CS594/690 Homework 1
Spring 2025
January 22, 2025
(Due 4:10pm, January 29, 2025)

Email homework assignments to ldojcsak@vols.utk.edu by the beginning of class time.

1. Like Königsberg, New York City is made up of islands (Manhattan, Long Island, and Staten Island), which are connected to New Jersey and the Bronx via dozens of bridges and tunnels. I've drawn in 19 of the major bridges and tunnels on the map below (simplified to avoid hyperedges and self-loops).

   Draw a graph to represent the NYC metro area using the 5 labeled land masses and 19 bridges. Read the Königsberg problem and apply similar reasoning to determine if it would be possible to start at some point, drive across all 19 NYC bridges and tunnels exactly once, and then return to your starting point. No cheating by taking a ferry. If it is possible, state the path. If it is not possible, defend your answer and determine how many new bridges you would need to build to make it possible.

2. Given the following adjacency matrix, draw the corresponding graph by hand and determine its minimum degree, maximum degree, and graph density.

	1	2	3	4	5	6	7	8	9
1	0	1	0	0	0	0	1	0	0
2	1	0	1	0	1	0	0	0	0
3	0	1	0	0	0	1	0	0	0
4	0	0	0	0	0	1	1	0	0
5	0	1	0	0	0	1	0	1	0
6	0	0	1	1	1	0	0	1	1
7	1	0	0	1	0	0	0	0	0
8	0	0	0	0	1	1	0	0	1
9	0	0	0	0	0	1	0	1	0

3. What is the minimum number of leaves a tree on n vertices may have? And the maximum? Now describe an algorithm (do not program it) that takes as input such a tree and returns its number of leaves. A leaf in a graph is a vertex with degree 1.

4. Write a program that takes as input a graph in modified DIMACS format and outputs the adjacency matrix representation of the given graph. You may use any programming language, as long as it is supported on the EECS Linux machines. The modified DIMACS format consists of a header that indicates the number of vertices followed by number of edges in the graph. Each following line consists of a tab-separated pair of vertices u v that indicate an edge between vertex u and v. Include a short README text file with instructions on how to invoke your program.

DIMACS Example:
```
5	7
0	1
0	2
0	4
1	2
1	4
2	3
3	4
```

Wikibook topics that may help you with this assignment:

- Königsberg problem

- Glossary of graph theory terms

- Adjacency matrix