Analysis and Control of Time-Varying and Perturbed Systems

Keno Bürger
Advanced Nonlinear Control
July 2022

Chair of Automatic Control Engineering
Technical University of Munich

Main Objective

- Nonlinear Control: Practical Analsysis concerned with exponential stability of time varying systems subject to perturbations (vanishing and non-vanishing)
- Nonlinear Systems: Rigorous proof of stability and boundedness theorems/lemmas while being more broadly applicable
- More pracitcal theorems for applying Lyapunov theory

Lyapunov Theory for Time-Varying Systems

- Definition of Uniform, Asymptotic and exponential stability [Muennighoff+ 2025]
- Application of Lyapunov Stability Theorems

Boundedness and Ultimate Boundedness

- Differences
- Build bridge to non vanishing and vanishing perturbations

Perturbation Model for Vanisihing Perturbation Models

- Exact Modelling rarley feasible due to modelling errors/external disturbances or parameter drift
- $\dot{x} = f(x) + g(x,t)$
  - f is locally Lipschitz
  - g is piecewise continuous in t and locally Lipschitz
  - generally unknown but bounded
- $g(0,t)$ and $g(x,t) = 0$ for $t \to \infty$

Lyapunov Stability Theorems

- Exponential Stability
- Highlight challenges with this approach

Comparison Functions

- Differences and Benefits of this approach
- Corollary 1

Exampe: Linear Time-Varying System

- $\dot{x} = [A(T) + B(t)]x$
- Lyapunov function $V(t, x)$ is positive definite and derivative negative definite
- $g(t, x) = B(t)x \Rightarrow \|g(t, x)\| \leq \|B(t)\| \cdot \|x\| = \gamma(t)\|x\|$
- $\int_{t_0}^{t} \gamma(\tau)d\tau < \epsilon(t - t_o) + \eta$

$\Rightarrow$ Exponetial stability of nominal system is preserved under vanishing perturbations

Perturbation Model for Non-Vanisihing Perturbations

- Impede the system's convergence towards the origin
- Analysing the behavior in terms of boundedness/ultimate boundedness
- Gurantee that the state will remain within a small neighborhood around the origin

Lyapunov Based Conditions for Boundednes

- Why only boundedness
- Lemma 2 for Ultimate Boundedness

Example: Non-Vanisihing Perturbation in a Nonlinear System

- As a special case of non-vanishing perturbations
- Stability Theorems
- Case distinctions

Conceptual Links Between Sections

- As in the report

Benefits and Drawbacks

- Benefits and Drawbacks
- Final Remarks

References I

Chaoqun He, Renjie Luo, Yuzhuo Bai, Shengding Hu, Zhen Leng Thai, Junhao Shen, Jinyi Hu, Xu Han, Yujie Huang, Yuxiang Zhang, Jie Liu, Lei Qi, Zhiyuan Liu and Maosong Sun.
OlympiadBench: A Challenging Benchmark for Promoting AGI with Olympiad-Level Bilingual Multimodal Scientific Problems. 2024. eprint: 2402.14008.

Andreas Hochlehnert, Hardik Bhatnagar, Vishaal Udandarao, Samuel Albanie, Ameya Prabhu and Matthias Bethge.
A Sober Look at Progress in Language Model Reasoning: Pitfalls and Paths to Reproducibility. Apr. 2025. DOI: 10.48550/arXiv.2504.07086.

Jie Huang and Kevin Chen-Chuan Chang. **Towards Reasoning in Large Language Models: A Survey**. May 2023. DOI: 10.48550/arXiv.2212.10403.

Naman Jain, King Han, Alex Gu, Wen-Ding Li, Fanjia Yan, Tianjun Zhang, Sida Wang, Armando Solar-Lezama, Koushik Sen and Ion Stoica.
LiveCodeBench: Holistic and Contamination Free Evaluation of Large Language Models for Code. June 2024. DOI: 10.48550/arXiv.2403.07974.

Niklas Muennighoff, Zitong Yang, Weijia Shi, Xiang Lisa Li, Li Fei-Fei, Hannaneh Hajishirzi, Luke Zettlemoyer, Percy Liang, Emmanuel Candès and Tatsunori Hashimoto. **s1: Simple test-time scaling**. Mar. 2025. DOI: 10.48550/arXiv.2501.19393.

Simon Ott, Konstantin Hebenstreit, Valentin Liévin, Christoffer Egeberg Hother, Milad Moradi, Maximilian Mayrhauser, Robert Praas, Ole Winther and Matthias Samwald. **ThoughtSource: A central hub for large language model reasoning data**.
In: *Scientific Data* 10.1 (Aug. 2023). ISSN: 2052-4463. DOI: 10.1038/s41597-023-02433-3.

References II

Fengli Xu, Qianyue Hao, Zefang Zong, Jingwei Wang, Yunke Zhang, Jingyi Wang, Xiaochong Lan, Jiahui Gong, Tianjian Ouyang, Fanjin Meng, Chenyang Shao, Yuwei Yan, Qinglong Yang, Yiwen Song, Sijian Ren, Xinyuan Hu, Yu Li, Jie Feng, Chen Gao and Yong Li. **Towards Large Reasoning Models: A Survey of Reinforced Reasoning with Large Language Models**. Jan. 2025. DOI: 10.48550/arXiv.2501.09686.

Kunhao Zheng, Jesse Michael Han and Stanislas Polu. **MiniF2F: a cross-system benchmark for formal Olympiad-level mathematics**. Feb. 2022. DOI: 10.48550/arXiv.2109.00110.