現代控制理論 HW1

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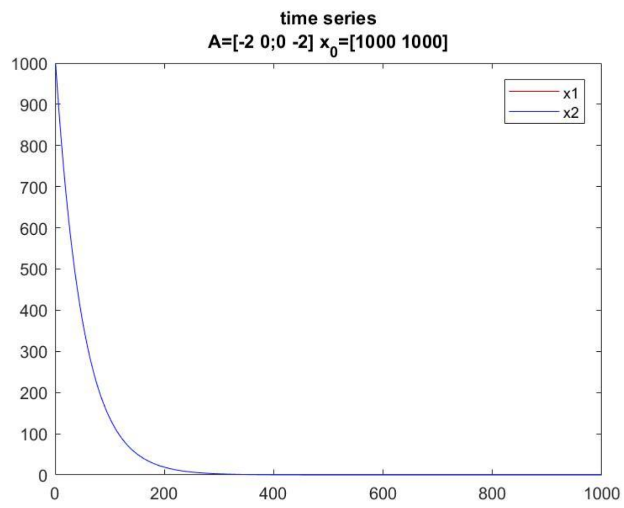
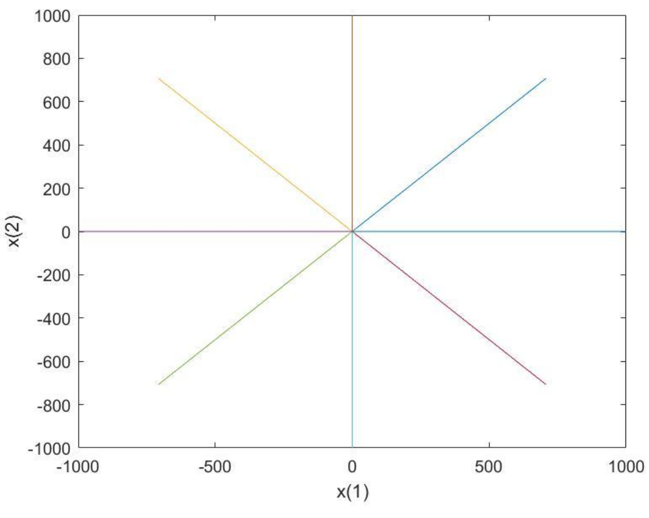
1.題目

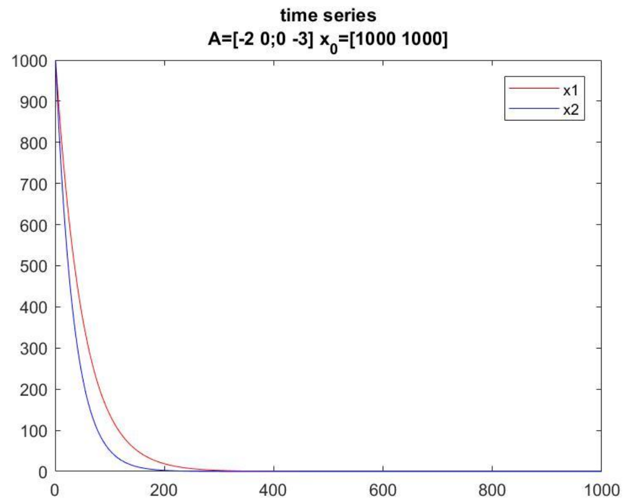
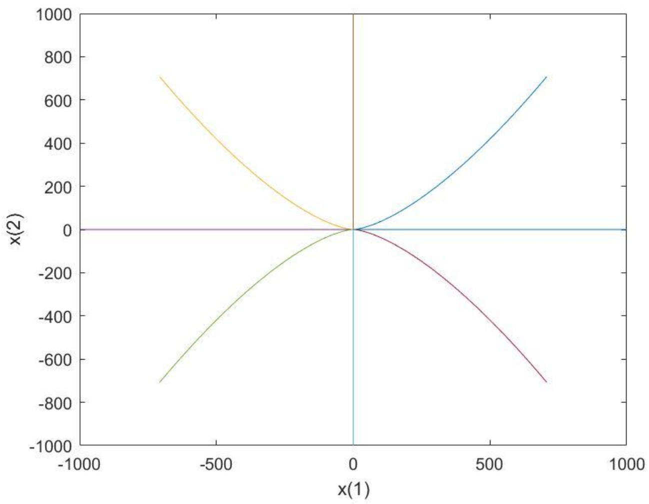
設計出自己的 linear state equation，x_dot = Ax。利用不同的 A，產生五種 type 的 state portrait，針對每種 type，必須畫出所有初始值 X0 = [cosθ , sinθ]^T (only for node, saddle point and star)

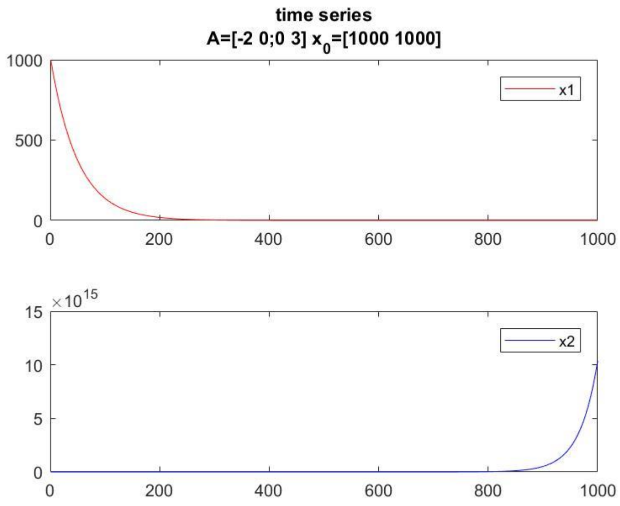
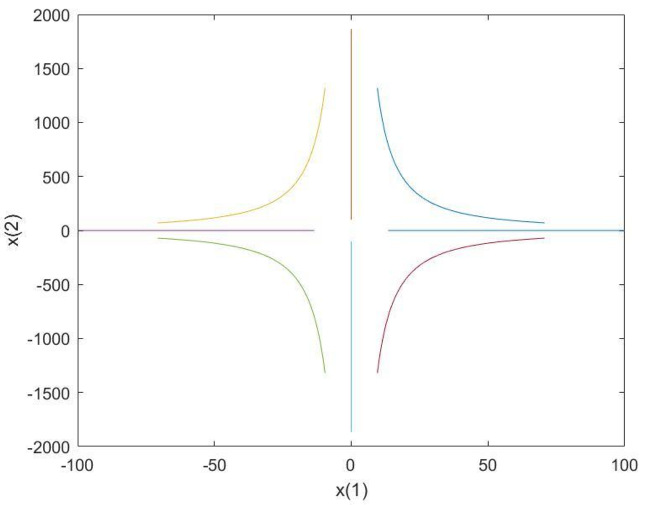
2. 執行結果截圖

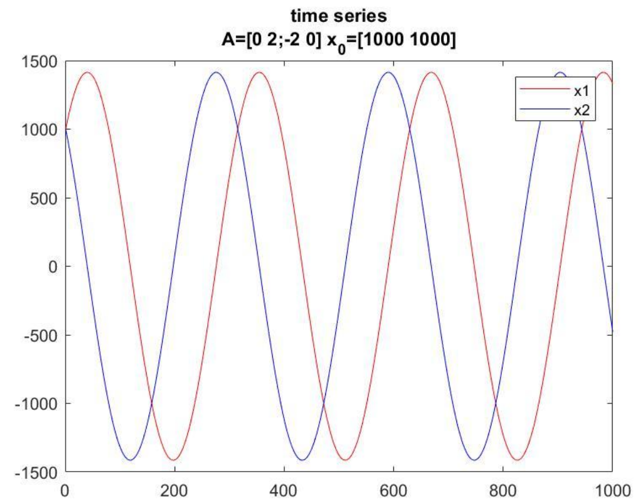
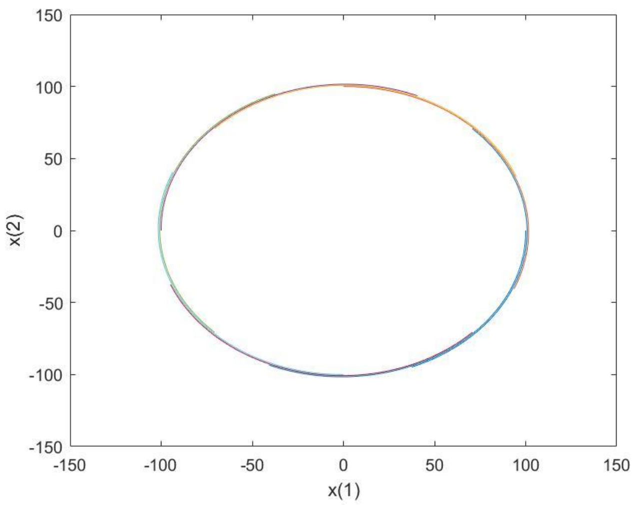
Phase Portrait 的部分都畫出 8 種初始值 X0 = [cosθ , sinθ]^1000，θ 分別為：

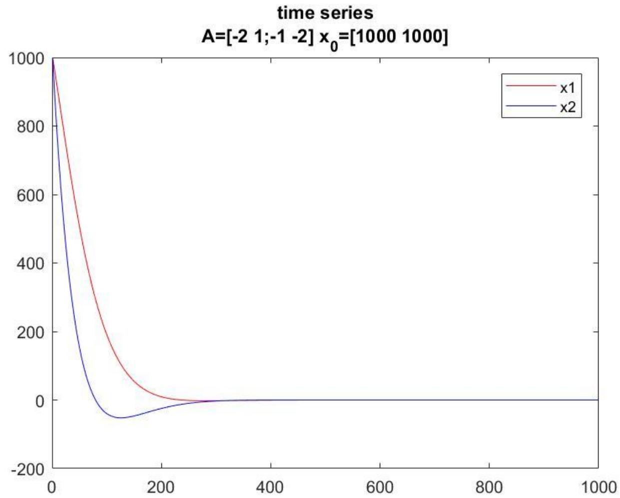
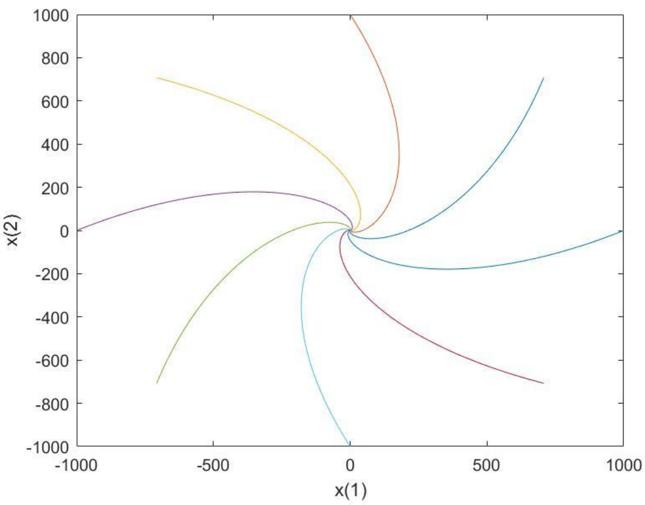
0,π /4, π /2, 3π /4, π , 5π /4, 3π /2, 7π /4。

<table>
<tr><td>star</td><td>$A=\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$</td></tr>
<tr><td>Time Series</td><td>
</td></tr>
<tr><td>Phase Portrait</td><td>
</td></tr>
<tr><td>備註</td><td></td></tr>
</table>

node	$A=\begin{bmatrix}-2 & 0\\ 0 & -3\end{bmatrix}$
Time Series	
Phase Portrait	
備註	

saddle	$A=\begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$
Time Series	
Phase Portrait	
備註	畫 Phase Portrait 的時候只疊代 100 次，否則衰減的那個變數相對於遞增的那個幾乎看不出變化。

center	$A=\begin{bmatrix}0 & 2\\ -2 & 0\end{bmatrix}$
Time Series	
Phase Portrait	
備註	

focus	$A=\begin{bmatrix}-2 & 1\\ -1 & -2\end{bmatrix}$
Time Series	
Phase Portrait	
備註	

3.程式碼

(1)函式 使用 Euler Method 近似：

```
function xNew=Euler(x,delta,A)
xDot=[0 0]';
xNew=xDot;
xDot(1)=A(1,1)*x(1)+A(1,2)*x(2);
xDot(2)=A(2,1)*x(1)+A(2,2)*x(2);
% fprintf('xDot=%f %f\n', xDot(1),xDot(1));
xNew(1)=x(1)+xDot(1)*delta;
xNew(2)=x(2)+xDot(2)*delta;
% fprintf('xNew=%f %f\n', xNew(1),xNew(1));
return;
```

(2)函式 使用Runge Kutta Method近似：

```
function xNew=RungeKutta(x_0,delta,A)
k1=[0 0]';k2=k1;k3=k1;k4=k1;tmp=k1;xNew=k1;

k1(1)=A(1,1)*x_0(1)+A(1,2)*x_0(2);
k1(2)=A(2,1)*x_0(1)+A(2,2)*x_0(2);
tmp(1)=x_0(1)+k1(1)*(delta/2);
tmp(2)=x_0(2)+k1(2)*(delta/2);
k2(1)=A(1,1)*tmp(1)+A(1,2)*tmp(2);
k2(2)=A(2,1)*tmp(1)+A(2,2)*tmp(2);
tmp(1)=x_0(1)+k2(1)*(delta/2);
tmp(2)=x_0(2)+k2(2)*(delta/2);
k3(1)=A(1,1)*tmp(1)+A(1,2)*tmp(2);
k3(2)=A(2,1)*tmp(1)+A(2,2)*tmp(2);
tmp(1)=x_0(1)+k3(1)*(delta);
tmp(2)=x_0(2)+k3(2)*(delta);
k4(1)=A(1,1)*tmp(1)+A(1,2)*tmp(2);
k4(2)=A(2,1)*tmp(1)+A(2,2)*tmp(2);

xNew(1)=x_0(1)+delta*(k1(1)+2*k2(1)+2*k3(1)+k4(1))/6;
xNew(2)=x_0(2)+delta*(k1(2)+2*k2(2)+2*k3(2)+k4(2))/6;
return;
```

(3)畫Phase Portrait

```
clear;clc;
% A=[-2 0;0 -2];%star
% A=[-2 0;0 -3];%node
A=[-2 0;0 3];%saddle
% A=[0 2;-2 0];%center
% A=[-2 1;-1 -2]%focus

num=8;theta=0; %total of the different kind of initial condition
datasize=100;
for j=1:8
    theta=j*(2*pi/num);
    x1array(1)=real(datasize*exp(1i*theta));
    x2array(1)=imag(datasize*exp(1i*theta));

    for i=1:(datasize-1)
        x(1)=x1array(i); x(2)=x2array(i);
        xNext=RungeKutta(x,0.01,A);
        x1array(i+1)=xNext(1);
        x2array(i+1)=xNext(2);
    end

    xlabel('x(1)');
    ylabel('x(2)');
    plot(x1array,x2array);
    hold on;
end
```

(4)畫Time Series

```
clear;clc;
% A=[-2 0;0 -2];%star
% A=[-2 0;0 -3];%node
% A=[-2 0;0 3];%saddle
% A=[0 2;-2 0];%center
A=[-2 1;-1 -2]%focus

datasize=1000;
x1array(1)=1000;
x2array(1)=1000;
for i=1:(datasize-1)
    x(1)=x1array(i); x(2)=x2array(i);
    xNext=RungeKutta(x,0.01,A);
    x1array(i+1)=xNext(1);
    x2array(i+1)=xNext(2);
end



plot(x1array,'r');
hold on;
plot(x2array,'b');
legend('x1','x2');
title({'time series';'A=[-2 1;-1 -2] x_0=[1000 1000]'});
```