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1.

1.  $f(x) = x+1, x>0$

$$\$| \rightarrow 1\$$$

$$\$\# \rightarrow |$$

$$\$||\# \rightarrow |\$||\# \rightarrow ||\$|\# \rightarrow |||\$ \# \rightarrow ||||$$

2.  $f(x) = 3*x, x>0$

$$\$| \rightarrow |||\$$$

$$\$\# \rightarrow \varepsilon$$

$$\$||\# \rightarrow |||\$||\# \rightarrow |||||\$|\# \rightarrow |||||\$ \# \rightarrow |||||\$$$

3.  $f(x,y) = x-y, x>0, y>0, \text{ and } x>y$

$$|\&| \rightarrow \&$$

$$\&\# \rightarrow \varepsilon$$

$$\$| \rightarrow |$$

$$\$|||\&||\# \rightarrow \$||\&|\# \rightarrow \$|\&\# \rightarrow \$| \rightarrow |$$

2.

1) Rule 1  $(0+0) \rightarrow 0$

Rule 2  $(0+1) \rightarrow 1$

Rule 3  $(1+0) \rightarrow 1$

Rule 4  $(0+2) \rightarrow 2$

Rule 5  $(2+0) \rightarrow 2$

Rule 6  $(1+1) \rightarrow 2$

Rule 7  $(1+2) \rightarrow 0$

Rule 8  $(2+1) \rightarrow 0$

Rule 9  $(2+2) \rightarrow 1$

2)  $((1+2)+0) \rightarrow \text{rule 7} \rightarrow (0+0) \rightarrow \text{rule 1} \rightarrow 0$

$(1+(2+2)) \rightarrow \text{rule 9} \rightarrow (1+1) \rightarrow \text{rule 6} \rightarrow 2$

3) NO, it is not always unique because in  $((1+2)+(0+1))$ , we can do rule 7 or rule 2 to rewrite the expression .

3.

1) All strings over the alphabet  $\{0,1\}$ , and also  $\varepsilon$ .

2) All strings over the alphabet  $\{0,1\}$  that have the property starts with 0 and ends with 101 or 111.

4.

$a^*((b|\varepsilon)a^*(c|\varepsilon)a^*(c|\varepsilon)a^*(c|\varepsilon))$

$|((c|\varepsilon)a^*(b|\varepsilon)a^*(c|\varepsilon)a^*(c|\varepsilon))$

$|((c|\varepsilon)a^*(c|\varepsilon)a^*(b|\varepsilon)a^*(c|\varepsilon))$

$|((c|\varepsilon)a^*(c|\varepsilon)a^*(c|\varepsilon)a^*(b|\varepsilon))a^*$

5.

1)

$(0|(1|2|3|4|5|6|7|8|9)(0|1|2|3|4|5|6|7|8|9)^*)$

$.(0|(0|1|2|3|4|5|6|7|8|9)^*(1|2|3|4|5|6|7|8|9))$

2)

