

# All Pictures

all

March 10, 2016

1 1\_DC

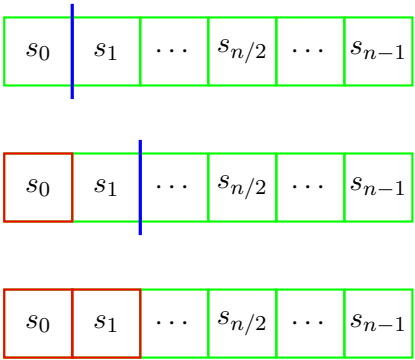


Figure 1: L5-incremental-dc1.png

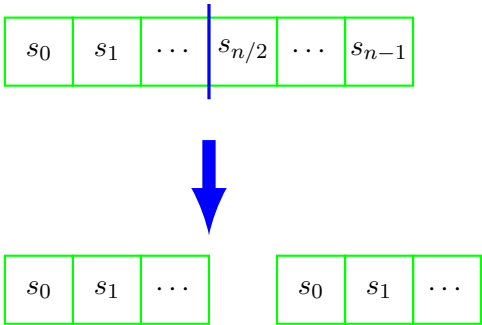


Figure 2: L5-incremental-dc2.png

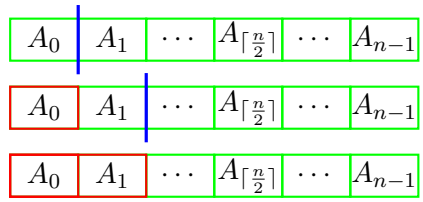


Figure 3: L5-incremental.eps

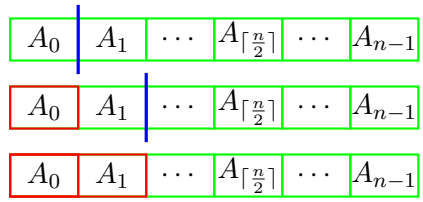
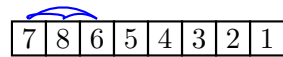
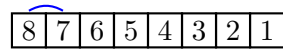
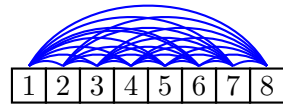


Figure 4: L5-incremental.eps



⋮



INSERTSORT: 28 ops

Figure 5: Analysis of insertionsort algorithm.eps

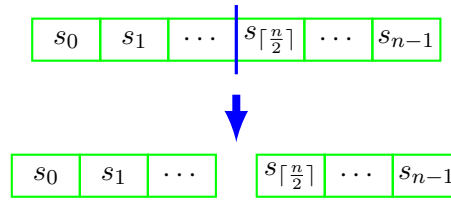


Figure 6: L5-dc.eps

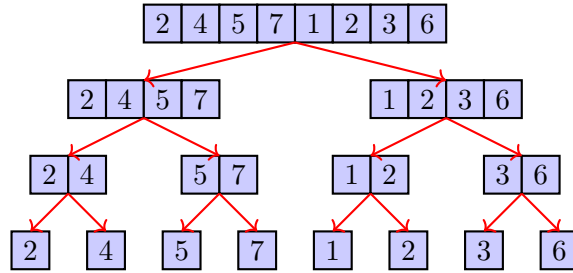


Figure 7: L5-merge-algo.eps

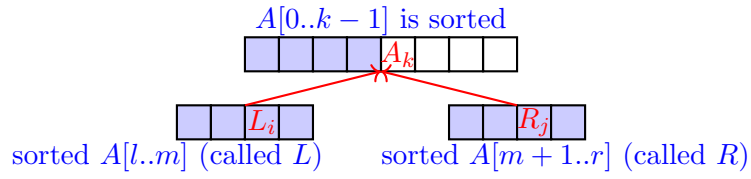


Figure 8: L5-mergesort-example-analysis0(p14).eps

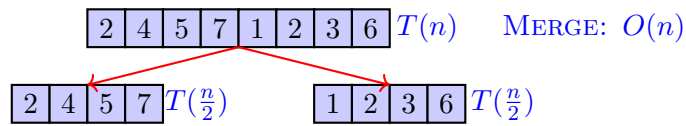


Figure 9: p20.eps

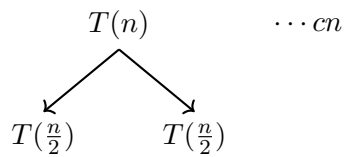


Figure 10: L5-unrolling-tree1.eps

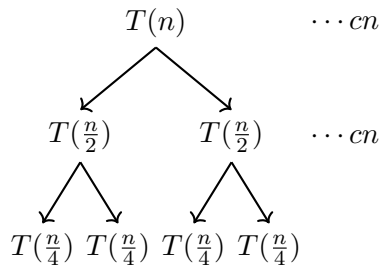


Figure 11: L5-unrolling-tree2.eps

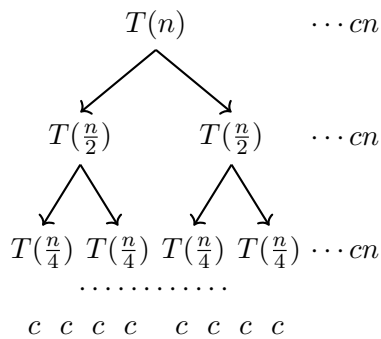


Figure 12: L5-unrolling-tree3.eps

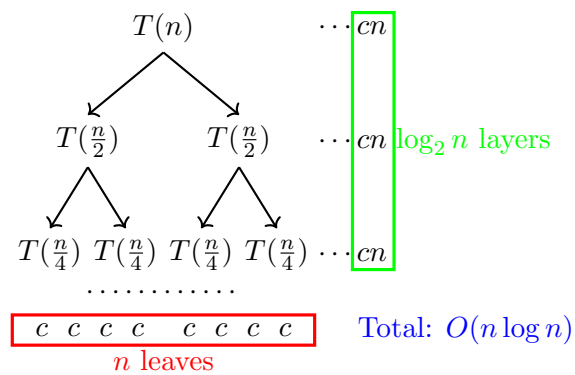
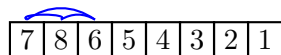
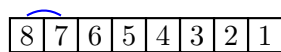
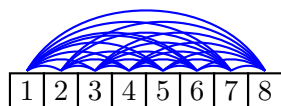


Figure 13: L5-unrolling-tree4.eps



⋮



INSERTSORT: 28 ops

Figure 14: L5-insertsort-left.png

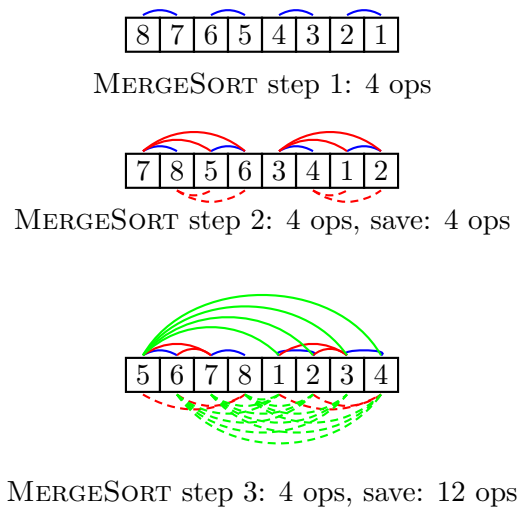
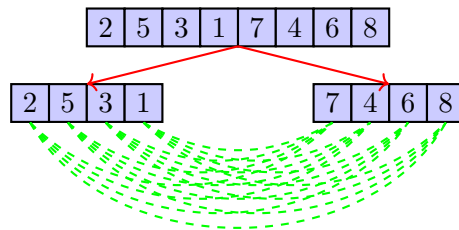
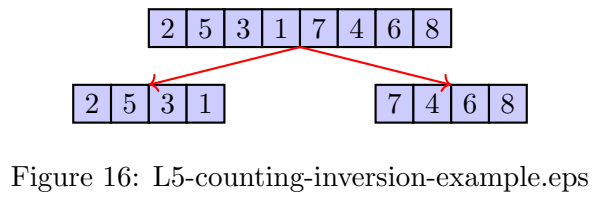


Figure 15: L5-mergesort-right.png



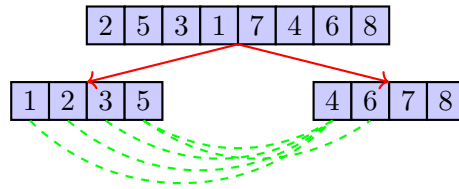


Figure 18: p37 .eps

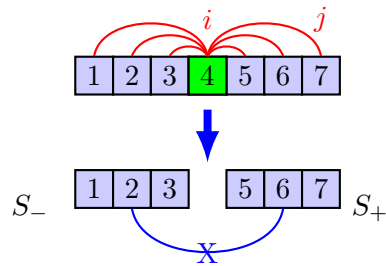


Figure 19: L5-randomizedquicksort.eps

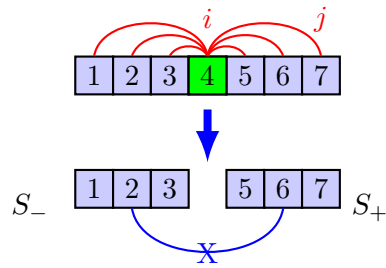


Figure 20: L5-randomizedquicksort.eps(p47) .eps

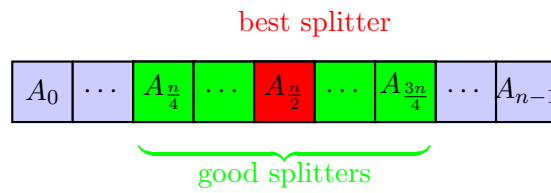


Figure 21: L5-quick-sort-pivot.eps

$\times$	$y_h$	$y_l$
$x_h$	$x_h y_h$	$x_h y_l$
$x_l$	$x_l y_h$	$x_l y_l$

Table 1: P57.eps

#Iteration	$x_i$	$\epsilon_i$
0	0.018700	-0.058223
1	0.032854	-0.044069
2	0.051676	-0.025247
3	0.068636	-0.008286
4	0.076030	-0.000892
5	0.076912	-1.03583e-05
6	0.076923	-1.39483e-09
7	0.076923	-2.77556e-17
8	...	...

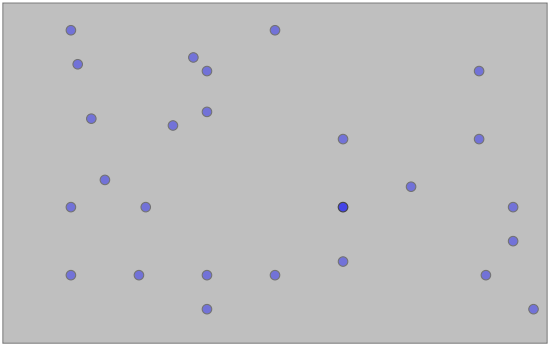


Figure 22: L5-clustering-closestpair.eps



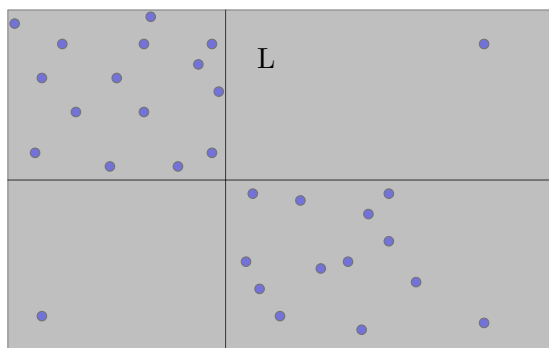


Figure 23: L5-closestpair-4subsets.eps

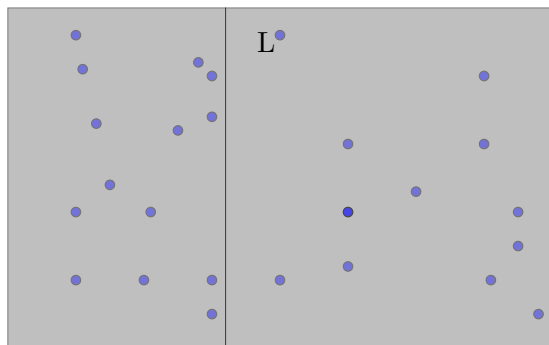


Figure 24: L5-closestpair.eps

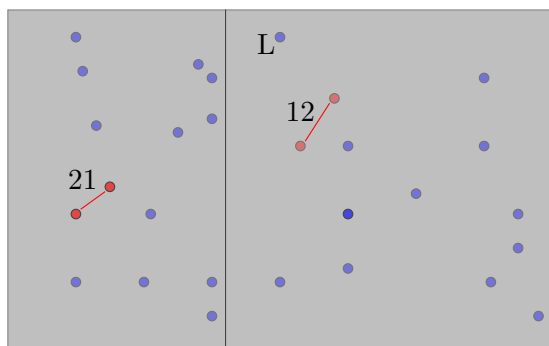


Figure 25: L5-closestpair-1221.eps

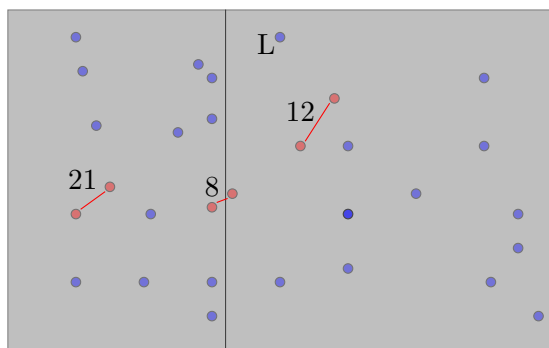


Figure 26: L5-closestpair-12218

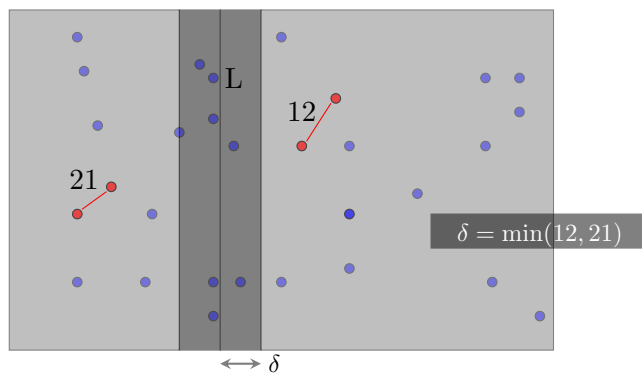


Figure 27: L5-closestpair-1221delta-strip.eps

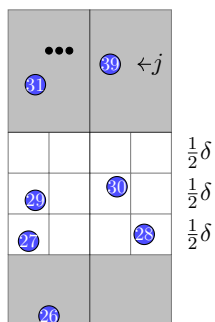
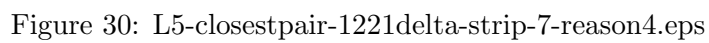
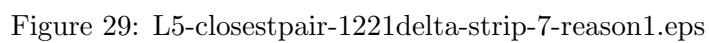


Figure 28: L5-closestpair-1221delta-strip-7-reason.eps



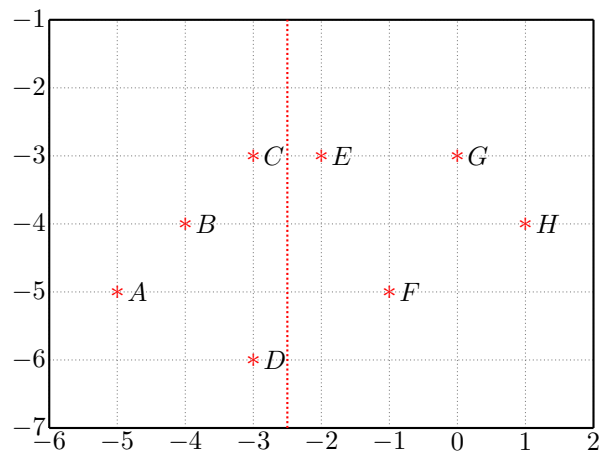


Figure 32: P97.eps

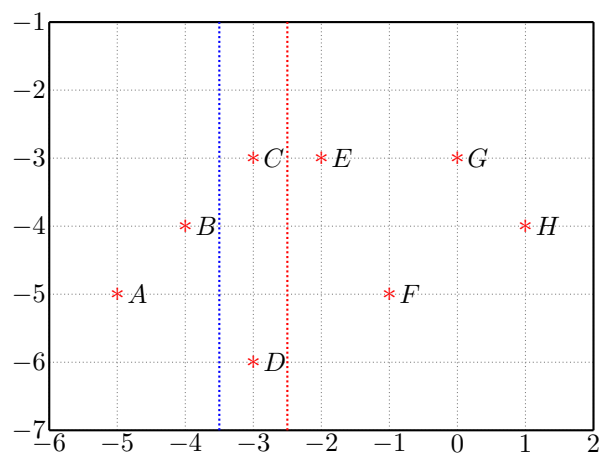


Figure 33: p98 .eps

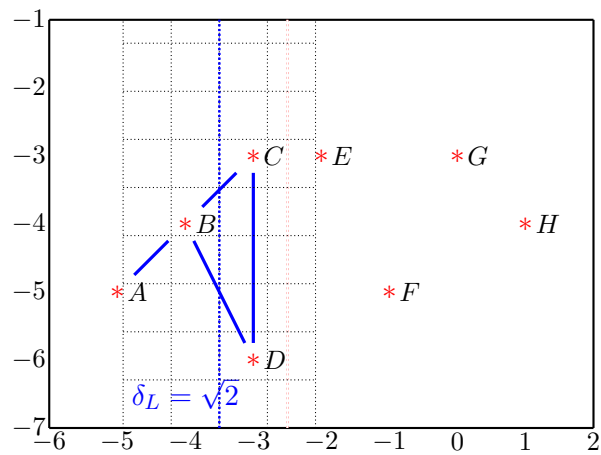


Figure 34: P99.eps

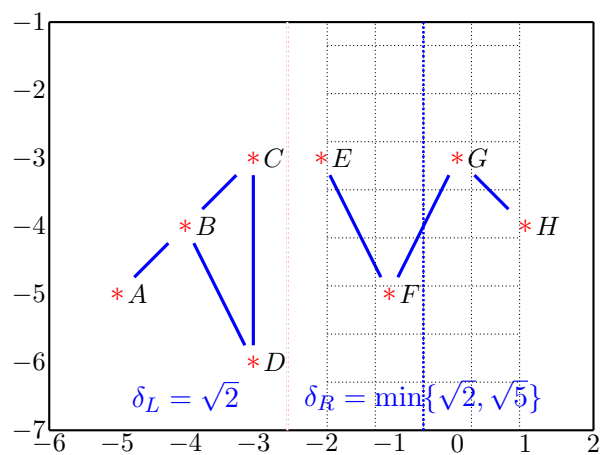


Figure 35: P101.eps

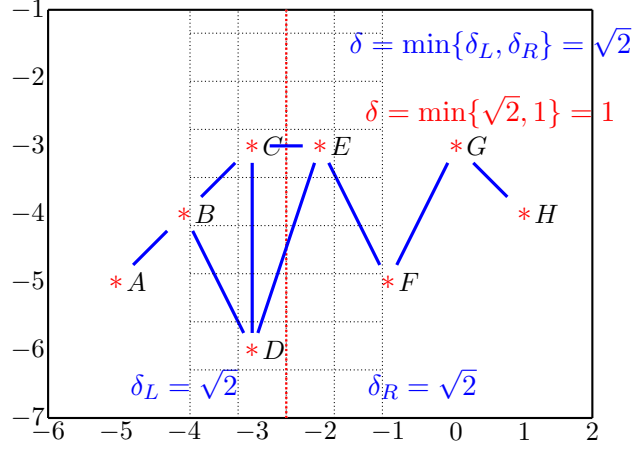


Figure 36: P102.eps

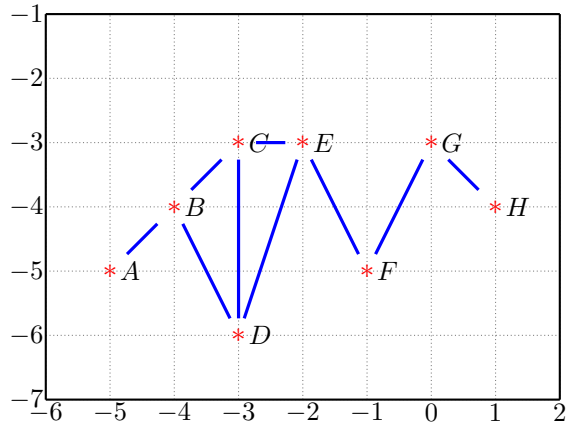


Figure 37: L5-closestpair-ABCDEFGH-solution.eps

## 2 2\_DP\_1

$$\begin{array}{ccccccc}
 A_1 = [1 & 2] & A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} & A_3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} & A_4 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix} \\
 1 \times 2 & & 2 \times 3 & & 3 \times 4 & & 4 \times 5
 \end{array}$$

Table 2: P4-1

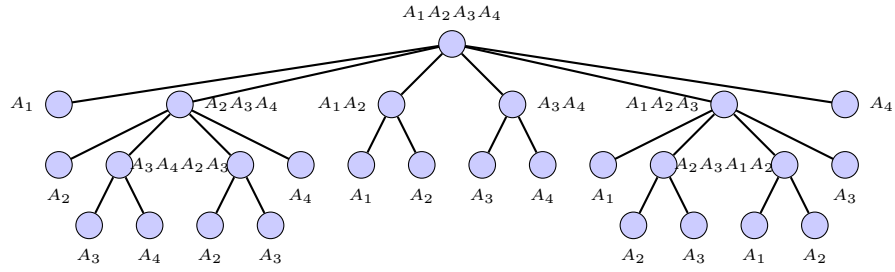


Figure 38: P8-1

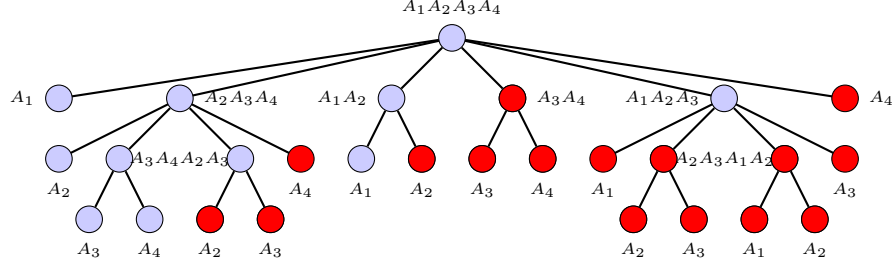


Figure 39: P10-1

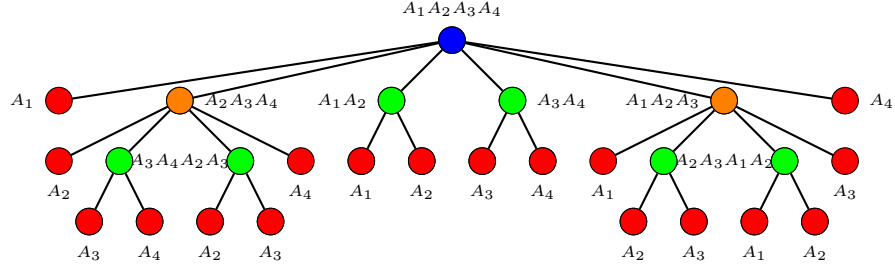


Figure 40: P13-1

OPT					
1	2	3	4		
0	6			1	
	0	24		2	
		0	60	3	
			0	4	

SPLITTER					
1	2	3	4		
	1			1	
		2		2	
			3	3	
				4	

Figure 41: P14-1

OPT					
1	2	3	4		
0	6	18		1	
	0	24	64	2	
		0	60	3	
			0	4	

SPLITTER					
1	2	3	4		
	1	2		1	
		2	3	2	
			3	3	
				4	

Figure 42: P15-1

OPT					
1	2	3	4		
0	6	18	38	1	
	0	24	64	2	
		0	60	3	
			0	4	

SPLITTER					
1	2	3	4		
	1	2	3	1	
		2	3	2	
			3	3	
				4	

Figure 43: P16-1



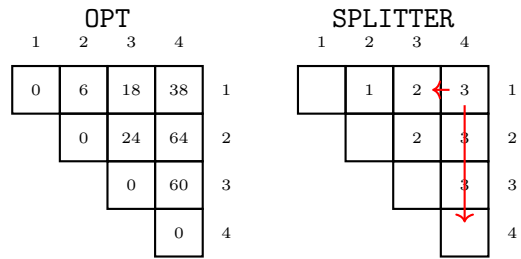


Figure 44: P18-1

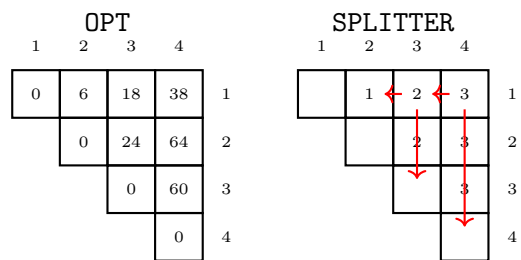


Figure 45: P18-2

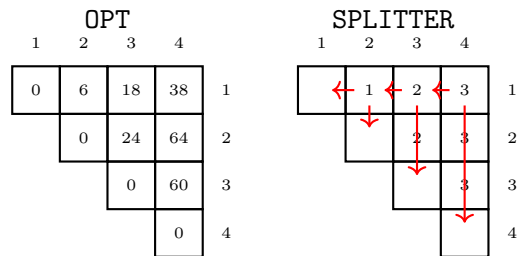


Figure 46: P19-1

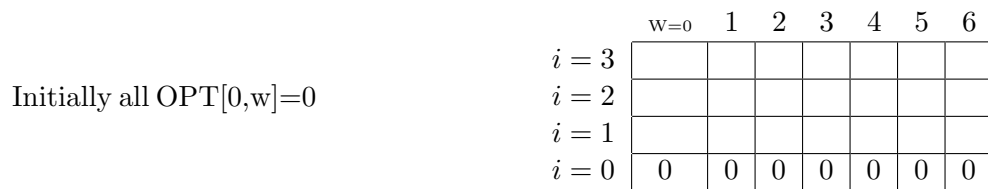


Figure 47: L5-Knapsackalgostep1.eps

$$\begin{aligned} \text{OPT}[1,2] = & \max\{ \\ & \text{OPT}[0,2](=0), \\ & \text{OPT}[0,0] + V_1(=0+2)\} \\ = & 2 \end{aligned}$$

	w=0	1	2	3	4	5	6
$i = 3$							
$i = 2$							
$i = 1$	0	0	2	2	2	2	2
$i = 0$	0	0	0	0	0	0	0

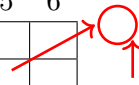


Figure 48: L5-Knapsackalgostep2.eps

$$\begin{aligned} \text{OPT}[2,4] = & \max\{ \\ & \text{OPT}[1,4](=2), \\ & \text{OPT}[1,2] + V_2(=2+2)\} \\ = & 4 \end{aligned}$$

	w=0	1	2	3	4	5	6
$i = 3$							
$i = 2$	0	0	2	2	4	4	4
$i = 1$	0	0	2	2	2	2	2
$i = 0$	0	0	0	0	0	0	0

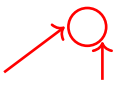


Figure 49: L5-Knapsackalgostep3.eps

$$\begin{aligned} \text{OPT}[3,3] = & \max\{ \\ & \text{OPT}[2,3](=2), \\ & \text{OPT}[2,0] + V_3(=0+3)\} \\ = & 3 \end{aligned}$$

	w=0	1	2	3	4	5	6
$i = 3$	0	0	2	3	4	5	5
$i = 2$	0	0	2	2	4	4	4
$i = 1$	0	0	2	2	2	2	2
$i = 0$	0	0	0	0	0	0	0

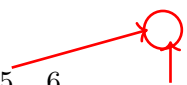



Figure 50: L5-Knapsackalgostep4.eps

Backtracking

$$\begin{aligned} \text{OPT}[3,6] = & \max\{ \\ & \text{OPT}[2,6](=4), \\ & \text{OPT}[2,3] + V_3(=2+3)\} \\ = & 5 \end{aligned}$$

	w=0	1	2	3	4	5	6
$i = 3$	0	0	2	3	4	5	5
$i = 2$	0	0	2	2	4	4	4
$i = 1$	0	0	2	2	2	2	2
$i = 0$	0	0	0	0	0	0	0



Decision: Select item 3

Figure 51: L5-Knapsackalgorithbacktrackstep1.eps

S': O C \_ U R R A \_ \_ \_ N C E

T': O C C U \_ P A T I O N \_ \_

Figure 52: L6-ocurrance-occupation-align.eps

S':O \_ C U R R A N C E

T':O C C U R R E N C E

Figure 53: L6-ocurrance-occurrence-align1.eps

S':O \_ C U R R \_ A N C E

T':O C C U R R E \_ N C E

Figure 54: L6-ocurrance-occurrence-align2.eps

S:	'	'	O	C	U	R	R	A	N	C	E	
T:	'	'	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
			-3	1	-2	-5	-8	-11	-14	-17	-20	-23
O			-6	-2	2	-1	-4	-7	-10	-13	-16	-19
C			-9	-5	-1	1	-2	-5	-8	-11	-12	-15
C			-12	-8	-4	0	0	-3	-6	-9	-12	13
U			-15	-11	-7	-3	1	1	-2	-5	-8	-11
R			-18	-14	-10	-6	-2	2	-	-3	-6	-9
R			-21	-17	-13	-9	-5	-1	1	-1	-4	-5
E			-24	-20	-16	-12	-8	-4	-2	2	-1	-4
N			-27	-23	-19	-15	-11	-7	-5	-1	3	0
C			-30	-26	-22	-18	-14	-10	-8	-4	0	4
E												

Figure 55: P36-1

$$\begin{aligned}
 \text{Score:} \quad & d("OC", "O") = \max \left\{ \begin{array}{ll} d("OC", "") & -3 \quad (= -9) \\ d("O", "") & -1 \quad (= -4) \\ d("O", "O") & -3 \quad (= -2) \end{array} \right. \\
 \text{Alignment:} \quad & S' = OC \\
 & T' = O-
 \end{aligned}$$

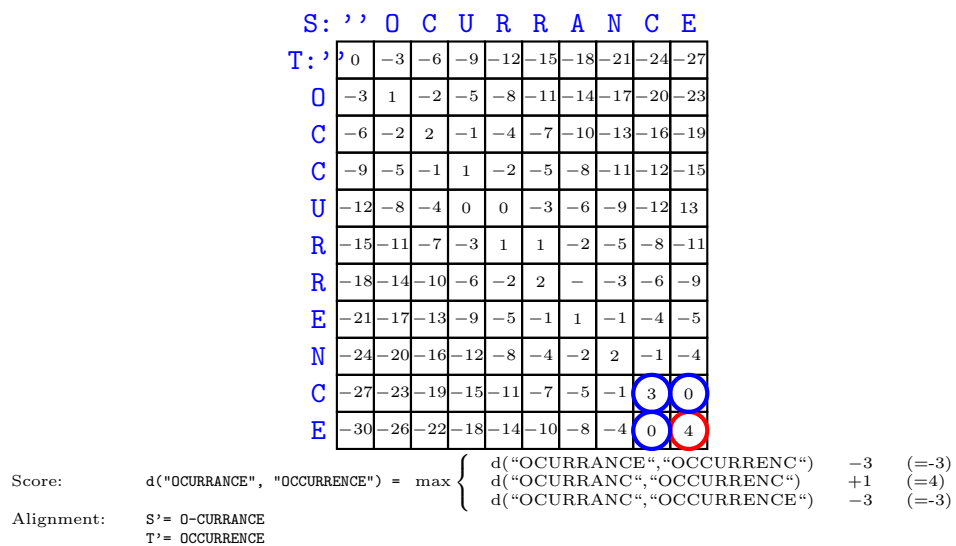
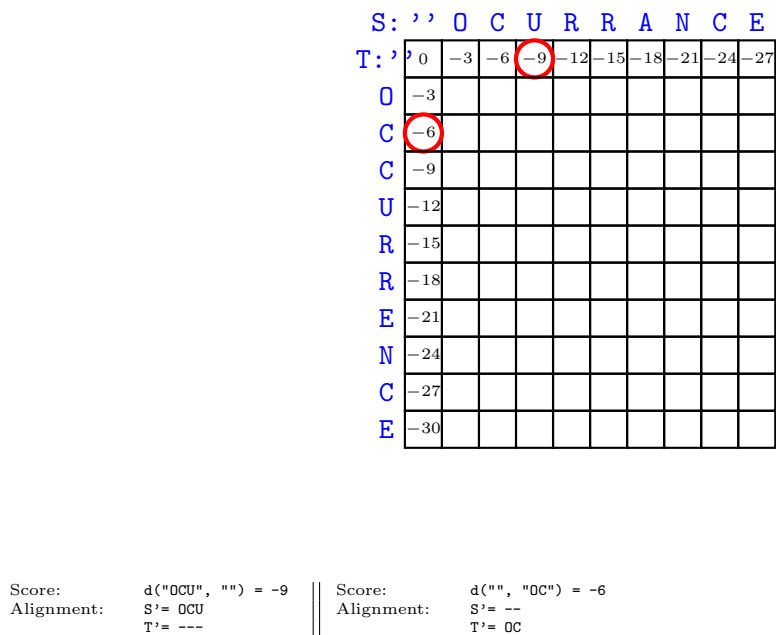


Figure 56: P37-1



### 3 3\_DP\_2

#### 3.1 Hirschberg

##### 3.1.1 First Observe

S:	'	'	O	C	U	R	R	A	N	C	E
T:	'										
	0	-3	-6	-9	-12	-15	-18	-21	-24	-27	
O	-3	1	-2	-5	-8	-11	-14	-17	-20	-23	
C	-6	-2	2	-1	-4	-7	-10	-13	-16	-19	
C	-9	-5	-1	1	-2	-5	-8	-11	-12	-15	
U	-12	-8	-4	0	0	-3	-6	-9	-12	13	
R	-15	-11	-7	-3	1	1	-2	-5	-8	-11	
R	-18	-14	-10	-6	-2	2	-	-3	-6	-9	
E	-21	-17	-13	-9	-5	-1	1	-1	-4	-5	
N	-24	-20	-16	-12	-8	-4	-2	2	-1	-4	
C	-27	-23	-19	-15	-11	-7	-5	-1	3	0	
E	-30	-26	-22	-18	-14	-10	-8	-4	0	4	

Figure 57:

S:	'	'	O	C	U	R	R	A	N	C	E
T:	'	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
O	-3	1	-2	-5	-8	-11	-14	-17	-20	-23	
C	-6	-2	2	-1	-4	-7	-10	-13	-16	-19	
C	-9	-5	-1	1	-2	-5	-8	-11	-12	-15	
U	-12	-8	-4	0	0	-3	-6	-9	-12	13	
R	-15	-11	-7	-3	1	1	-2	-5	-8	-11	
R	-18	-14	-10	-6	-2	2	-	-3	-6	-9	
E	-21	-17	-13	-9	-5	-1	1	-1	-4	-5	
N	-24	-20	-16	-12	-8	-4	-2	2	-1	-4	
C	-27	-23	-19	-15	-11	-7	-5	-1	3	0	
E	-30	-26	-22	-18	-14	-10	-8	-4	0	4	

Figure 58:

;

S:	'	'	O	C	U	R	R	A	N	C	E	
T:	'	'	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
O			-3	1	-2	-5	-8	-11	-14	-17	-20	-23
C			-6	-2	2	-1	-4	-7	-10	-13	-16	-19
C			-9	-5	-1	1	-2	-5	-8	-11	-12	-15
U			-12	-8	-4	0	0	-3	-6	-9	-12	13
R			-15	-11	-7	-3	1	1	-2	-5	-8	-11
R			-18	-14	-10	-6	-2	2	-	-3	-6	-9
E			-21	-17	-13	-9	-5	-1	1	-1	-4	-5
N			-24	-20	-16	-12	-8	-4	-2	2	-1	-4
C			-27	-23	-19	-15	-11	-7	-5	-1	3	0
E			-30	-26	-22	-18	-14	-10	-8	-4	0	4

Figure 59:

;

### 3.1.2 Second Observe

4	0	-4	-10	-12	-16	-18	-22	-26	-30	O
5	3	-1	-7	-9	-13	-15	-19	-23	-27	C
3	6	2	-4	-6	-10	-12	-16	-20	-24	C
-1	2	5	-1	-3	-7	-9	-13	-17	-21	U
-5	-2	1	4	0	-4	-6	-10	-14	-18	R
-9	-6	-3	0	3	-1	-3	-7	-11	-15	R
-13	-10	-7	-4	-1	2	0	-4	-8	-12	E
-15	-12	-9	-6	-3	0	3	-1	-5	-9	N
-19	-16	-13	-10	-7	-4	-1	2	-2	-6	C
-23	-20	-17	-14	-11	-8	-5	-2	1	-3	E
-27	-24	-21	-18	-15	-12	-9	-6	-3	0	''
O	C	U	R	R	A	N	C	E	''	S

Figure 60:

;

### 3.1.3 Third Observe

$$\frac{m}{2}$$

S: O C U R R A N C E

T: O C C U R R E N C E

$$1 \leq q \leq n$$

Figure 61:

;

$$OPT(S, T) = OPT(S[1..\frac{m}{2}], T[1..q]) + OPT(S[\frac{m}{2} + 1..m], T[q + 1..n])$$



3.1.4 Algorithm

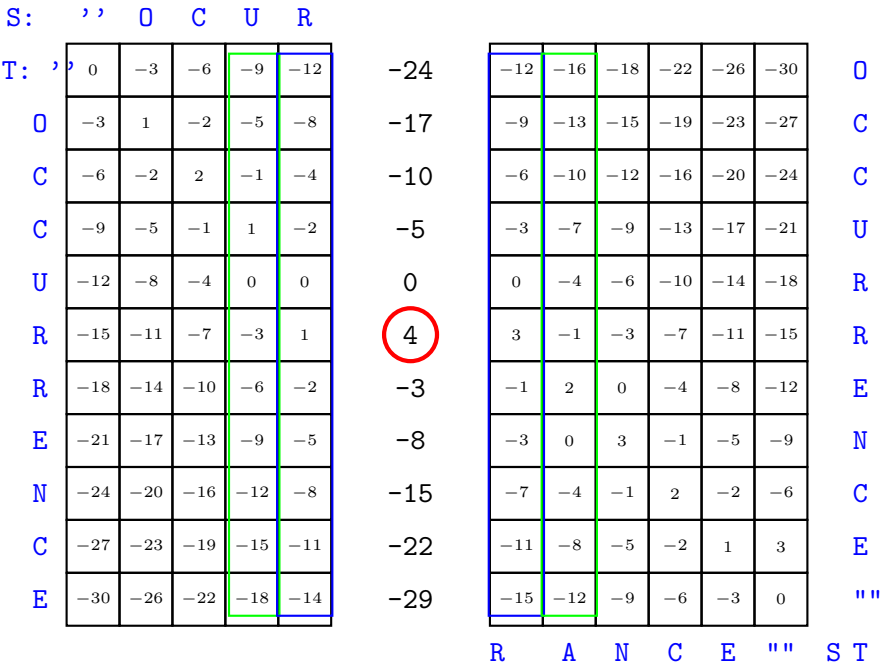


Figure 62: 13.png  
;

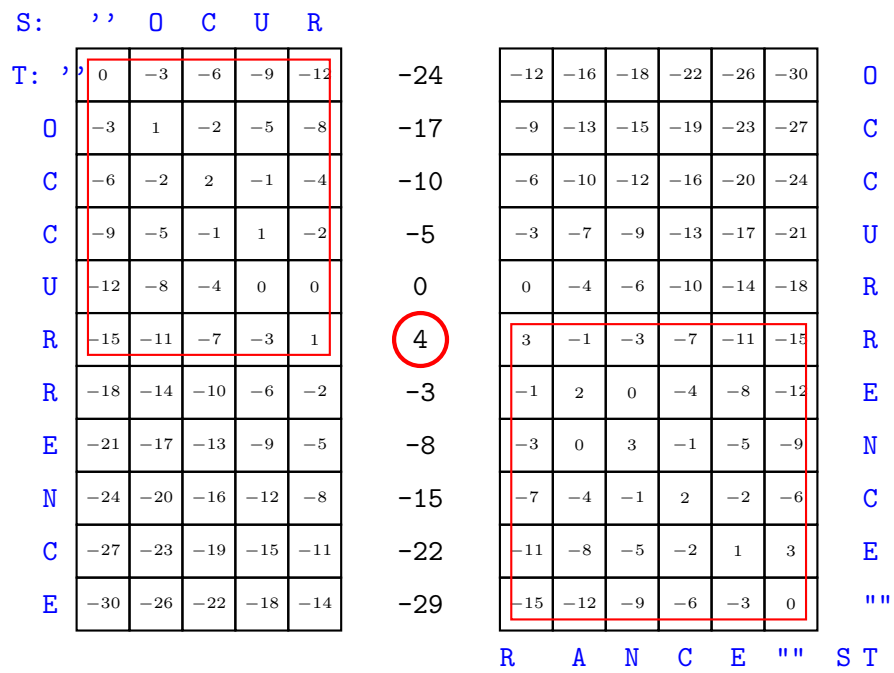


Figure 63: 14.png  
;

### 3.2 Recursive on figure

#### 3.2.1 TSP

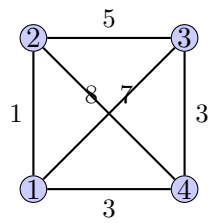


Figure 64:  
;

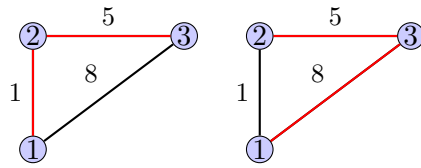


Figure 65:  
;

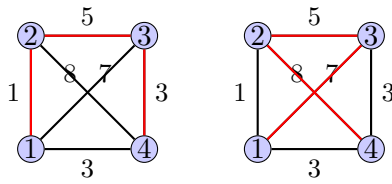


Figure 66:  
;

### 3.2.2 Single source shortest path

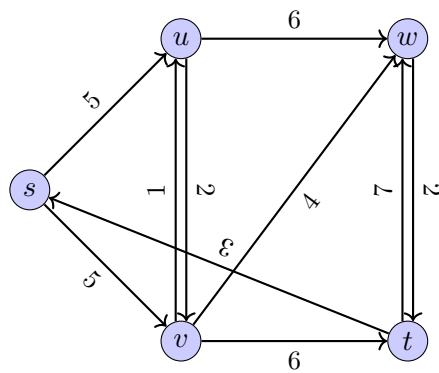


Figure 67: sp.png  
;

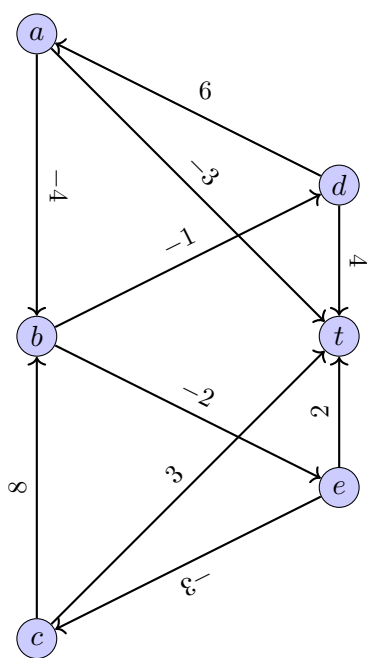


Figure 68: example.png

;

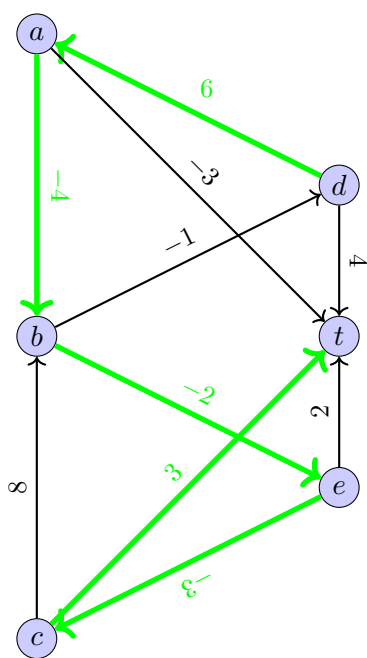


Figure 69: tree.png

;

## 4 4\_Greedy\_1

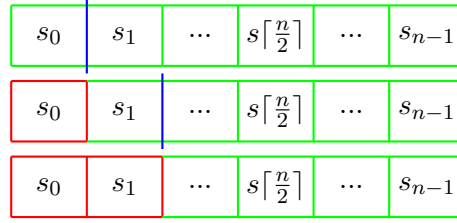


Figure 70: L5-incremental-dc1.png

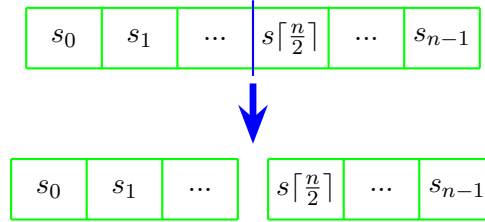


Figure 71: L5-incremental-dc2.png

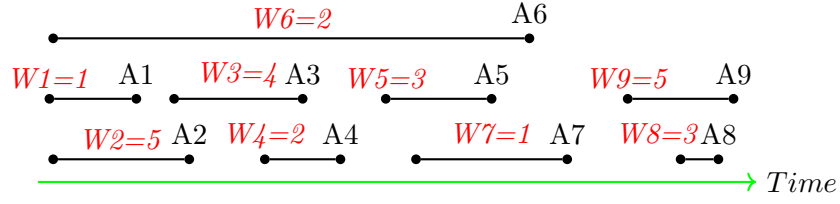


Figure 72: L7-intervalschedulingexample.eps

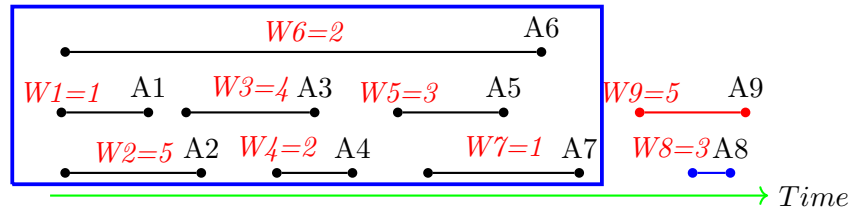


Figure 73: L7-intervalschedulingexamplek1.eps

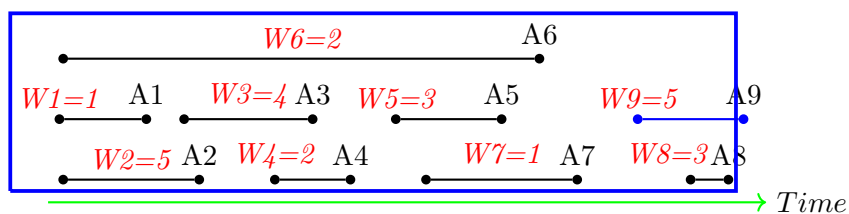


Figure 74: L7-intervalschedulingexamplek2.eps

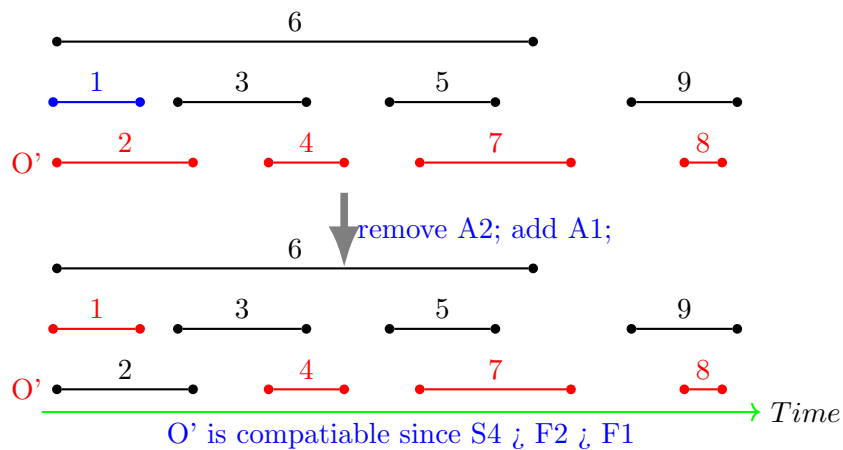


Figure 75: L7-intervalschedulingexampleall1am.eps

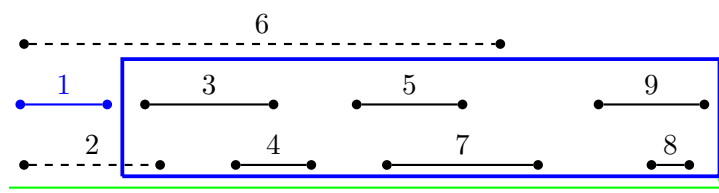


Figure 76: L7-intervalschedulingexamplegreedystep1.eps

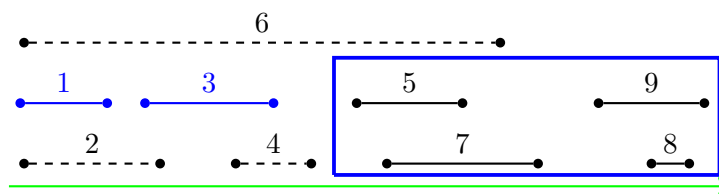


Figure 77: L7-intervalschedulingexamplegreedystep2.eps

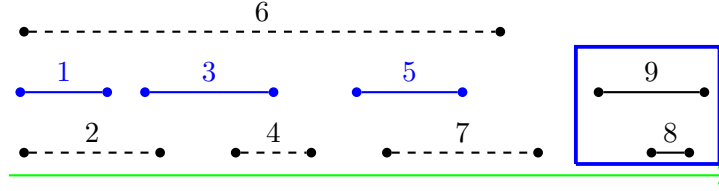


Figure 78: L7-intervalschedulingexamplegreedystep3.eps

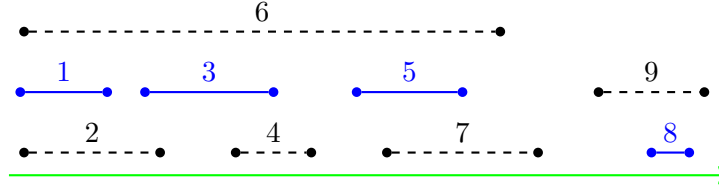


Figure 79: L7-intervalschedulingexamplegreedystep4.eps

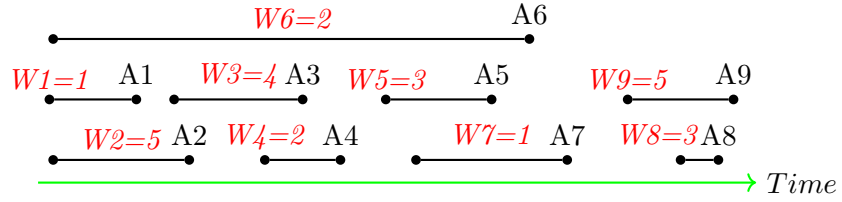


Figure 80: L7-intervalschedulingexample.eps

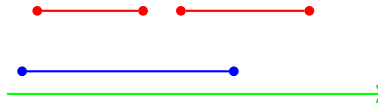


Figure 81: L7-intervalschedulingexample-error2.eps

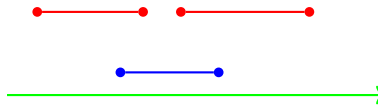


Figure 82: L7-intervalschedulingexample-error1.eps

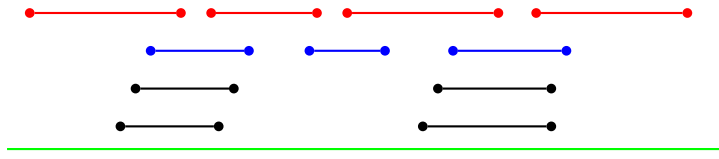


Figure 83: L7-intervalschedulingexample-error3.eps



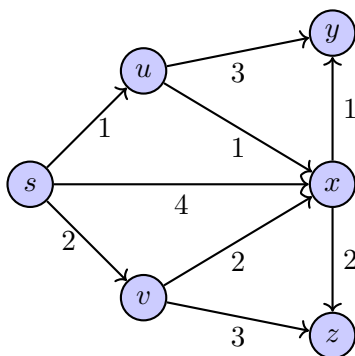


Figure 84: L7-shortestpathexample.png

	k=0	1	2	3	4	5
S	0	0	0	0	0	0
U	-	1	1	1	1	1
V	-	2	2	2	2	2
X	-	4	2	2	2	2
Y	-	-	4	3	3	3
Z	-	-	5	4	4	4

Figure 85: L7-Dijkstraexample.png

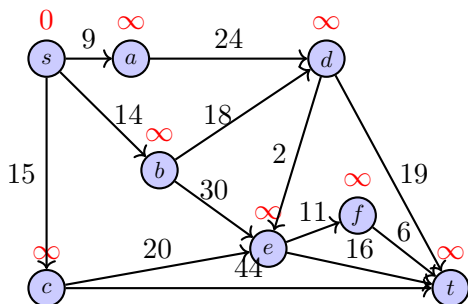


Figure 86: Dijkstra\_demo.png

Operation	Linked List
INSERT	$O(1)$
EXTRACTMIN	$O(n)$
DECREASEKEY	$O(1)$
UNION	$O(1)$

Table 3: L7-heaptablelinkedlist.png

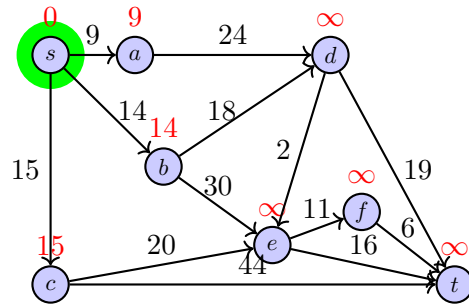


Figure 87: Dijkstra\_demo\_1.png

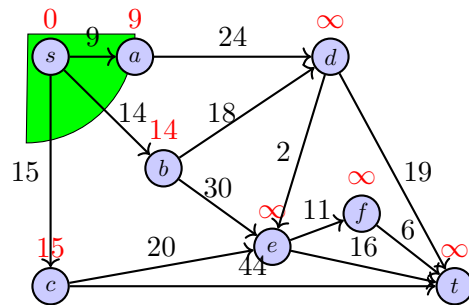


Figure 88: Dijkstra\_demo\_2.png

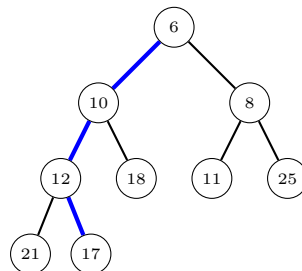


Figure 89: P16-1

Operation	Linked List	Binary Heap	Binomial Heap
<b>INSERT</b>	$O(1)$	$O(\log n)$	$O(\log n)$
<b>EXTRACTMIN</b>	$O(n)$	$O(\log n)$	$O(\log n)$
<b>DECREASEKEY</b>	$O(1)$	$O(\log n)$	$O(\log n)$
<b>UNION</b>	$O(1)$	$O(n)$	$O(\log n)$

Table 4: L7-heaptablebinomialheap.png

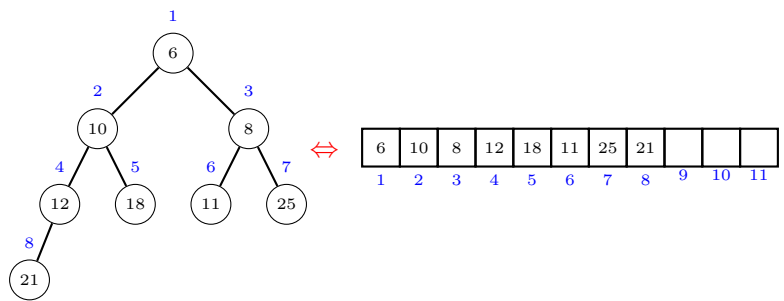


Figure 90: P16-2

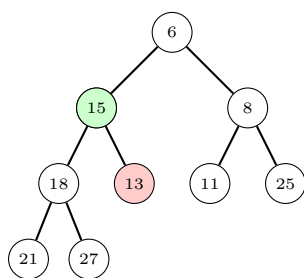


Figure 91: P16-3

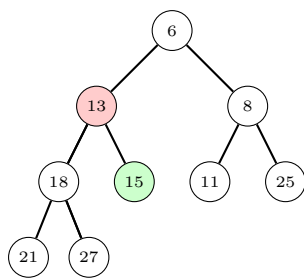


Figure 92: P16-4

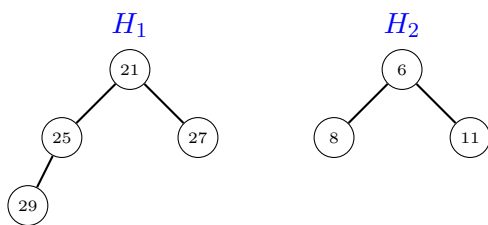


Figure 93: P17-1

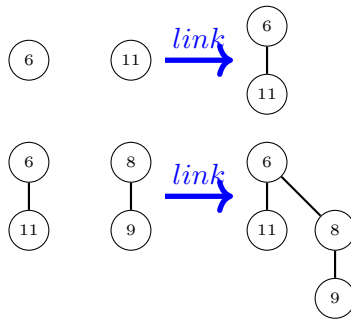


Figure 94: P18-1

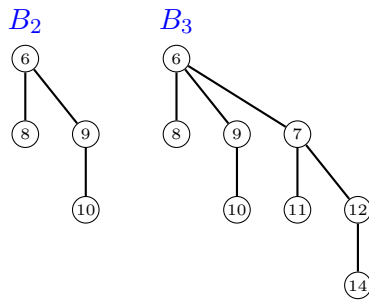


Figure 95: P18-2

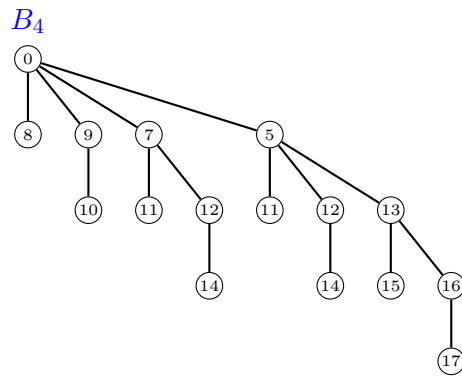


Figure 96: P18-3

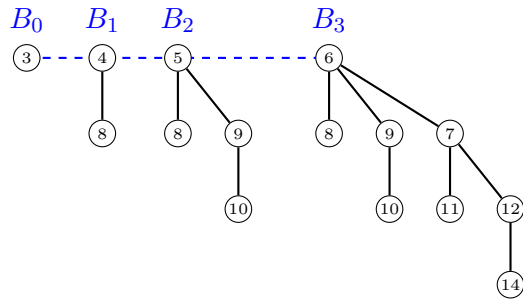


Figure 97: P19-1

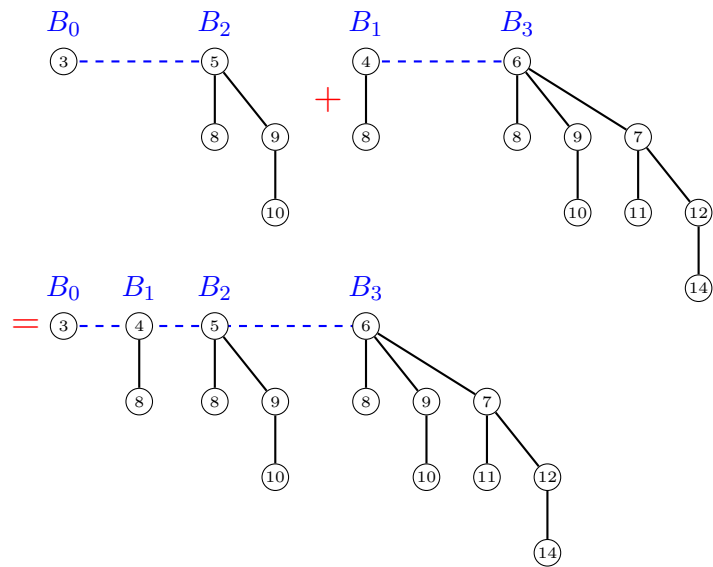


Figure 98: P19-2

## 5 5\_Greedy\_2

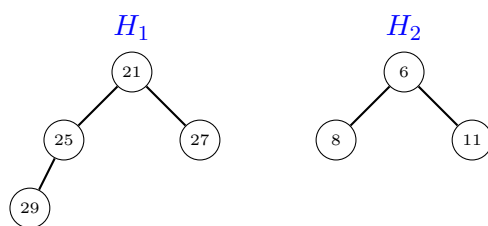


Figure 99: P2-1

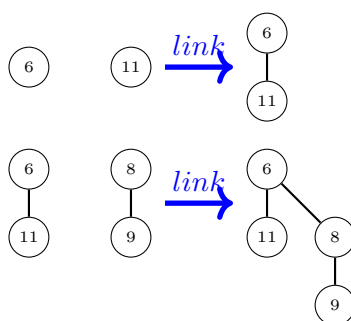


Figure 100: P3-1

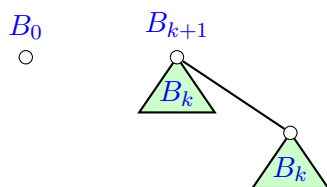


Figure 101: P3-2



Figure 102: P3-3



Figure 103: P3-4

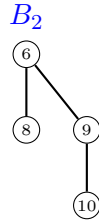


Figure 104: P4-1

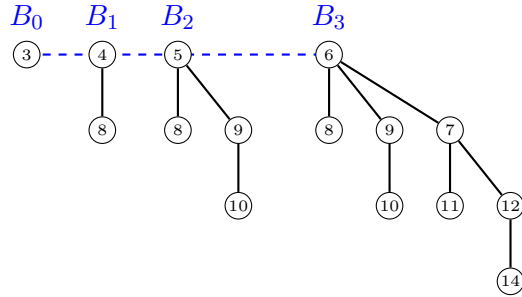


Figure 105: P4-2

Operation	Linked List	Binary Heap	Binomial Heap
<b>INSERT</b>	$O(1)$	$O(\log n)$	$O(\log n)$
<b>EXTRACTMIN</b>	$O(n)$	$O(\log n)$	$O(\log n)$
<b>DECREASEKEY</b>	$O(1)$	$O(\log n)$	$O(\log n)$
<b>UNION</b>	$O(1)$	$O(n)$	$O(\log n)$

Table 5: P7-1

Operation	Linked List	Binary Heap	Binomial Heap	Binomial Heap*
<b>INSERT</b>	$O(1)$	$O(\log n)$	$O(\log n)$	$O(1)$
<b>EXTRACTMIN</b>	$O(n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
<b>DECREASEKEY</b>	$O(1)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
<b>UNION</b>	$O(1)$	$O(n)$	$O(\log n)$	$O(1)$

Table 6: P7-2

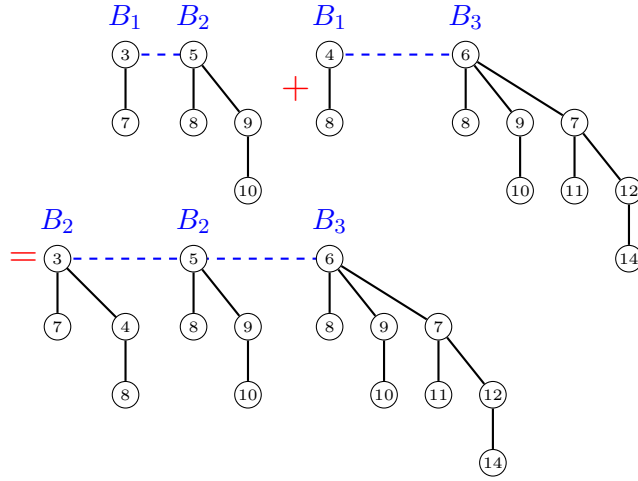


Figure 106: P5-1

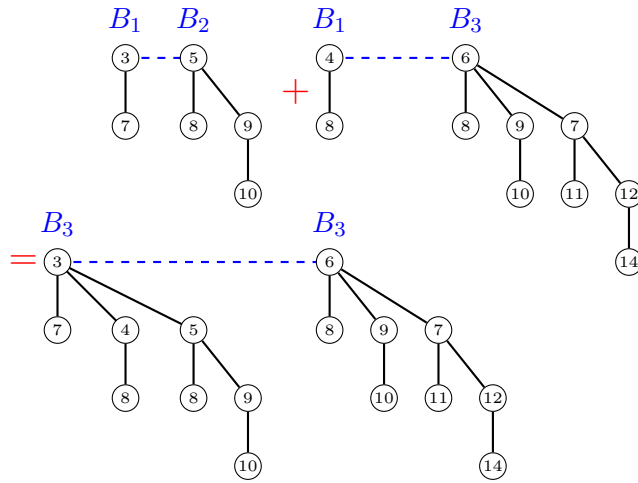


Figure 107: P5-2

Operation	Linked List	Binary Heap	Binomial Heap	Binomial Heap*	Fibonacci Heap*
<b>INSERT</b>	$O(1)$	$O(\log n)$	$O(\log n)$	$O(1)$	$O(1)$
<b>EXTRACTMIN</b>	$O(n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(\log n)$
<b>DECREASEKEY</b>	$O(1)$	$O(\log n)$	$O(\log n)$	$O(\log n)$	$O(1)$
<b>UNION</b>	$O(1)$	$O(n)$	$O(\log n)$	$O(1)$	$O(1)$

Table 7: P18-1



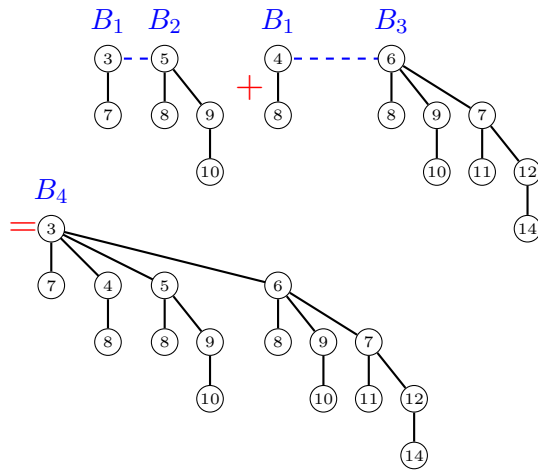


Figure 108: P6-1

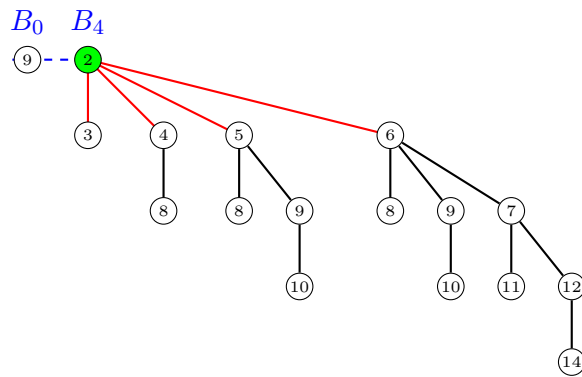


Figure 109: P6-2

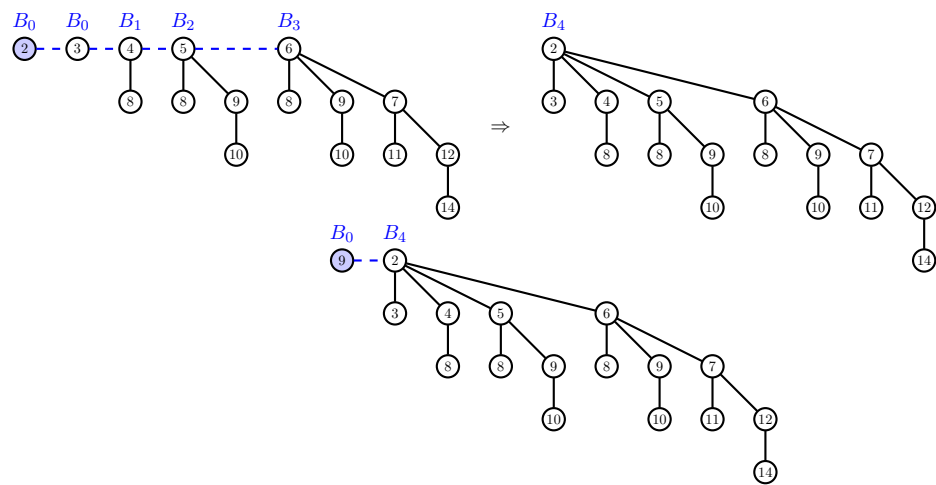


Figure 110: P8-1

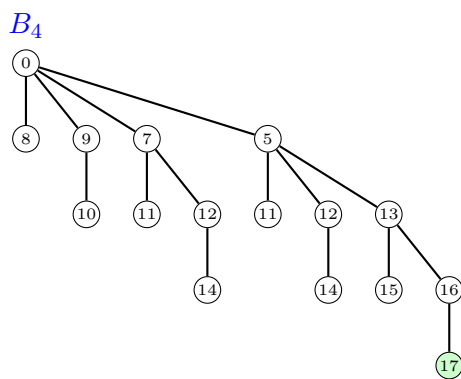


Figure 111: P10-1

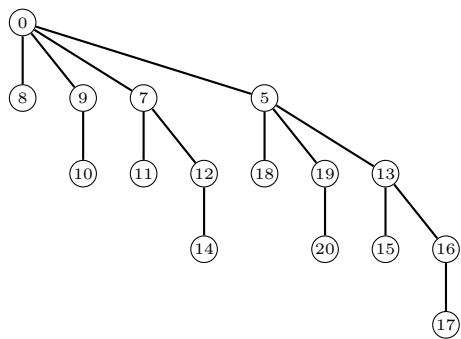


Figure 112: P11-1

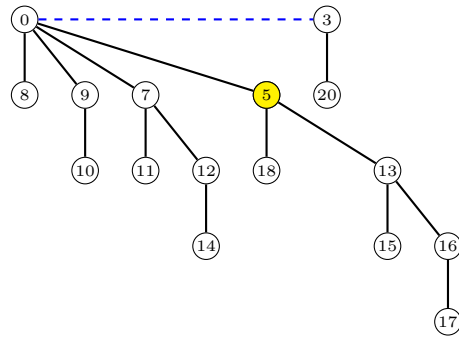


Figure 113: P11-2

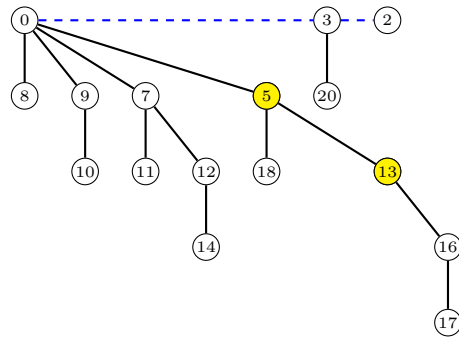


Figure 114: P11-3

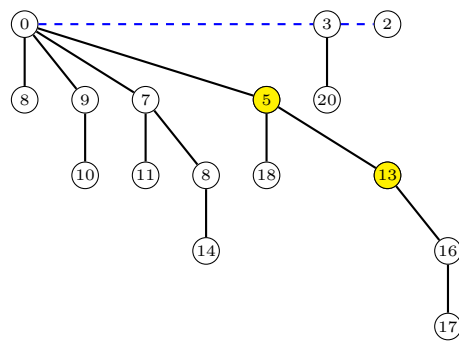


Figure 115: P12-1

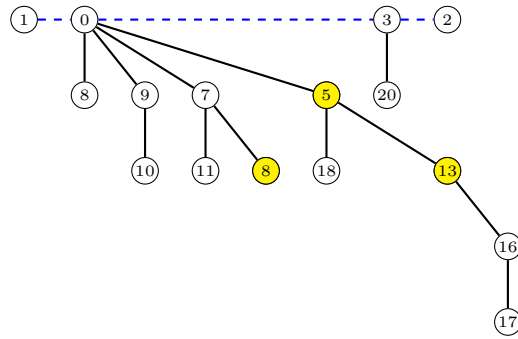


Figure 116: P12-2

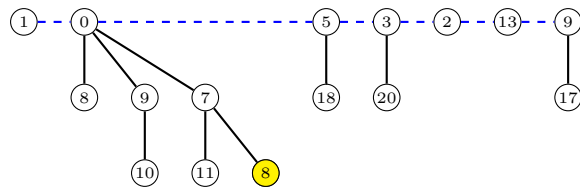


Figure 117: P12-3

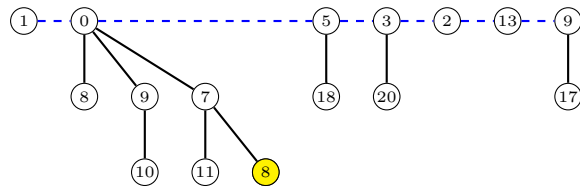


Figure 118: P13-1

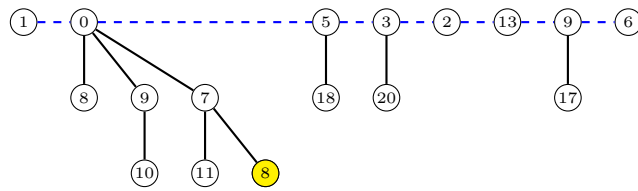


Figure 119: P13-2

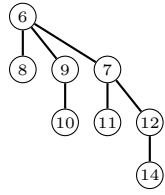


Figure 120: P14-1

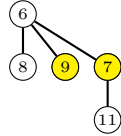


Figure 121: P15-1

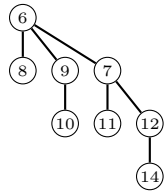


Figure 122: P15-2

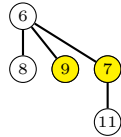


Figure 123: P15-3

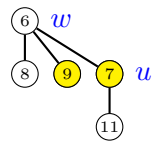


Figure 124: P16-1



Figure 125: P16-2



Figure 126: P16-3

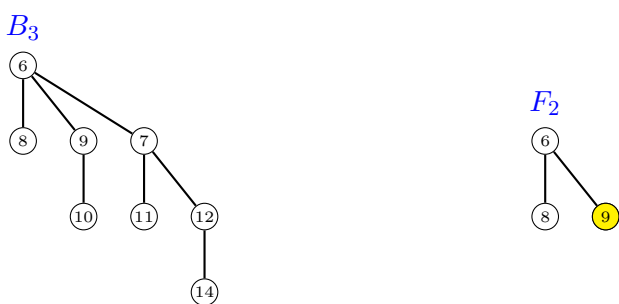


Figure 127: P17-1

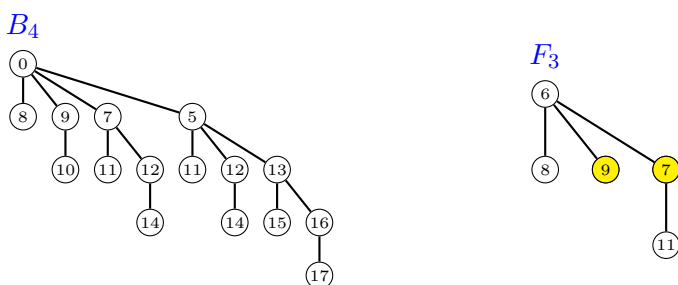


Figure 128: P17-2

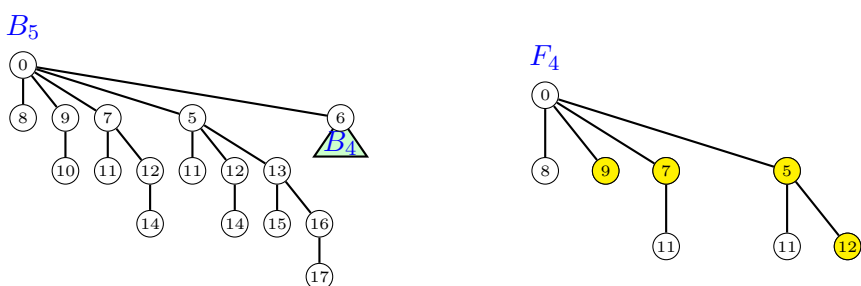


Figure 129: P17-3

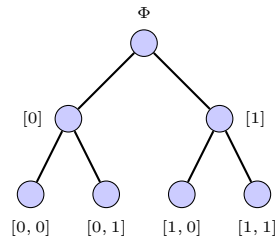


Figure 130: P19-1

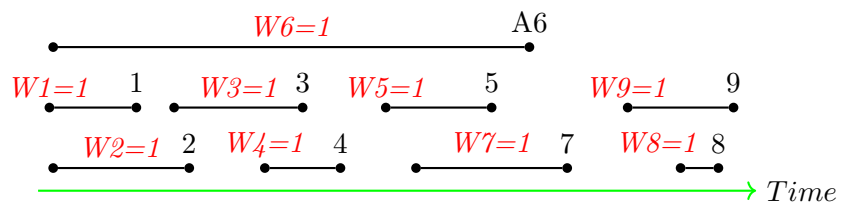


Figure 131: L7-intervalschedulingexampleall1.eps

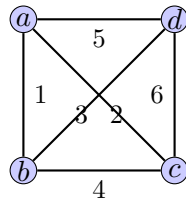


Figure 132: P19-3

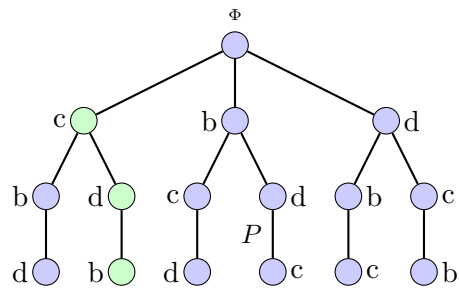


Figure 133: P20-1

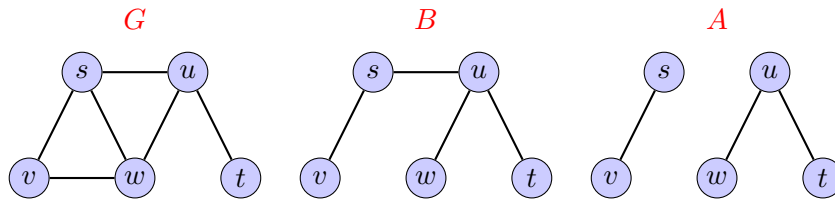


Figure 134: P22-1

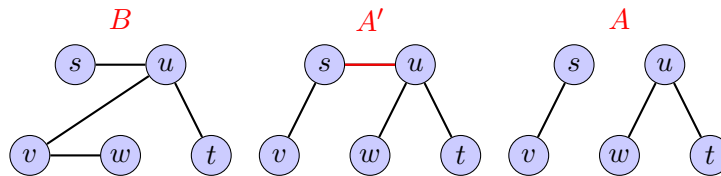


Figure 135: P22-2

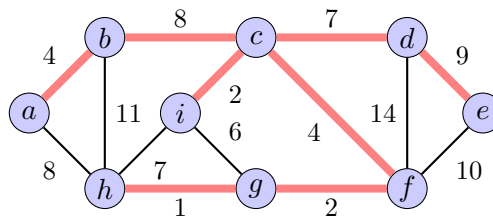


Figure 136: P23-1

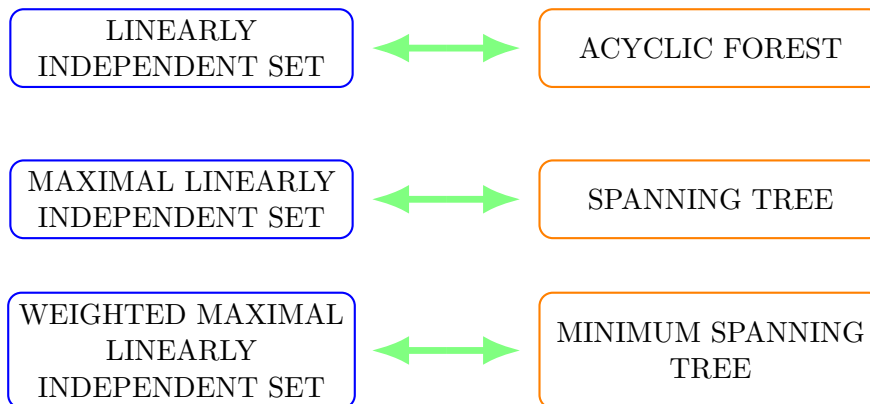


Figure 137: P23-2



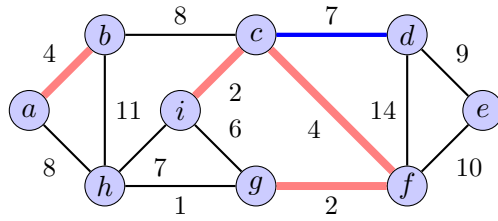


Figure 138: P23-3

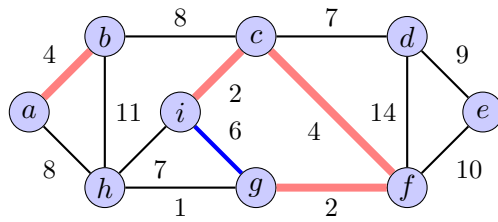


Figure 139: P24-1

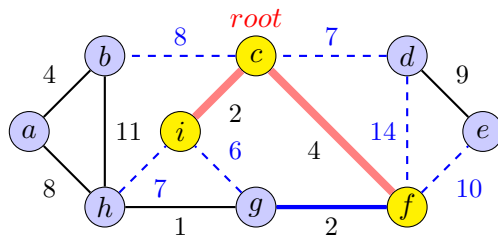


Figure 140: P25-1

## 6 LP\_1-3

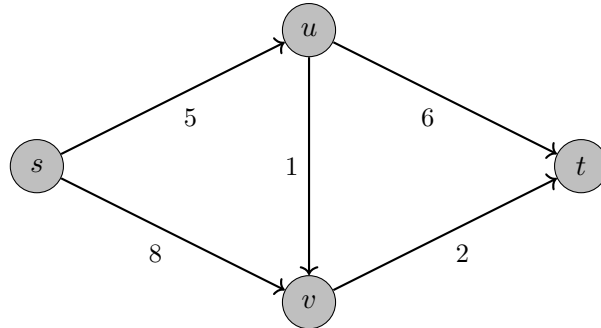


Figure 141: L8-networkflowexample.png

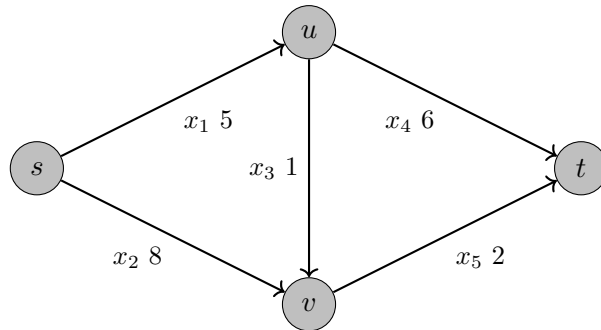


Figure 142: L8-networkflowexampleLP.eps

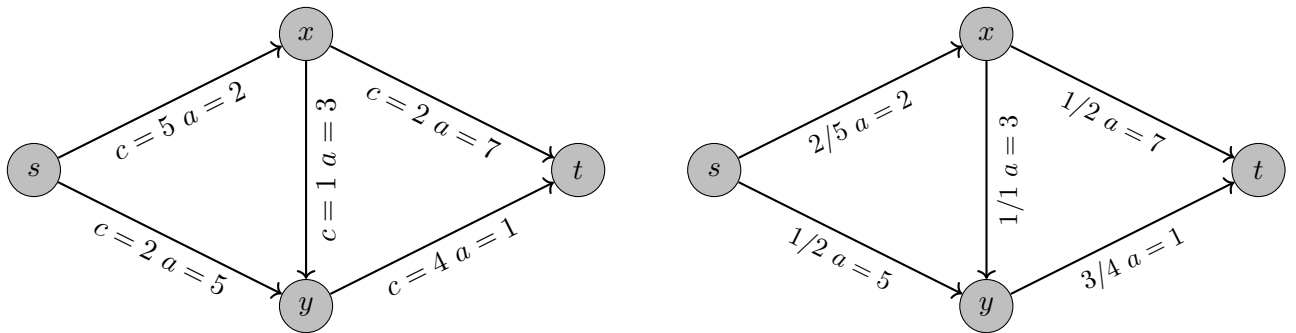


Figure 143: Pic3.eps:L8-LPminimumcostflow1.eps + L8-LPminimumcostflow2.eps

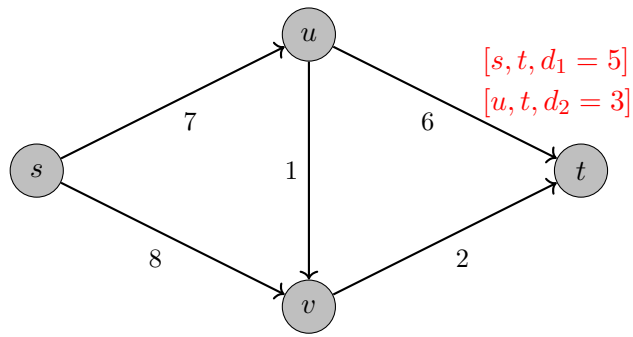


Figure 144: L8-multicommodityflowexample.eps

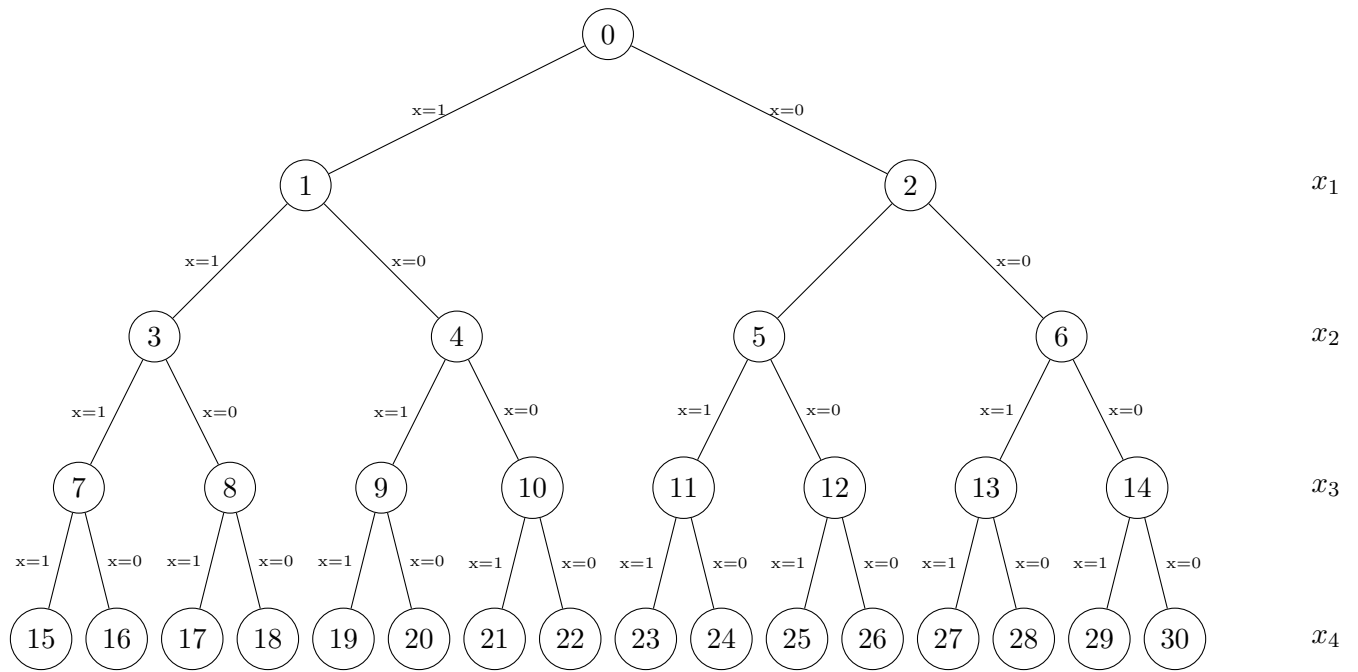


Figure 145: Branch-and-bound1.png

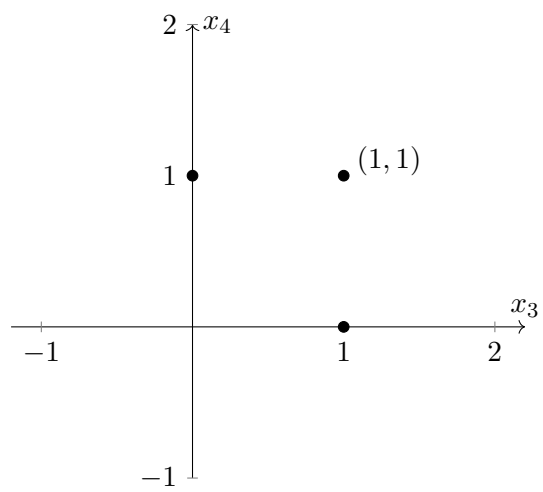


Figure 146: L8-LEexample.png

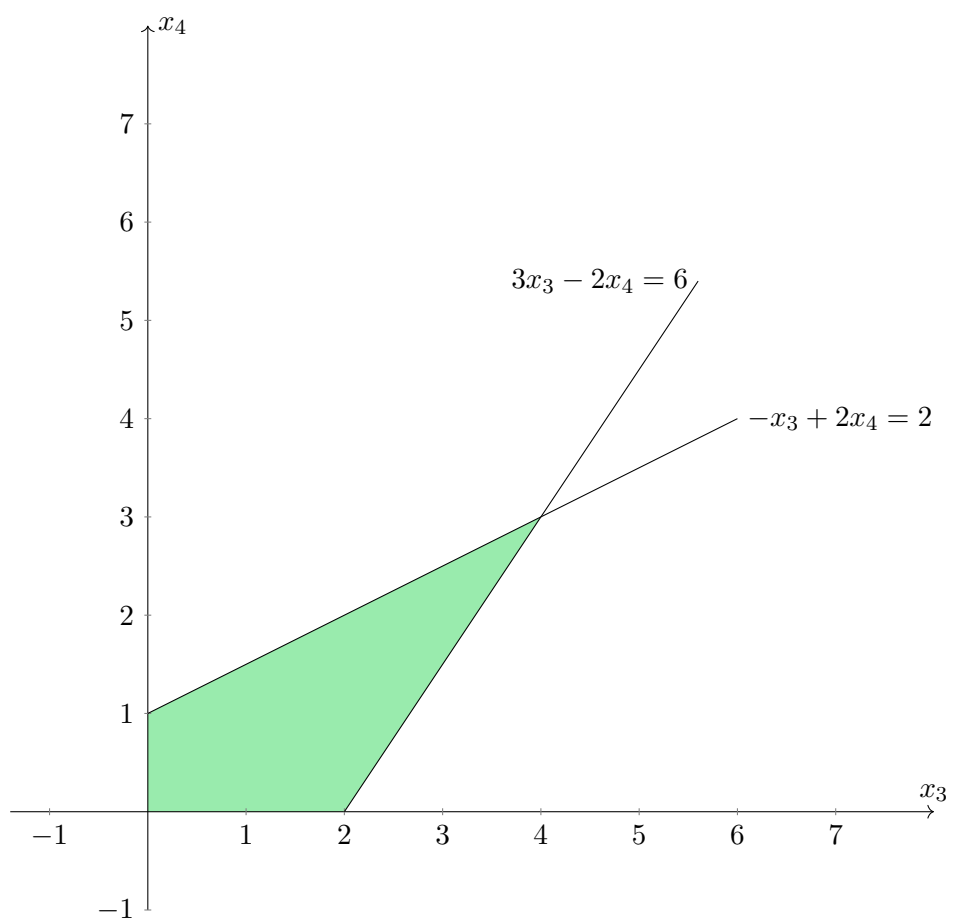


Figure 147: L8-LP-GE.png

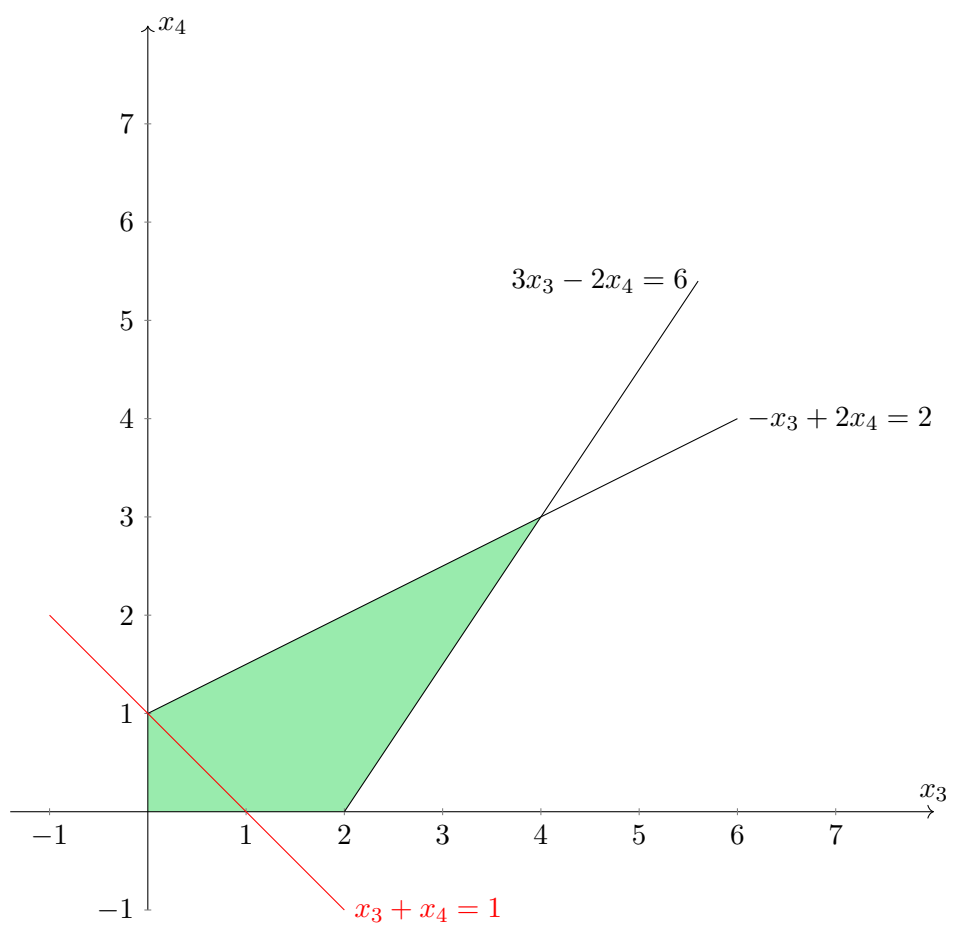


Figure 148: L8-LP-GE1.png

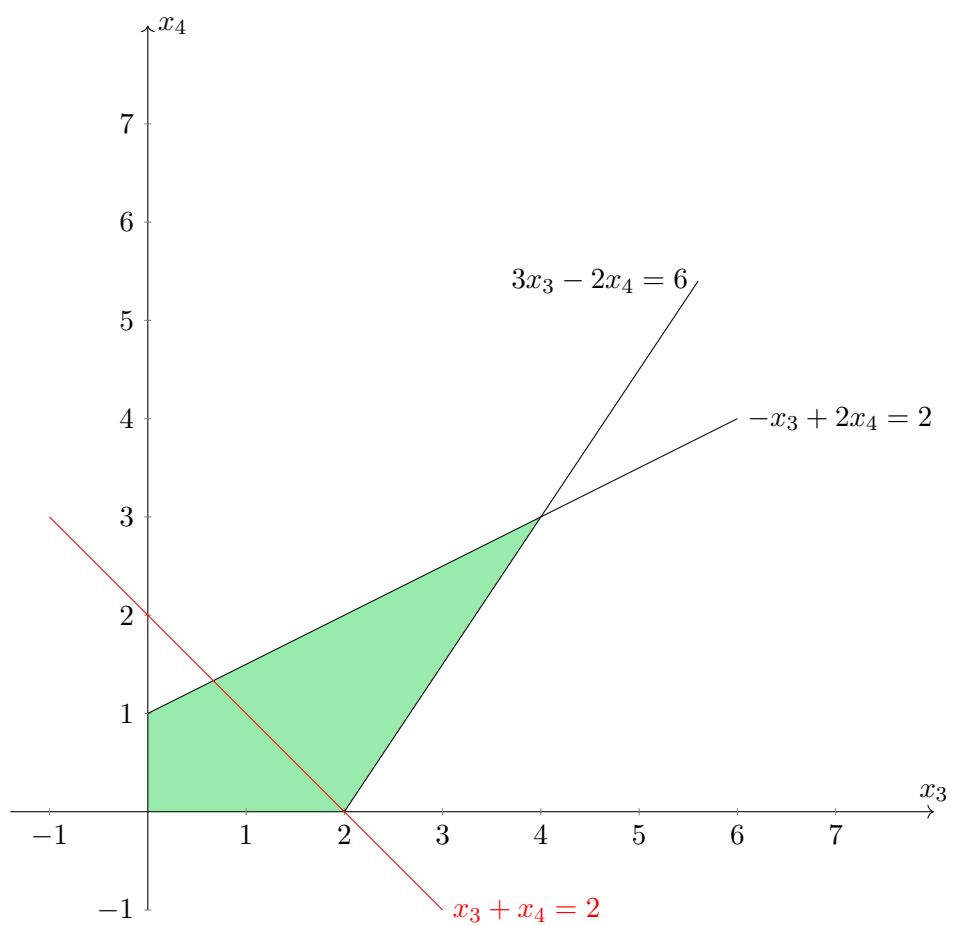


Figure 149: L8-LP-GE2.png

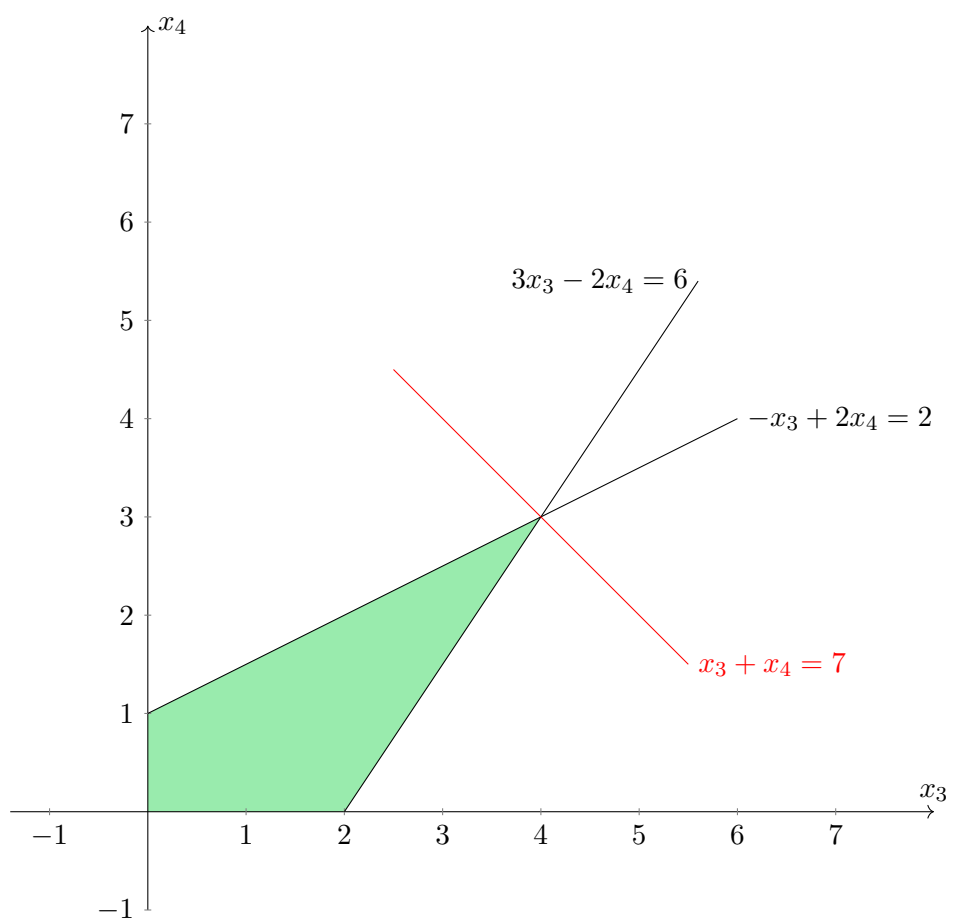


Figure 150: L8-LP-GE3.png



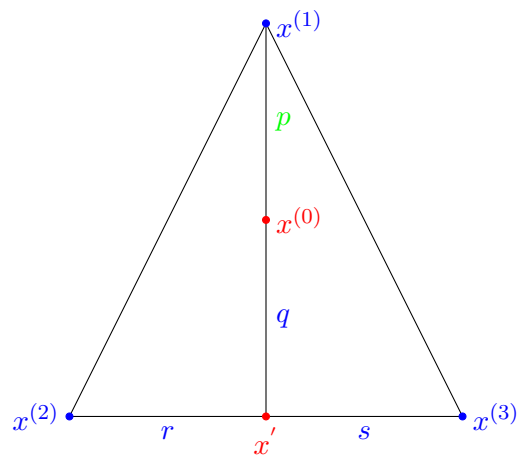


Figure 151: L8-x1x2x3.eps

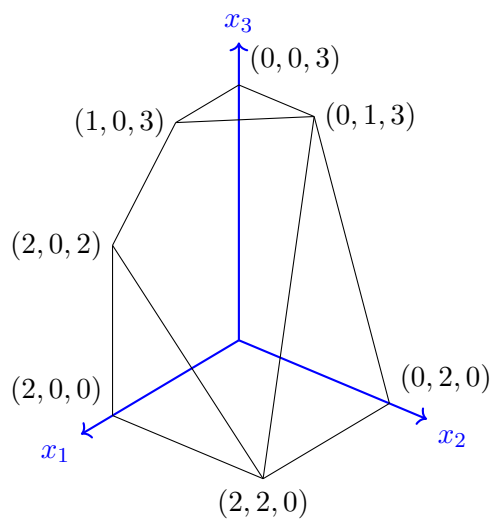


Figure 152: L8-LPexample3D.eps

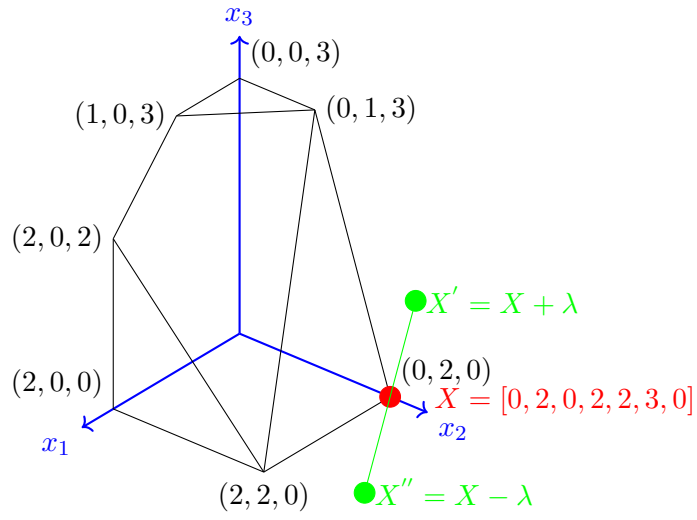


Figure 153: L8-LPexample3Dvertex.eps

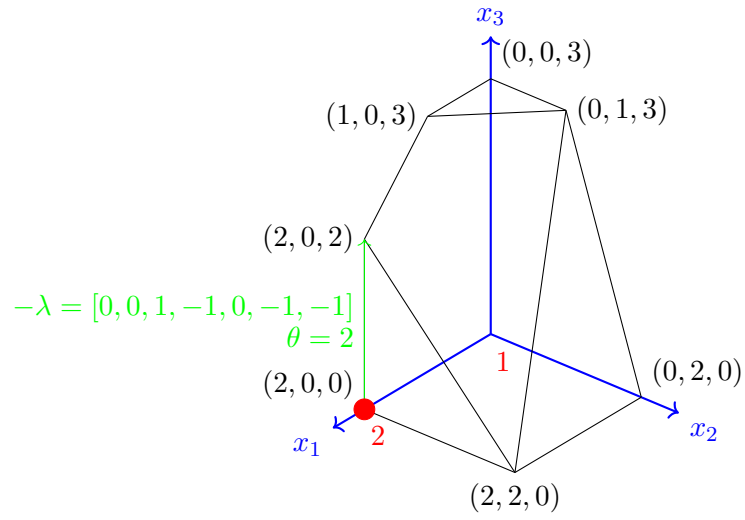


Figure 154: L8-LPexample3Dstep2.eps

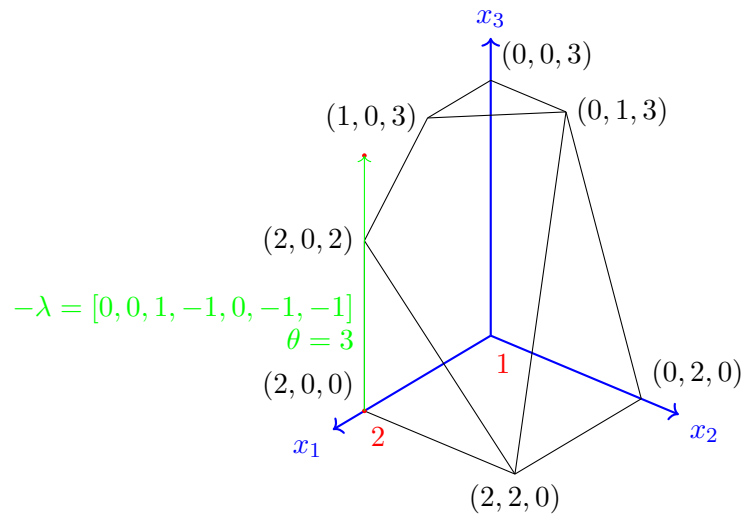


Figure 155: L8-LPexample3Dstep2theta3.eps

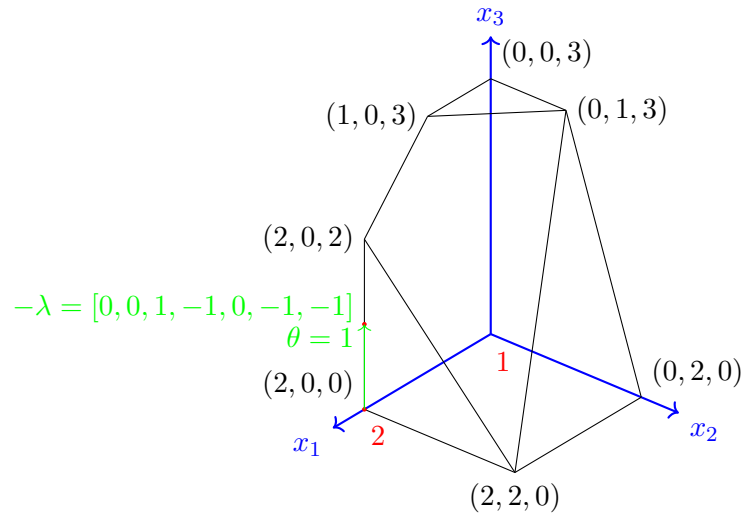


Figure 156: L8-LPexample3Dstep2theta1.eps

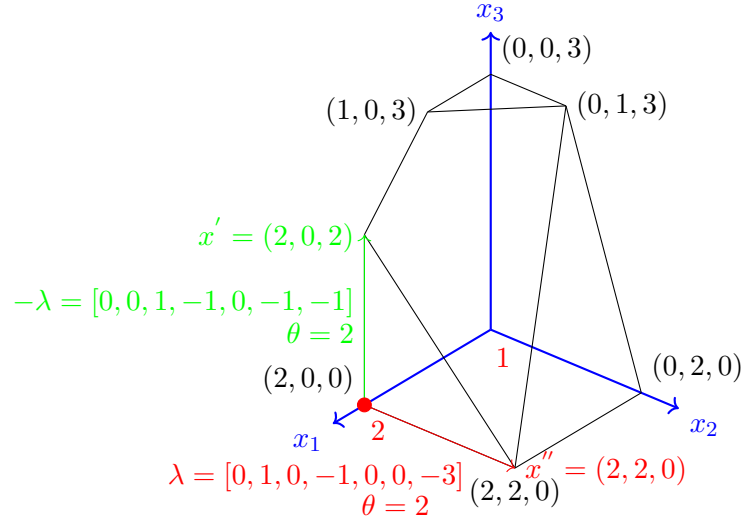


Figure 157: L8-LPexample3Dstep2twoedges.eps

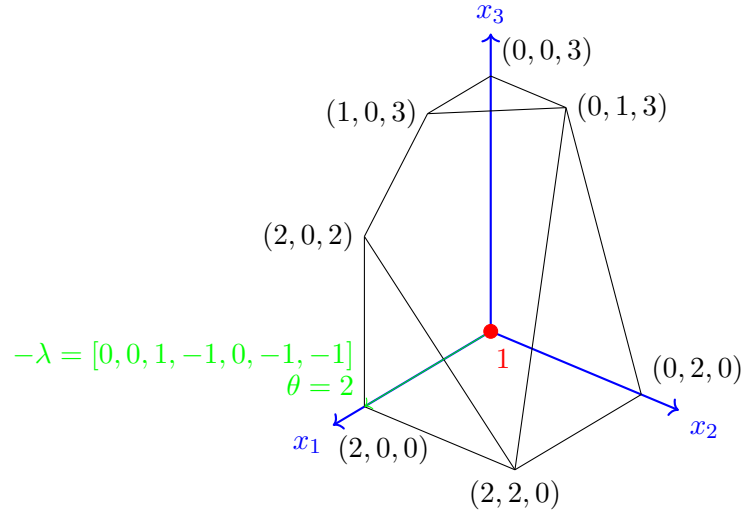


Figure 158: L8-LPexample3Dstep1.eps

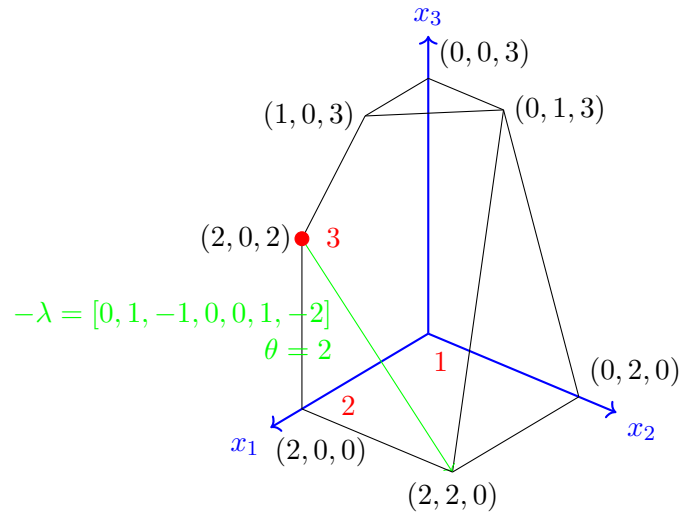


Figure 159: L8-LPexample3Dstep3.eps

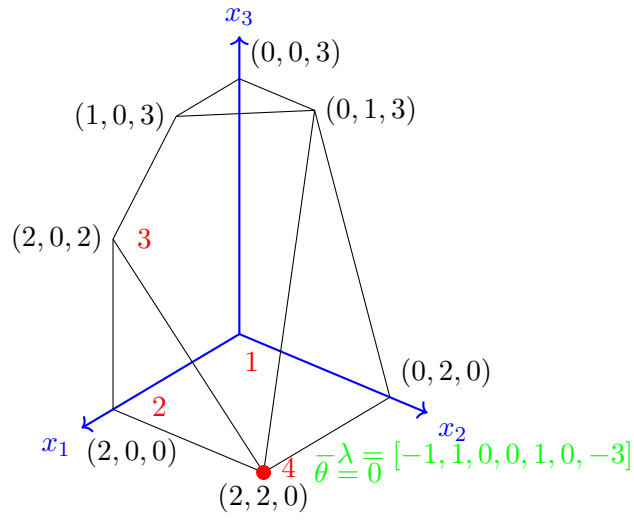


Figure 160: L8-LPexample3Dstep4.eps

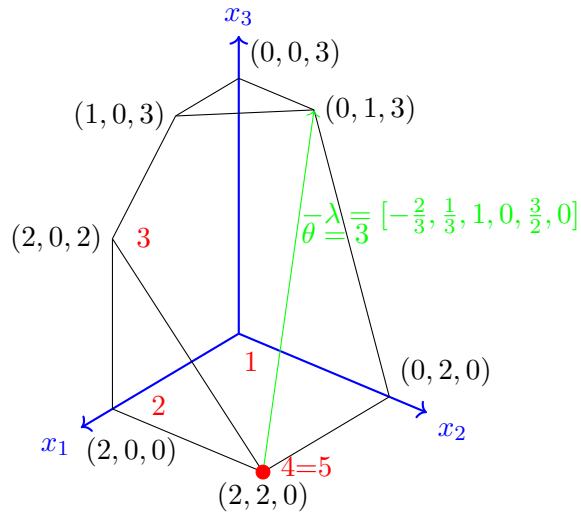


Figure 161: L8-LPexample3Dstep5.eps

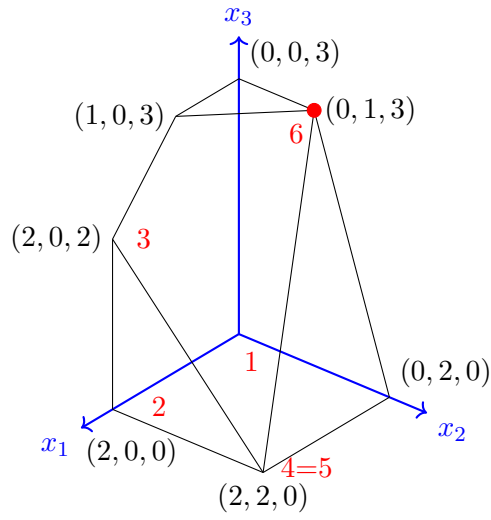


Figure 162: L8-LPexample3Dstep6.eps

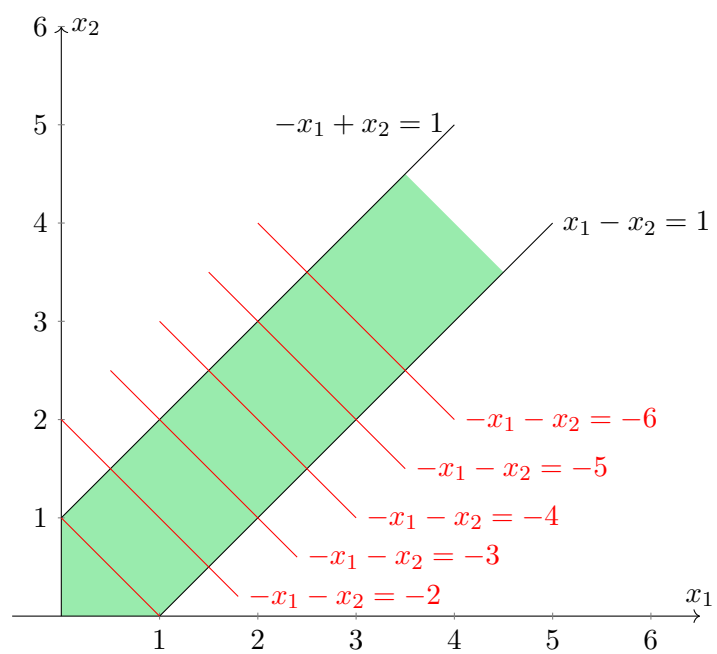


Figure 163: L8-unboundedlp.png

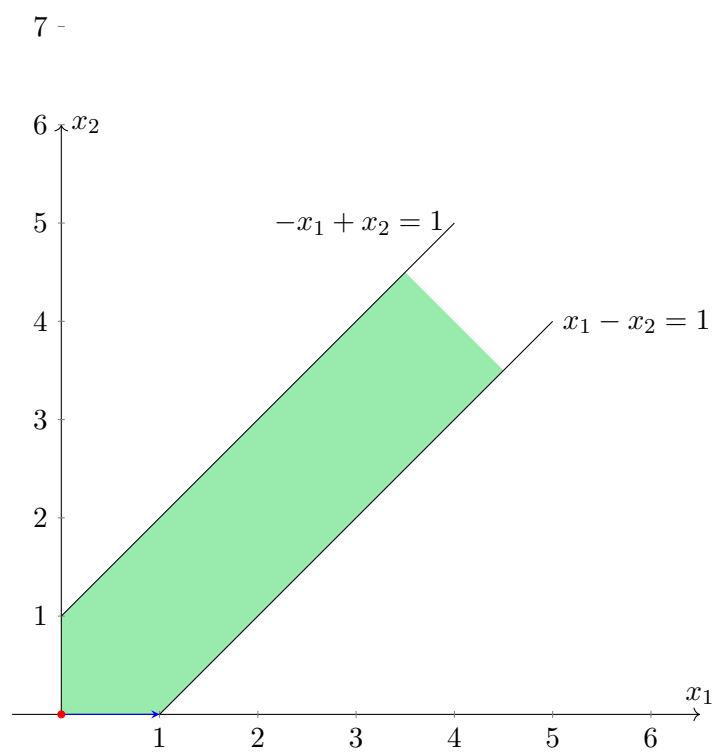


Figure 164: L8-unboundedlpstep1.png



7 -

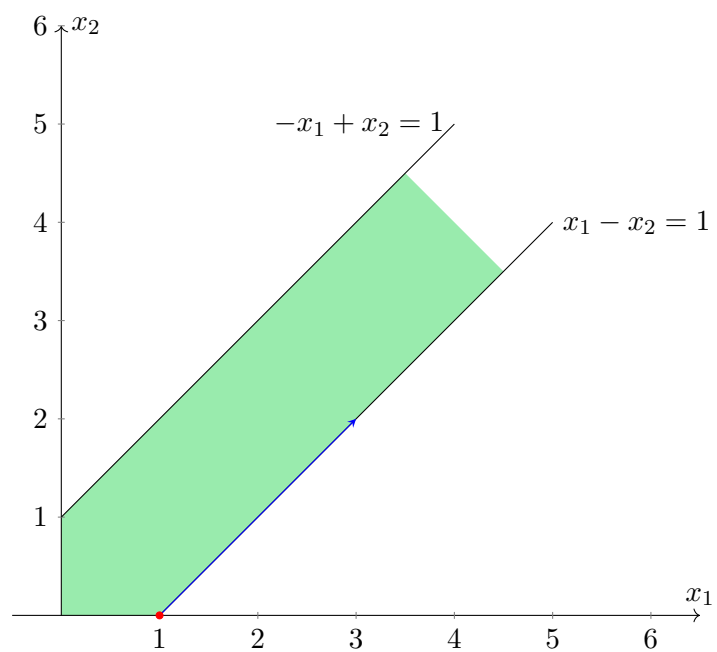


Figure 165: L8-unboundedlpstep2.png

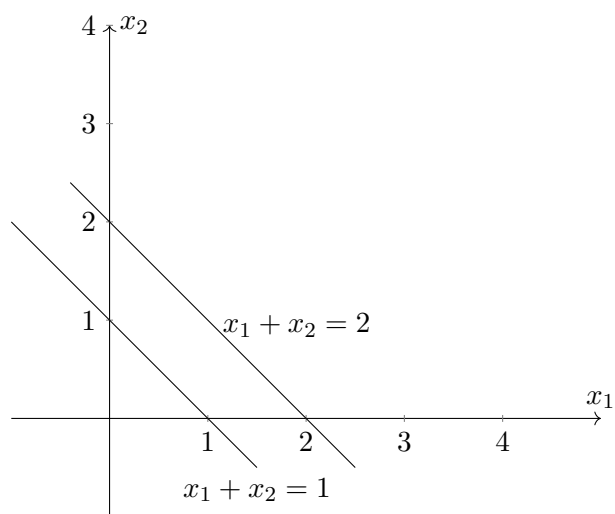


Figure 166: L8-LPinitialsolutionexample2.png

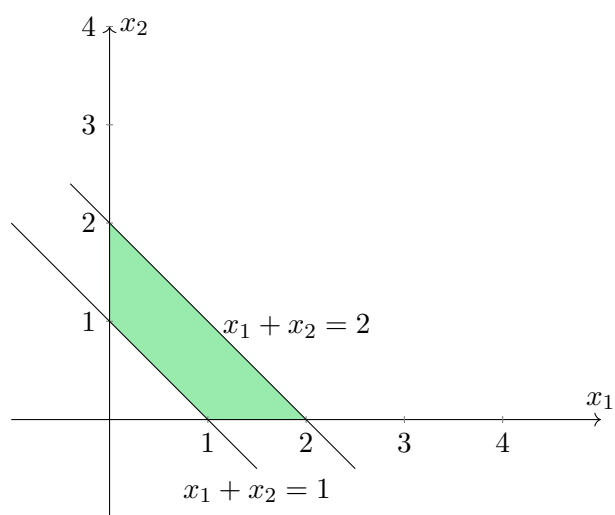


Figure 167: L8-LPinitialsolutionexample1.png

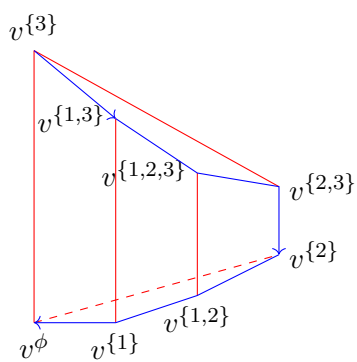


Figure 168: L8-kleemintycube1.png

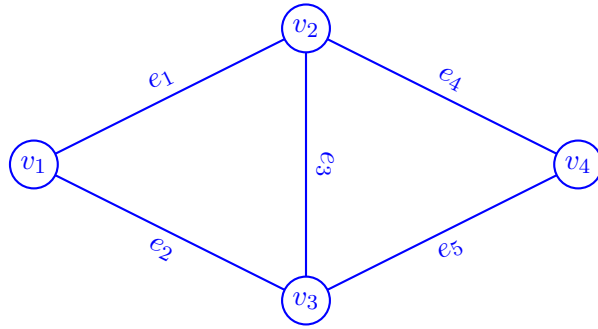


Figure 169: mingfuexample.eps

$$\mathbf{x} = \begin{bmatrix} 0 & 2 & 0 & 2 & 2 & 3 & 0 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 170: L8-LPexample3Dvertexmatrix.png

$$\mathbf{x} = \begin{bmatrix} 2 & 0 & 0 & 2 & 0 & 3 & 6 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 171: L8-LPexample3Dedgematrix.png

$$\mathbf{x} = \begin{bmatrix} 2 & 0 & 0 & 2 & 0 & 3 & 6 \end{bmatrix}^T$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 172: L8-LPexample3Dstep2twoedgesmatrix.png

$$\begin{array}{ll}
\min & \boxed{\mathbf{c}_B^T \mathbf{x}_B} + \boxed{\mathbf{c}_N^T \mathbf{x}_N} \\
s.t. & \left[ \begin{array}{|c|} \hline \mathbf{B} \\ \hline \end{array} \right] \left[ \begin{array}{|c|} \hline \mathbf{x}_B \\ \hline \mathbf{x}_N \\ \hline \end{array} \right] = \left[ \begin{array}{|c|} \hline \mathbf{b} \\ \hline \end{array} \right]
\end{array}$$

Figure 173: L8-simplextable.png

$$\left( \begin{array}{c|cccccc} 0 & \cdots & 0 & \cdots & c_e & \cdots \\ b_1 & \cdots & 0 & \cdots & a_{1e} & \cdots \\ b_2 & \cdots & 0 & \cdots & a_{2e} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_l & \cdots & 1 & \cdots & a_{le} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_m & \cdots & 0 & \cdots & a_{me} & \cdots \end{array} \right) \Rightarrow \left( \begin{array}{c|cccccc} -\frac{a_{me}}{a_{le}}b_l & \cdots & -\frac{c_e}{a_{le}} & \cdots & 0 & \cdots \\ b_1 - \frac{a_{1e}}{a_{le}}b_l & \cdots & -\frac{a_{1e}}{a_{le}} & \cdots & 0 & \cdots \\ b_2 - \frac{a_{2e}}{a_{le}}b_l & \cdots & -\frac{a_{2e}}{a_{le}} & \cdots & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{a_{le}}b_l & \cdots & \frac{1}{a_{le}} & \cdots & 1 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_m - \frac{a_{me}}{a_{le}}b_l & \cdots & -\frac{a_{me}}{a_{le}} & \cdots & 0 & \cdots \end{array} \right)$$

Figure 174: L8-LPpivoting.png

$$\left( \begin{array}{c|cc|cc} 0 & \overbrace{c_1 \ c_2 \ \cdots \ c_m}^{c_B^T} & \overbrace{\cdots \ c_n}^{c_N^T} \\ b_1 & a_{11} & a_{12} & \cdots & a_{1m} & \cdots & a_{1n} \\ b_2 & a_{21} & a_{22} & \cdots & a_{2m} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_m & a_{m1} & a_{m2} & \cdots & a_{mm} & \cdots & a_{mn} \\ \hline b & \underbrace{\phantom{a_{m1} \ a_{m2} \ \cdots \ a_{mm}}}_B & \underbrace{\phantom{\cdots \ a_{mn}}}_N \end{array} \right) \xrightarrow[B^{-1}x]{\text{RowOperation}} \left( \begin{array}{c|cc|cc} -c_B^T B^{-1}b & 0 & 0 & \cdots & 0 & \overbrace{\cdots}^{\bar{c}^T} & \overbrace{c_N^T - c_B^T B^{-1}N} \\ B^{-1}b & 1 & 0 & \cdots & 0 & \cdots & \cdots \\ 0 & 1 & \cdots & 0 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & \cdots & \cdots & \cdots \\ \hline & \underbrace{\phantom{1 \ 0 \ \cdots \ 0}}_{B^{-1}B} & \underbrace{\phantom{\cdots}}_{B^{-1}N} \end{array} \right)$$

Figure 175: L8-LPABC1-ABC2.png

## 7 LP\_1-3

$$\begin{array}{ll}
 \text{Primal:} & \text{Dual:} \\
 \min & c^T x \quad \text{same sign} \quad \max \quad y^T b \\
 \text{s.t.} & Ax \leq b \quad \text{s.t.} \quad y \leq 0 \\
 & x \geq 0 \quad \text{change sign} \quad y^T A \leq c^T
 \end{array}$$

Figure 176: L9-primaldual-case1.png

$$\begin{array}{ll}
 \text{Primal:} & \text{Dual:} \\
 \min & c^T x \quad \text{no constraint} \quad \max \quad y^T b \\
 \text{s.t.} & Ax = b \quad \text{s.t.} \quad y \leq 0 \\
 & x \geq 0 \quad \text{change sign} \quad y^T A \leq c^T
 \end{array}$$

Figure 177: L9-primaldual-case2.png

Dual \ Primal	Bounded Optimal Objective Value	Unbounded Optimal Objective Value	Infeasible
Bounded Optimal Objective Value	Possible	Impossible	Impossible
Unbounded Optimal Objective Value	Impossible	Impossible	Possible
Infeasible	Impossible	Possible	Possible

Table 8: L9-primaldual-table.png

## 8 9\_LP\_4

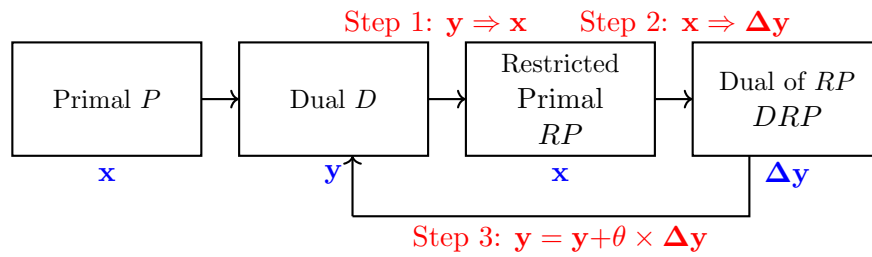


Figure 178: L9-primaldualflowchart.eps(Lec9\_P66\_Pic1)

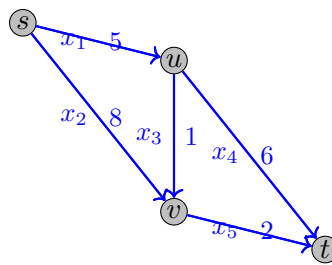


Figure 179: Lec9\_P80:the shortest path from city  $s$  to  $t$

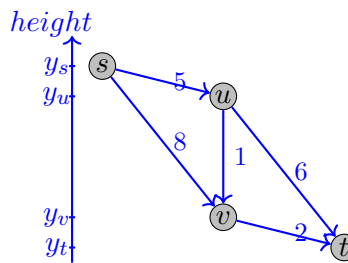


Figure 180: Lec9\_P81:DUAL PROBLEM:set variables for cities

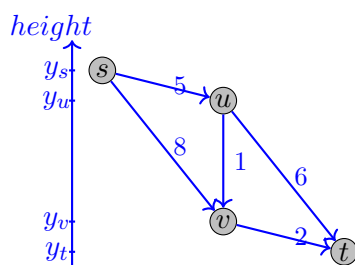


Figure 181: Lec9\_P82:DUAL PROBLEM: simplify by setting  $y_t = 0$

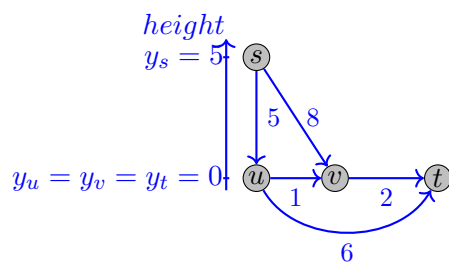


Figure 182: Lec9\_P85:After Iteration 1

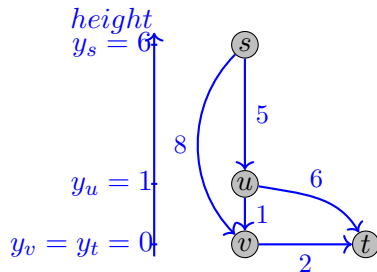


Figure 183: Lec9\_P88:After Iteration 2

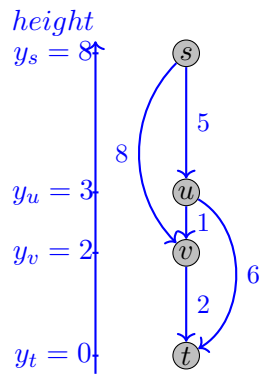


Figure 184: Lec9\_P91:After Iteration 3

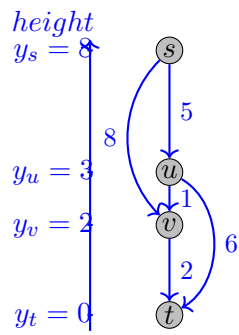


Figure 185: Lec9\_P95:After Iteration 4

## 9 9\_NF\_1

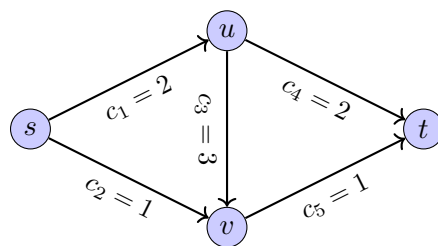


Figure 186: transport as many as commodity from  $s$  to  $t$



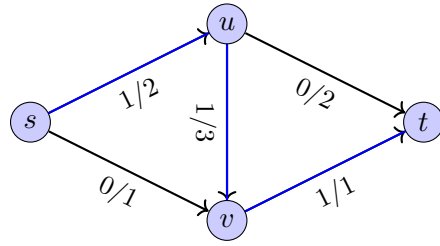


Figure 187: Pic2.eps

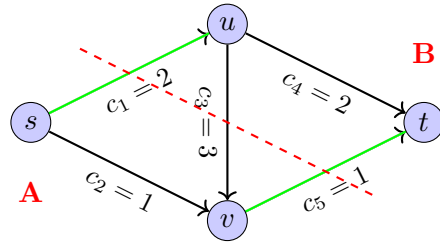


Figure 188:  $C(A,B)=3$

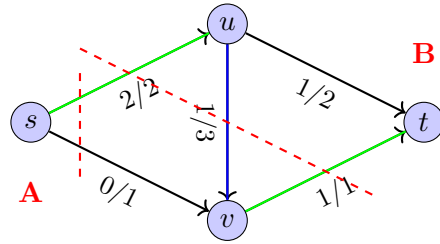


Figure 189: Pic4.eps

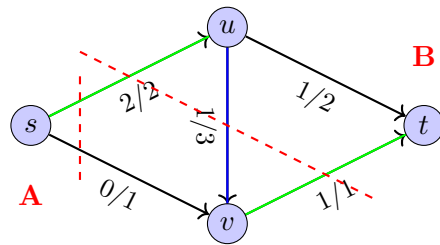


Figure 190: Pic5.eps

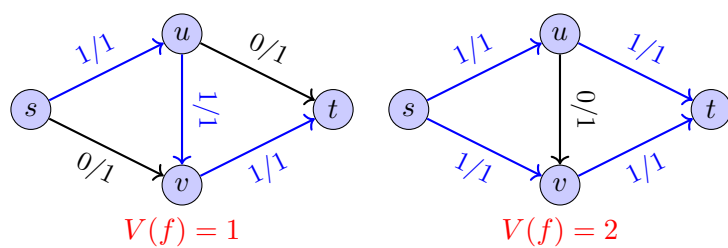


Figure 191: Pic6.eps

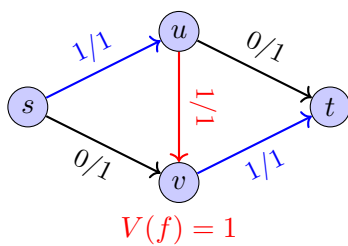


Figure 192: Pic7.eps

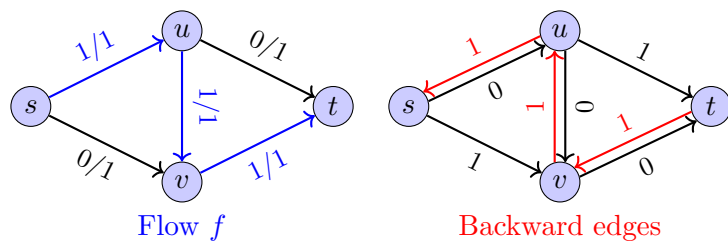


Figure 193: Pic8.eps

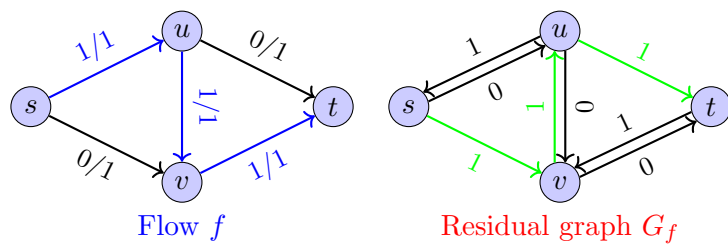


Figure 194: Pic9.eps

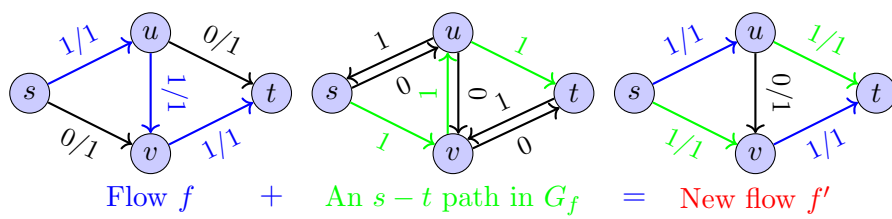


Figure 195: Pic10.eps

10 11\_NF\_3

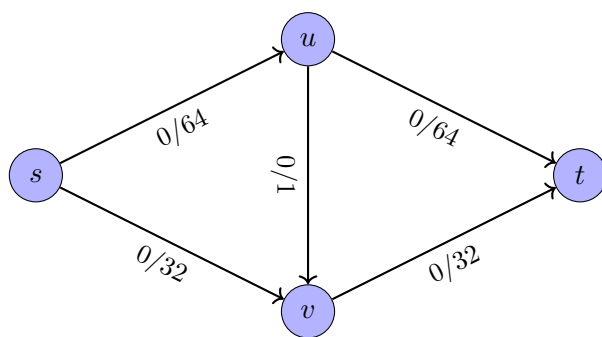


Figure 196: Flow  $f$ :  $V(f) = 0$

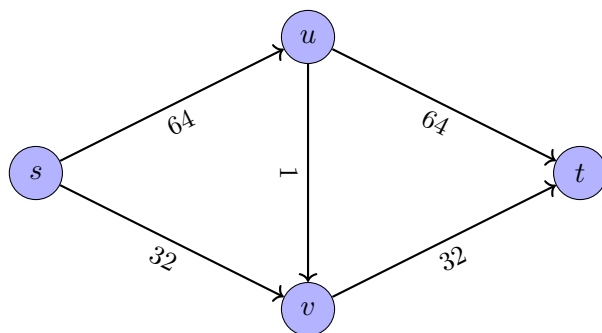


Figure 197: No  $s \rightarrow t$  path in  $G_f$

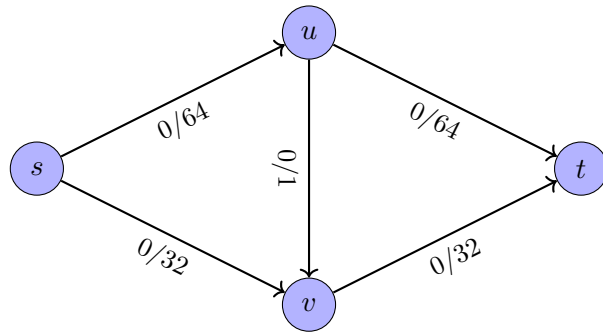


Figure 198: Flow  $f$ :  $V(f) = 0$

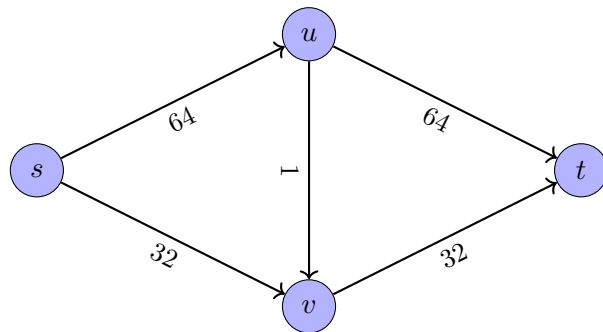


Figure 199: Have  $s \rightarrow t$  path in  $G_f$

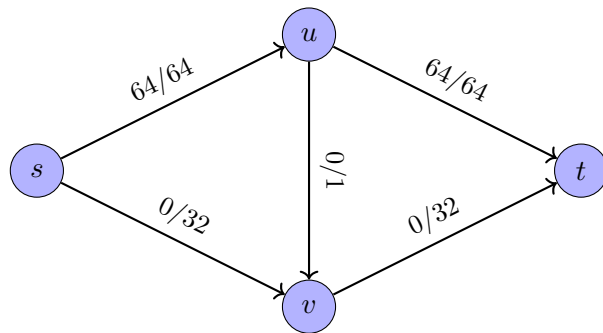


Figure 200: Flow  $f$ :  $V(f) = 64$

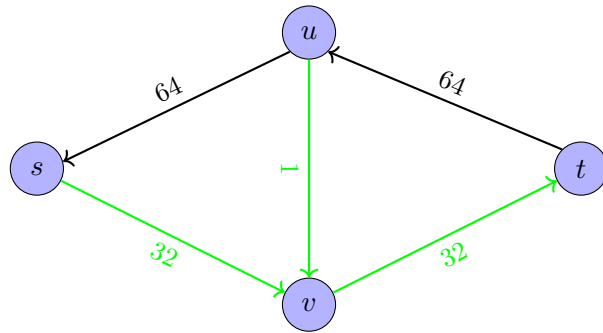


Figure 201: No  $s \rightarrow t$  path in  $G_f$

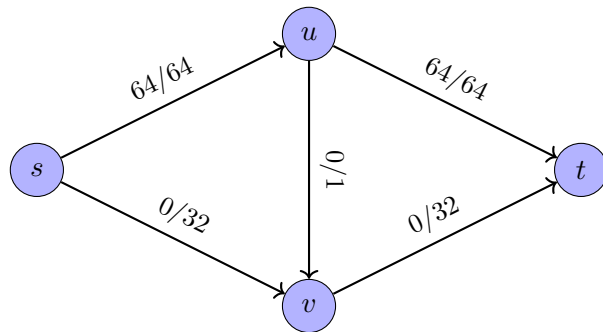


Figure 202: Flow  $f$ :  $V(f) = 64$

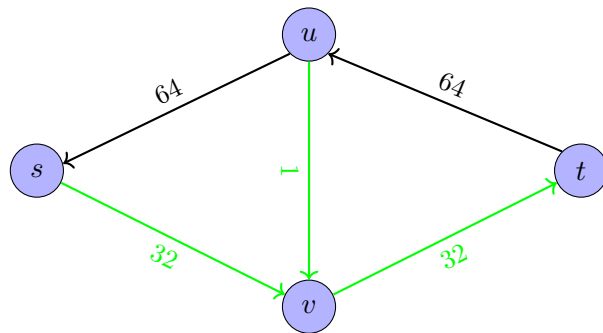


Figure 203: An  $s \rightarrow t$  path in  $G_f$

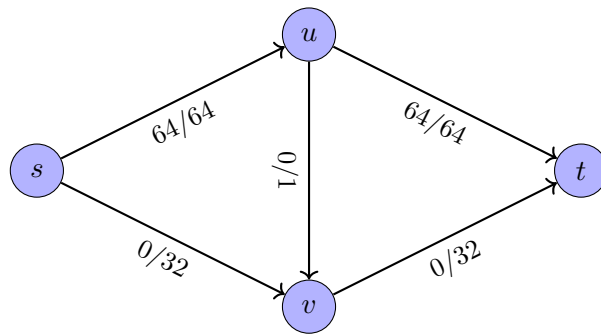


Figure 204: Flow  $f$ :  $V(f) = 96$

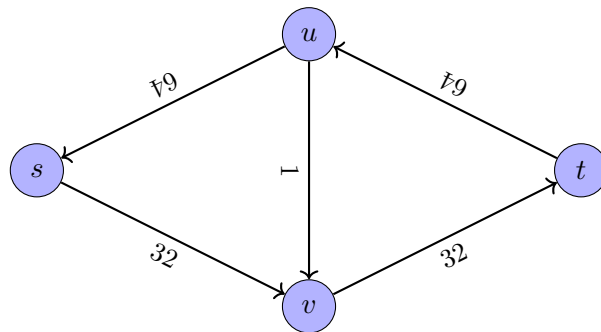


Figure 205: No  $s \rightarrow t$  path in  $G_f$

## 11 12\_NP\_1-2

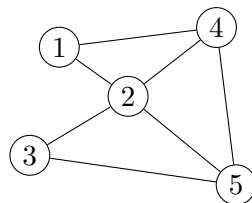


Figure 206: Pic1 .eps

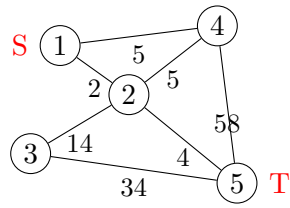


Figure 207: Pic2.eps

Problem Instance

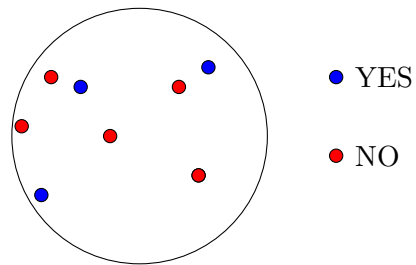


Figure 208: Pic3.eps

Problem A

Problem B

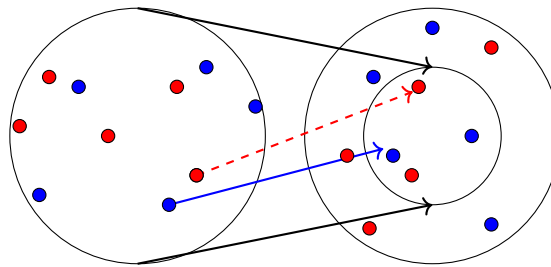


Figure 209: reduction.eps

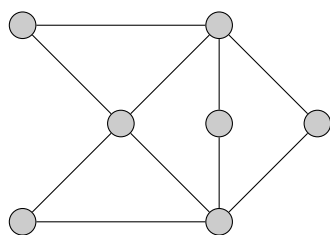


Figure 210: Pic5.eps

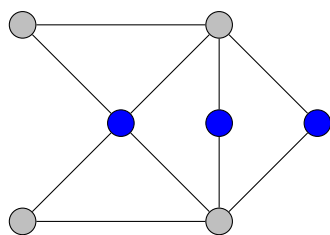


Figure 211: Pic6.eps

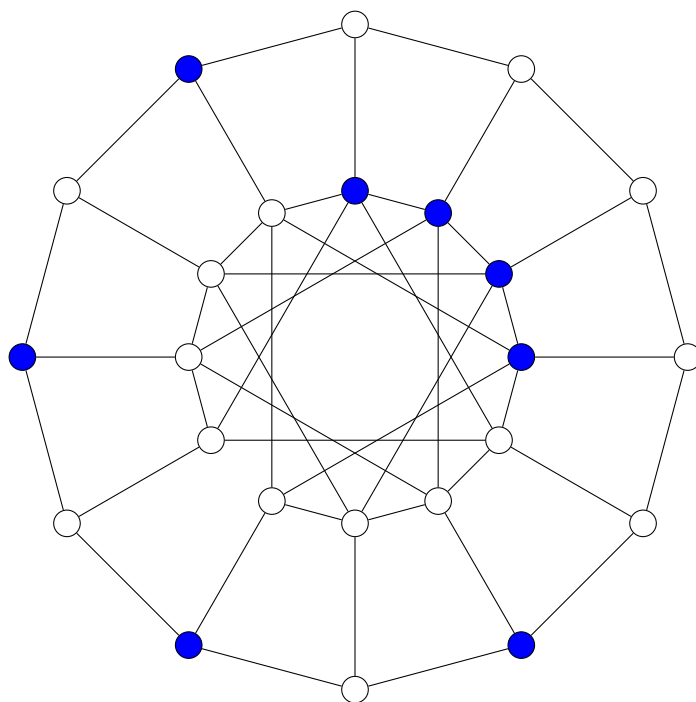


Figure 212: independentsetgraph24-8.eps



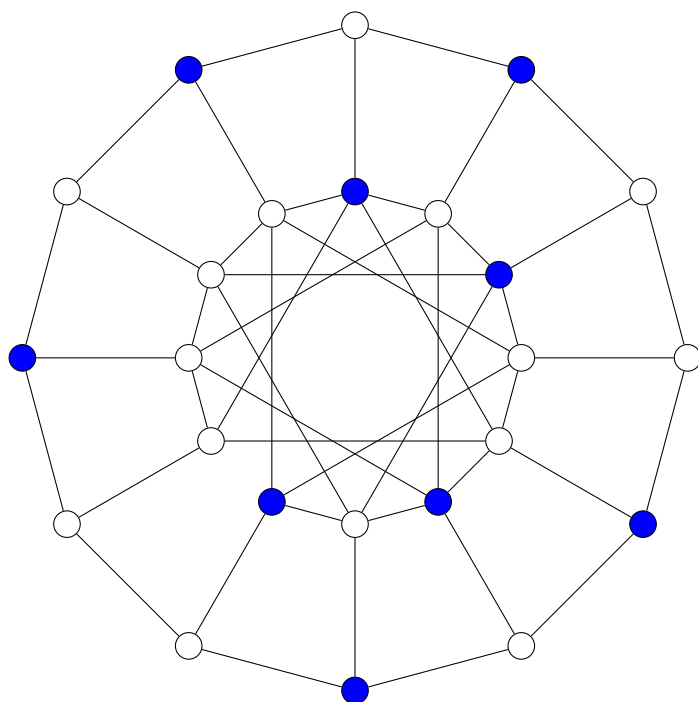


Figure 213: independentsetgraph24-9.eps

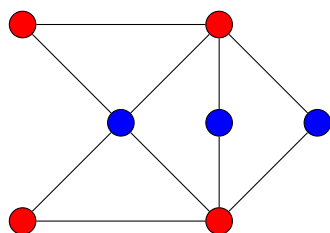


Figure 214: Pic9.eps

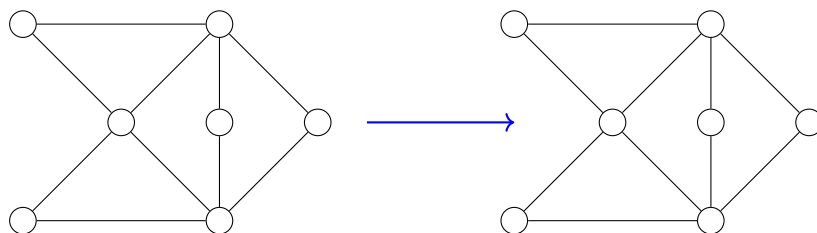


Figure 215: L3-independentsettovertexcover.eps

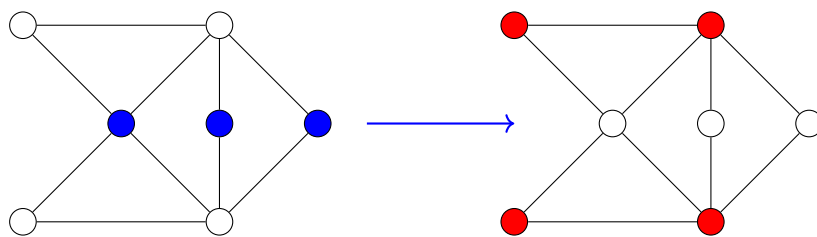


Figure 216: Pic11.eps

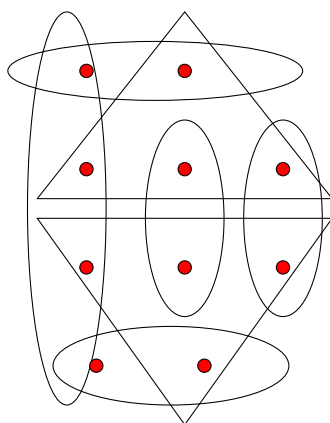


Figure 217: setcover1.png

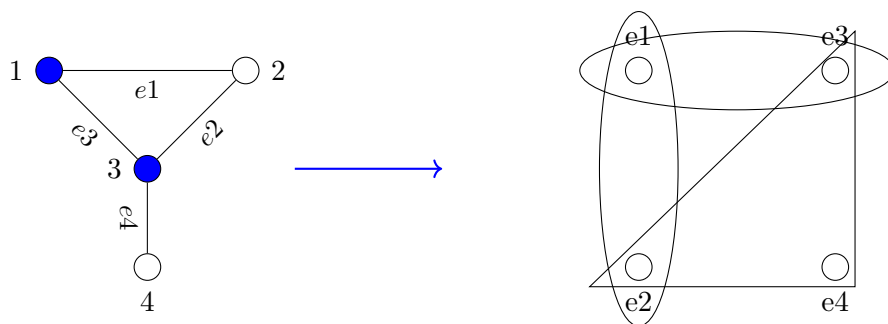


Figure 218: L3-vertexcoversetcover.eps

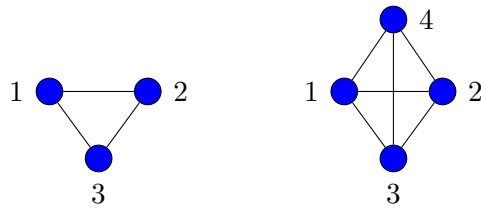


Figure 219: L3-clique.eps

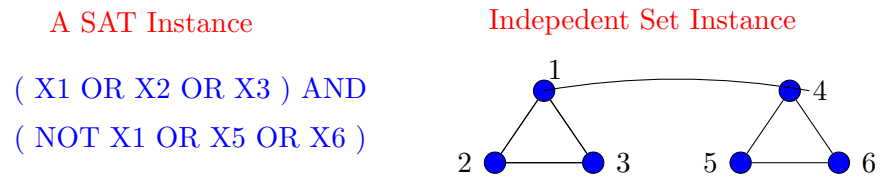


Figure 220: L3-satindependentset.eps

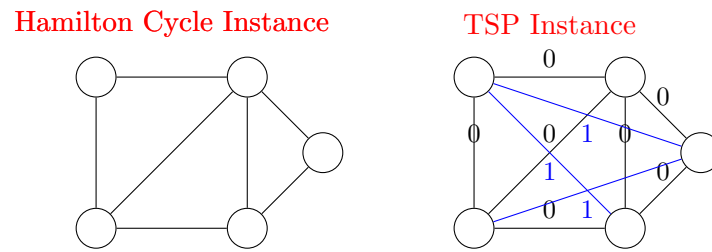


Figure 221: Pic17.eps

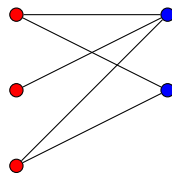


Figure 222: L3-2coloring.png

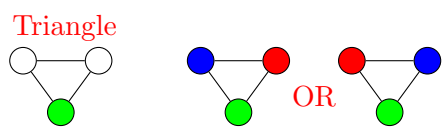


Figure 223: L3-coloringgadgettriangle.eps

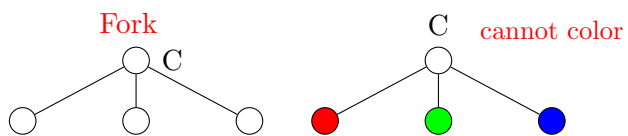


Figure 224: L3-coloringgadgetfork.eps

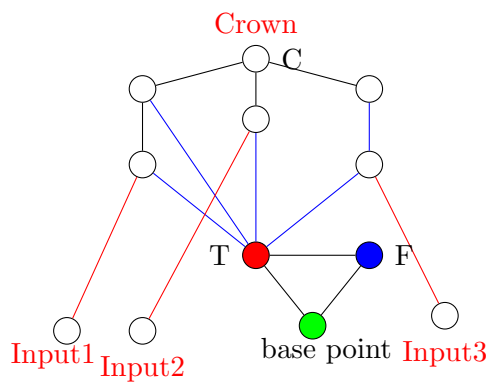


Figure 225: L3-coloringgadgetcrown.eps

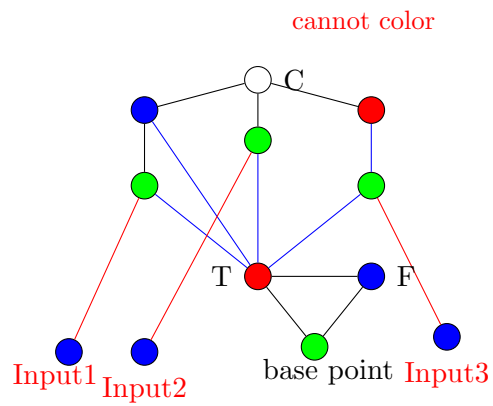


Figure 226: L3-coloringclausecrownBBB.eps

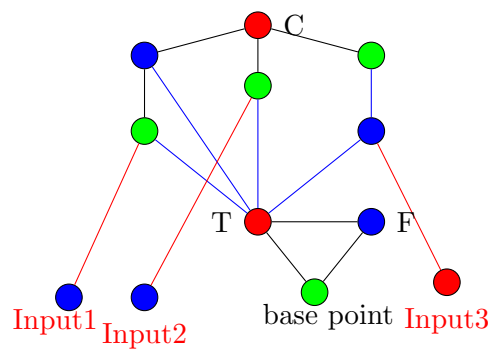


Figure 227: L3-coloringclausecrownBBR.eps

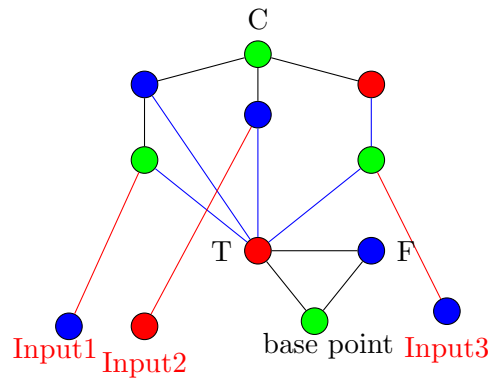


Figure 228: L3-coloringclausecrownBRB.eps

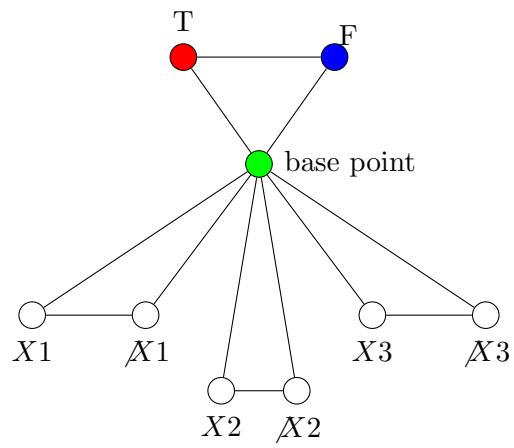


Figure 229: L3-coloringvariables.eps

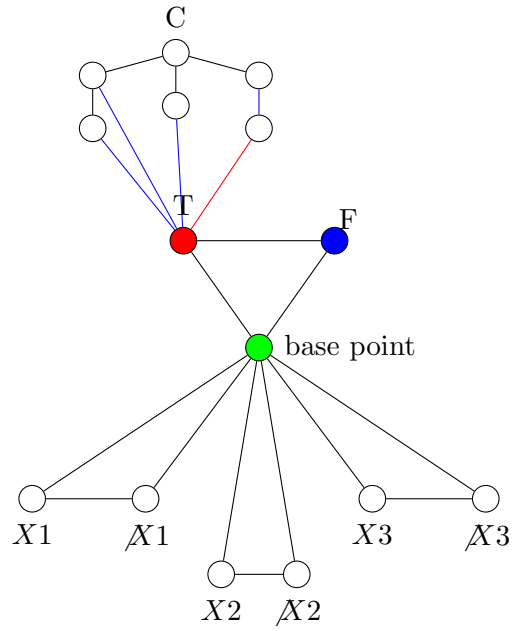


Figure 230: L3-coloringclause.eps

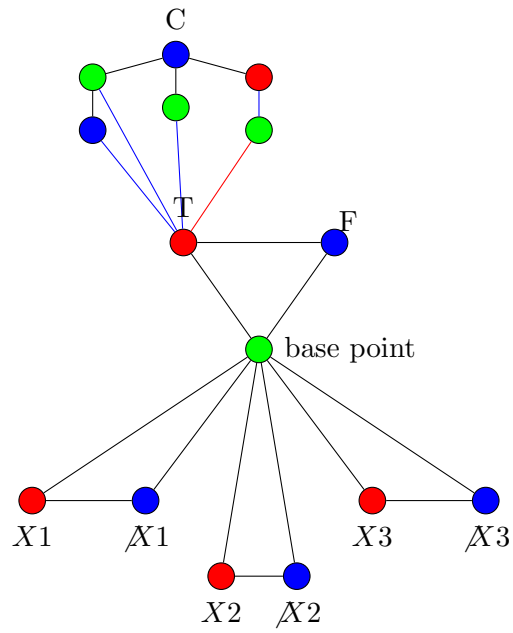


Figure 231: L3-coloringclause-satcase1.eps

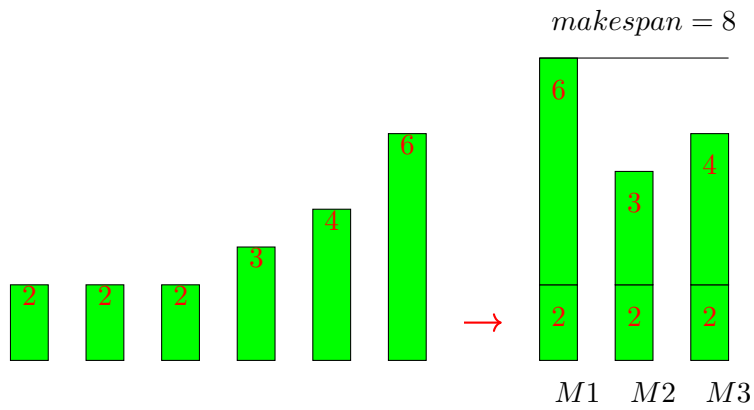


Figure 233: L11-makespanexample.eps

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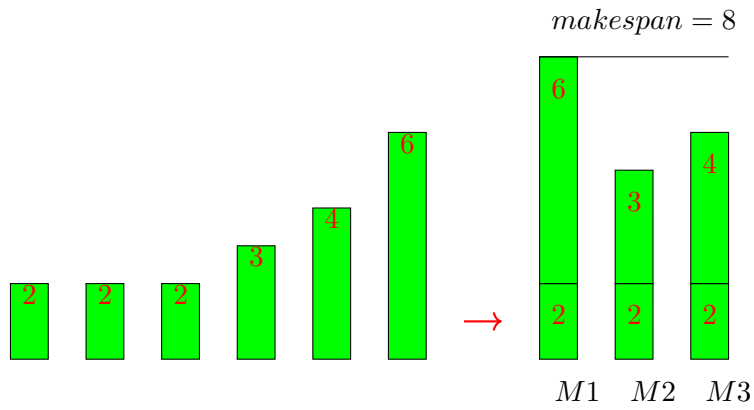


Figure 232: L11-makespanexample.eps

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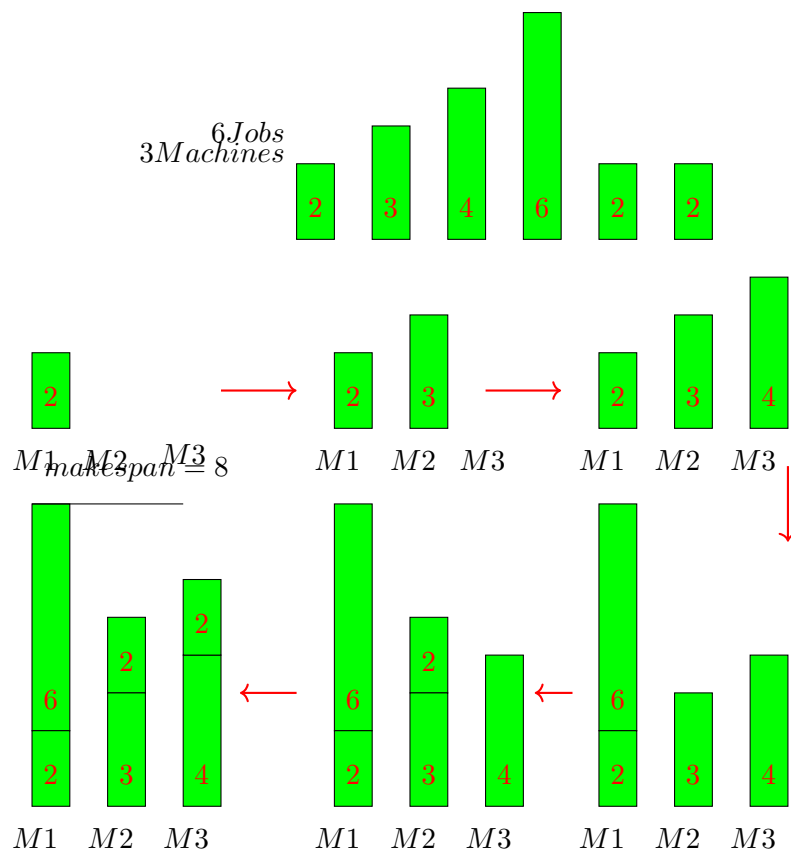


Figure 234: L11-makespanalgo1example.eps

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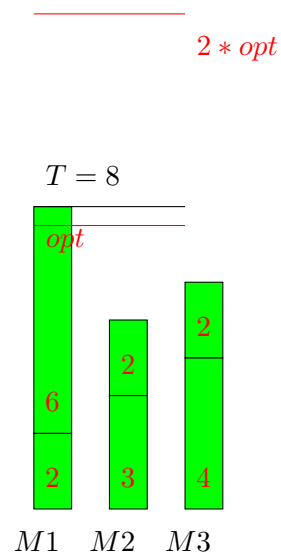


Figure 235: L11-makespanalgo1analysis2OPT.eps  
;

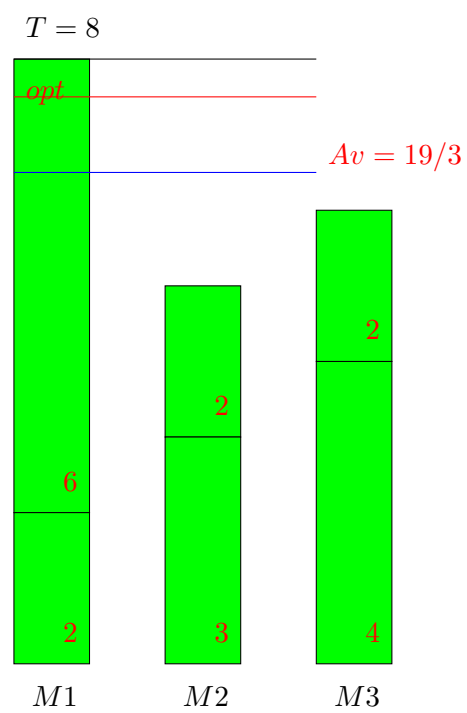


Figure 236: L11-makespanalgo1analysisLB.eps

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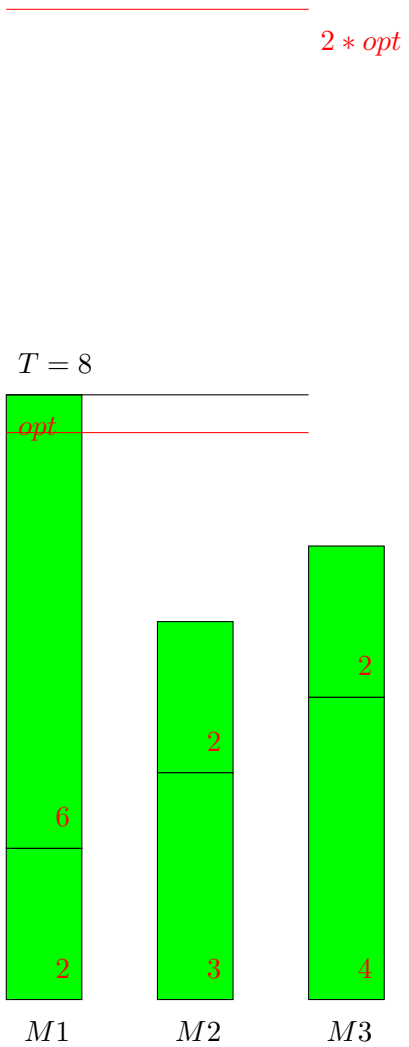


Figure 237: L11-makespanalgo1analysis2OPT.eps  
;

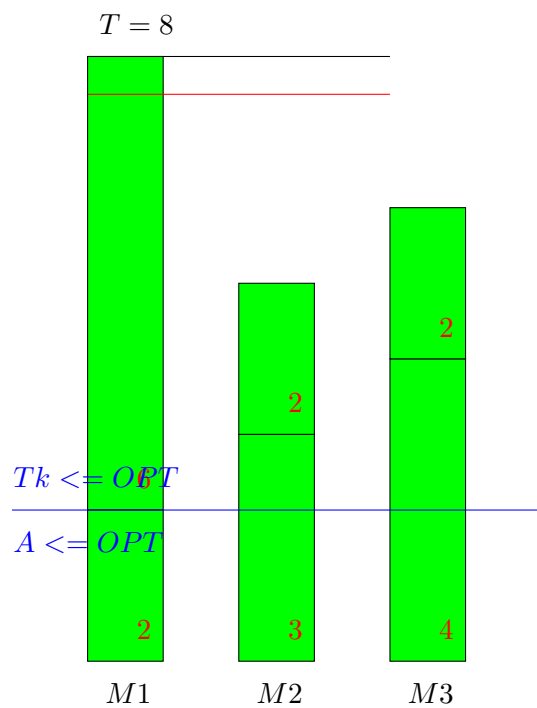


Figure 238: L11-makespanalgo1analysis.eps  
;

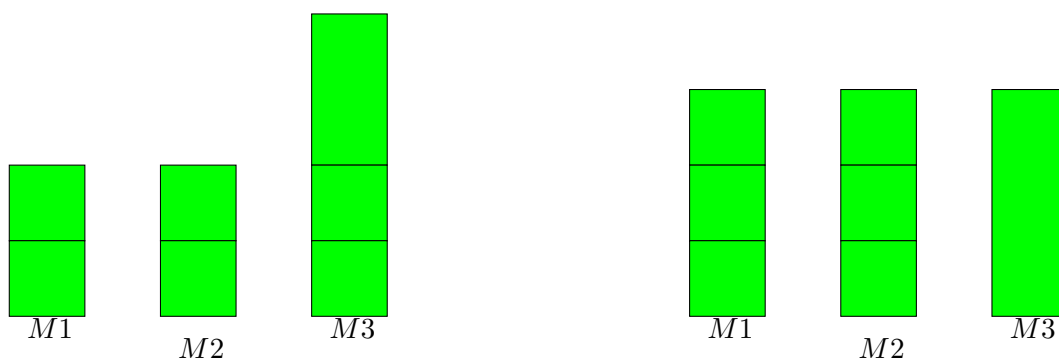


Figure 239: L11-makespanalgo1tightexample.eps  
;

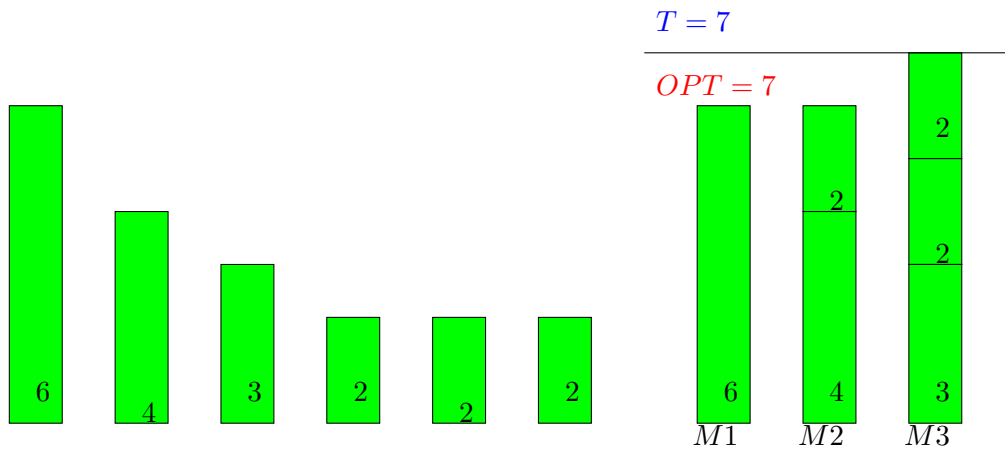


Figure 240: L11-makespanalgo2example.eps  
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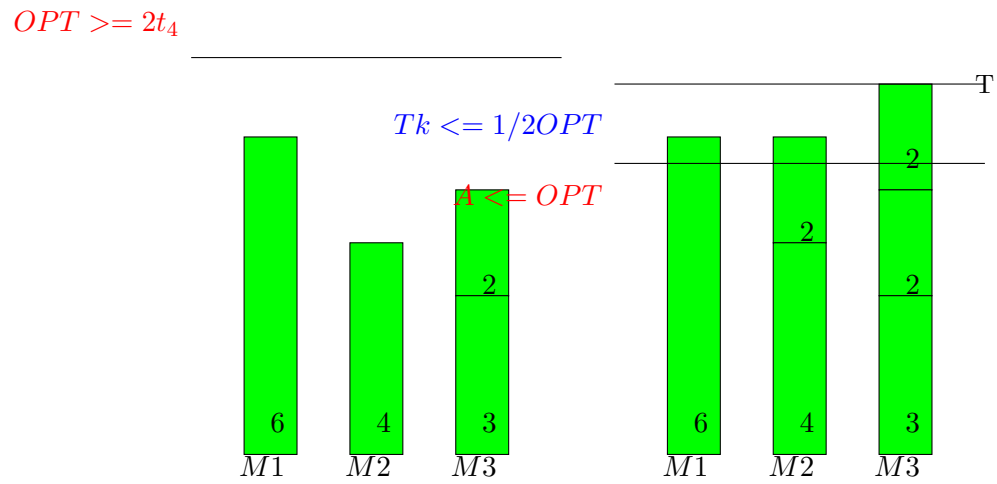


Figure 241: L11-makespanalgo2analysis.eps  
;

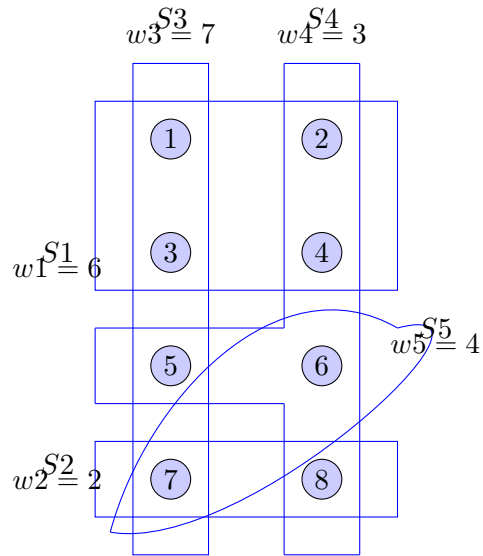


Figure 242: L11-setcoveralgorithm4.eps  
;

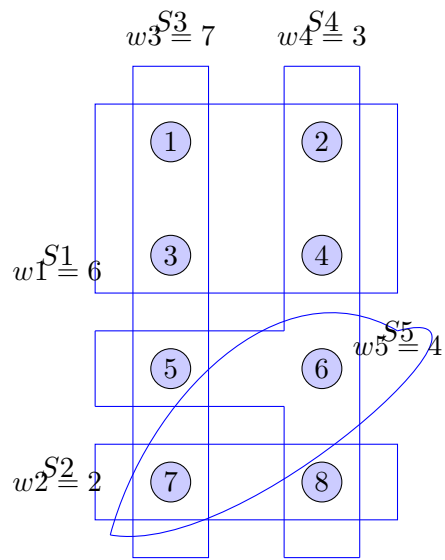


Figure 243: L11-setcoverexamplelognstep1.eps  
;

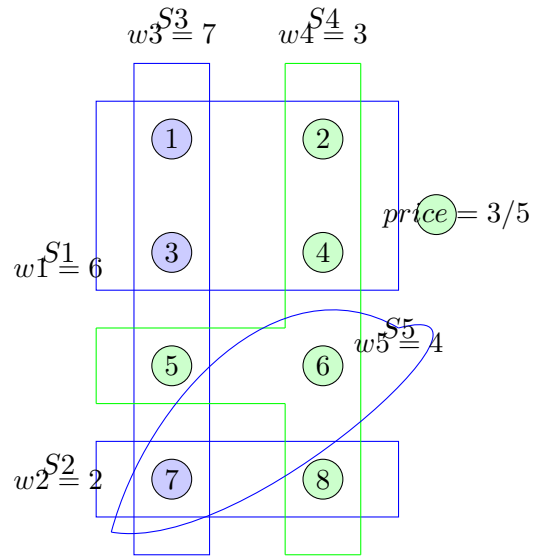


Figure 244: L1-setcoverexamplelognstep2.eps  
;

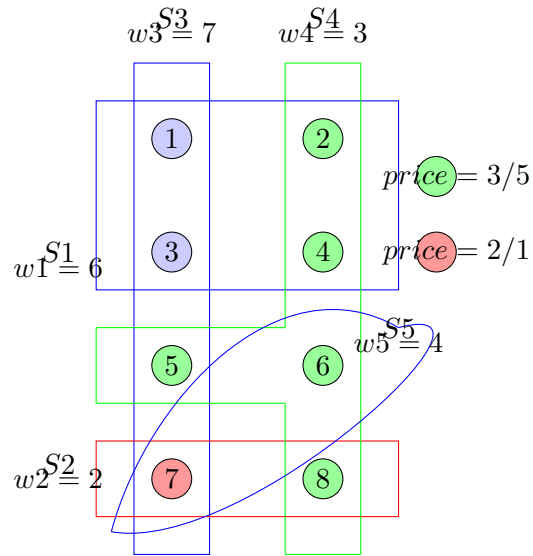


Figure 245: L1-setcoverexamplelognstep3.eps  
;



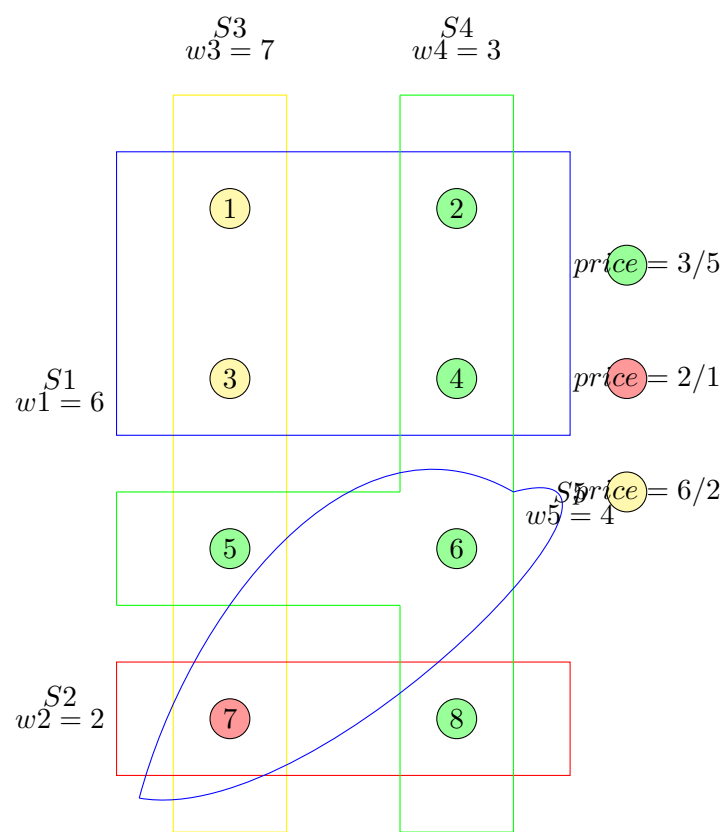


Figure 246: L11-setcoverexamplelogstep4.eps

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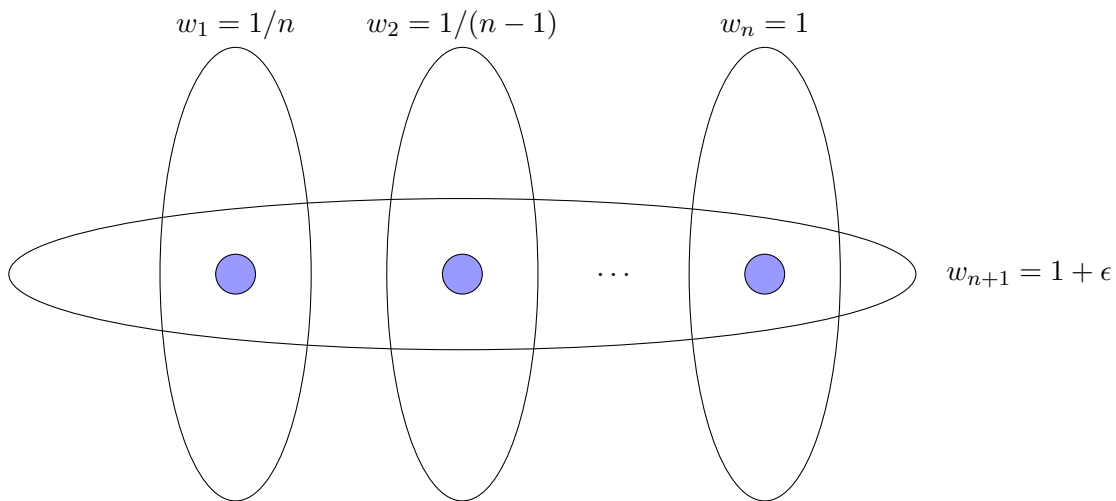


Figure 247: L11-setcoveralgo4example.eps  
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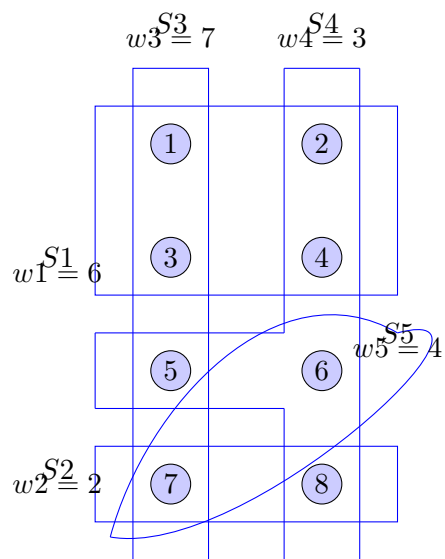
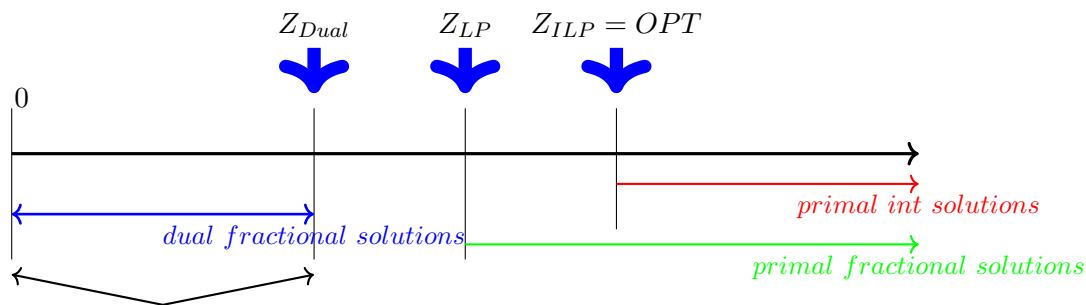


Figure 248: L11-setcoverexamplelognstep4.eps  
;



can be used as a lower bound

Figure 249: L11-LPlowerbounding.eps

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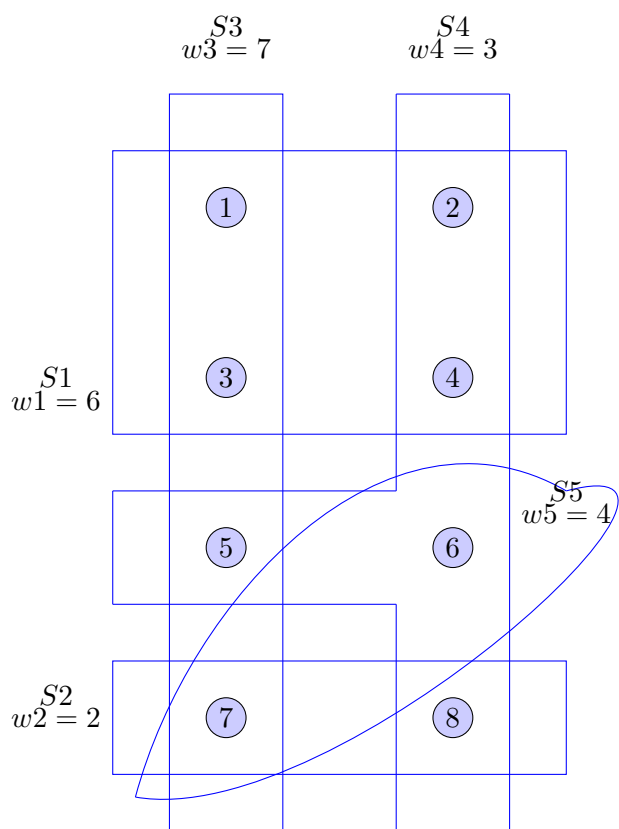
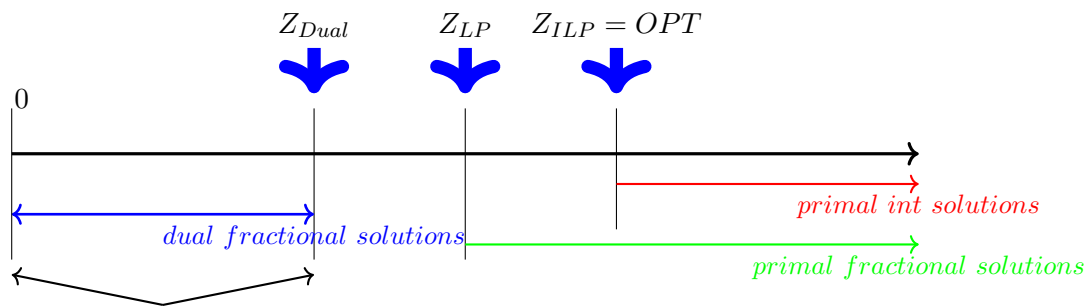


Figure 250: L11-setcoverexamplelognstep1.eps

;



can be used as a lower bound

Figure 251: L11-LPlowerbounding.eps

;

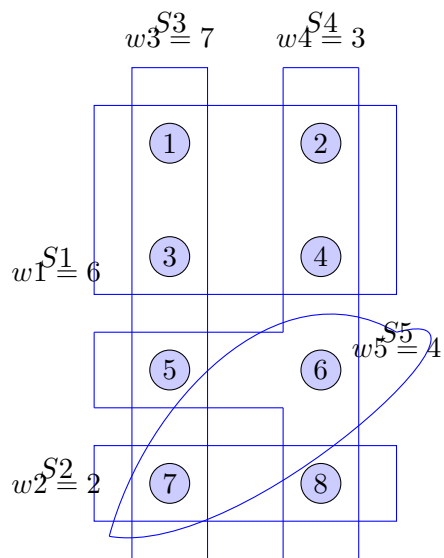


Figure 252: L11-setcoverexamplendualstep1.eps

;

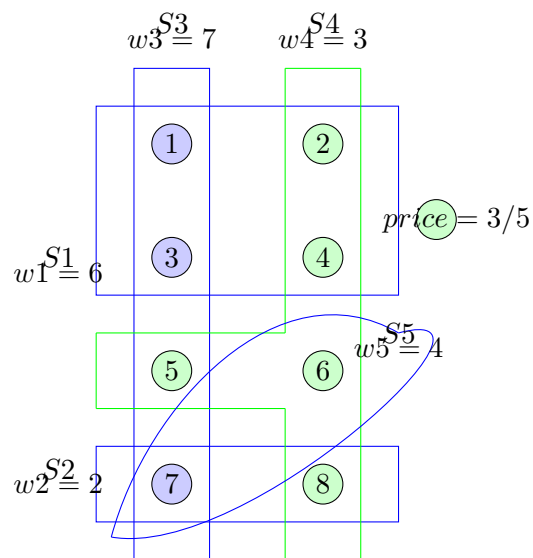


Figure 253: L1-setcoverexampledualstep2.eps

;

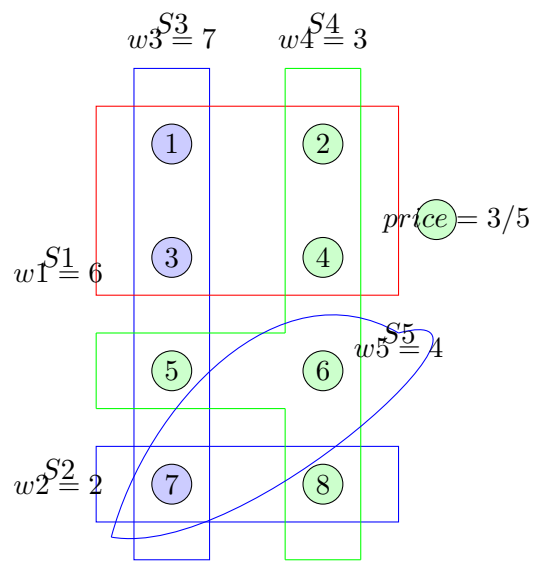


Figure 254: L11-setcoverexampledualstep3.eps  
;

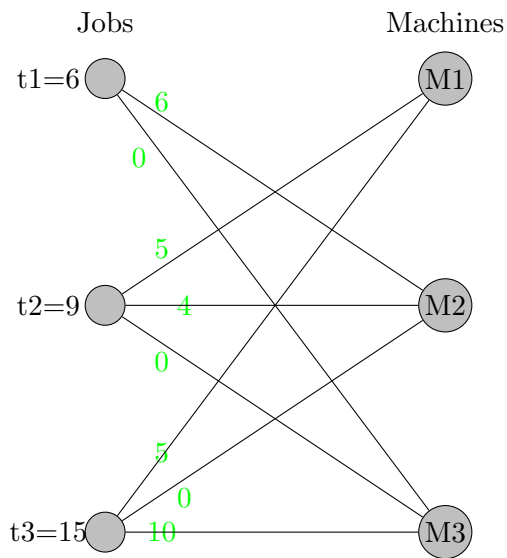


Figure 258: L11-makespanLPalgosolution.eps  
;

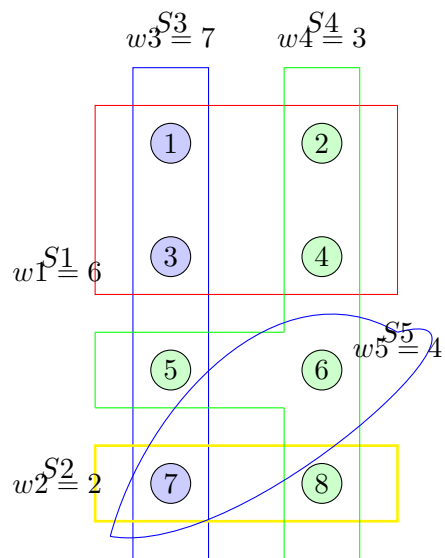


Figure 255: L11-setcoverexamplendualstep4.eps  
;

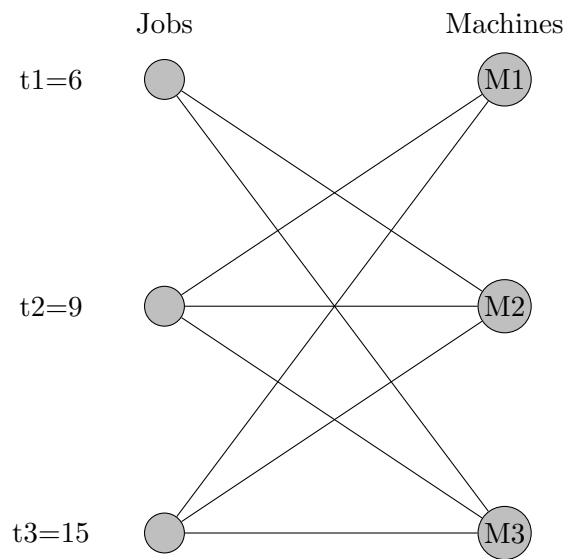


Figure 256: L11-makespanLP algoinput.eps  
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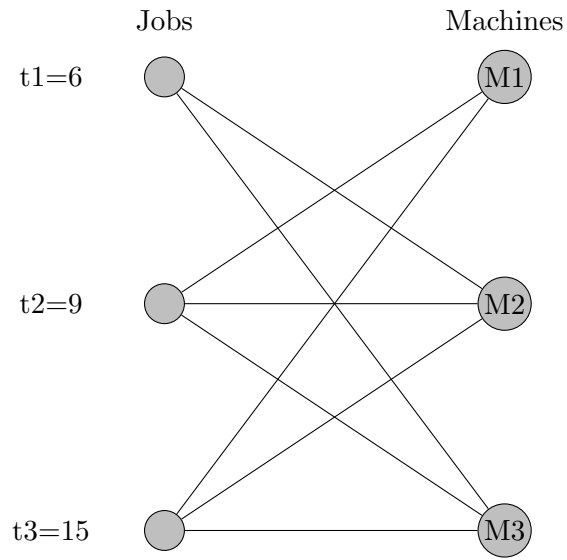


Figure 257: L11-makespanLPalgoinput.eps  
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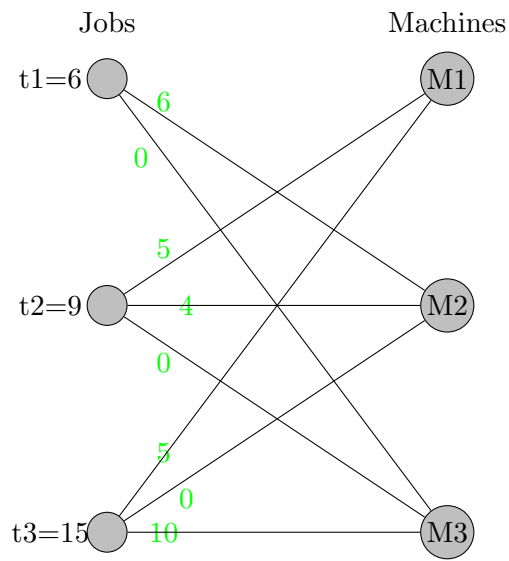


Figure 259: L11-makespanLPalgosolution.eps  
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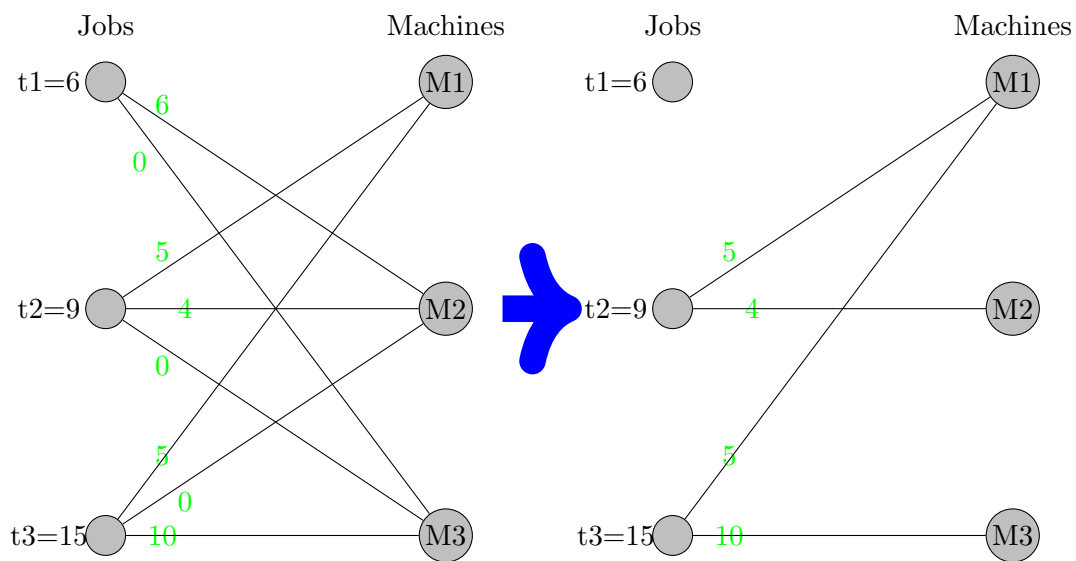


Figure 260: L11-makespanLPalgosolutionfractionaljobs.eps

;

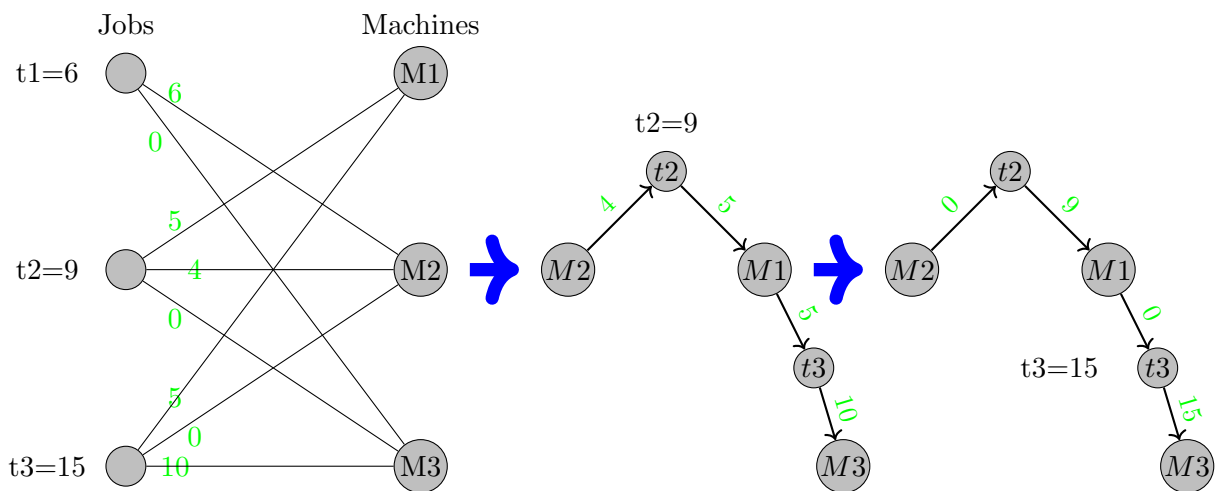


Figure 261: L11-makespanLPalgosolutionfractionaljobstree.eps

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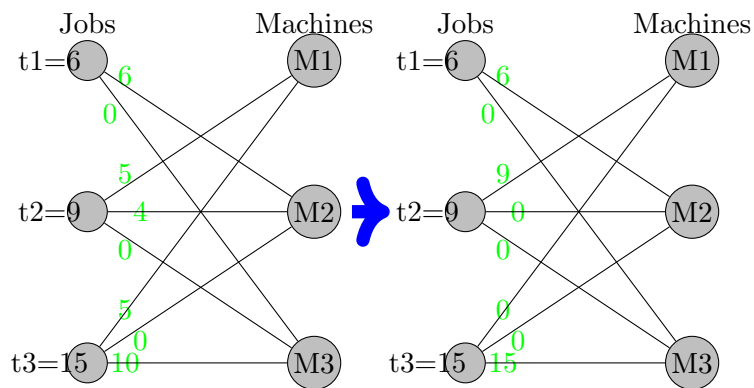


Figure 262: L11-makespanLPalgosolutioncase1.eps

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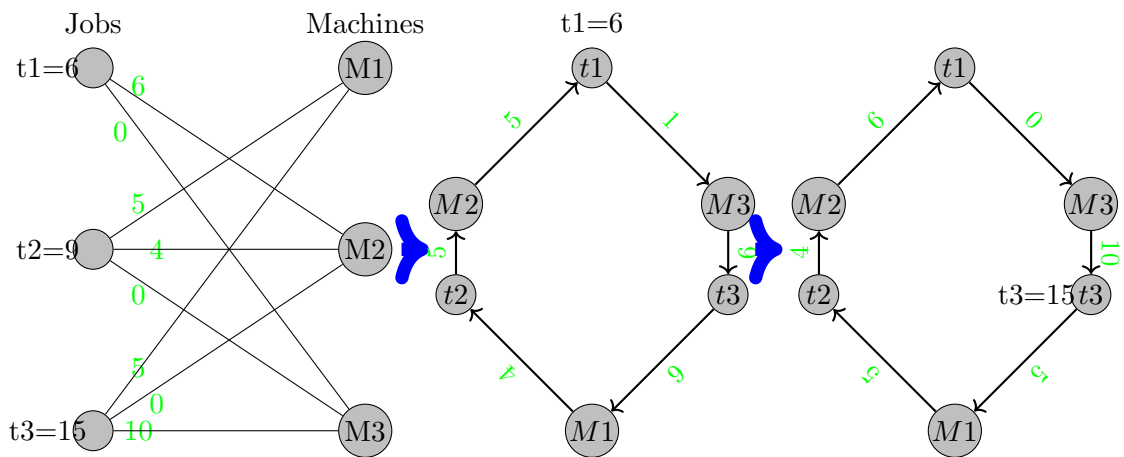


Figure 263: L11-makespanLPalgosolutioncase2tree.eps

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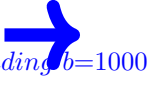

v1	3278		$\overline{v1}$	4000		$\widehat{v1}$	4
v2	1956		$\overline{v2}$	2000		$\widehat{v2}$	2
v3	4123		$\overline{v3}$	5000		$\widehat{v3}$	5
v4	2233		$\overline{v4}$	3000		$\widehat{v4}$	3

Figure 264: L11-KnapsackDP2rounding.eps

;

b	$\overline{v}$	$\epsilon$	$W$	#OP	Time (ms)
1	2223975	0.001	1768	889590000	18352.128
3	741325	0.010	1768	98843333	5990.893
5	444800	0.028	1768	35584000	3649.624
10	222400	0.112	1768	8896000	1836.567
30	74125	1.011	1768	988333	620.822
50	44475	2.810	1768	355800	381.982
100	22250	11.236	1768	89000	183.707
300	7425	101.010	1768	9900	60.422
500	4450	280.899	1768	3560	38.340
1000	2225	1123.6	1768	890	17.943
3000	750	10000	1809	100	6.872
5000	450	27777.8	1809	36	4.059
10000	225	111111	1809	9	3.134

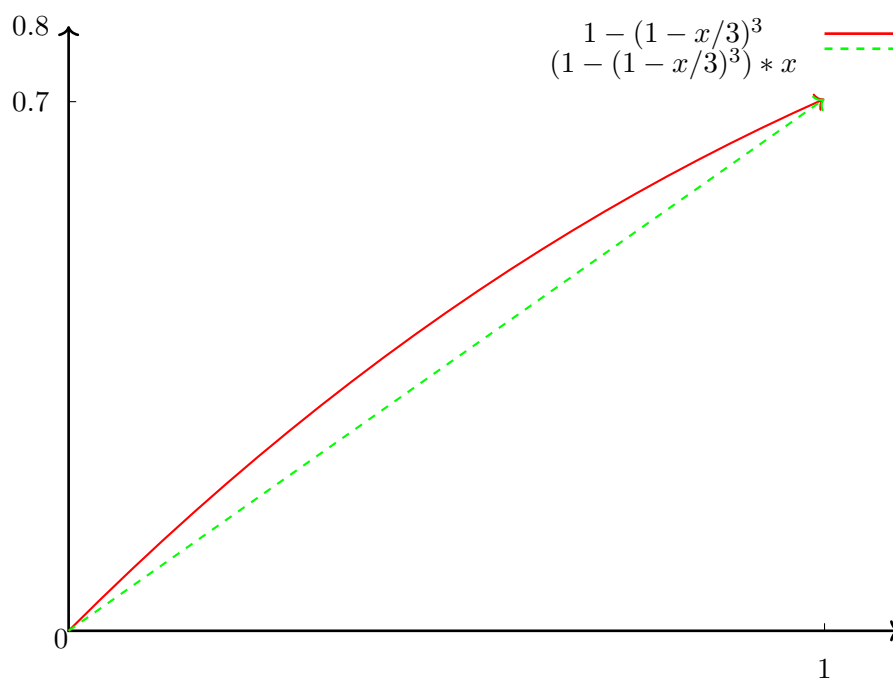


Figure 265: