Homework 14

2019.06.06

Note: When the exercise asks you to "design an algorithm for…," it always means that "designs an EFFICIENT algorithm for … and ANALYZES your algorithm". You should keep this in mind when writing solutions.

1. Exercises 34.2-11

Let G be a connected, undirected graph with at least 3 vertices, and let G^3 be the graph obtained by connecting all pairs of vertices that are connected by a path in G of length at most 3. Prove that G^3 is hamiltonian. (Hint: Construct a spanning tree for G, and use an inductive argument.)

2. Exercises 34.4-7

Let 2-CNF-SAT be the set of satisfiable Boolean formula in CNF with exactly 2 literals per clause. Show that 2-CNF-SAT $\in$ **P**. Make your algorithm as efficient as possible. (***Hint***: Observe that $x \vee y$ is equivalent to $\neg x \rightarrow y$. Reduce 2-CNF-SAT to an efficiently solvable problem on a directed graph.)

**** Note: To show a problem is NP-complete, you have to show the problem is NP first.**

3. Exercises 34.2-6

A ***hamiltonian path*** in a graph is a simple path that visits every vertex exactly once.

a. Show that the language HAM-PATH = { (G, u, v): there is a hamiltonian path from u to v in graph G } belongs to NP.

b. Show that the Hamiltonian path problem is NP-complete. (***Hint***: Reduce from HAM-CYCLE.)

4. Exercises 34.5-1

The ***subgraph-isomorphism problem*** takes two undirected graphs G_1 and G_2, and it asks whether G_1 is isomorphic to a subgraph of G_2. Show that the subgraph - isomorphism problem is NP-complete.

5. Exercises 34.5-2

Given an integer $m \times n$ matrix A, and an integer m-vector b, the ***0-1 integer-programming problem*** asks whether there exists an integer n-vector x with elements in the set $\{0, 1\}$ such that $Ax \leq b$. Prove that 0-1 integer programming problem is NP-complete. (***Hint***: Reduce from 3-CNF-SAT.)

6. Exercises 34.5-7

The ***longest-simple-cycle problem*** is the problem of determining a simple cycle (no repeated vertices) of maximum length in a graph. Formulate a related decision problem, and Show that the decision problem is NP-complete.

7. Exercises 35.1-4

Give an efficient greedy algorithm that finds an optimal vertex cover for a tree in linear time.