Exercises for Lecture 10

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10.10.1. Exercise 10.1

Write a library of functions to determine the parameter τ of an exponential distribution from a list of numbers filled with pseudo-random numbers distributed according to an exponential probability density distribution.

- Compare the result obtained with the mean of the numbers saved in the list.
- How does the result depend on the initial interval passed to the `sezione_aurea_max_LL` function?

10.10.2. Exercise 10.2

- Plot the profile of the likelihood function and the point identified as its maximum.

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10.10.3. Exercise 10.3

- Modify the `sezione_aurea_max_LL` function, adding the printing of the interval endpoint values at each iteration, to observe the narrowing of the interval during program execution.

10.10.4. Exercise 10.4

- Modify the `loglikelihood` function to calculate the logarithm of the product of the values of the probability density function, rather than the sum of individual logarithms. How does the algorithm's behavior change?

10.10.5. Exercise 10.5

Graphically show that as the available sample size increases, the profile of the logarithm of the likelihood function becomes narrower.

- To simplify visualization, use the logarithm of the ratio between the likelihood function and its maximum value:

$$LLR\left(\theta\right) = \log\left(\frac{\mathcal{L}(\theta)}{\mathcal{L}(\hat{\theta})}\right)$$

10.10.6. Exercise 10.6

Use the bisection method to find the two points τ - σ_τ and τ + σ_τ related to Exercise 1.

- Plot the log-likelihood profile, the estimator values, and the confidence interval along with the horizontal segment used for its determination.

10.10.7. Exercise 10.7

Using the toy experiments technique, plot the probability distribution of the τ estimator.

- Overlay the generated histogram with the plot of the estimator and the confidence interval

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- Compare the value of σ_τ obtained in the previous exercise with the one calculated from the distribution of the numbers saved in the list.

10.10.8. Exercise 10.8

In the asymptotic regime, the distribution of the differences $(\tau - \tau_{true}) / \sigma_\tau$ follows a Normal distribution.

- Use the toy experiments method to fill the histogram of the differences, given a number of events per toy experiment.
- Calculate the mean and sigma of the distribution of differences, and plot their values as a function of the number of events available for estimation, showing the trend on a graph with the number of events available on the horizontal axis and the parameter value on the vertical axis.

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10. Parameter Estimate using the Maximum Likelihood Method

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11. Least-Squares Fitting