

BUFN 640 – Problem Set 1

Due on **Monday, September 26 at 11:59 pm ET**

This assignment will show you how to estimate and test the Capital Asset Pricing Model (CAPM) in Python and evaluate its predictions. Please use the template Python notebook (*Problem Set 1 – template.ipynb*) in the shared Google Drive folder or on Canvas to load the data. Implement the rest of the file yourself.

Important: Please submit your homework using Canvas. Your submission needs to include **two** files: a PDF (or Word; it is okay to include your responses in the Python notebook instead, if you wish) document with all your responses **AND** a copy of your Python notebook (*.ipynb* Jupyter notebook file). To produce the latter, please click *File* → *Download .ipynb* in Google Colab, then save and upload the file on Canvas.

Each student has to submit his/her individual assignment and show all work. Legibly handwritten and scanned submissions are allowed, but they need to be submitted as a single document. Please do not submit photographs of pages in separate files.

Part I: Multiple-choice questions (25 points)

1. (5 points) Assume that the relationship between a company's stock price (y) and dividends paid per share (x) is linear. If the slope of the equation is 0.50 and the intercept is 30, what would be the expected stock price if the dividend paid was 3?
 - (a) 33
 - (b) 30.50
 - (c) 31.5
 - (d) 30.
2. (5 points) Data that have both time series and cross-sections is referred to as
 - (a) Cross-sectional data
 - (b) Time-cross-sectional data
 - (c) Time-series data
 - (d) Panel data.

3. (5 points) An individual has \$10000 capital to invest in the stock market. He invests 30% of his capital in stock A, 45% in stock B and 25% in Stock C. What is the return on his/her portfolio assuming that the simple returns on stocks A, B and C are 5%, 10% and 20%, respectively? Use Python as a calculator to answer this question.
- (a) 9.0%
 - (b) 10.0%
 - (c) 11.0%
 - (d) 12.0%.
4. (5 points) What does a positive linear relationship between x and y in a simple regression imply?
- (a) Increases in the independent variable are usually accompanied by increases in the regressor
 - (b) The relationship between x and y cannot be explained by a straight line
 - (c) Decreases in the independent variable is usually accompanied by increases in the regressors
 - (d) Increases in the regressor are usually accompanied by increases in the dependent variable.
5. (5 points) Which of the following statements is correct concerning the conditions required for OLS to be a usable estimation technique?
- (a) The model must be linear in the parameters
 - (b) The model must be linear in the variables
 - (c) The model must be linear in the variables and the parameters
 - (d) The model must be linear in the residuals.

Part II: The CAPM (75 points)

The Capital Asset Pricing Model (CAPM) can be written as

$$E(R_i) = \beta_i E(R_M),$$

where R_i is the excess return on a stock i (in excess of the risk-free rate), R_M is the excess return on the aggregate market index (e.g., S&P 500), and β_i is a measure of stock's systematic risk – also known as stock's *beta* – given by:

$$\beta_i = \frac{\text{cov}(R_i, R_M)}{\text{var}(R_M)}.$$

The first step in using the CAPM is to estimate the stock's beta using the *market model*. The market model can be written as:

$$R_{i,t} = \alpha_i + b_i R_{M,t} + u_{i,t},$$

where $R_{i,t}$ is the excess return for security i at time t , $R_{M,t}$ is the excess return on the aggregate market index (e.g., S&P 500) at time t , and $u_{i,t}$ is an iid (independent and identically distributed) random disturbance term.

1. (5 points) What does the mean of $u_{i,t}$, that is, $E(u_{i,t})$, equal to?
2. (5 points) What does the CAPM imply about the intercept, α_i ?
3. (10 points) Write down a formula for the slope estimate, \hat{b}_i . Explain why \hat{b}_i can be used as an estimator for the CAPM's β .
4. (5 points) Explain why the market model is a valid way of estimating the CAPM. (Hint: Recall that regression is a tool for modeling conditional means).
5. (15 points) In your own words, explain the intuitive meaning of the term “unbiased estimator.” Use derivations on the slides to argue that \hat{b}_i is an unbiased estimator of the stock's β . Write down formulas. Explain every single step in detail.
6. (5 points) In your own words, explain the intuitive meaning of the term “consistent estimator.” What does it imply practically about our ability to recover the true value of β ?

7. Use Python and Google Colab to answer the following questions. You may write your text responses either in the document or in the Colab (Jupyter) notebook, or both. Please use the template Python file (*Problem Set 1 – template.ipynb*) in the shared Google Drive folder or on Canvas to load the data and guide you through the questions:
- (a) (5 points) Download data on risk-free rates, S&P 500 index returns, and returns on Microsoft (MSFT) from FRED and Yahoo Finance. Construct excess returns. Start your sample on 2005/1/1 and end it on 2019/1/1. Plot the time-series of returns. Use the template file in the shared Google Drive folder (or on Canvas) to help you with this step.
 - (b) (5 points) Produce a scatter plot with realized excess returns on MSFT on the y -axis vs. realized excess returns on the S&P500 index on the x -axis. How well can a regression line fit the data? Do you expect to get a high R^2 ?
 - (c) (5 points) Estimate the market model. Interpret the numbers we talked about in class, such as coefficients, their t -statistics and p -values, as well as R^2 . (You may focus on either the Adjusted or simple R^2).
 - (d) (10 points) A city analyst has told you that Microsoft's return closely follows the market, but that it is no more risky, on average, than the market, in terms of its systematic risk – β . This can be tested by the null hypothesis that the value of beta is one. Test this hypothesis against a one-sided alternative that the security is more risky than the market, at the 5% level. Write down the null and alternative hypothesis. What do you conclude? Are the analyst's claims empirically verified?
 - (e) (5 points) Recall that the CAPM predicts that α_i is zero. The CAPM can be empirically tested by verifying this hypothesis. Test this hypothesis against a two-sided alternative at the 10% level. Write down the null and alternative hypothesis. What do you conclude? Is there statistical evidence that Microsoft is a good/bad investment option (i.e., that it is under or overpriced) currently?