介面說明（開發平台、如何執行 ...）

1.開發平台

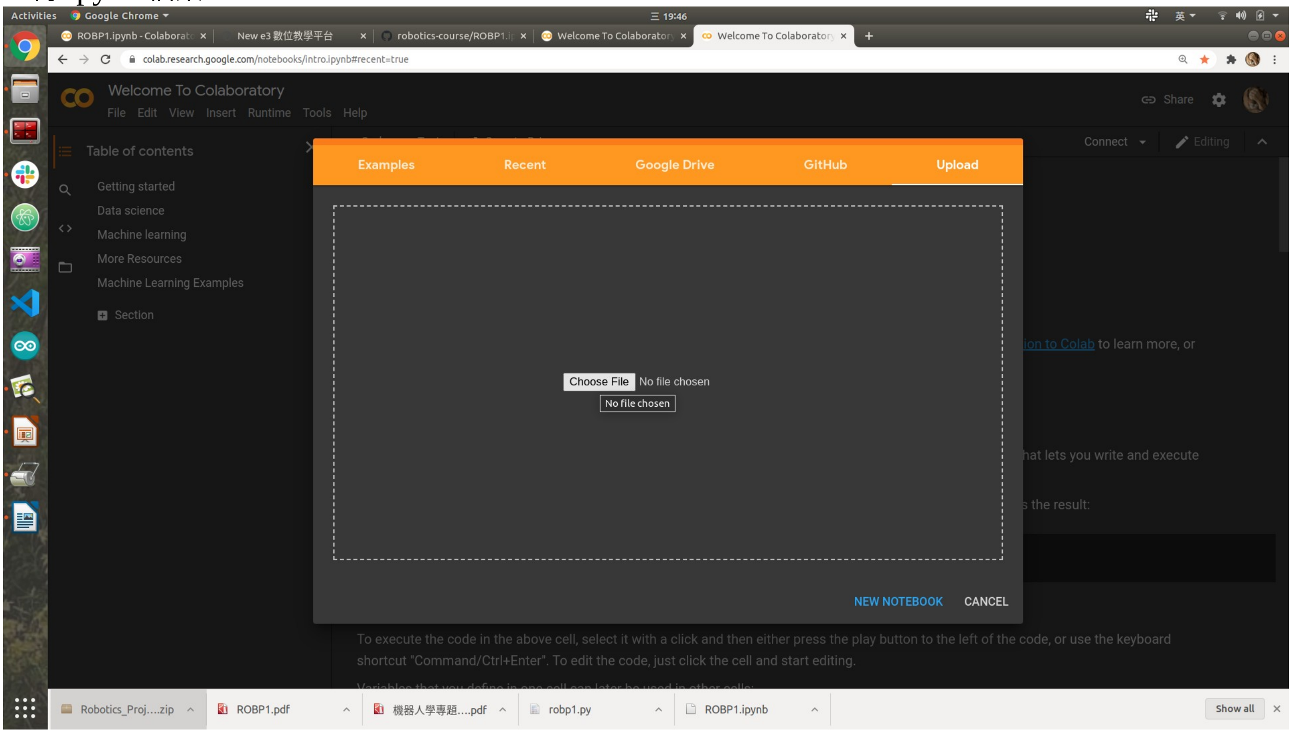
python3+jupyter notebook ,能於 Google Colab 上執行

使用到 numpy 和 math 函式庫

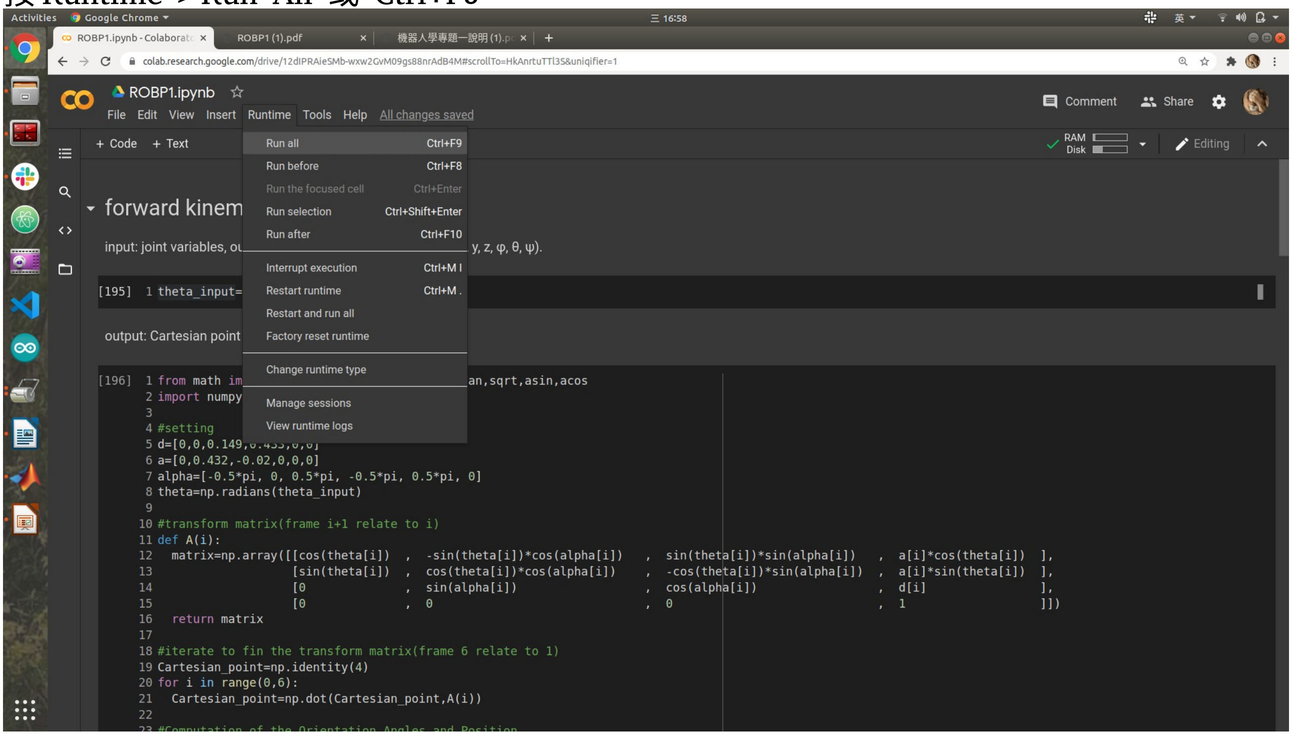
2.一鍵執行

瀏覽器進入 https://colab.research.google.com/

上傳.ipynb 檔案



按 Runtime->Run All 或 Ctrl+F9



3.輸入

分別於 notebook 中的這兩個 cell 中輸入數值

forward kinematics

input: joint variables, output: Cartesian point (n, o, a, p) and (x, y, z, φ, θ, ψ).

```
[195]  theta_input=[50,50,50,50,50,50]
```

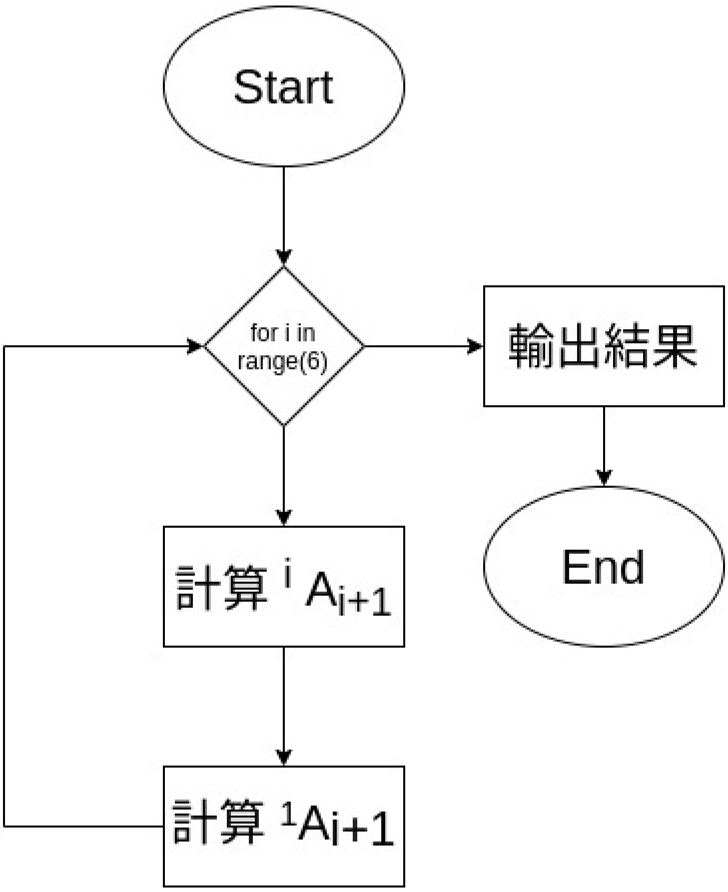
inverse kinematics

input: Cartesian point (n, o, a, p)*italicized text*

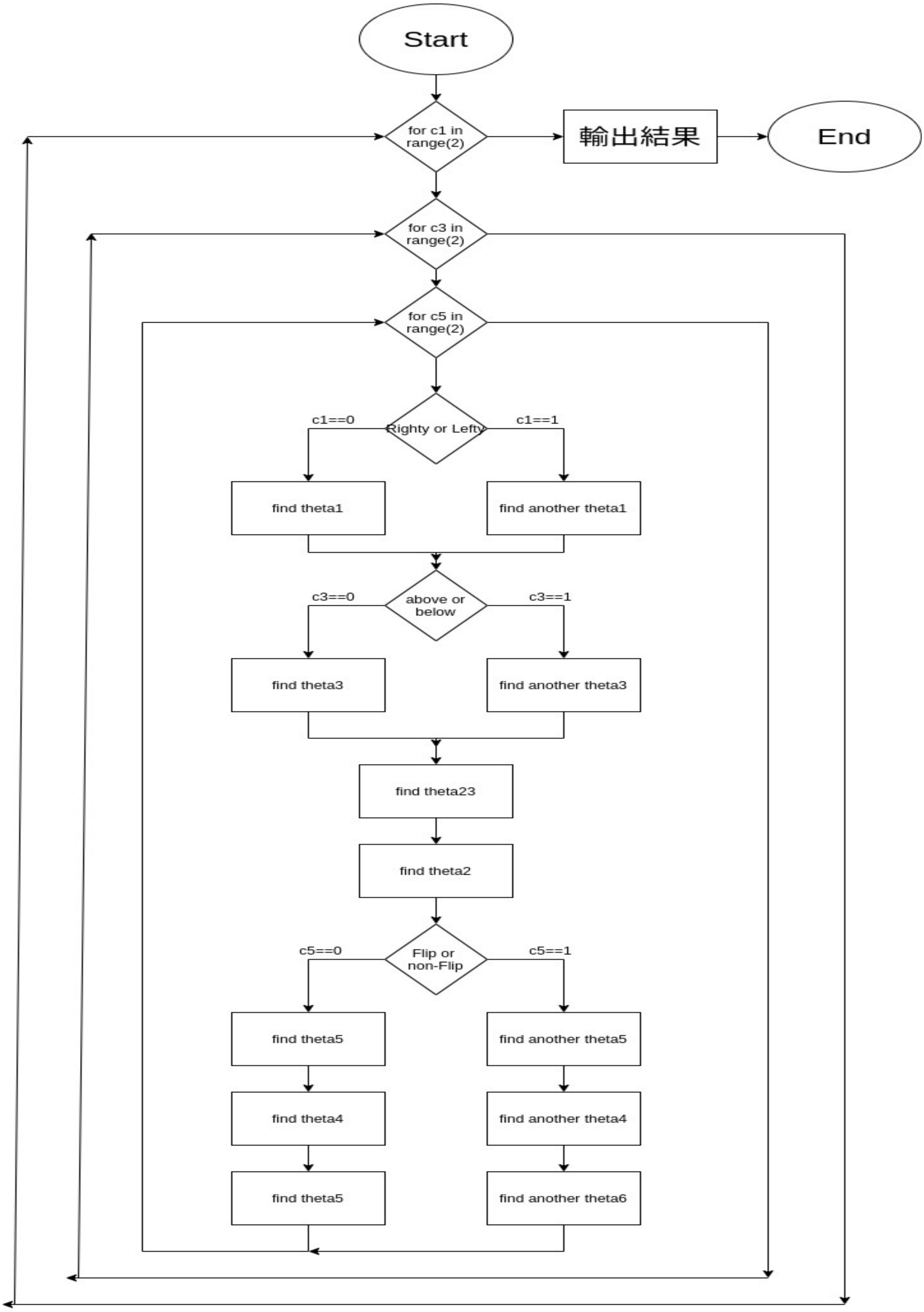
```
[197]  Cartesian_point=np.array([[-0.895511    ,0.43420623  ,-0.09759607  ,0.34068237],
                                 [ 0.19121987  ,0.57343036  ,0.79662575   ,0.63781229],
                                 [ 0.40186441  ,0.69472482  ,-0.59654205  ,-0.38642471],
                                 [ 0.          ,0.          ,0.           ,1.        ]] )
```

程式架構說明（程式運行流程、核心程式碼說明 ...）

1.forward kinematics



2.inverse kinematics



數學運算說明

θ_1 :

$$A_1^{-1} \cdot T_6 = {}^1T_6$$

$$\begin{bmatrix} c_1 & s_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} & P_x \\ f_{21} & f_{22} & f_{23} & P_y \\ f_{31} & f_{32} & f_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \times & \times & \times & S_{23}d_4 + C_{23}a_3 + a_2C_2 \\ \times & \times & \times & -C_{23}d_4 + S_{23}a_3 + a_2S_2 \\ \times & \times & \times & d_3 \\ \times & \times & \times & 1 \end{bmatrix}$$

$\boxed{-S_1P_x + c_1P_y = d_3}$, 令 $P_x = \rho\cos\phi$, $P_y = \rho\sin\phi$

$$\rho = \sqrt{P_x^2 + P_y^2}, \quad \phi = A\tan2(P_y, P_x)$$

$$c_1 S\phi - s_1 C\phi = \frac{d_3}{\rho}$$

$$\sin(\phi - \theta_1) = \frac{d_3}{\rho}$$

$$\therefore \cos(\phi - \theta_1) = \pm\sqrt{1 - \frac{d_3^2}{\rho^2}}$$

$$\therefore \phi - \theta_1 = A\tan2\left[\frac{d_3}{\rho}, \pm\sqrt{1 - \frac{d_3^2}{\rho^2}}\right]$$

$$\theta_1 = A\cdot\tan2(P_y, P_x) - A\tan2\left(d_3, \pm\sqrt{P_x^2 + P_y^2 - d_3^2}\right)$$

θ_3 :

$$\boxed{c_1P_x + s_1P_y = S_{23}d_4 + C_{23}a_3 + a_2C_2}$$

$$\boxed{-P_z = -C_{23}d_4 + S_{23}a_3 + a_2S_2}$$

$$\boxed{-s_1P_x + c_1P_y = d_3}$$

$$\Rightarrow M = a_3C_3 + d_4S_3 = \frac{P_x^2 + P_y^2 + P_z^2 - a_2^2 - a_3^2 - d_3^2 - d_4^2}{2a_2}$$

$$\theta_3 = A\tan2\left(M, \pm\sqrt{a_3^2 + d_4^2 - M^2}\right) - A\tan2(a_3, d_4)$$

θ_2 :

$$T_3^{-1} T_6 = {}^3T_6$$

$$[n \quad o \quad a \quad p]$$

$$|||$$

$$\begin{bmatrix} c_1 C_{23} & S_1 C_{23} & -S_{23} & -a_3 - a_2 C_3 \\ -s_1 & C_1 & 0 & -d_3 \\ c_1 S_{23} & s_1 S_{23} & C_{23} & -a_2 S_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} & P_x \\ f_{21} & f_{22} & f_{23} & P_y \\ f_{31} & f_{32} & f_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_4 C_5 C_6 - S_4 S_6 & -C_4 C_5 S_6 - S_4 C_6 & C_4 S_5 & 0 \\ S_4 C_5 C_6 + C_4 S_6 & -S_4 C_5 S_6 + C_4 C_6 & S_4 S_5 & 0 \\ -S_5 C_6 & S_5 S_6 & C_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} \boxed{C_1 C_{23} P_x + S_1 C_{23} P_y - S_{23} P_z = a_3 + a_2 C_3} \\ \boxed{C_1 S_{23} P_x + S_1 S_{23} P_y + C_{23} P_z = d_4 + a_2 S_3} \end{cases}$$

$$\Rightarrow \begin{bmatrix} C_1 P_x + S_1 P_y & -P_z \\ P_z & C_1 P_x + S_1 P_y \end{bmatrix} \begin{bmatrix} C_{23} \\ S_{23} \end{bmatrix} = \begin{bmatrix} a_3 + a_2 C_3 \\ d_4 + a_2 S_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} C_{23} \\ S_{23} \end{bmatrix} = \begin{bmatrix} C_1 P_x + S_1 P_y & -P_z \\ P_z & C_1 P_x + S_1 P_y \end{bmatrix}^{-1} \begin{bmatrix} a_3 + a_2 C_3 \\ d_4 + a_2 S_3 \end{bmatrix}$$

$$\Rightarrow \theta_{23} = \mathrm{Atan2}\left(S_{23} , C_{23} \right)$$

$$\theta_2 = \theta_{23} - \theta_3$$

θ_5 :

$$\boxed{c_1 s_{23} f_{13} + s_1 s_{23} f_{23} + c_{23} f_{33} = c_5}$$

$$\theta_5 = \cos^{-1}\left(c_1 s_{23} f_{13} + s_1 s_{23} f_{23} + c_{23} f_{33}\right)$$

θ_6 & θ_4 :

$$\boxed{c_4 s_5 = c_1 c_{23} f_{13} + s_1 c_{23} f_{23} - s_{23} f_{33}}$$

$$\boxed{s_4 s_5 = -s_1 f_{13} + c_1 f_{23}}$$

$$s_5 c_6 = -\left(c_1 s_{23} f_{11} + s_1 s_{23} f_{21} + c_{23} f_{31}\right)$$

$$\boxed{s_5 s_6 = c_1 s_{23} f_{12} + s_1 s_{23} f_{22} + c_{23} f_{32}}$$

$$\theta_4 = \mathrm{Atan2}\left(s_4 s_5,\ c_4 s_5\right)$$

$$\theta_6 = \mathrm{Atan2}\left(s_5 s_6,\ s_5 c_6\right)$$

討論兩種逆向運動學(代數法，幾何法)的優缺點

代數法比較簡單，只是較暴力且計算過程繁複。

幾何法難度較高，需要很好的空間概念才能把圖畫出來。