain of the exponential function = $(-\infty, \infty)$

Example 4.4.4 (5: Find the domain of each logarithmic function:)

1. $f(x) = \log_2(x+3) \rightarrow x+3 > 0$

- $x > -3$ or $(-3, \infty)$

2. $g(x) = \log_b\left(\frac{1+x}{1-x}\right) \rightarrow \frac{1+x}{1-x} > 0$

- $x \neq 1, -1$. Now use a number line to find out where it applies. In this case it is $-1 < x < 1$ or $(-1, 1)$ or $x|x \neq 1, -1$

3. $h(x) = \log_{\frac{1}{2}}|x| \rightarrow |x| > 0$

- **Domain** = \mathbb{R} where $x \neq 0$, or All Real Numbers where $x \neq 0$, or $x|x \neq 0$

4.4.1 Natural Logarithm

Theorem 4.4.3 If the base of a logarithmic function is the number e , then we have the natural logarithm function. That is,

1. $y = \ln x$ if and only if $x = ey$
2. $y = \ln x$ and $y = ex$ are inverse functions

Theorem 4.4.4 Common Logarithm Function

If the base of a logarithmic function is the number 10, then we have the common logarithm function. If the base a of the logarithmic function is not indicated, it is understood to be 10. That is,

- $y = \log_x$ if and only if $x = 10^y$

Example 4.4.5 (6 & 7: Determine the domain, range, and vertical asymptote of each logarithmic function. List any transformations.)

a. $f(x) = \ln(x) \rightarrow g(x) = -\ln(x+2)$

- Domain: $x > -2$
- Range: $(-\infty, \infty)$
- Vertical Asymptote: $x \neq -2$

Note:-

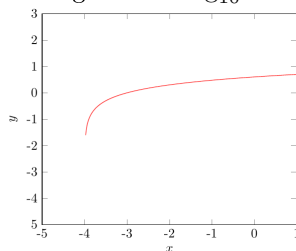
The negative applied to the natural log, seen in the equation $-\ln(x+2)$, is causing it to reflect over the x-axis.

b. $f(x) = \log(x) \rightarrow g(x) = 3\log(-x) - 1$

Theorem 4.4.5 Equations that contain logarithms are called logarithmic equations. Be sure to check each solution in the original equation and discard any extraneous solutions. Remember in $\log_a M$, a and M are positive and $a \neq 1$.

1. Change the logarithmic equation to an exponential equation and solve for x

Figure 4.4: $\log_{10} x$



2. If the exponential equation has base e , change it to the natural logarithm function
3. If the exponential equation has base 10, change it to the common logarithm function

Example 4.4.6 (8: Solve for x)

1. $\log_3(4x - 7) = 2$

- $3^2 = 4x - 7$
- $9 = 4x - 7$
- $x = 4$

2. $\log_x(64) = 2$

- $x^2 = 64$
- $x = \sqrt[2]{64}$
- $x = 8$ Note: -8 does not work as a solution as base values for a logarithm must be greater than 1.

Example 4.4.7 (8.5: Solve for x . Give the exact solution then use your calculator to give the approximate solution.)

1. $e^{2x} = 5$

- $\log_e 5 = 2x$
- $\ln 5 = 2x$
- $\frac{\ln 5}{2} = x$

Example 4.4.8 (Additional Example:)

1. $10^{x^2+2x+1} = 50$

- $\log(50) = x^2 + 2x + 1$
- $\pm\sqrt{\log(50)} = \sqrt{(x+1)^2}$
- $\pm\sqrt{\log(50)} = x + 1$
- $x = \pm\sqrt{\log(50)} + 1$

Example 4.4.9 (10: The concentration of alcohol in a person's blood is measurable. Recent medical research suggests that the risk R (given as a percent) of having an accident while driving a car can be modeled by the equation $6e^{kx}$ where x is the variable concentration of alcohol in the blood and k is a constant.

1. Suppose that a concentration in the blood of 0.04 results in a 10% risk ($R=10$) of an accident. Find the constant k in the equation. Graph $R = 6e^{kx}$ using the k value.
 - do stuff so that $k=20.62$. She literally used her calc
2. Using the value of k , what is the risk if the concentration is 0.17?
 - uhhhh she didn't do this.
3. Using the same value of k , what concentration of alcohol corresponds to a risk of 100%?
 - didn't do this one either. apparently D is the most important.
4. If the law asserts that anyone with a risk of having an accident of 20% or more should not have driving privileges, at what concentration of alcohol in the blood should a driver be arrested and charged with a DUI?
 - $20 = 6e^{12.77x}$
 - $\frac{10}{3} = e^{12.77x}$
 - $\ln\left(\frac{10}{3}\right) = 12.77x$
 - $x = 0.94$

)

4.5 Properties of Logarithms

Note:-

Properties of logarithms:

1. Identity $\rightarrow \log_a 1 = 0$ or $\log_a a = 1$
2. Inverse $\rightarrow \log_b b^x = x$ or $b^{\log_b(x)} = x$
3. Product $\rightarrow \log_a xy = \log_a x + \log_a y$
4. Quotient $\rightarrow \log_a \frac{x}{y} = \log_a x - \log_a y$
5. Equality $\rightarrow \log_b a = \log_b c \Rightarrow a = c$
6. Change of Base Formula $\rightarrow \log_a b = \frac{\log_c b}{\log_c a}$

Example 4.5.1 (1: Use properties of logarithms to find the exact value of each expression. Do not use a calculator.)

1. $\ln e^{\sqrt{2}}$
 - $\log_e e^{\sqrt{2}}$
 - $\sqrt{2} \times \log_e e$
 - $\sqrt{2} \times \ln e$
2. $\log_8 16 - \log_8 2$

- $\log_8 \frac{16}{2}$
- $\log_8 8$
- 1

Example 4.5.2 (3: Write the expression as a sum of logarithms. Express all powers as factors.)

1. $\log_a (x\sqrt{x^2+1})$
 - $\log_a x + \log_a \sqrt{x^2+1}$
 - $\log_a x + \log_a (x^2+1)^{\frac{1}{2}}$
 - $\log_a x + \frac{1}{2} \log_a (x^2+1)$

Example 4.5.3 (4: Write the expression as a difference in logarithms. Express all powers as factors.)

1. $\ln \left(\frac{x^2}{(x-1)^3} \right)$
 - $\ln(x^2) - \ln(x-1)^3$
 - $2 \ln(x) - 3 \ln(x-1)$

Example 4.5.4 (6: Write each of the following as a single logarithm.)

1. $\log_a 7 + 4 \log_a 3$
 - $\log_a 7 + \log_a 3^4$
 - $\log_a (7 \times 3^4)$
 - $\log_a 567$
2. $\frac{2}{3} \ln 8 - \ln(3^4 - 8)$
 - $\ln 8^{\frac{2}{3}} - \ln(3^4 - 8)$
 - $\ln \left(\frac{8^{\frac{2}{3}}}{3^4 - 8} \right)$
 - $\ln \left(\frac{4}{7^3} \right)$
3. $\log_a x + \log_a 9 + \log_a (x^2 + 1) - \log_a 5$
 - blah

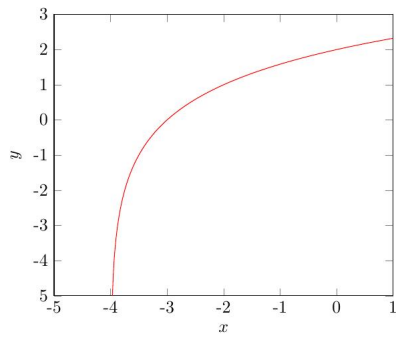
Example 4.5.5 (7: Approximate the following. Round answers to four decimal places.)

1. $\log_2 27$
 - $\frac{\log_{10} 27}{\log_{10} 2}$ Note: 10 is the common base, thus it can be omitted.
 - Another answer could be, $\frac{\ln 27}{\ln 2}$

Example 4.5.6 (9: Use a graphing utility to graph the following)

1. $y = \log_2 x$

Figure 4.5: $\log_2 x$



4.6

Example 4.6.1 (1: Solve.)

1. $2 \log_5 x = \log_5 9$

- $x^2 = 9$
- Ergo, $x = \pm 3$

Example 4.6.2 (2: Solve.)

1. $\log_4(x + 3) + \log_4(2 - x) = 1$

- $\log_4(x + 3)(2 - x) = 1$
- $4^1 = (x + 3)(2 - x)$
- $4 = x^2 - x + 6$
- $x^2 + x - 2$
- $(x + 2)(x - 1)$
- $x = -2, 1$

4.7 Interest

Definition 4.7.1: Interest Formulas

Note:-

I = amount of interest, A = final amount, P = principal, r = interest rate (as a decimal),
t = time (in years), n = number of times compounded per year)

Formulas:

1. Simple Interest:

$$I = Prt$$

or

$$A = P + I = P + Prt = P(1 + rt)$$

2. Compound Interest:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

3. Continuous Compounding:

$$A = Pe^{rt}$$

Example 4.7.1 (1: A credit union pays interest of 8% per annum compounded quarterly on a certain savings plan. If \$100 is deposited in such a plan and the interest is left to accumulate, how much will be in the account after a year?)

1. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. $A = 100\left(1 + \frac{0.08}{4}\right)^{4 \cdot 1}$
3. $A = \$108.24$

Example 4.7.2 (2 & 3 : Determine the final amount you invest \$1000 at an annual rate of 10% for 5 years, compounding at the amounts shown below.)

1. $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. $P = \$1000$
3. $r = 0.1$
4. $t = 5$
 - Annually:
 - \$1610.51
 - Semiannually
 - \$1628.89
 - Quarterly
 - \$1638.12
 - Monthly
 - \$1645.31
 - Daily
 - \$1648.61
 - Continuously
 - New Formula: Pe^{rt}
 - \$1648.72

Note:-

Calculate Effective rates of return:

The effective rate of interest is the equivalent annual simple rate of interest that would yield the same amount of compounding after 1 year.

To find the effective rate of interest:

1. Using the appropriate formula, find the final amount A .
2. Subtract the principal P from the final amount A to get the interest earned I .
3. Using the simple interest formula, $I = Prt$, plug in values and solve for r .

Example 4.7.3 (4: On January 2, 2004, \$2000 is placed in an Individual Retirement Account (IRA) that will pay interest of 7% per annum compounded continuously.)

- a. What will the IRA be worth on January 1, 2024?:

$$Pe^{rt} = 2000e^{0.07 \cdot 20} = \$8110.40$$

- b. Effective Rate of Interest?: (use the last note for guidance)

$$A = Pe^{rt} = 2000e^{0.07 \cdot 1} = 2145.02$$

$$2145.02 - 2000 = 145.02 = I$$

$$I = Prt = 145.02 = 2000r \rightarrow r = 0.07251 \text{ or } 7.25\%$$

Definition 4.7.2: Determine the Present Value of a Lump Sum of Money

Present Value Formulas are the compound interest and continuous compounding formulas solved for the principal P . So,

$$P = A\left(1 + \frac{r}{n}\right)^{-nt}$$

or

$$Ae^{-rt}$$

Example 4.7.4 (5: A zero-coupon (non-interest bearing) bond can be redeemed in 10 years for \$1000. How much should you be willing to pay for it now if you want a return of:)

1. 8% compounded monthly?

•

$$P = 1000\left(1 + \frac{0.08}{12}\right)^{-12 \cdot 10}$$

•

$$\$450.52$$

2. 7% compounded continuously?

•

$$P = 1000e^{-0.07 \cdot 10}$$

- blah, its higher.

Note:-

To determine the time required to double or triple lump sums of money, if we want to double P, A will be equal to 2p. Triple is the same thing.

Example 4.7.5 (6: What rate of interest compounded annually should you seek if you want to double your investment in 5 years?)

$$\begin{aligned}
 A &= P\left(1 + \frac{r}{n}\right)^{nt} \\
 2P &= P\left(1 + \frac{r}{1}\right)^{1 \times 5} \\
 \text{Divide by } P & 2 = \left(1 + \frac{r}{1}\right)^{1 \times 5} \\
 \text{5th Root } \sqrt[5]{2} &= 1 + r \\
 \text{ANS : } r &= \sqrt[5]{2} - 1 \\
 \text{Or : } r &= 14.9\%
 \end{aligned}$$

Example 4.7.6 (7:)

1. How long will it take for an investment to double in value if it earns 5% interest compounded continuously?

$$\begin{aligned}
 A &= Pe^{rt} \\
 2P &= Pe^{0.05t} \\
 2 &= e^{0.05t} \\
 \ln(2) &= 0.05t \\
 t &= \frac{\ln(2)}{0.05} \\
 t &= 13.86
 \end{aligned}$$

2. How long will it take to triple in value?

$$\begin{aligned}
 3P &= Pe^{0.05t} \\
 t &= 21.97
 \end{aligned}$$

Thanks for reading

