

Homework 2

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Problem 1

A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

| Dosage | | Observations | | |
|--------|----|--------------|----|----|
| 20g | 24 | 28 | 37 | 30 |
| 30g | 37 | 44 | 31 | 35 |
| 40g | 42 | 47 | 52 | 38 |

Complete the ANOVA table:

```
nt <- 3 * 4
a <- 3
df_tr <- ( a - 1 )
df_tr
```

```
[1] 2
```

```
df_e <- nt - a
df_e
```

```
[1] 9
```

```
SStr <- 450.7
SST <- 783.9
SSe <- SST - SStr
SSe
```

```
[1] 333.2
```

```
MStr <- SStr / df_tr  
MStr
```

```
[1] 225.35
```

```
MSe <- SSe / df_e  
MSe
```

```
[1] 37.02222
```

```
F_v <- MStr / MSe  
F_v
```

```
[1] 6.086885
```

```
P_v <- pf(F_v, df_tr, df_e, lower.tail = F)  
P_v
```

```
[1] 0.02128136
```

| | df | SS | MS | F-Value | P-Value |
|----------------|-----------|--------------|---------------|-------------|--------------|
| Bet__ | <u>2</u> | 450.7 | <u>225.35</u> | <u>6.09</u> | <u>0.021</u> |
| Dossage | | | | | |
| Error | <u>9</u> | <u>333.2</u> | <u>37.02</u> | | |
| Total | <u>11</u> | <u>783.9</u> | | | |

a.

Is there evidence to indicate that dosage level affects bioactivity?

R/

Assuming an $\alpha = 0.05$, There is strong evidence to conclude that the at least one of the mean bioactivities is different from the others (P-value $< \alpha$).

b.

Compute a 95 percent interval estimate of the mean of dosage level is 30g.

R/

```
dos20 <- c(24, 25, 37, 30)
dos30 <- c(37, 44, 31, 35)
dos40 <- c(42, 47, 52, 38)
x2 <- mean(dos30)
x2
```

```
[1] 36.75
```

```
s2 <- sd(dos30)
s2
```

```
[1] 5.439056
```

```
n2 <- 4
n2
```

```
[1] 4
```

```
t2_star <- qt(.975, n2-1)
t2_star
```

```
[1] 3.182446
```

```
SE2 <- s2 / sqrt(n2)
SE2
```

```
[1] 2.719528
```

```
Upper <- x2 + t2_star * SE2
Upper
```

```
[1] 45.40475
```

```
Lower<- x2 - t2_star * SE2  
Lower
```

```
[1] 28.09525
```

We are 95% confident that the true mean bioactivity of the new drug at a dosage of 30 mg is between (28.1, 45.4).

c.

Compute a 99 percent interval estimate of the mean difference between dosages levels 20g and 40g.

R/

```
x1 <- mean(dos20)  
x1
```

```
[1] 29
```

```
x3 <- mean(dos40)  
x3
```

```
[1] 44.75
```

```
s1 <- sd(dos20)  
s1
```

```
[1] 5.944185
```

```
s2 <- sd(dos40)  
s2
```

```
[1] 6.075909
```

```
n1 = n3 = 4  
PEs <- x3 - x1  
PEs
```

```
[1] 15.75
```

```
df <- n1+n3-2
df
```

```
[1] 6
```

```
SE <- sqrt(MSe / n1 + MSe / n3)
SE
```

```
[1] 4.302454
```

```
t_star <- qt(0.995, df, lower.tail = T)
t_star
```

```
[1] 3.707428
```

```
Upper1 <- PEs + (SE * t_star)
Upper1
```

```
[1] 31.70104
```

```
Lower1 <- PEs - (SE * t_star)
Lower1
```

```
[1] -0.2010388
```

We are 99% confident that the bioactivity of the new drug at a dosage of 40 mg is between 31.7 higher, and -0.2) lower than the dosage of 20 mg.

d.

Compute simultaneous 95% CIs for the difference in means for dosage levels 20g and 40g, based on Tukey's, Bonferroni's and Scheffe's Methods:

R/

```
#Tukey's HSD
Q <- qtukey(0.95, 3, 12-3)/sqrt(2)
Q
```

```
[1] 2.792006
```

```
Up_Tu <- PEs + Q * SE  
Up_Tu
```

```
[1] 27.76248
```

```
Lo_Tu <- PEs - Q * SE  
Lo_Tu
```

```
[1] 3.737524
```

```
# Bonferroni  
g = 3 * (3 - 1)/2  
dfe = 12 - 3  
t_bf = qt(1 - (0.05/(2*g)), dfe)  
t_bf
```

```
[1] 2.933324
```

```
Up_bf <- PEs + t_bf * SE  
Up_bf
```

```
[1] 28.37049
```

```
Lo_bf <- PEs - t_bf * SE  
Lo_bf
```

```
[1] 3.129508
```

```
# Scheffe  
Sh <- sqrt((3 - 1) * qf(0.95, 3-1, dfe))  
Sh
```

```
[1] 2.917703
```

```
Up_sh <- PEs + Sh * SE
Up_sh
```

```
[1] 28.30328
```

```
Lo_sh <- PEs - Sh * SE
Lo_sh
```

```
[1] 3.196718
```

Tukey's HSD 95% confidence interval = (3.74, 27.76)
Bonferroni's 95% confidence interval = (3.13, 28.37)
Scheffe's 95% confidence interval = (3.2, 28.3)

e.

Use Tukey's HSD to compare pairs of treatment means.

R/

When $|x_a - x_b| > \text{Tukey's HSD}$ we can conclude that there is a difference mean of a is different of the mean of b

```
# Tukey's HSD for all the comparisons
T_HSD <- Q * SE
T_HSD
```

```
[1] 12.01248
```

```
abs(x1 - x2) > T_HSD #Comparison for dosage 20g to dosage 30g
```

```
[1] FALSE
```

```
abs(x1 - x3) > T_HSD #Comparison for dosage 20g to dosage 40g
```

```
[1] TRUE
```

```
abs(x2 - x3) > T_HSD #Comparison for dosage 30g to dosage 40g
```

[1] FALSE

For these comparisons, the only comparison in which the mean bioactivity of the new drug is significantly different, is when it is compared 20g and 40 g of dosage.

Problem 2

The tensile strength of Portland cement is being studied. Four different mixing techniques can be used economically. A completely randomized experiment was conducted, and the following data were collected.

| Mixing Technique | Tensile Strength (lb/in ²) | | | |
|---------------------|--|------|------|------|
| 1 | 3129 | 3000 | 2865 | 2890 |
| 2 | 3200 | 3300 | 2975 | 3150 |
| 3 | 2800 | 2900 | 2985 | 3050 |
| 4 | 2600 | 2700 | 2600 | 2765 |

You are given the following information:

$$\bar{Y}_{1.} = 2971.00, s(\bar{Y}_{1.}) = 56.65 \text{ and, } MSE = 12825.7$$

$$\bar{Y}_{2.} = 3156.25, s(\bar{Y}_{2.}) = 56.65$$

$$\bar{Y}_{3.} = 2933.75, s(\bar{Y}_{3.}) = 56.65$$

$$\bar{Y}_{4.} = 2666.25, s(\bar{Y}_{4.}) = 56.65$$

a.

Test the hypothesis that mixing techniques affect the strength of the cement. Test the hypothesis:

R/

$$H_o : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_a : \text{At least one } \mu_i \neq \mu_j ; \text{ for } i \neq j$$


```

dft <- 4-1
dfe <- 16-4
y1 <- 2971
y2 <- 3156.25
y3 <- 2933.75
y4 <- 2666.25
s1 = s2 = s3 = s4 = 56.65
n1 = n2 = n3 = n4 = 4
MSE <- 12825.7
y_bar = 4 * (y1 + y2 + y3 + y4) / 16
SSt = 4 * ((y1 - y_bar)^2 + (y2 - y_bar)^2 + (y3 - y_bar)^2 + (y4 - y_bar)^2) / dft
SSt

```

```
[1] 163246.7
```

```

MSt <- SSt/dft
MSt

```

```
[1] 54415.58
```

```

SSe <- MSE*dfe
SSe

```

```
[1] 153908.4
```

```

F_v <- MSt / MSE
F_v

```

```
[1] 4.242698
```

```

P_v <- pf(F_v, dft, dfe, lower.tail = F)
P_v

```

```
[1] 0.02923341
```

| | df | SS | MS | F-Value | P-Value |
|-------------------------|-----------|-----------------|----------------|-------------|--------------|
| Mixing technique | <u>3</u> | <u>163246.7</u> | <u>54415.6</u> | <u>4.24</u> | <u>0.029</u> |
| Error | 12 | 153908.4 | 12825.7 | | |
| Total | <u>15</u> | <u>317155.1</u> | | | |

As our P-value obtained is less than our $\alpha = 0.05$, we reject the null hypothesis, and we have enough evidence to conclude that at least one of the mean tensile strengths is different from the others when a different mixing technique is applied.

b.

Find a 95 percent confidence interval on the mean tensile strength of the Portland cement produced by mixing technique 3.

R/

```
t_star <- qt(.975, n3-1)
t_star
```

```
[1] 3.182446
```

```
SE3 <- s3 / sqrt(n3)
SE3
```

```
[1] 28.325
```

```
Up3 <- y3 + t_star * SE3
Up3
```

```
[1] 3023.893
```

```
Lo3 <- y3 - t_star * SE3
Lo3
```

```
[1] 2843.607
```

We are 95% confident that the true mean tensile strength of the Portland cement produced by mixing technique 3 lies between (2843.61, 3023.89) lb/in^2 .

c.

Find a 95 percent confidence interval on the difference in means for techniques 1 and 3 & 3 and 4.

```
PE31 <- y3 - y1  
PE31
```

```
[1] -37.25
```

```
PE34 <- y3 - y4  
PE34
```

```
[1] 267.5
```

```
df31 <- n1 + n3 - 2  
df31
```

```
[1] 6
```

```
df34 <- n3 + n4 - 2  
df34
```

```
[1] 6
```

```
SE <- sqrt( 2 * MSE / 4)  
SE
```

```
[1] 80.08027
```

```
t_star <- qt(0.975, df34, lower.tail = T)  
t_star
```

```
[1] 2.446912
```

```
Up31 <- PE31 + (SE * t_star)  
Up31
```

```
[1] 158.6994
```

```
Lo31 <- PE31 - (SE * t_star)
Lo31
```

```
[1] -233.1994
```

```
Up34 <- PE34 + (SE * t_star)
Up34
```

```
[1] 463.4494
```

```
Lo34 <- PE34 - (SE * t_star)
Lo34
```

```
[1] 71.55063
```

We are 95% confident that the true difference in mean tensile strength for mixing technique 3 and 1 is between $(-233.2, 158.7)$ lb/in^2 . That is the tensile strength for mixing technique 3 is between $158.7 lb/in^2$ higher and $233.2 lb/in^2$ lower than mixing technique 1.

We are 95% confident that the true difference in mean tensile strength for mixing technique 3 and 4 is between $(71.55, 463.45)$ lb/in^2 . That is the tensile strength for mixing technique 3 is between $71.55 lb/in^2$ and $463.45 lb/in^2$ higher than mixing technique 1.

d.

Compute simultaneous 95% CIs for the difference in means for techniques 1 and 3, based on Tukey's, Bonferroni's and Scheffe's Methods:

R/

```
#Tukey's HSD
Q <- qtkey(0.95, 4, 16-4)/sqrt(2)
Q
```

```
[1] 2.968901
```

```
Up_Tu <- PE31 + Q * SE
Up_Tu
```

```
[1] 200.5004
```

```
Lo_Tu <- PE31 - Q * SE
Lo_Tu
```

```
[1] -275.0004
```

```
# Bonferroni
g = 4 * (4 - 1)/2
dfe = 16 - 4
t_bf = qt(1 - (0.05/(2*g)), dfe)
t_bf
```

```
[1] 3.152681
```

```
Up_bf <- PE31 + t_bf * SE
Up_bf
```

```
[1] 215.2176
```

```
Lo_bf <- PE31 - t_bf * SE
Lo_bf
```

```
[1] -289.7176
```

```
# Scheffe
Sh <- sqrt((4 - 1) * qf(0.95, 4-1, dfe))
Sh
```

```
[1] 3.235875
```

```
Up_sh <- PE31 + Sh * SE
Up_sh
```

```
[1] 221.8797
```

```
Lo_sh <- PE31 - Sh * SE
Lo_sh
```

```
[1] -296.3797
```

Tukey's HSD 95% confidence interval = (-275, 200.5)
Bonferroni's 95% confidence interval = (-289.72, 215.22)
Scheffe's 95% confidence interval = (-296.38, 221.88)

e.

Compute Bonferroni's Minimum Significant Difference

```
# Bonferroni
g <- 4 * (4 - 1)/2
dfe <- 16 - 4
t_bf <- qt(1 - (0.05/(2*g)), dfe)
t_bf
```

```
[1] 3.152681
```

```
Bf_MSD <- t_bf * sqrt( 2 * MSE / 4)
Bf_MSD
```

```
[1] 252.4676
```

f.

Compute Tukey's Honest Significant Difference

```
#Tukey's HSD
Tu_HSD <- (qtukey(0.95, 4, 16-4)/sqrt(2)) * sqrt( 2 * MSE / 4)
Tu_HSD
```

```
[1] 237.7504
```

g.

Compute Scheffe's Minimum Significant Difference

R/

```
Sh <- sqrt((4 - 1) * qf(0.95, 4-1, dfe))
Sh_MSD <- Sh * sqrt( 2 * MSE / 4)
Sh_MSD
```

```
[1] 259.1297
```

Problem 3

A study is conducted to compare 4 menus in terms of numbers of calories ordered by restaurant customers (in 100s of calories). The treatments (menus) are (consider them increasing in order of information provided):

1. No Calories Reported
2. Calories Reported
3. Rank-Ordered Calories
4. Color-Coded Calories

The sample sizes are all based on samples of $r = 20$ customers per menu. The sample means and estimated variance are:

$$\bar{Y}_{1.} = 17.6$$

$$\bar{Y}_{2.} = 16.8$$

$$\bar{Y}_{3.} = 16.0$$

$$\bar{Y}_{4.} = 14.4$$

$$\bar{Y}_{..} = 16.2$$

$$s^2 = MSE = 196.0$$

Give 3 orthogonal contrasts, and their estimates and estimated standard errors.

Contrast 1: Menu 1 vs Menus {2, 3, 4}

Contrast 1:

```
Lhat_1 <- 3 * 17.7 - 16.8 - 16.0 - 14.4  
Lhat_1
```

```
[1] 5.9
```

```
MSE <- 196  
s_ci <- ( 3 ^ 2 /20) + (( -1) ^ 2 /20) + (( -1) ^ 2 /20) + (( -1) ^ 2 /20)  
s_ci
```

```
[1] 0.6
```

```
s_L1 <- sqrt( MSE * s_ci)  
s_L1
```

```
[1] 10.84435
```

$$\begin{aligned}H_o : L_1 &= \mu_1 = \frac{\mu_2 + \mu_3 + \mu_4}{3} \\L_1 &= 3\mu_1 + (-1)\mu_2 + (-1)\mu_3 + (-1)\mu_4 = 0 \\ \hat{L}_1 &= 3\bar{y}_1. + (-1)\bar{y}_2. + (-1)\bar{y}_3. + (-1)\bar{y}_4. = 5.9 \\sd(\hat{L}_1) &= 10.84\end{aligned}$$

Contrast 2: Menu 2 vs Menus {3, 4}

```
Lhat_2 <- 0 * 17.7 + 2 * 16.8 - 16.0 - 14.4
Lhat_2
```

```
[1] 3.2
```

```
MSE <- 196
s_ci <- ( 0 ^ 2 /20) + ( 2 ^ 2 /20) + (( -1) ^ 2 /20) + (( -1) ^ 2 /20)
s_ci
```

```
[1] 0.3
```

```
s_L2 <- sqrt( MSE * s_ci)
s_L2
```

```
[1] 7.668116
```

$$\begin{aligned}H_o : L_2 &= \mu_2 = \frac{\mu_3 + \mu_4}{2} \\L_2 &= (0)\mu_1 + 2\mu_2 + (-1)\mu_3 + (-1)\mu_4 = 0 \\ \hat{L}_2 &= (0)\bar{y}_1. + 2\bar{y}_2. + (-1)\bar{y}_3. + (-1)\bar{y}_4. = 3.2 \\sd(\hat{L}_2) &= 7.67\end{aligned}$$

Contrast 3: Menu 3 vs Menu 4

```
Lhat_3 <- 0 * 17.7 + 0 * 16.8 + 16.0 - 14.4
Lhat_3
```

```
[1] 1.6
```



```
MSE <- 196
s_ci <- ( 0 ^ 2 /20) + ( 0 ^ 2 /20) + (1 ^ 2 /20) + (( -1) ^ 2 /20)
s_ci
```

```
[1] 0.1
```

```
s_L3 <- sqrt( MSE * s_ci)
s_L3
```

```
[1] 4.427189
```

$$H_o : L_3 = \mu_3 = \mu_4$$

$$L_3 = (0)\mu_1 + (0)\mu_2 + (1)\mu_3 + (-1)\mu_4 = 0$$

$$\hat{L}_3 = (0)\bar{y}_{1.} + (0)\bar{y}_{2.} + \bar{y}_{3.} + (-1)\bar{y}_{4.} = 1.6$$

$$sd(\hat{L}_3) = 4.43$$

Problem 4 R

An article in Environment International (Vol. 18, No. 4, 1992) describes an experiment in which the amount of radon released in showers was investigated. Radon enriched water was used in the experiment and six different Orifice diameters were tested in shower heads. The data from the experiment are shown in the following table.

| Orifice Dia. | Radon Released (%) | | | |
|--------------|--------------------|----|----|----|
| 0.37 | 80 | 83 | 83 | 85 |
| 0.51 | 75 | 75 | 79 | 79 |
| 0.71 | 74 | 73 | 76 | 77 |
| 1.02 | 67 | 72 | 74 | 74 |
| 1.40 | 62 | 62 | 67 | 69 |
| 1.99 | 60 | 61 | 64 | 66 |

a.

Write a suitable model (Factor effects) for this experiment. Define all terms in the model. Include all assumptions of the model.

R/

```
radon_release<-c(80,83,83,85,75,75,79,79,74,73,76,77,67,72,74,74,62,62,67,69,60,61,64,66)
diameter<-c(rep("0.37",4),rep("0.51",4),rep("0.71",4),rep("1.02",4),rep("1.40",4),rep("1.99",4))
radon_data<-data.frame(radon_release,diameter)
radon_data
```

| | radon_release | diameter |
|----|---------------|----------|
| 1 | 80 | 0.37 |
| 2 | 83 | 0.37 |
| 3 | 83 | 0.37 |
| 4 | 85 | 0.37 |
| 5 | 75 | 0.51 |
| 6 | 75 | 0.51 |
| 7 | 79 | 0.51 |
| 8 | 79 | 0.51 |
| 9 | 74 | 0.71 |
| 10 | 73 | 0.71 |
| 11 | 76 | 0.71 |
| 12 | 77 | 0.71 |
| 13 | 67 | 1.02 |
| 14 | 72 | 1.02 |
| 15 | 74 | 1.02 |
| 16 | 74 | 1.02 |
| 17 | 62 | 1.40 |
| 18 | 62 | 1.40 |
| 19 | 67 | 1.40 |
| 20 | 69 | 1.40 |
| 21 | 60 | 1.99 |
| 22 | 61 | 1.99 |
| 23 | 64 | 1.99 |
| 24 | 66 | 1.99 |

```
str(radon_data)
```

```
'data.frame':  24 obs. of  2 variables:
 $ radon_release: num  80 83 83 85 75 75 79 79 74 73 ...
 $ diameter     : chr  "0.37" "0.37" "0.37" "0.37" ...
```

```
model <- aov(radon_release ~ diameter, data = radon_data)
summary(model)
```

| | Df | Sum Sq | Mean Sq | F value | Pr(>F) |
|-----------|----|--------|---------|---------|--------------|
| diameter | 5 | 1133.4 | 226.68 | 30.85 | 3.16e-08 *** |
| Residuals | 18 | 132.2 | 7.35 | | |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

The assumptions for this model are:

- Independence of observations
- Equal variance between each level evaluated
- The residuals follow an approximately normal distribution
- Linearity

Terms of the model:

- 5 are the degrees of freedom due to the treatments $(a-1) = (6-1)$
- 18 are the degrees of freedom for the error $(n-a) = (24-6)$
- 1133.4 is the sum of squares for the treatments
- 132.2 is the sum of squares for the error
- 226.68 is the variation explained by the treatments
- 7.35 is the variation explained by the error
- 30.85 is the F-value of our observations under the assumption that the null hypothesis is true
- 3.16×10^{-8} is the probability to obtain the values observed under the assumption of the null hypothesis (all means are equal).

b.

Obtain point estimates and standard deviation of the radon released at each level.

R/

```
tapply(radon_release, diameter, mean) #Mean (Point estimates)
```

```
0.37 0.51 0.71 1.02 1.40 1.99
82.75 77.00 75.00 71.75 65.00 62.75
```

```
tapply(radon_release, diameter, sd) #Standard deviation
```

```
      0.37      0.51      0.71      1.02      1.40      1.99
2.061553 2.309401 1.825742 3.304038 3.559026 2.753785
```

c.

Does the size of the orifice affect the mean percentage of radon released? Use $\alpha = 0.05$

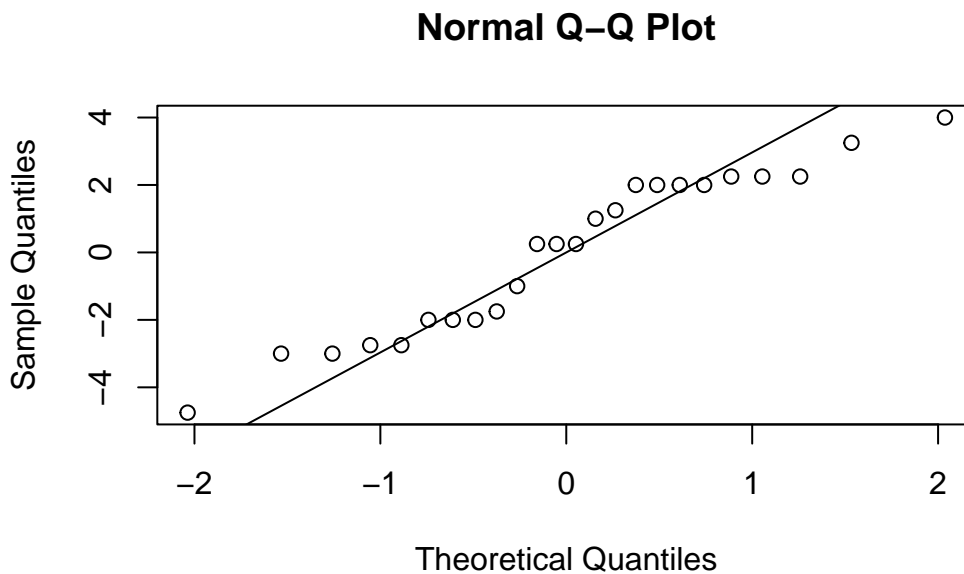
R/

Given our alpha value = 0.05, and that the P-value obtained in the ANOVA is 3.16e-8, we have strong evidence to conclude that the orifice diameter does affect the mean percentage of radon released (at least one the means is different from the others)

d.

Construct a normal probability plot of the residuals. What conclusion would you draw about the validity of the normality assumption?

```
qqnorm(residuals(model))
qqline(residuals(model))
```

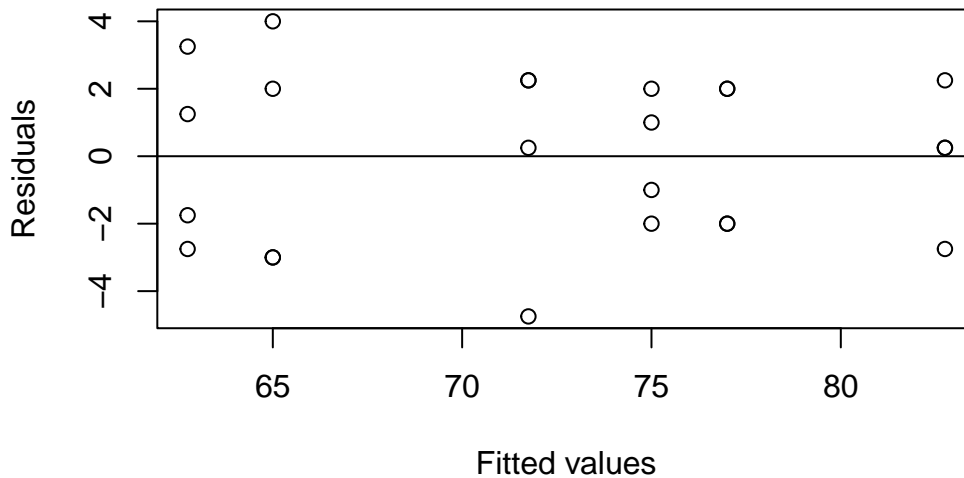


Based on the normal Q-Q plot, we can conclude that the data follows the normality assumption, since the residuals from our data follows a similar linear trend against the theoretical normal distributed residuals.

e.

Plot the residuals versus the predicted radon release. Comment on the plot.

```
plot(fitted.values(model), residuals(model),  
     xlab = "Fitted values", ylab = "Residuals")  
abline(h = 0)
```



Based on the residuals vs fitted values plot, we can confirm our assumption regarding equal variance since there is not a funnel shape trend, also there is no specific pattern on each point that may make us think about the lack of linearity within the data.

f.

Test by means of the Modified Leven's (Brown-Forsythe) and Bartlett's test whether the treatment error variances for the response variable are equal.

```
#It is testing Ho: all group variances are equal  
library(car)
```

Loading required package: carData

```
leveneTest(radon_release, diameter)
```

Warning in leveneTest.default(radon_release, diameter): diameter coerced to factor.

```
Levene's Test for Homogeneity of Variance (center = median)
      Df F value Pr(>F)
group  5  0.8605 0.5261
      18
```

Based on the P-value (Big P-value) we fail to reject the null hypothesis and we conclude that the variance for each treatment is approximately equal to the others.

```
#It is testing Ho: all group variances are equal
library(mvoutlier)
```

Loading required package: sgeostat

```
bartlett.test(radon_release ~ diameter)
```

Bartlett test of homogeneity of variances

```
data: radon_release by diameter
Bartlett's K-squared = 1.8214, df = 5, p-value = 0.8733
```

Based on the P-value (Big P-value) we fail to reject the null hypothesis and we conclude that the variance for each treatment is approximately equal to the others.

g.

Test the normality assumption using Shapiro-Wilk test.

```
#It is testing Ho: the data comes from a normal distribution
tapply(radon_release, diameter, shapiro.test)
```

```
$`0.37`
```

Shapiro-Wilk normality test

```
data: X[[i]]
W = 0.92614, p-value = 0.5719
```

\$`0.51`

Shapiro-Wilk normality test

data: X[[i]]

W = 0.72863, p-value = 0.02386

\$`0.71`

Shapiro-Wilk normality test

data: X[[i]]

W = 0.94971, p-value = 0.7143

\$`1.02`

Shapiro-Wilk normality test

data: X[[i]]

W = 0.80781, p-value = 0.117

\$`1.40`

Shapiro-Wilk normality test

data: X[[i]]

W = 0.83785, p-value = 0.1892

\$`1.99`

Shapiro-Wilk normality test

data: X[[i]]

W = 0.93927, p-value = 0.6499

Given the P-value for each of the treatments, we fail to reject the null hypothesis for all of the treatments, and conclude that the data follows an approximate normal distribution. Except for the 0.51 orifice size in which the P-value is lower than our assumed $\alpha = 0.05$, we reject the

null hypothesis here and conclude that the residuals for this treatment do not follow a normal distribution.

h.

Use the Tukey's and Bonferroni's methods to make comparisons among the five treatments to determine specifically which treatments differ in radon release.

```
TukeyHSD(model)
```

```
Tukey multiple comparisons of means
 95% family-wise confidence level
```

```
Fit: aov(formula = radon_release ~ diameter, data = radon_data)
```

```
$diameter
      diff      lwr      upr    p adj
0.51-0.37 -5.75 -11.841234  0.3412336 0.0707511
0.71-0.37 -7.75 -13.841234 -1.6587664 0.0084181
1.02-0.37 -11.00 -17.091234 -4.9087664 0.0002404
1.40-0.37 -17.75 -23.841234 -11.6587664 0.0000004
1.99-0.37 -20.00 -26.091234 -13.9087664 0.0000001
0.71-0.51 -2.00  -8.091234  4.0912336 0.8968057
1.02-0.51 -5.25 -11.341234  0.8412336 0.1153360
1.40-0.51 -12.00 -18.091234 -5.9087664 0.0000841
1.99-0.51 -14.25 -20.341234 -8.1587664 0.0000089
1.02-0.71 -3.25  -9.341234  2.8412336 0.5513482
1.40-0.71 -10.00 -16.091234 -3.9087664 0.0007059
1.99-0.71 -12.25 -18.341234 -6.1587664 0.0000650
1.40-1.02  -6.75 -12.841234 -0.6587664 0.0249971
1.99-1.02  -9.00 -15.091234 -2.9087664 0.0021152
1.99-1.40  -2.25  -8.341234  3.8412336 0.8432736
```

```
pairwise.t.test(radon_release, diameter, p.adj="bonferroni")
```

```
Pairwise comparisons using t tests with pooled SD
```

```
data: radon_release and diameter
```


| | | | | | |
|------|---------|---------|---------|---------|---------|
| | 0.37 | 0.51 | 0.71 | 1.02 | 1.40 |
| 0.51 | 0.11528 | - | - | - | - |
| 0.71 | 0.01143 | 1.00000 | - | - | - |
| 1.02 | 0.00029 | 0.20222 | 1.00000 | - | - |
| 1.40 | 4.3e-07 | 1.0e-04 | 0.00087 | 0.03654 | - |
| 1.99 | 6.9e-08 | 1.0e-05 | 7.7e-05 | 0.00270 | 1.00000 |

P value adjustment method: bonferroni

Based on the P-value adjusted in Tukey's HSD, we can conclude that the following pairs differ in the mean radon release percentage:

- 0.71-0.37
- 1.02-0.37
- 1.40-0.37
- 1.99-0.37
- 1.40-0.51
- 1.99-0.51
- 1.40-0.71
- 1.99-0.71
- 1.40-1.02
- 1.99-1.02

Which match with the comparisons that are considered significant for the Bonferroni's adjusted P-values.