

1.

$((\text{lambda}(x)$

$((\text{lambda}(y)$

$((\text{lambda}(z) e) v3)) v2)) v1)$

2.

$$((\lambda x. [x]) (\lambda x. 1)) (\lambda y. y)$$

$$= ((\lambda x. [1]) (\lambda y. y))$$

$$= 1$$

1)

$$((\lambda z. [(\lambda y. \overset{\text{free}}{z}) ((\lambda x. (x x)) (\lambda x. (x x)))] 5) 5)$$

$$= ((\lambda y. [5]) ((\lambda x. (x x)) (\lambda x. (x x))))$$

$$= 5$$

2)

For 1), it does not have other orders to apply beta reduction

However, for 2) if we choose to apply beta reduction on $((\lambda x. (x x))(\lambda x. (x x)))$ first, it will never end because it will give same one(which is $((\lambda x. (x x))(\lambda x. (x x)))$) after do beta reduction.

3.

$$\begin{aligned}
& (((\lambda x. ((\lambda y. ((\overset{\text{free}}{\cancel{x}} + \text{true}) y))) + \text{true}) \text{false})) \\
&= ((\lambda y. ((\text{true} + \text{true}) \overset{\text{free}}{y})) \text{false}) \\
&= (\text{true} + \text{true}) \text{false} \\
&= (((\lambda a. ((\lambda b. \overset{\text{free}}{a}))) + \text{true}) \text{false}) \\
&= ((\lambda b. [\text{true}]) \text{false}) \\
&= \text{true}
\end{aligned}$$

1)

2) XOR = $(\lambda x. \lambda y. ((x ((y \text{ false}) \text{ true})) y))$

x	y	$(\lambda x. \lambda y. (x ((y \text{ false}) \text{ true}) y))$
true	True	$((\text{true } ((\text{true false}) \text{ true})) \text{ true})$ $= ((\text{true false}) \text{ true})$ $= \text{false } (\checkmark)$
True	False	$((\text{true } ((\text{false false}) \text{ true})) \text{ false})$ $= ((\text{true true}) \text{ false})$ $= \text{true } (\checkmark)$
False	True	$((\text{false } ((\text{true false}) \text{ true})) \text{ true})$ $= ((\text{false false}) \text{ true})$ $= \text{true } (\checkmark)$
false	false	$((\text{false } ((\text{false false}) \text{ true})) \text{ false})$ $= ((\text{false true}) \text{ false})$ $= \text{false } (\checkmark)$

4.

$$\begin{aligned}
& ((S K) K) \\
&= (((\lambda x y z. ((x z) (y z))) (\lambda x y. x)) K) \\
&= ((\lambda y z. ((\lambda x y. x) z) (y z))) K \\
&= ((\lambda y z. ((\lambda y. z) (y z))) K) \\
&= ((\lambda y z. z) K) \\
&= (\lambda z. z) \\
&= I
\end{aligned}$$