Approximation Algorithm

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set cover problem

greedy algorithm: (weight $\equiv 1$)

1° $\max|S_i|$: choose S_1

2° $\underline{\max|S_i - S_1|}$: choose S_2

$\max|S_i - S_1 - S_2|$

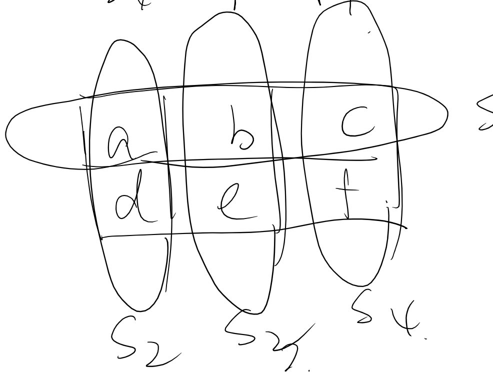
$U = \{a, b, c, d, e, f\}$

$S_1 = \{a, b, c\}$

$S_2 = \{a, d\}$

$S_3 = \{b, e\}$

$S_4 = \{c, f\}$



Algo: $S_1, S_2, \ldots, S_k$ $|Algo| = k$

OPT: $T_1, T_2, \ldots, T_l$ $|OPT| = l$

$$\begin{cases} |S_1| \geq |T_j| \quad \forall j \\ |S_2 - S_1| \geq |T_j - S_1| \quad \forall j \\ \vdots \\ |S_i - (S_1 \cup \cdots \cup S_{i-1})| \geq |T_j - (S_1 \cup \cdots \cup S_{i-1})| \quad \forall j \end{cases}$$

$$|S_1| \geq \frac{|T_1| + \cdots + |T_l|}{l} \geq \frac{|T_1 \cup \cdots \cup T_l|}{l} \geq \frac{n}{l} \Leftarrow$$

$$|S_2 - S_1| \geq \frac{|T_1 - S_1| + \cdots + |T_l - S_1|}{l} \geq \frac{n - |S_1|}{l}$$

$$\frac{1}{|S_1|} \leq \frac{l}{n}$$

$$|S_i - (S_1 \cup \cdots \cup S_{i-1})| \geq \frac{n - |S_1 \cup \cdots \cup S_{i-1}|}{l}$$

$S_1 = u_1 \cdots u_{|S_1|}$ weight $= \frac{1}{|S_1|}$

$S_2 - S_1 = u_{|S_1|+1}, \cdots u_{|S_1 \cup S_2|} \Leftarrow \frac{1}{|S_2 - S_1|}$

1 ~ 总 weight = $\frac{1}{|S_1|} + \frac{1}{|S_1|} + \cdots + \frac{1}{|S_1|} + \frac{1}{|S_2 - S_1|} + \cdots + \frac{1}{|S_2 - S_1|} + \cdots$

k = 总 weight =
$$\underbrace{\frac{1}{|S_1|}+\frac{1}{|S_1|}+\cdots+\frac{1}{|S_1|}}_{|S_1|个}+\underbrace{\frac{1}{|S_2-S_1|}+\cdots+\frac{1}{|S_2-S_1|}}_{|S_2-S_1|}+\cdots$$

$$\leq \underbrace{\frac{l}{n}+\frac{l}{n}+\cdots+\frac{l}{n}}_{|S_1|}+\underbrace{\frac{l}{n-|S_1|}+\cdots+\frac{l}{n-|S_1|}}_{|S_2-S_1|}+\cdots$$

$$= l\left(\underbrace{\frac{1}{n}+\cdots+\frac{1}{n}}_{|S_1|}+\underbrace{\frac{1}{n-|S_1|}+\cdots+\frac{1}{n-|S_1|}}_{|S_2-S_1|}+\cdots\right)$$

$$\leq l\left(\underbrace{\frac{1}{n}+\frac{1}{n-1}+\frac{1}{n-2}+\cdots+\frac{1}{n-|S_1|+1}}_{|S_1|个}+\underbrace{\frac{1}{n-|S_1|}+\cdots+\frac{1}{n-|S_1+S_2|+1}}_{|S_2-S_1|个}\right)\cdots$$

$$= l\cdot\boxed{\sum_{i=1}^{n}\frac{1}{i}} \leq l(\log n+1)$$