Array Signal Processing

Lab Course Exercises

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Exercise 1: Beampattern of ULA

A uniform linear array (ULA) with $N = 10$ sensors, uniform weighs $w_n = \frac{1}{N}$ and sensor spacing $d = \frac{\lambda}{2}$ is considered. The beampattern in the Θ-domain is given by

$$B_\Theta(\Theta) = \boldsymbol{w}^H \boldsymbol{v}_\Theta(\Theta) = \sum_{n=0}^{N-1} w_n^* e^{j\,(n-n_0)\,\frac{2\pi d}{\lambda}\cos\Theta}; \quad 0 \le \Theta \le \pi\,.$$

a) Plot the magnitude of the beampattern in the k_z-, Ψ-, u- and Θ-space in a single MATLAB figure (using the function `subplot`). Plot the beampatterns in a second figure for $N = 11$ and compare the results.
Hint: $\psi = -k_z\, d = \frac{2\,\pi}{\lambda}\, d\, \cos\Theta = \frac{2\,\pi}{\lambda}\, u_z d$

b) Perform a polar plot of the powerpattern in the Θ-space for $N = 10$ and $N = 11$. Which significant difference can be observed?
Hint: Use the provided MATLAB-function `polardb` and consider a magnitude range of -40 to 0 dB.

c) Plot the powerpattern in the Θ-space for $0.001 \le \frac{d}{\lambda} \le 1$. Compare the plots for broadside and endfire orientation w.r.t. mainlobe width and grating lobes.
Hint: Use the MATLAB-function `imagesc` for this three-dimensional plot.

d) Perform the previous plots in the Θ-domain for steering angles $\Theta_T \in \{0°, 30°, 60°, 90°\}$. What is the effect of the array steering on the main beam?

Due to sensor failure, $w_n = 0$ for $n \in \{\,3,\, 5,\, 6\,\}$.

e) Plot the powerpattern for the array in the ψ-domain with and without sensor failure in a single figure. Discuss the effect of the sensor failure?

Exercise 2: Non-uniform Weighting

An ULA with $N = 11$ sensors and $d = \frac{\lambda}{2}$ is considered. Two different weights are applied

$$w_I(n) = \frac{1}{N}$$
$$w_{II}(n) = \sin\left(\frac{\pi}{2\,N}\right)\cos\left(\frac{\pi\,n}{N}\right)$$

with

$$n = -\frac{N-1}{2}, -\frac{N+1}{2}, \dots, \frac{N-1}{2}.$$

Plot in a single figure the $u-$domain powerpatterns in dB for both weights ($0 \le u \le 1$). What is the effect of the non-uniform weighting?

Exercise 3: Half-Power Bandwidth

The angle for the left and right half-power point for a steering angle Θ_t is given by

$$|B_\Theta(\Theta_\mathrm{l}, \Theta_\mathrm{t})|^2 = |B_\Theta(\Theta_\mathrm{r}, \Theta_\mathrm{t})|^2 = \frac{1}{2}\,.$$

The half-power beamwidth in the Θ-space is given by

$$\Theta_\mathrm{h} = \Theta_\mathrm{r} - \Theta_\mathrm{l} = \cos^{-1}\left(\cos\theta_\mathrm{t} - c_0\frac{\lambda}{N\,d}\right) - \cos^{-1}\left(\cos\theta_\mathrm{t} + c_0\frac{\lambda}{N\,d}\right)$$

with

$$c_0 = 0.443 \quad \text{and} \quad \Theta_t^{(l)} \le \Theta_t \le \frac{\pi}{2}\,.$$

The half-power beamwidth for the point where $\Theta_\mathrm{l} = 0$ is given by (scan limit)

$$\begin{aligned}\Theta_\mathrm{h}^{(l)} &= \cos^{-1}\left(\cos\Theta_\mathrm{t}^{(l)} - c_0\frac{\lambda}{N\,d}\right)\\ &= \cos^{-1}\left(1 - 2\,c_0\frac{\lambda}{N\,d}\right)\,.\end{aligned}$$

These half-power bandwidths should be plotted in a single figure for different steering angles and $1 \le \frac{N\,d}{\lambda} \le 1000$. The $x-$axis is given by $\frac{N\,d}{\lambda}$ and the $y-$axis by the half-power bandwidth in degrees. Both axes should be plotted on a logarithmic scale. The provided MATLAB script `Ex_HPBW_template` plots the half-power bandwidth for the endfire array. Extend this script to plot (in dependence of $\frac{N\,d}{\lambda}$) the

a) half-power beamwidth for the scan limit $\Theta_\mathrm{h}^{(l)}$

b) half-power beamwidth for the steering angles $\Theta_\mathrm{t} \in \{2.5^\circ, 5^\circ, 10^\circ, 20^\circ, 30^\circ, 45^\circ, 90^\circ\}$.

What can be observed?

Exercise 4: Beampattern of MVDR Beamformer

An MVDR beamformer with $N = 10$ sensors, $d = \lambda/2$ and broadside orientation in the presence of a single discrete interference source and isotropic noise is considered. The weights of this MVDR beamformer are given by

$$\boldsymbol{w}_o^H = \frac{\boldsymbol{v}_s^H \boldsymbol{S}_n^{-1}}{\boldsymbol{v}_s^H \boldsymbol{S}_n^{-1} \boldsymbol{v}_s}$$

with

$$\boldsymbol{S}_n = \sigma_w^2 \boldsymbol{I} + \sigma_1^2 \boldsymbol{v}_1 \boldsymbol{v}_1^H \ .$$

The interference-to-noise ratio is given by

$$\sigma_I^2 = \frac{\sigma_1^2}{\sigma_w^2} \ .$$

a) Plot the powerpattern in the u-domain for an interferer at $u_1 = 0.3$ and $u_1 = 0.004$ for an interference-to-noise ratio (INR) of 70 dB.

b) Perform a 3-dimensional plot of the powerpattern for varying interferer directions with $0.001 \leq u_1 \leq 0.5$ and an INR of 70 dB. Redo this plot for an INR of 0 dB.

How does the direction of the interferer and the interference-to-noise ratio influence the powerpattern?

Exercise 5: Delay-and-Sum Beamformer for Narrowband Signals

The processing of a narrowband signal by a delay-and-sum beamformer (DSB) with $N = 10$ sensors and a sensor spacing of $d = 0.05$ m should be investigated. A single narrowband source signal with a carrier frequency $f_0 = 2\frac{1}{3}$ kHz ($c = 342$ m/s) impinges on the array from different directions $0° \leq \Theta \leq 90°$.

The behavior of the beamformer should be investigated by calculating the ratio of the input signal power to the output signal power of the beamformer (output-to-input ratio, OIR). Consider an endfire and broadside orientation and different directions-of-arrival $\Theta = 0°, 5°, \ldots, 90°$.

The implementation of the system should be done in the following steps:

- Generate a narrowband source signal by the provided MATLAB function `NB_signal.m`.

- Generate the sensor input signals by delaying the source signal by the provided MATLAB function `frac_delay`. This function delays a signal by a rational number of sampling instances for a given delay (in seconds) and sampling frequency (in Hz).
- Implement the DSB and process the sensor signals for different directions-of-arrival Θ.
- Calculate and plot the OIR in dependence of Θ.

Compare the calculated OIR with the beampattern of the beamformer. What can be observed?