Problem Set 8- Questions 5, 7 and 9

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1 Question 5

How does your estimate compare with the true value β in (1)?

Table 1: Comparison of True and Estimated Beta Coefficients

Index	True_Beta	Estimated_Beta	Difference
1	1.50	1.5010518	0.0010518190
2	-1.00	-1.0008296	-0.0008296258
3	-0.25	-0.2516480	-0.0016480100
4	0.75	0.7490406	-0.0009593796
5	3.50	3.5005531	0.0005531240
6	-2.00	-2.0008185	-0.0008185346
7	0.50	0.4987148	-0.0012852344
8	1.00	1.0028269	0.0028268510
9	1.25	1.2465102	-0.0034898039
10	2.00	2.0010012	0.0010011821

2 Question 7

Do it again using the Nelder-Mead algorithm. Do your answers differ?

Table 2: Comparison of Beta Estimates Across Different Methods

Index	True_Beta	OLS_Beta	GD_Beta	LBFGS_Beta	NM_Beta
1	1.50	1.5010518	1.5010515	1.5010518	1.5010518
2	-1.00	-1.0008296	-1.0008295	-1.0008296	-1.0008296
3	-0.25	-0.2516480	-0.2516479	-0.2516480	-0.2516480
4	0.75	0.7490406	0.7490405	0.7490406	0.7490406
5	3.50	3.5005531	3.5005526	3.5005531	3.5005531
6	-2.00	-2.0008185	-2.0008182	-2.0008185	-2.0008185
7	0.50	0.4987148	0.4987147	0.4987148	0.4987148
8	1.00	1.0028269	1.0028268	1.0028269	1.0028269
9	1.25	1.2465102	1.2465100	1.2465102	1.2465102
10	2.00	2.0010012	2.0010009	2.0010012	2.0010012

Comparison of Estimation Methods The table above compares the beta coefficient estimates obtained using different optimization methods: the closed-form OLS solution, gradient descent (GD), L-BFGS algorithm, and Nelder-Mead algorithm. All methods converged to essentially identical solutions, with differences only in the seventh decimal place or beyond. The closed-form OLS solution, L-BFGS, and Nelder-Mead produced identical results to the precision shown. The answers of NM and L-BFGS do not differ. Nelder-Mead Algorithm is a derivative free approach. However, L-BFGS uses gradient information to find the minimum.

3 Question 9

In your .tex file, tell me about how similar your estimates of $\hat{\beta}$ are to the "ground truth" β that you used to create the data in (1).

Table 3: Comparison of Estimated Coefficients to Ground Truth

Parameter	**True Value**	**Estimated Value**	**Difference**
Intercept	1.5000	1.5011	0.0011
X1	-1.0000	-1.0008	-0.0008
X2	-0.2500	-0.2516	-0.0016
X3	0.7500	0.7490	-0.0010
X4	3.5000	3.5006	0.0006
X5	-2.0000	-2.0008	-0.0008
X6	0.5000	0.4987	-0.0013
X7	1.0000	1.0028	0.0028
X8	1.2500	1.2465	-0.0035
X9	2.0000	2.0010	0.0010

Comparison of Estimated Coefficients to Ground Truth The estimates are very close to the ground truth values, which is expected given the large sample size (N = 100,000). The differences are due to the random error term that was added during data generation. With this large sample size, the Law of Large Numbers ensures that our estimates converge to the true parameter values.

Table 4: OLS Regression Results

	OLS Model
X1	1.501^{***}
	(0.002)
X2	-1.001^{***}
	(0.002)
X3	-0.252^{***}
	(0.002)
X4	0.749^{***}
	(0.002)
X5	3.501^{***}
	(0.002)
X6	-2.001^{***}
	(0.002)
X7	0.499^{***}
	(0.002)
X8	1.003^{***}
	(0.002)
X9	1.247^{***}
	(0.002)
X10	2.001^{***}
	(0.002)
R^2	0.991
Adjusted R^2	0.991
Observations	100000

$+p < 0.1 * p < 0.05 * * p < 0.01 * * * p < 0.001$