

# CS612 Assignment 4

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Notice:

1. Due Dec. 3, 2009.
2. Please send your answer to wangchao1987@ict.ac.cn, shaomingfu@gmail.com, yuanxiongying@ict.ac.cn
3. You can arbitrarily choose two problems from Problems 1-5.

## 1 Dynamic Programming (10 marks)

Professor Stewart is consulting for the president of a corporation that is planning a company party. The company has a hierarchical structure; that is, the supervisor relation forms a tree rooted at the president. The personnel office has ranked each employee with a conviviality rating, which is a real number. In order to make the party fun for all attendees, the president does not want both an employee and his or her immediate supervisor to attend.

Professor Stewart is given the tree that describes the structure of the corporation, using the left-child, right-sibling representation. Each node of the tree holds, in addition to the pointers, the name of an employee and that employee's conviviality ranking. Describe an algorithm to make up a guest list that maximizes the sum of the conviviality ratings of the guests. Analyze the running time of your algorithm.

## 2 Dynamic Programming (10 marks)

Suppose you have one machine and a set of  $n$  jobs  $a_1, a_2, \dots, a_n$  to process on that machine. Each job  $a_j$  has a processing time  $t_j$ , a profit  $p_j$ , and a deadline  $d_j$ . The machine can process only one job at a time, and job  $a_j$  must run uninterruptedly for  $t_j$  consecutive time units. If job  $a_j$  is completed by its deadline  $d_j$ , you receive a profit  $p_j$ , but if it is completed after its deadline, you receive a profit of 0. Give an algorithm to find the schedule

that obtains the maximum amount of profit, assuming that all processing times are integers between 1 and  $n$ . What is the running time of your algorithm?

### 3 Dynamic Programming (10 marks)

To assess how “well-connected” two nodes in a directed graph are, one can not only look at the length of the shortest path between them, but can also count the *number* of shortest paths.

This turns out to be a problem that can be solved efficiently, subject to some restrictions on the edge costs. Suppose we are given a directed graph  $G = (V, E)$ , with costs on the edges; the costs may be positive or negative, but every cycle in the graph has strictly positive cost. We are also given two nodes  $v, w \in V$ . Give an efficient algorithm that computes the number of shortest  $v - w$  paths in  $G$ .

### 4 Dynamic Programming (10 marks)

You have a set of  $n$  integers each in the range  $0, \dots, K$ . Partition these integers into two subsets such that you minimize  $|S_1 - S_2|$ , where  $S_1$  and  $S_2$  denote the sums of the elements in each of the two subsets.

### 5 Dynamic Programming (10 marks)

Given a sequence of  $n$  real numbers  $a_1, \dots, a_n$ , determine a subsequence (not necessarily contiguous) of maximum length in which the values in the subsequence form a strictly increasing sequence.