

1. 给定如下训练数据集,

$$\mathbf{x}^1=[3 \ 3], \mathbf{x}^2=[4 \ 3], y^1=1, y^2=1$$

$$\mathbf{x}^3=[1 \ 1], y^3=-1$$

通过求解 SVM 的原始问题来求解最大间隔的分离超平面。

2. 给定如下训练数据集,

$$\mathbf{x}^1=[3 \ 3], \mathbf{x}^2=[4 \ 3], y^1=1, y^2=1$$

$$\mathbf{x}^3=[1 \ 1], y^3=-1$$

通过求解 SVM 的对偶问题来求解最大间隔的分离超平面。

3. 推导软间隔SVM的对偶形式。

4. Show that, irrespective of the dimensionality of the data space, a data set consisting of just two data points (call them $\mathbf{x}(1)$ and $\mathbf{x}(2)$, one from each class) is sufficient to determine the maximum-margin hyperplane. Fully explain your answer, including giving an explicit formula for the solution to the hard margin SVM (i.e., \mathbf{w}) as a function of $\mathbf{x}(1)$ and $\mathbf{x}(2)$.

5. Gaussian kernel takes the form:

$$K(\mathbf{x}, \mathbf{z}) = \exp\left(-\frac{\|\mathbf{x} - \mathbf{z}\|^2}{2}\right)$$

Try to show that the Gaussian kernel can be expressed as the inner product of an infinite-dimensional feature vector.

Hint: Making use of the following expansion, and then expanding the middle factor as a power series.

$$K(\mathbf{x}, \mathbf{z}) = \exp\left(-\frac{\mathbf{x}^T \mathbf{x}}{2\sigma^2}\right) \exp\left(-\frac{\mathbf{x}^T \mathbf{z}}{2\sigma^2}\right) \exp\left(-\frac{\mathbf{z}^T \mathbf{z}}{2\sigma^2}\right)$$