Exercise 1 - Homework

03.01.2024

Due date: 10.01.2024, 14:00

Part I
Complex Numbers

1

Express the following complex number in an exponential form ($z = re^{i\varphi}$):

$$z = 3 + 4i \tag{1}$$

2

What is the natural logarithm of $z = re^{i\varphi}$? Don't forget all possible values of φ.

Part II
Linear Algebra

3

Write the following linear system in the canonical representation $a\vec{X} = \vec{b}$:

$$\begin{aligned} 4 - 2x + 2z &= 0 \\ y + z - 4x &= 4 \\ x + y &= z \end{aligned} \tag{2}$$

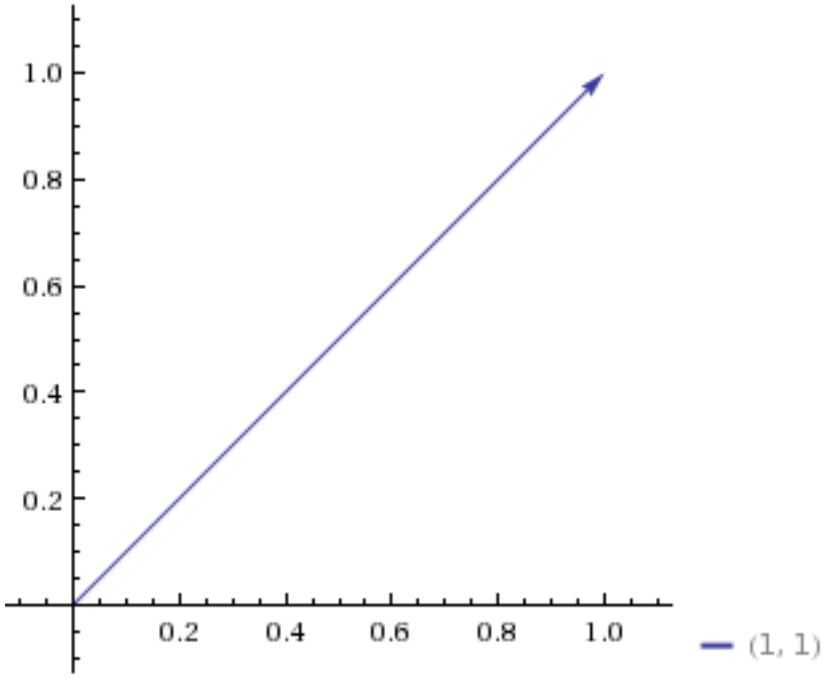

Figure 1: A vector point to $(x, y) = (1, 1)$.

4

To achieve a deeper understanding of matrices, we'll see how it's used as a **transformation in space**. Assuming we have a vector on the real plane $\vec{v}_1$, we can represent its coordinates as a linear algebra vector by defining the first number of the vector to be the x-axis coordinates, and the second - y-axis coordinates (figure 1). The vector in the figure points to $(x, y) = (1, 1)$.

4.1

What is the mathematical operator that can transform this vector so that it points to $(x, y) = (2, 3)$? *Hint:* $\mathrm{A}\vec{v}_1 = \vec{v}_2$
comment: there is more than one operator, try to find the general solution.

4.2

What is the mathematical operator that can rotate this vector to $(x, y) = (-1, 1)$? Try to find the most general form that solves all questions of this type.

Part III
Statistics and Probability

5

In a football game, a specific player has a probability of 0.5 to not score any goals in a match. He also has a 0.25 probability to score 1 goal, 0.15 probability to score 2, and a probability of 0.1 to score 3 goals. What is the expected value of goals in the coming football season, assuming the player will play 30 games?

6

Bob goes to the gym each day of the week with a probability of 40%. Alice promised to go to a movie with him only if he visited the gym at least 5 times in the past week. What are the chances they'll see each other?