

COSC581 Homework 1

Due Thursday, January 30, 2025.

1. Prove that $(f = O(g) \text{ and } f = \Omega(g)) \Leftrightarrow f = \Theta(g)$. (This can absolutely be a direct proof, allow yourself to use common sense and definitions. Just practice writing formal and concise work)
2. Show $x < 2^x$ for $x \geq 1$ using induction.
3. If r is a real number such that $r^2 = 2$, then r is an irrational number. (prove via contradiction)
4. For the following set of functions, tell me which f/g pairs demonstrate which of the 6 asymptotic behaviors we discussed in class today. SHOW YOUR WORK.
 - a. $f = n^3 + 4$, $g = n^3 + 8n + \log(n)$
 - b. $f = 2^n$, $g = n$
 - c. $f = \log(2n)$, $g = n$
 - d. $f = n \log(n)$, $g = n^n$
 - e. $f = \log n^{\log 17}$, $g = \log 17^{\log n}$
5. Using the formal definition, justify why f can NOT be $o(g)$ and $\Omega(g)$.
6. A little fun with pseudo-randomness*. Suppose we start with an edgeless graph of order, say, 100. Then we begin uniformly generating edges (pairs of integers between 1 and 100) without replacement, stopping as soon as we produce in our graph a P_5 , a C_5 , or a K_5 subgraph. Which of these three events is most likely and why?

**If this question seems too complex, turn your brain off. You're thinking too hard. (That's the magic of this question) Don't ask ChatGPT.*