

Homework 7

2019.04.18

Note: When the exercise asks you to “design an algorithm for...,” it always means that “designs an EFFICIENT algorithm for ... and ANALYZES your algorithm”. You should keep this in mind when writing solutions.

1. Exercise 16.5-2

Show how to use property 2 of Lemma 16.12 to determine in time $O(|A|)$ whether or not a given set A of tasks is independent.

Lemma 16.12 (in the textbook, you can also check the slide Unit 6 Page 30.)

For any set of tasks A , the following statements are equivalent.

1. The set A independent.
2. For $t = 0, 1, 2, \dots, n$, we have $N_t(A) \leq t$.
3. If the tasks in A are scheduled on order of monotonically increasing deadlines, then no task is late.

2. Problem 16-2: Scheduling to minimize average completion time

Suppose you are given a set $S = \{a_1, a_2, \dots, a_n\}$ of tasks, where task a_i requires t_i units of processing time to complete, once it has started. You have one computer on which to run these tasks, and the computer can run only one task at a time. Let c_i be the *completion time* of task a_i , that is, the time at which task a_i completes processing. Your goal is to minimize the average completion time, that is, to minimize $\sum_{i=1}^n c_i$. Give an algorithm that schedules the tasks so as to minimize the average completion time. Each task must run non-preemptively, that is, once task a_i starts, it must run continuously for t_i units of time. Prove that your algorithm minimizes the average completion time, and state the running time of your algorithm.

3. EXT 6-3

Suppose you have one machine and a set of n jobs a_1, a_2, \dots, a_n to process on that machine. Each job a_j has a processing time t_j , the same profit 1, and a deadline d_j . The machine can process only one job at a time, and job a_j must run uninterruptedly for t_j consecutive time units. If job a_j is completed by its deadline d_j , you receive a unit profit, but if it is completed after its deadline, you receive a profit of 0. Give an algorithm to find the schedule that obtains the maximum amount of profit.

4. 寫出Euler Circuit的充分必要條件，並設計一演算法找出Euler Circuit。

5. Exercise 22.1-1

Given an adjacency-list representation of a directed graph, how long does it take to compute the out-degree of every vertex? How long does it take to compute the in-degrees?

6. Exercises 22.1-5

The **square** of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$ such that $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v . Describe efficient algorithms for computing G^2 from G for both the adjacency-list and adjacency-matrix representations of G . Analyze the running times of your algorithms.

7. Exercises 22.2-7 (塗兩色問題)

There are two types of professional wrestlers: “babyfaces” (“good guys”) and “heels” (“bad guys”). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n + r)$ -time algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.