

Precalculus

Me. I am Him.

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Chapter 1

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Chapter 2

Chapter 3

3.1 Properties of functions and Complex Zeros

3.1.1 3.1 - Completing the square

Definition 3.1.1: Completing the square

In form $ax^2 + bx + c$, you half b, and then square.

Note:-

Forms:

Standard: $ax^2 + bx + c ; a \neq 0$

Vertex: $a(x - h)^2 + k ; a \neq 0$

Note:-

Vertices:

Standard: $(-b/2a , f(-b/2a))$

Vertex: (h , k)

Note:-

Axis of Symmetry:

Standard: $x = -b/2a$

Vertex: $x = h$

Note:-

y-Intercept

Standard: $(0 , c)$

Vertex: *Plug in 0 for x and solve*

Note; There is always one.

Note:-

x-Intercept

Standard: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, otherwise known as the quadratic formula.

If the inside of the quadratic is < 0 , there are no x-intercepts.

If the inside of the quadratic is $= 0$, there is exactly one x-intercept.

If the inside of the quadratic is > 0 , there are exactly two x-intercepts.

Vertex: *Plug in 0 for y and solve*

Chapter 4

4.1 4: Composite Functions

4.1.1

Note:-

11/28/2022 - Didn't really do much. Just reviewed what $f \cdot g$ or $f(g(x))$ was.

4.1.2

4.1.3 Exponential Functions

Definition 4.1.1: Are you prepared? MAKE NEW COMMAND

- $4^3 = 8$
- $8^{\frac{2}{3}} = 4$
- $3^{-2} = \frac{1}{9}$

Note:-

In a^n , a is known as the base whereas n is known as the exponent, index, or power.

Note:-

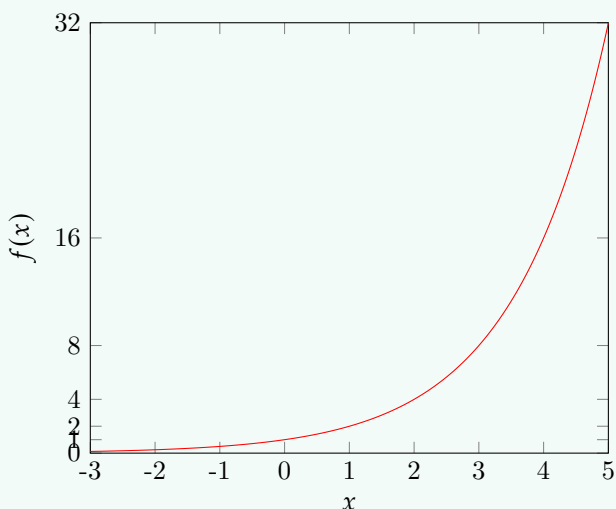
Law of Exponents:

- (1) $a^m \cdot a^n = a^{m+n}$ Example: $3^2 \cdot 3^5 = 3^{2+5} = 3^7 = 2187$
- (2) $(a^m)^n = a^{mn}$ Example: $(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$
- (3) $(ab)^m = a^m b^m$ Example: $(5x)^3$
- (4) $1^n = 1$ Example: $1^{1001} = 1$
- (5) $a^{-n} = \frac{1}{a^n}$ Example: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$
- (6) $a^0 = 1$ Example: $7^0 = 1$
- (7) $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ Example: $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 2^2 = 4$

Definition 4.1.2: Exponential Function

A function of the form $f(x) = a^x$ where x is a positive real number ($a > 0$) and $a \neq 1$. The domain of f is \mathbb{R}

Example 4.1.1 (2: Graph the exponential function: $f(x) = 2^x$)

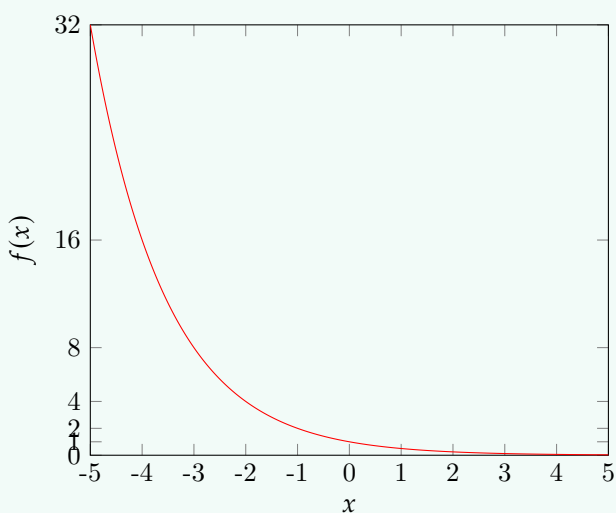


Note:-

Properties of the Exponential Function: $f(x) = a^x$, where $a > 1$

1. The *domain* is the set of all real numbers. The *range* is the set of all positive real numbers.
2. There are no *x-intercepts*. The *y-intercept* is 1.
3. The *x-axis* ($y=0$) is a horizontal asymptote as $x \rightarrow -\infty$
4. The function is an increasing function and is one-to-one.
5. The graph of f contains the points $(0,1)$, $(1,a)$, and $(-1,1/a)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.2 (3: Graph the exponential function: $f(x) = (\frac{1}{2})^x$)



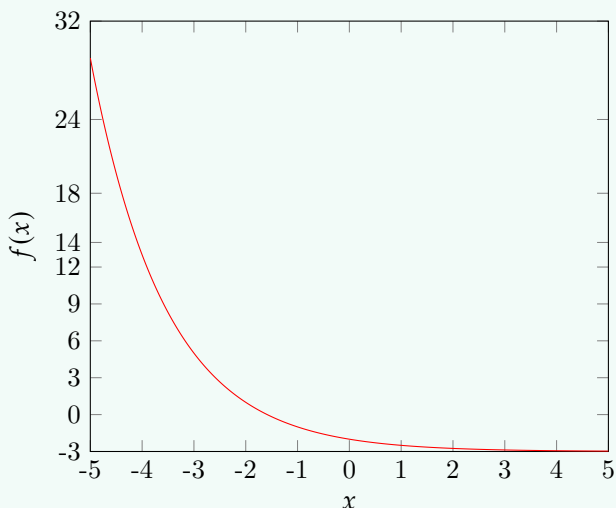
Note:-

Properties of Exponential Function: $f(x) = a^x$, where $0 < a < 1$.

1. The domain is the set of all real numbers; the range is the set of positive real numbers.
2. There are no *x-intercepts*; the *y-intercept* is 1.

3. The x-axis ($y = 0$) is a horizontal asymptote as $x \rightarrow \infty$.
4. The function is an decreasing function and is one-to-one.
5. The graph of f contains the points $(0, 1)$, $(1, a)$, and $(-1, 1/a)$.
6. The graph of f is smooth and continuous, with no corners or gaps.

Example 4.1.3 (Graph $f(x) = 2^{-x} - 3$ and determine the domain, range, and horizontal asymptote of f .)



- Domain: $x|x \in \mathbb{R}$ or $[-\infty, \infty]$
- Range: $y|y > -3$ or $[-3, \infty]$
- Horizontal Asymptote: $y = -3$

Example 4.1.4 (Explain the transformation of $g(x)$ from $f(x) = e^x$)

- $g(x) = -e^{x-3}$
- $g(x) = 3e^{-x} - 5$

Example 4.1.5 (6: Solve $3^{x+1} = 81$)

- $3^{x+1} = 3^4$
- $x + 1 = 4$
- $x = 3$

Example 4.1.6 (7: Solve $e^{-x^2} = (e^{x^2} \cdot \frac{1}{e^3})$)

- $e^{-x^2} = e^{2x} \cdot e^{-3}$
- $e^{-x^2} = e^{2x-3}$
- $-x^2 = 2x - 3$
- $x^2 + 2x - 3$
- $(x + 3)(x - 1) = 0$
- $x = -3, 1$

Example 4.1.7 (8: Between 9 AM and 10 PM cars arrive at burger king's drive-thru at the rate of 12 cars per hour (0.2 cars per minute). The following formula from statistics can be used to determine the probability that a car will arrive within t minutes of 9 PM)

$$F(t) = 1 - e^{-2t}$$

- (a) 63%
- (b) 99.7%
- (c) graph
- (d) other thing

4.2 Logarithmic Functions

Definition 4.2.1: Logarithmic Function:

The opposite to an exponential function. The logarithmic function to the base a, where $a > 0$ and $a \neq 1$, is denoted and defined by $y = \log_a x$ if and only if $x = a^y$

Note:-

You can remember the format by thinking log-base-answer-exponent.

Example 4.2.1 (2: Change each exponential expression to an equivalent expression involving a logarithm.)

1. $1.2^3 \rightarrow$

Example 4.2.2 (3: Change each logarithmic expression to an equivalent expression involving an exponent.)

- 1. $\log_a 4 = 5 \rightarrow a^5 = 4$
- 2. $\log_b e = -3 \rightarrow b^{-3} = e$
- 3. $\log_3 5 = c \rightarrow 3^c = 5$

Theorem 4.2.1

Get that exponential theorem from slides

Example 4.2.3 (4: Find the exact value of:)

- 1. $\log_2 16 = x \rightarrow x = 4$
- 2. $\log_3 \frac{1}{27} = x \rightarrow x = -3$ Convert to exponential then use the rules of exponents.
- 3. $\log_4 2 = x \rightarrow x = \frac{1}{2}$

Theorem 4.2.2 Determine the Domain of a logarithmic function:

- Domain of the logarithmic function = range of the exponential function = $(0, \infty)$

- Range of the logarithmic function = domain of the exponential function = $(-\infty, \infty)$

Example 4.2.4 (5: Find the domain of each logarithmic function:)

1. $f(x) = \log_2(x + 3) \rightarrow x + 3 > 0$

- $x > -3$ or $(-3, \infty)$

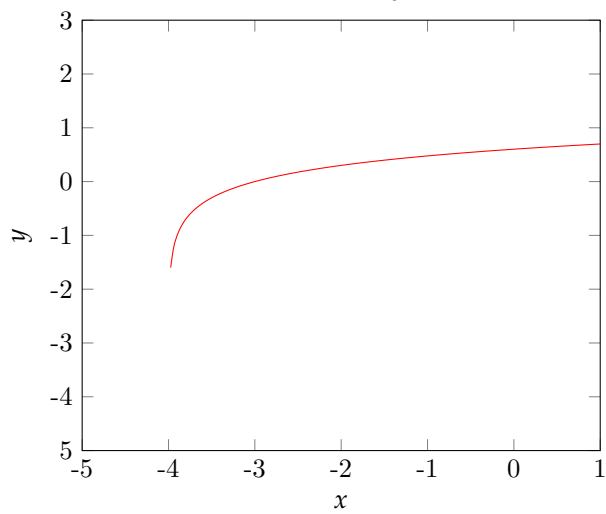
2. $g(x) = \log_b\left(\frac{1+x}{1-x}\right) \rightarrow \frac{1+x}{1-x} > 0$

- $x \neq 1, -1$. Now use a number line to find out where it applies. In this case it is $-1 < x < 1$ or $(-1, 1)$ or $x|x \neq 1, -1$

3. $h(x) = \log_{\frac{1}{2}}|x| \rightarrow |x| > 0$

- **Domain** = \mathbb{R} where $x \neq 0$, or All Real Numbers where $x \neq 0$, or $x|x \neq 0$

Figure 4.1: $\log_{10} x$



Thanks for reading

