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[ ] I completed this assignment without assistance or external resources.
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I. Formal Proofs

Assuming $p \rightarrow q$ and $p \rightarrow r$, prove $p \rightarrow (q \text{ AND } r)$.

1. $p \rightarrow q$	Given
2. $p \rightarrow r$	Given
3. p	Given (I assume that p must exist in order to prove that p $\rightarrow q \wedge r$)
4. q	1, 3, modus ponens
5. r	2, 3, modus ponens
6. q AND r	4, 5, conjunction
7. $p \rightarrow q$ AND r	3, 6, conditional introduction

Assuming $p \rightarrow (q \text{ OR } r)$ and $p \rightarrow (q \text{ OR NOT } r)$, prove $p \rightarrow q$.

1. $p \rightarrow (q \text{ OR } r)$	Given
2. $p \rightarrow (q \text{ OR NOT } r)$	Given
3. p	Given (I assume that p must exist in order to prove that $p \rightarrow q$)
4. q OR r	1, 3, modus ponens
5. q OR NOT r	2, 3, modus ponens
6. $(q \text{ OR } r) \text{ AND } (q \text{ OR NOT } r)$	4, 5, conjunction
7. $q \text{ OR } (r \text{ AND NOT } r)$	6, distributive property
8. $q \text{ OR } (F)$	7, negation
9. q	8, identity
10. $p \rightarrow q$	3, 9, conditional introduction

II. Truth Tables

Assuming $p \rightarrow q$ and $p \rightarrow r$, prove $p \rightarrow (q \text{ AND } r)$.

p	q	r	q AND r
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Style note: Canonical order of the rows should be binary order.
The highlighted row indicates the row where I've proved $p \rightarrow (q \text{ AND } r)$.

III. Tautology, Satisfiable, Contradiction

(p AND q AND r) → (p OR q)

p	q	r	p AND q AND r	p OR q
0	0	0	0	0
0	0	1	0	0
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

This is satisfiable. There is only one row in the truth table that renders both statements true; when *p*, *q*, and *r* are all true.

((p → q) AND (q → r)) → (p → r)

p	q	r	(p → q) AND (q → r)	p → r
0	0	0	1	1
0	0	1	0	0
0	1	0	0	1
0	1	1	0	0
1	0	0	0	0
1	0	1	0	1
1	1	0	0	0
1	1	1	1	1

This is satisfiable. There are two rows in the truth table that render both statements true; when *p*, *q*, and *r* have the same values (all 0s or all 1s).

(p → q) → p

p	q	p → q	(p → q) → p
0	0	1	0
0	1	0	1
1	0	0	0
1	1	1	1

This is satisfiable. There are two rows in the truth table that render both statements true; when *p* and *q* both have the same values (all 0s or all 1s).

(p OR q OR r) AND ((NOT p) OR (NOT q) OR (NOT r)) AND (p OR (NOT q)) AND (q OR (NOT r)) OR (r OR (not p))

			1	2	3	4	5	6	7	8	9
p	q	r	p OR q OR r	(NOT p) OR (NOT q) OR (NOT r)	1 AND 2	p OR (NOT q)	q OR (NOT r)	r OR (NOT p)	3 AND 4	7 AND 5	8 OR 6
0	0	0	0	1	0	1	1	1	0	0	1
0	0	1	1	1	1	1	0	1	1	0	1
0	1	0	1	1	1	0	1	1	0	0	1
0	1	1	1	1	1	0	1	1	0	0	1
1	0	0	1	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	0	1	1	0	1
1	1	0	1	1	1	1	1	0	1	1	1
1	1	1	1	0	0	1	1	1	0	0	1

This is a tautology. For any value assigned to *p*, *q*, and *r*, the result is 1 (true).

IV. CNF

p OR (q AND NOT (r AND (s → t)))	
p V (q Λ ~(r Λ (s → t)))	Rewriting expression
p V (q Λ ~r Λ ~(s → t))	de Morgan’s
p V (q Λ ~r Λ ~(~s V t))	Eliminating implication
p V (q Λ ~r Λ (s V ~t))	de Morgan’s
p V (q Λ (~r Λ s) V (~r Λ ~t))	Distributive property
(p V q) Λ (p V ((~r Λ s) V (~r Λ ~t)))	Distributive property
(p V q) Λ (p V (~r Λ (s V ~t)))	Distributive property
(p V q) Λ (p V ~r) Λ (p V s V ~t)	Distributive property

V. Short Certificate

p OR (q AND NOT (r AND (s → t)))

This formula is satisfied when *p* is true, when *q AND NOT (r AND (s → t))* is true, and when both are true.

p	*q*	*r*	*s*	*t*	*s → t*	*r AND (s → t)*	*q AND NOT (r AND (s → t))*	*p OR (q AND NOT (r AND (s → t)))*
0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	1	1	0	0
0	1	0	0	0	1	0	1	1
0	1	0	0	1	0	0	1	1
0	1	0	1	0	0	0	1	1
0	1	1	0	0	1	0	1	1
0	1	1	0	1	0	0	1	1
0	1	1	1	1	1	1	0	0
1	0	0	0	0	1	0	0	1
1	0	0	0	1	0	0	0	1
1	0	0	1	0	0	0	0	1
1	0	1	0	0	1	1	0	1
1	1	0	0	0	1	0	1	1
1	1	0	0	1	0	0	1	1
1	1	0	1	0	0	0	1	1
1	1	1	0	0	1	1	0	1
1	1	1	0	1	0	0	1	1

I used short certificate and exhaustive enumeration to make this determination. By short certificate, I see that if p is true, then the whole formula is satisfied. Otherwise, if p is not true, I have to check if *(q AND NOT (r AND (s → t)))* is true. In the truth table, I highlighted all of the results that satisfy the formula. The green highlighted area indicates the results I get when p is true; the yellow highlighted area indicates the results I get when p is not true, but *(q AND NOT (r AND (s → t)))* is true. Therefore, this formula is definitely satisfiable.