現代控制理論 HW4

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甲.

a.

$x_1\_dot=x_1^2+x_2$

$x_2\_dot=-x_1+u$

取$z_1=x_1, z_2=z_1_dot=x_1_dot=x_1^2+x_2$

則$z_2_dot=2x_1x_1_dot+x_2_dot=2x_1(x_1^2+x_2)+(-x_1+u)\equiv \alpha(x)+\beta(x)u$

$\alpha(x)=2x_1^3+2x_1x_2-x_1$

$\beta(x)=1$

b.

$zdot=\begin{bmatrix}0 & 1\\ k1 & k2\end{bmatrix}z$，目標極點位置s=-1+9i, -1-9i

$\det\left(\begin{bmatrix}0-s & 1\\ k1 & k2-s\end{bmatrix}\right)=s^2-k_2s-k_1=s^2+2s+82$，經比較係數得

$k_1=-82, k_2=-2$

設計

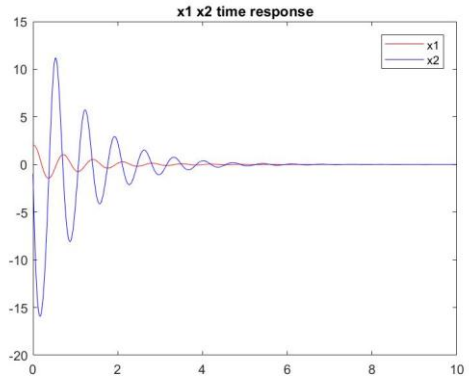
$u=(-\alpha(x)+(k_1z_1+k_2z_2))/\beta(x)$

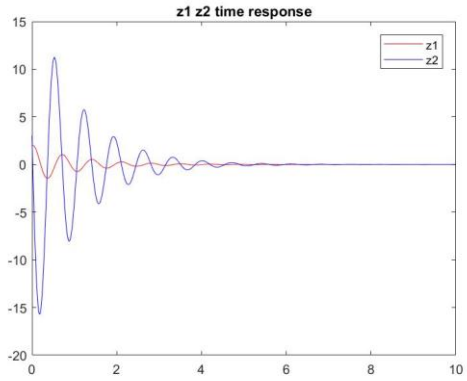
$=-(2x_1^3+2x_1x_2-x_1)+(-82)z_1+(-2)z_2$

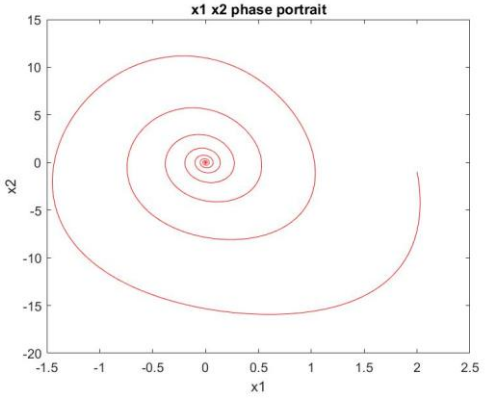
$=-(2x_1^3+2x_1x_2-x_1)+(-82)x_1+(-2)(x_1^2+x_2)$

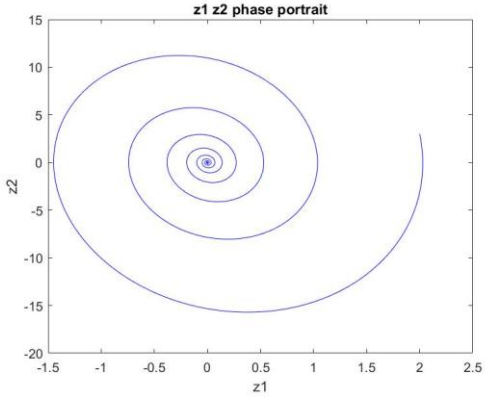
c.模擬結果：

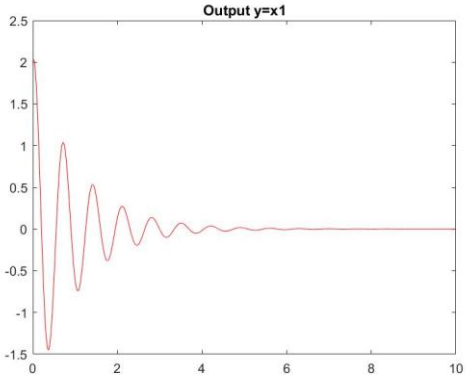
註：初值$x_1=2, x_2=-1$











乙.

$$zdot=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ k1 & k2 & k3\end{bmatrix}x$$，目標極點位置s=-1+9i, -1-9i, -1

$$\det\left(\begin{bmatrix}0-s & 1 & 0\\ 0 & 0-s & 1\\ k1 & k2 & k3-s\end{bmatrix}\right)=-s^3+k_3s^2+k_2s+k_1=-(s^3+3s^2+84s+82)$$，比較係數得

k_1=-82,k_2=-84,k_3=-3

$x_1\_dot=x_2+x_1^3$

$x_2\_dot=x_3$

$x_3\_dot=u$

取$z_1=x_1$

$z_2=z_1\_dot$

$z_3=z_2\_dot$

z_3_dot

$=(x_1)'''=(x_2+x_1^3)''=(x_2\_dot+3x_1^2x_1\_dot)'=((x_3)+3x_1^2(x_2+x_1^3))'$

$= u +3(2x_1(x_2+x_1^3)x_2+x_1^2(x_3))+15x_1^4(x_2+x_1^3) \equiv \alpha(x)+\beta(x)u$

$\alpha(x)=3(2x_1x_1\_dot(x_2+x_1^3)+x_1^2(x_2\_dot+3x_1^2x_1\_dot))$;

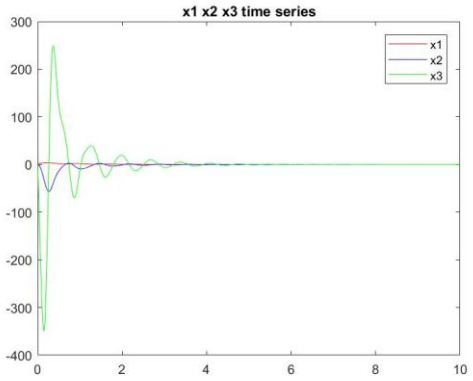
$\beta(x)=1$;

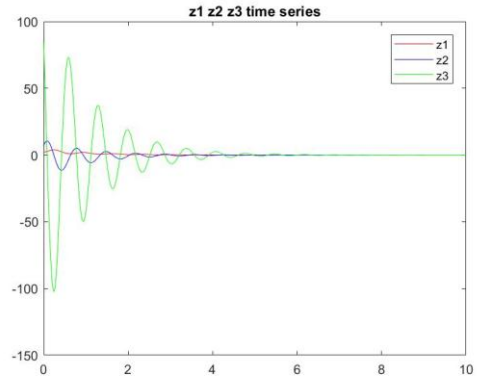
設計

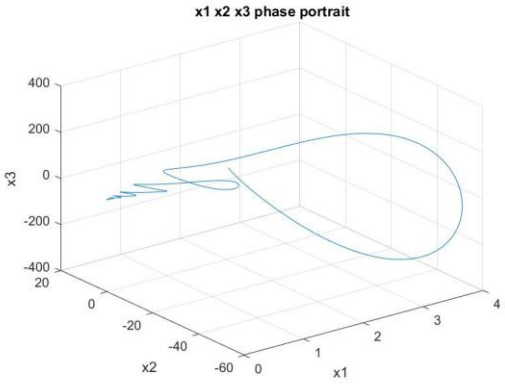
$u=(-\alpha(x)+(k_1z_1+k_2z_2+k_3z_3))/\beta(x)$

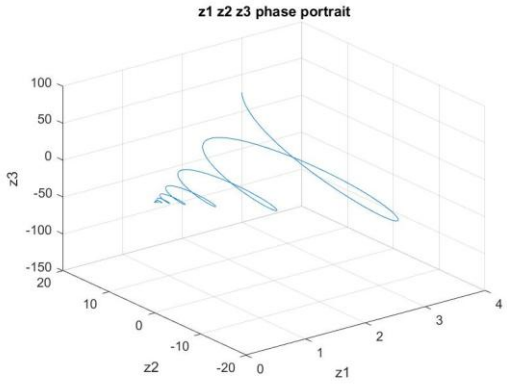
模擬結果：

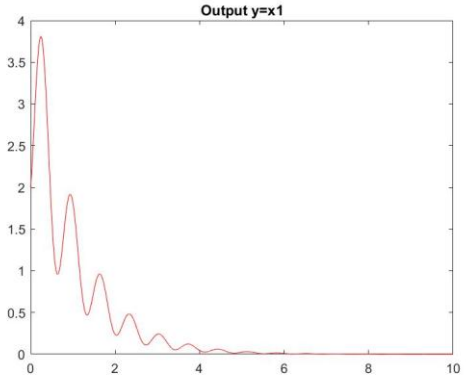
註：初值$x_1=2, x_2=-1, x_3=1$











丙.

zdot=$\begin{bmatrix} 0 & 1 \\ k1 & k2 \end{bmatrix}$z，目標極點位置s=-1+9i, -1-9i

$\det\left(\begin{bmatrix} 0-s & 1 \\ k1 & k2-s \end{bmatrix}\right)=s^2-k_2s-k_1=s^2+2s+82$，經比較係數得

$k_1=-82, k_2=-2$

$x_1\_dot=0.5x_1^3+7x_2$

$x_2\_dot=2x_1x_2+u$

取$z_1=x_1$

$z_2= z_1\_dot=x_1\_dot=0.5x_1^3+7x_2$

$z_2\_dot=1.5x_1^2(0.5x_1^3+7x_2)+7(2x_1x_2+u) \equiv \alpha(x)+\beta(x)u$

$\alpha(x)= 7.5\ x_1^5+10.5\ x_1^2\ x_2+14x_1x_2$

$\beta(x)=7$

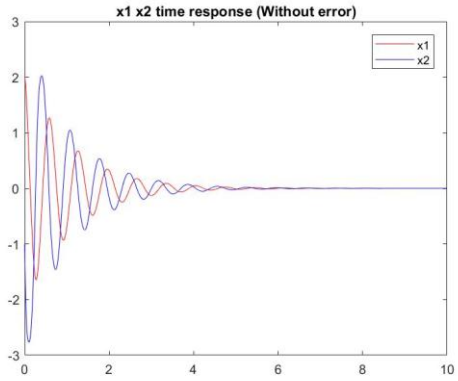
設計

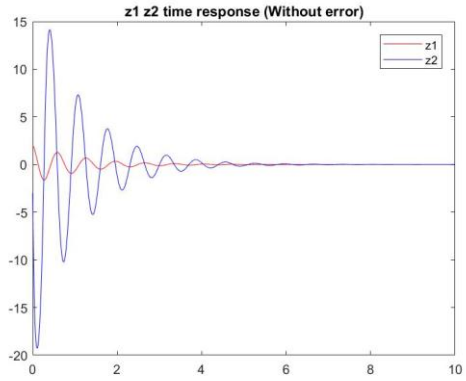
$u=(-\alpha(x)+(k_1z_1+k_2z_2))/\beta(x)$

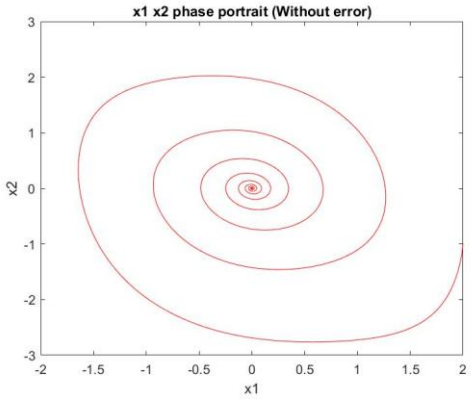
$=(-(7.5\ x_1^5+10.5\ x_1^2\ x_2+14x_1x_2)-82\ x_1-2(0.5x_1^3+7x_2))/7$

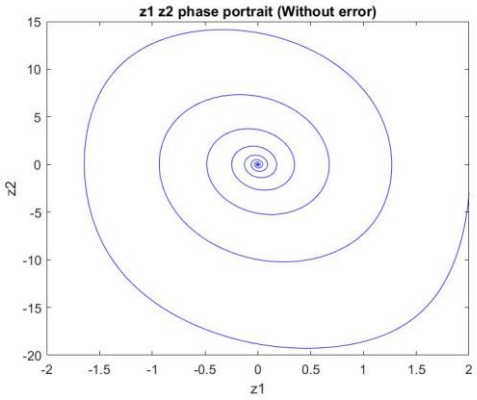
模擬結果1(無誤差)：

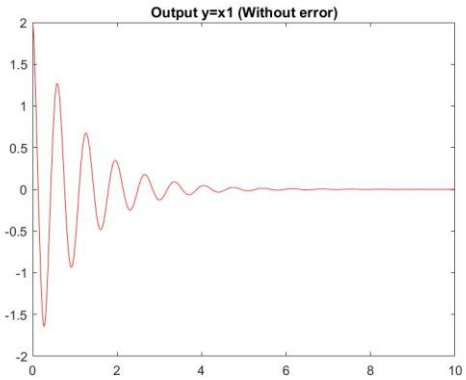
註：初值$x_1=2, x_2=-1$





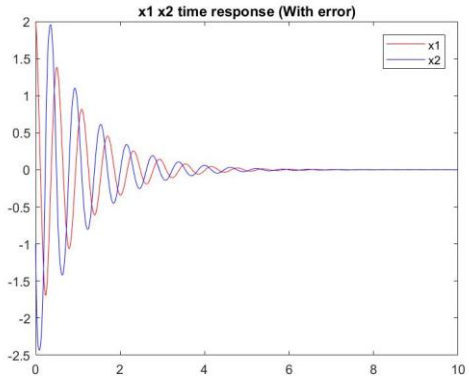


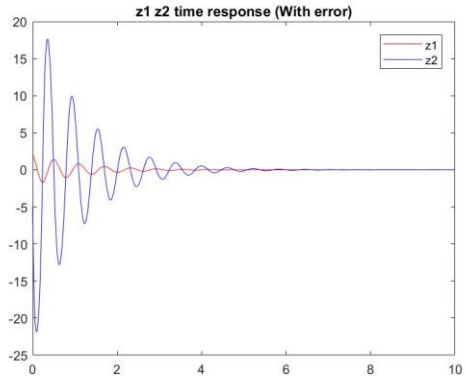


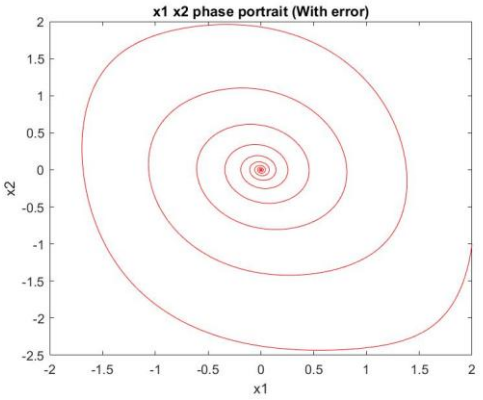


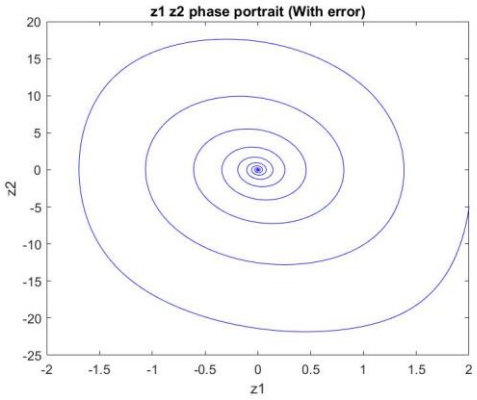
模擬結果2(有誤差)：

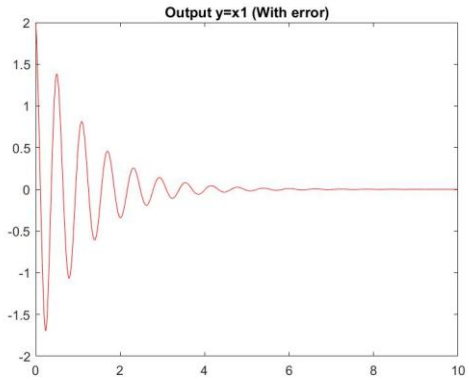
註：初值$x_1=2, x_2=-1$



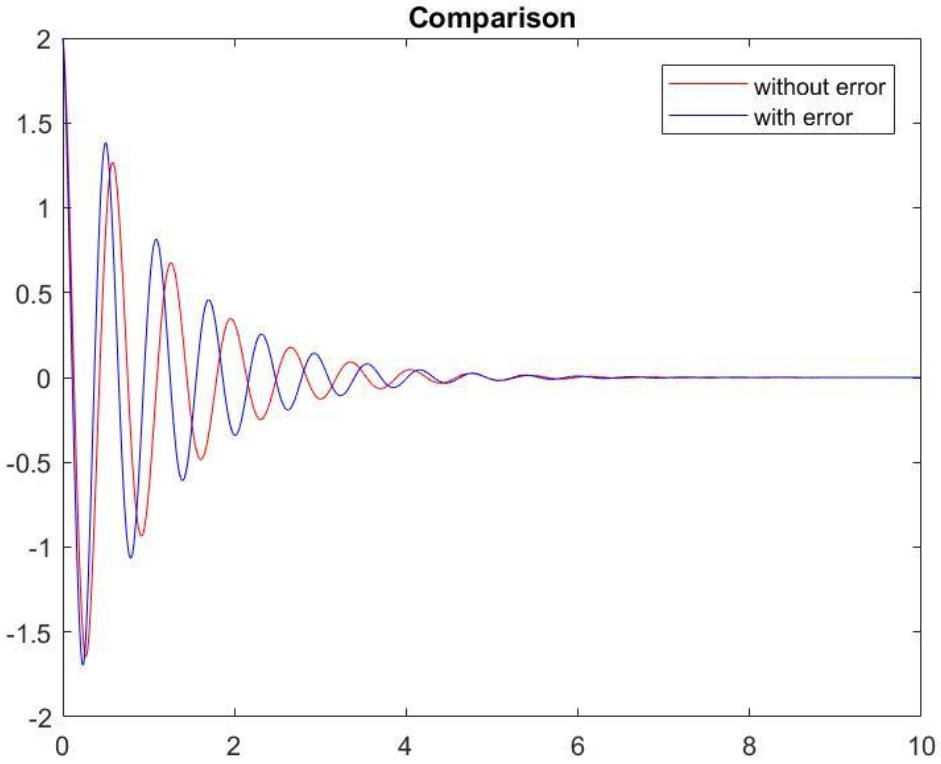








比較：相同控制器，誤差對輸出(x_1)之影響：



附錄(程式碼)

甲.

```
clear;clc;

delta=0.001;
totalTime=10;
totalStep=totalTime/delta;
x1array=[1:totalStep]*0;x2array=x1array;
z1array=x1array;z2array=x1array;

x1array(1)=2;x2array(1)=-1;%init condition
for i=1:totalStep
    x1=x1array(i);x2=x2array(i);
    u=-(2*x1^3+2*x1*x2-x1)+(-82)*x1+(-2)*(x1^2+x2);
    x1_dot=x1^2+x2;
    x2_dot=-x1+u;
    z1array(i)=x1;
    z2array(i)=x1_dot;

    x1array(i+1)=x1+x1_dot*delta;
    x2array(i+1)=x2+x2_dot*delta;
end

figure(1);
plot([0:1:totalStep]*delta,x1array,'r');
hold on;
plot([0:1:totalStep]*delta,x2array,'b');
legend('x1','x2');
title('x1 x2 time response');

figure(2);
plot([0:1:totalStep-1]*delta,z1array,'r');
hold on;
plot([0:1:totalStep-1]*delta,z2array,'b');
legend('z1','z2');
title('z1 z2 time response');
```

```
figure(3);
plot(x1array,x2array,'r');
title('x1 x2 phase portrait');
xlabel('x1');ylabel('x2');

figure(4);
plot(z1array,z2array,'b');
title('z1 z2 phase portrait');
xlabel('z1');ylabel('z2');

figure(5);
plot([0:1:totalStep]*delta,x1array,'r');
title('Output y=x1');
```

乙.

```
clear;clc;

delta=0.001;
totalTime=10;
totalStep=totalTime/delta;
x1array=[1:totalStep]*0;x2array=x1array;x3array=x1array;
z1array=x1array;z2array=x1array;z3array=x1array;

x1array(1)=2;x2array(1)=-1;x3array(1)=1;%init condition
for i=1:totalStep
    x1=x1array(i);x2=x2array(i);x3=x3array(i);

    x1_dot=x2+x1^3;
    x2_dot=x3;
    z1=x1;
    z2=x2+x1^3;
    z3=x2_dot+3*x1^2*x1_dot;
    k1=-82;k2=-84;k3=-3;
    arpha=3*(2*x1*x1_dot*(x2+x1^3)+x1^2*(x2_dot+3*x1^2*x1_dot));
    u=-1*arpha+(k1*z1+k2*z2+k3*z3);
    x3_dot=u;
```

```
    z1array(i)=z1;
    z2array(i)=z2;
    z3array(i)=z3;
    x1array(i+1)=x1+x1_dot*delta;
    x2array(i+1)=x2+x2_dot*delta;
    x3array(i+1)=x3+x3_dot*delta;
end

figure(1);
plot([0:1:totalStep]*delta,x1array,'r');
hold on;
plot([0:1:totalStep]*delta,x2array,'b');
hold on;
plot([0:1:totalStep]*delta,x3array,'g');
legend('x1','x2','x3');
title('x1 x2 x3 time series');

figure(2);
plot([0:1:totalStep-1]*delta,z1array,'r');
hold on;
plot([0:1:totalStep-1]*delta,z2array,'b');
hold on;
plot([0:1:totalStep-1]*delta,z3array,'g');
legend('z1','z2','z3');
title('z1 z2 z3 time series');

figure(3);
plot3(x1array,x2array,x3array);
xlabel('x1');ylabel('x2');zlabel('x3');
title('x1 x2 x3 phase portrait');
grid on;

figure(4);
plot3(z1array,z2array,z3array);
xlabel('z1');ylabel('z2');zlabel('z3');
title('z1 z2 z3 phase portrait');
```

```
grid on;

figure(5);
plot([0:1:totalStep]*delta,x1array,'r');
title('Output y=x1');
```

丙.

```
clear;clc;

delta=0.001;
totalTime=10;
totalStep=totalTime/delta;
x1array=[1:totalStep]*0;x2array=x1array;
z1array=x1array;z2array=x1array;

x1array(1)=2;x2array(1)=-1;%init condition
for i=1:totalStep
    x1=x1array(i);x2=x2array(i);
    u=(-(7.5*x1^5+10.5*x1^2*x2+14*x1*x2)-82*x1-2*(0.5*x1^3+7*x2))/7;
%     x1_dot=0.5*x1^3+7*x2;
%     x2_dot=2*x1*x2+u;
    x1_dot=0.5*x1^3+9*x2;
    x2_dot=1.5*x1*x2+u;
    z1array(i)=x1;
    z2array(i)=x1_dot;

    x1array(i+1)=x1+x1_dot*delta;
    x2array(i+1)=x2+x2_dot*delta;
end

figure(1);
plot([0:1:totalStep]*delta,x1array,'r');
hold on;
plot([0:1:totalStep]*delta,x2array,'b');
legend('x1','x2');
title('x1 x2 time response (With error)');
```

```
figure(2);
plot([0:1:totalStep-1]*delta,z1array,'r');
hold on;
plot([0:1:totalStep-1]*delta,z2array,'b');
legend('z1','z2');
title('z1 z2 time response (With error)');

figure(3);
plot(x1array,x2array,'r');
title('x1 x2 phase portrait (With error)');
xlabel('x1');ylabel('x2');

figure(4);
plot(z1array,z2array,'b');
title('z1 z2 phase portrait (With error)');
xlabel('z1');ylabel('z2');

figure(5);
plot([0:1:totalStep]*delta,x1array,'r');
title('Output y=x1 (With error)');
```