1. Pasos intermedios para trapecio simple, ec. (1.74)

$$P_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$$

$$I = \int_a^b P_1(x)dx = \int_a^b \frac{x-b}{a-b} f(a) + \int_a^b \frac{x-a}{b-a} f(b) dx$$

$$= \left[\frac{f(a)}{a-b} \int_a^b x-b\, dx\right] + \left[\frac{f(b)}{b-a} \int_a^b x-a\, dx\right]$$

$$= \left[\frac{f(a)}{a-b} \cdot \left(\frac{b^2-a^2}{2} - b^2 + ba\right)\right] + \left[\frac{f(b)}{b-a} \cdot \left(\frac{b^2-a^2}{2} - ab - a^2\right)\right]$$

$$= \left[f(a) \cdot \frac{b-a}{2}\right] + \left[f(b) \cdot \frac{-a+b}{2}\right]$$

$$= \frac{b-a}{2}\left[f(a) - f(b)\right] \checkmark$$

2. Encontrar el error para trapecio simple, ec. (1.77)

$$E(x) = \frac{f''(\xi)}{2}(x-a)(x-b)$$

$$E = \int_a^b E(x)dx = \frac{f''(\xi)}{2} \cdot \int_a^b x^2 - bx - ax + ab \qquad \to \frac{x^2(-b-a)}{2}$$

$$= \frac{f''(\xi)}{2} \cdot \left[\frac{x^3}{3} - \frac{bx^2}{2} - \frac{ax^2}{2} + abx\right]\Big|_a^b$$

$$= \frac{f''(\xi)}{2} \cdot \left(\frac{a^3 - b^3 + 3b^2a - 3ba^2}{6}\right) \to (a-b)^3$$

$$= \frac{f''(\xi)}{12}(a-b)^3 \qquad \cdot h = (b-a) \rightsquigarrow -h = (a-b)$$

$$= -\frac{f''(\xi)}{12} \cdot h^3 \checkmark$$

3. Pasos intermedios para encontrar regla Simpson, ec.(1.87)

$$P_2(x) = \frac{(x-b)(x-x_m)}{(a-b)(a-x_m)} f(a) + \frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} f(x_m) + \frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} f(b) \qquad \cdot x_m = \frac{a+b}{2}$$

$$I = \int_a^b P_2(x)\,dx = \int_a^b \text{Término 1} + \int_a^b \text{Termino 2} + \int_a^b \text{Termino 3}$$

$$= \frac{b-a}{6} + \left(-\frac{2(a-b)}{3}\right) + \left(\frac{(b-a)^3}{6(b-a)^2}\right) \quad \} \; f(a), f(x_m), f(b) \text{ son CTE respecto } dx$$

$$= \frac{b-a}{3}\left[\frac{f(a)}{2} + 2f(x_m) + \frac{f(b)}{2}\right] \cdot 2$$

$$= \frac{h}{3}\left(f(a) + 4f(x_m) + f(b)\right) \checkmark$$

4. Verificar resultado de ec.(1.89)

$$E = \int_a^b \left(\overbrace{\frac{f'''(\xi)}{4!}}^{\text{CTE}}\right)(x-a)(x-b)\left(x-\frac{(a+b)}{2}\right)dx = 0$$

$$= \int_a^b (x^2 - bx - ax + ab)\left(x - \frac{(a+b)}{2}\right)dx$$

$$= \int_a^b x^3 - bx^2 - ax^2 + abx - \frac{x^2(a+b)}{2} + \frac{bx(a+b)}{2} + \frac{ax(a+b)}{2} - \frac{ab(a+b)}{2}\,dx$$

$$= \int_a^b x^3 - bx^2 - ax^2 + abx - \frac{(a+b)}{2}(x^2 + bx + ax - ab)\,dx$$

$$= \frac{(b-a)^4}{4} - \frac{b(b-a)^3}{3} - \frac{a(b-a)^3}{3} + \frac{ab(b-a)^2}{2} - \frac{(a+b)}{2} \qquad \text{Factorizando, se cancelan términos}$$

$$= 0$$