Assignment 7: Implementing a Single-Layer Neural Network for Binary Classification

Question 1: Design and Initialize Neural Network

1.1 Network Design and Truth Table Creation

- Creating a truth table and corresponding binary output

Step 1.1 Network Design and Truth Table Creation

The logical function (x1 AND x2) OR (x3 AND x4)

x1	x2	x3	x4	Output
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

1.2 Manual Forward Propagation Calculation

- I choose example set of inputs from my truth are x1=0, x2=0, x3=0, x4=0 and x1=0, x2=0, x3=1, x4=1 and x1=1, x2=1, x3=0, x4=0 and x1=1, x2=0, x3=1, x4=1
- Using sigmoid activation function and weight=0.5, bias = 1

Step 1.2 Manual Forward Propagation Calculation

$w = 0.5$, $b = 1$, z = sigmoid, $f(z) = \frac{1}{1+e^{-z}}$

$x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $x_4 = 0$

$z = (0.5 \times 0) + (0.5 \times 0) + (0.5 \times 0) + (0.5 \times 0) + 1$

$z = 0 + 0 + 0 + 0 + 1$

$z = 1$

$f(1) = \frac{1}{1+e^{-1}} = 0.73$, $y_{pred} = 0.73$

$x_1 = 0$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$

$z = (0.5 \times 0) + (0.5 \times 0) + (0.5 \times 1) + (0.5 \times 1) + 1$

$z = 0 + 0 + 0.5 + 0.5 + 1$

$z = 2$

$f(2) = \frac{1}{1+e^{-2}} = 0.88$, $y_{pred} = 0.88$

$x_1 = 1$, $x_2 = 1$, $x_3 = 0$, $x_4 = 0$

$z = (0.5 \times 1) + (0.5 \times 1) + (0.5 \times 0) + (0.5 \times 0) + 1$

$z = 0.5 + 0.5 + 0 + 0 + 1$

$z = 2$

$f(2) = \frac{1}{1+e^{-2}} = 0.88$, $y_{pred} = 0.88$

$x_1 = 1$, $x_2 = 0$, $x_3 = 1$, $x_4 = 1$

$z = (0.5 \times 1) + (0.5 \times 0) + (0.5 \times 1) + (0.5 \times 1) + 1$

$z = 0.5 + 0 + 0.5 + 0.5 + 1$

$z = 2.5$

$f(2.5) = \frac{1}{1+e^{-2.5}} = 0.923$, $y_{pred} = 0.923$

1.3 Manual Backpropagation Calculation

Step 1.3 Manual Backpropagation Calculation

Error Calculation

Error $= t - y_{pred}$

$x_1 = 1, \; x_2 = 1, \; x_3 = 0, \; x_4 = 0$

$w = 0.5$

$b = 1$

$t = 1$

$y_{pred} = 0.88$

Error $= 1 - 0.88 = 0.12$

Gradient Computation

$$E(O) = (t - y_{pred})^2$$

$$\frac{\partial E}{\partial w_i} = \frac{\partial}{\partial w_i} (t - y_{pred})^2$$

$$= -2 . (t - y_{pred}) . \frac{\partial (t - y_{pred})}{\partial w_i}$$

$$= -2 . (t - y_{pred}) . \frac{\partial y_{pred}}{\partial w_i}$$

$$y_{pred} = \sigma(z)$$

Using the derivation of the sigmoid function

$$\frac{\partial \sigma(z)}{\partial z} = \sigma(z) . (1 - \sigma(z))$$

Thus

$$\frac{\partial y_{pred}}{\partial w_i} = \frac{\partial \sigma(z)}{\partial z} . \frac{\partial z}{\partial w_i}$$

$$\frac{\partial z}{\partial w_i} = x_i$$

Therefore:

$$\frac{\partial y_{pred}}{\partial w_i} = \sigma(z) . (1 - \sigma(z)) . x_i$$

Error = 0.12

$$\sigma'(z) = \sigma(z) \cdot (1 - \sigma(z))$$

$$\sigma'(z) = 0.88 \cdot (1 - 0.88)$$
$$= 0.1056$$

$$\frac{\partial y_{pred}}{\partial w_i} = \sigma'(z) \cdot x_i$$

For each weight:

$$x_1 = 1, \quad \frac{\partial y_{pred}}{\partial w_i} = 0.1056 \times 1 = 0.1056$$

$$x_2 = 1, \quad \frac{\partial y_{pred}}{\partial w_i} = 0.1056 \times 1 = 0.1056$$

$$x_3 = 0, \quad \frac{\partial y_{pred}}{\partial w_i} = 0.1056 \times 0 = 0$$

$$x_4 = 0, \quad \frac{\partial y_{pred}}{\partial w_i} = 0.1056 \times 0 = 0$$

$$\frac{\partial E}{\partial w_i} = -2 \cdot \text{error} \cdot \frac{\partial y_{pred}}{\partial w_i}$$

$$\frac{\partial E}{\partial w_1} = -2 \times 0.12 \times 0.1056 = -0.0253$$

$$\frac{\partial E}{\partial w_2} = -2 \times 0.12 \times 0.1056 = -0.0253$$

$$\frac{\partial E}{\partial w_3} = 0$$

$$\frac{\partial E}{\partial w_4} = 0$$

Gradient of the loss function with respect to the bias (b):

$$\frac{\partial E}{\partial b} = -2 \cdot (t - y_{pred}) \cdot \sigma'(z)$$

$$= -2 \times 0.12 \times 0.1056$$

$$= -0.0253$$

Gradient value = -0.0253

Weight Update

$$w_i = w_i - \eta \cdot \frac{\partial E}{\partial w_i}$$

learning rate, $\eta = 0.001$

$w_1 = 0.5 - 0.001 \times (-0.0253) = 0.5000253$

$w_2 = 0.5 - 0.001 \times (-0.0253) = 0.5000253$

$w_3 = 0.5 - 0.001 \times 0 = 0.5$

$w_4 = 0.5 - 0.001 \times 0 = 0.5$

bias update:

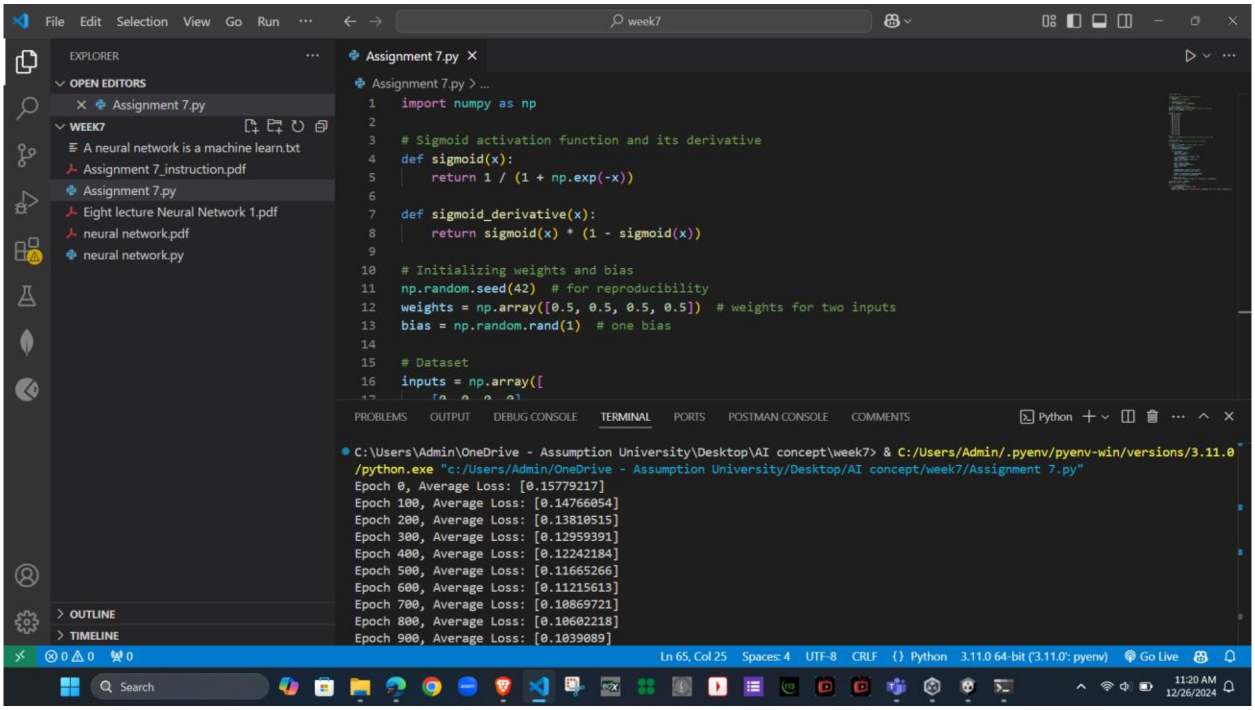
$$b = b - \eta \cdot \frac{\partial E}{\partial b}$$

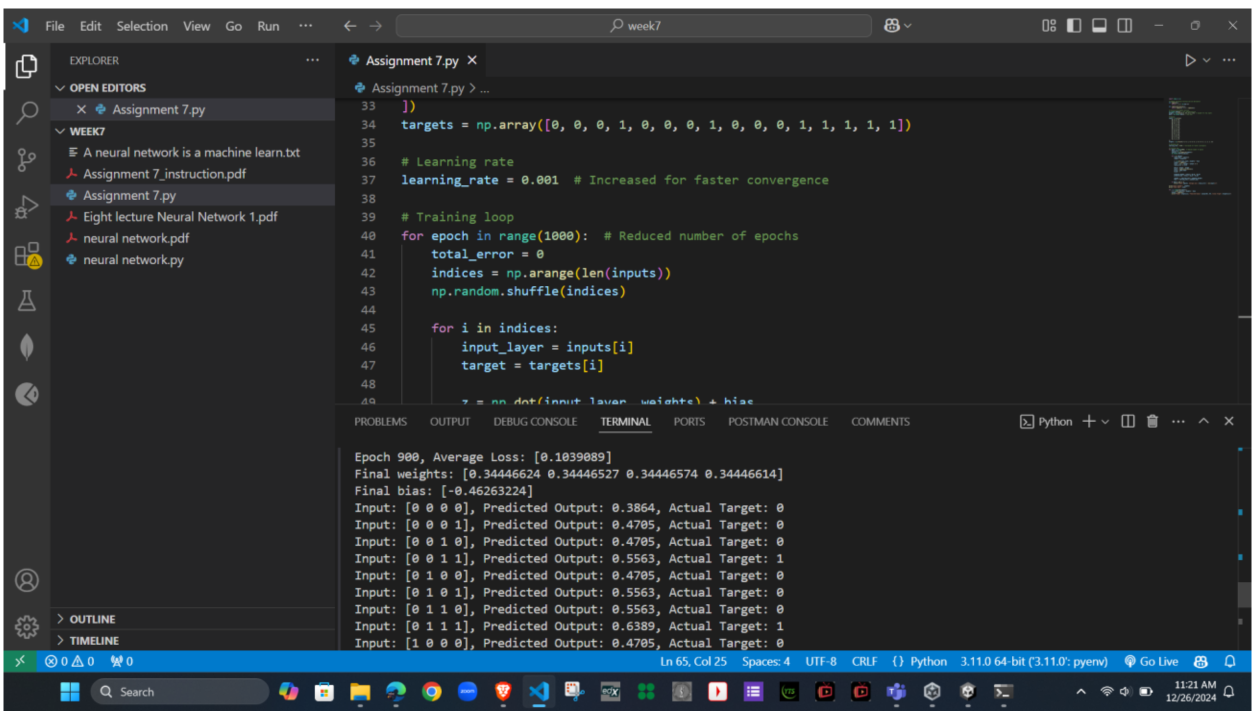
$b = 1 - 0.01 \times (-0.0253)$

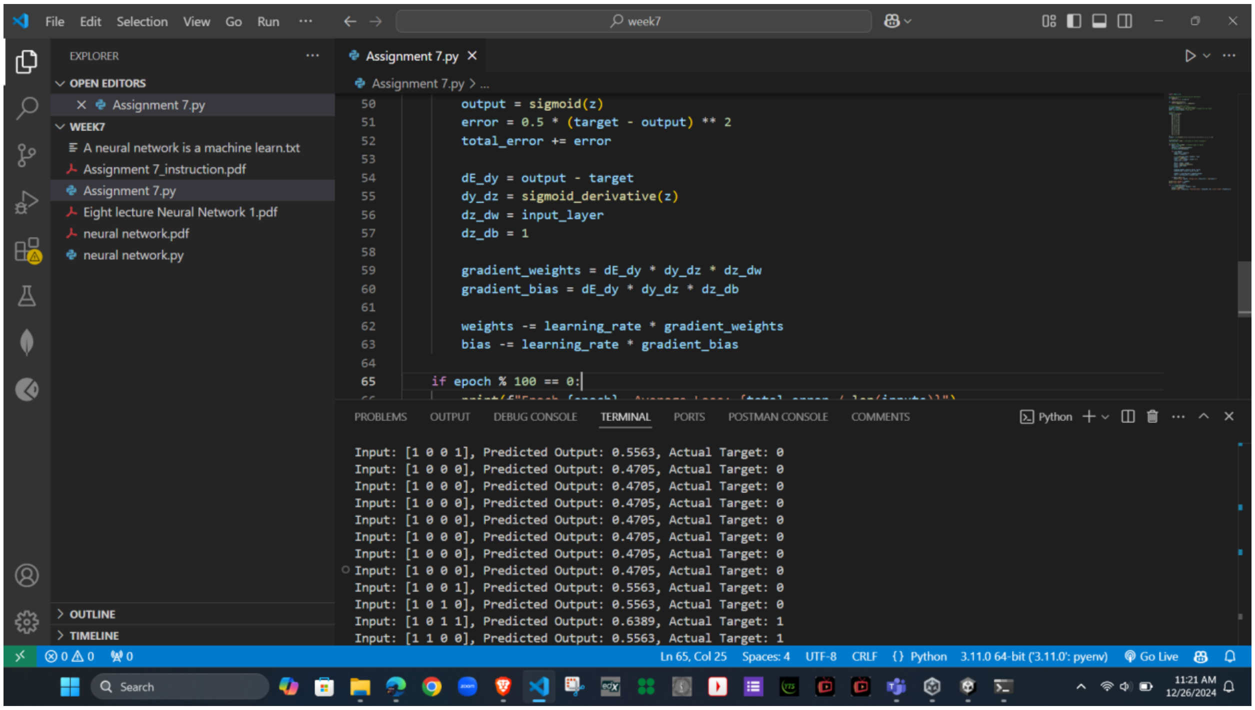
$b = 1.0000253$

weight update value using learning rate 0.001

1.4 Implementation and Training in Python



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week7
EXPLORER
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Assignment 7.py
WEEK7
A neural network is a machine learn.txt
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neural network.pdf
neural network.py
OUTLINE
TIMELINE
Assignment 7.py
        output = sigmoid(z)
        error = 0.5 * (target - output) ** 2
        total_error += error

        dE_dy = output - target
        dy_dz = sigmoid_derivative(z)
        dz_dw = input_layer
        dz_db = 1

        gradient_weights = dE_dy * dy_dz * dz_dw
        gradient_bias = dE_dy * dy_dz * dz_db

        weights -= learning_rate * gradient_weights
        bias -= learning_rate * gradient_bias

    if epoch % 100 == 0:
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS POSTMAN CONSOLE COMMENTS
Python
Input: [1 0 0 1], Predicted Output: 0.5563, Actual Target: 0
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
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Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 1], Predicted Output: 0.5563, Actual Target: 0
Input: [1 0 1 0], Predicted Output: 0.5563, Actual Target: 0
Input: [1 0 1 1], Predicted Output: 0.6389, Actual Target: 1
Input: [1 1 0 0], Predicted Output: 0.5563, Actual Target: 1
Ln 65, Col 25 Spaces: 4 UTF-8 CRLF {} Python 3.11.0 64-bit ('3.11.0': pyenv) Go Live
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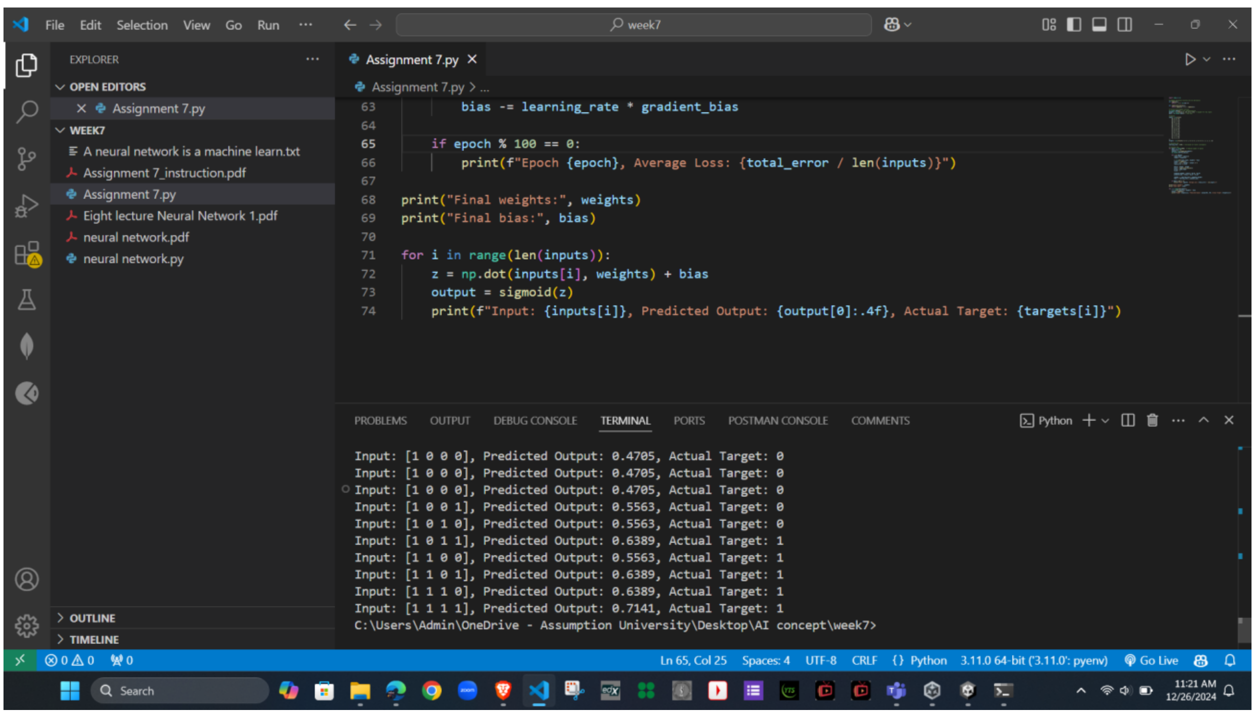
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Eight lecture Neural Network 1.pdf
neural network.pdf
neural network.py
OUTLINE
TIMELINE
Assignment 7.py
        bias -= learning_rate * gradient_bias

    if epoch % 100 == 0:
        print(f"Epoch {epoch}, Average Loss: {total_error / len(inputs)}")

print("Final weights:", weights)
print("Final bias:", bias)

for i in range(len(inputs)):
    z = np.dot(inputs[i], weights) + bias
    output = sigmoid(z)
    print(f"Input: {inputs[i]}, Predicted Output: {output[0]:.4f}, Actual Target: {targets[i]}")
PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL PORTS POSTMAN CONSOLE COMMENTS
Python
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 0], Predicted Output: 0.4705, Actual Target: 0
Input: [1 0 0 1], Predicted Output: 0.5563, Actual Target: 0
Input: [1 0 1 0], Predicted Output: 0.5563, Actual Target: 0
Input: [1 0 1 1], Predicted Output: 0.6389, Actual Target: 1
Input: [1 1 0 0], Predicted Output: 0.5563, Actual Target: 1
Input: [1 1 0 1], Predicted Output: 0.6389, Actual Target: 1
Input: [1 1 1 0], Predicted Output: 0.6389, Actual Target: 1
Input: [1 1 1 1], Predicted Output: 0.7141, Actual Target: 1
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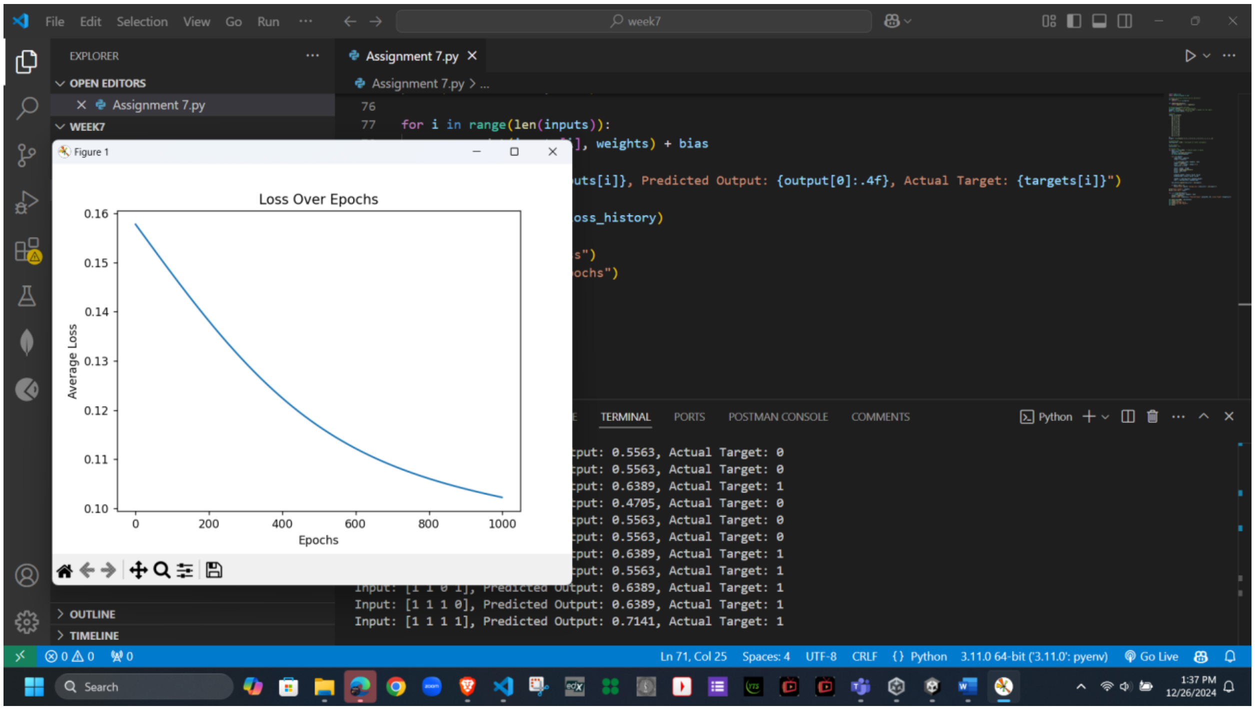

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Assignment 7.py
WEEK7
Assignment 7.py
Figure 1
Loss Over Epochs
Average Loss
Epochs
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0
200
400
600
800
1000
for i in range(len(inputs)):
], weights) + bias
uts[i]}, Predicted Output: {output[0]:.4f}, Actual Target: {targets[i]}")
oss_history)
s")
ochs")
TERMINAL
PORTS
POSTMAN CONSOLE
COMMENTS
tput: 0.5563, Actual Target: 0
tput: 0.5563, Actual Target: 0
tput: 0.6389, Actual Target: 1
tput: 0.4705, Actual Target: 0
tput: 0.5563, Actual Target: 0
tput: 0.5563, Actual Target: 0
tput: 0.6389, Actual Target: 1
tput: 0.5563, Actual Target: 1
Input: [1 1 0 1], Predicted Output: 0.6389, Actual Target: 1
Input: [1 1 1 0], Predicted Output: 0.6389, Actual Target: 1
Input: [1 1 1 1], Predicted Output: 0.7141, Actual Target: 1
OUTLINE
TIMELINE
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Spaces: 4
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CRLF
Python
3.11.0 64-bit ('3.11.0': pyenv)
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