

Homework 1

2019.02.27

Note: When the exercise asks you to “design an algorithm for...,” it always means that “designs an EFFICIENT algorithm for ... and ANALYZES your algorithm”. You should keep this in mind when writing solutions.

1. Prove that each of the following sorting algorithms is stable or show that it is unstable by giving a counter example; moreover, determine whether it is in place: bubble sort, insertion sort, selection sort, quick sort, merge sort, heap-sort.
2. Given a positive integer n , design an algorithm for computing $\lfloor \sqrt{n} \rfloor$.
3. How to implement merge-sort such that the extra space used is about $n/2$ where n is the number of input elements?
4. Design a data structure to represent a set with elements being positive integers, and then design algorithms for the following operations:
Compute the union of two sets.
Compute the intersection of two sets.
Determine if a given element is in a given set.
5. Given two sorted arrays $x[1] \dots x[m]$, $y[1] \dots y[n]$, design an algorithm to compute $\min_i |x[i] - y[j]|$.
6. Solve the recurrence $T(n) = 2T(n/2) + n - 1$ where $n = 2^k$ is assumed. And assume that $T(n)$ is constant if $n \leq 2$.
7. Analyze best-case, average-case, and worst-case performance of the following pseudocode which describes a sorting algorithm. Append your analyzing process or reasons.

```
i = 2
while i <= size of array
    if i == 1 or array[i] >= array[i - 1]
        i += 1
    else
        swap array[i], array[i - 1]
        i -= 1
```