

# History and Development of Dynamic Programming

R. E. Bellman and E. S. Lee

## A Memorial to Richard E. Bellman



August 26, 1920–March 19, 1984  
E. S. Lee

The sudden death of Professor Richard Bellman on March 19, 1984, is not only a personal loss to his family and friends but is also a tragic and tremendous loss to the scientific community. Professor Bellman's pioneering work in pure and applied mathematics has influenced and literally created many different fields and directions of research in such diversified areas as mathematical physics, stability theory, control theory, operations research, the various engineering fields, economics, water resources, psychology, artificial intelligence, and energy. He was a member of both the National Academy of Engineering and the National Academy of Sciences. He has 621 research papers in technical journals and 40 books to his credit. His work has been translated into various languages. The existence of areas such as dynamic programming, invariant imbedding, and quasilinearization, just to mention a few, are completely the creation of Professor Bellman.

Although Professor Bellman's scientific contributions are well known and well documented (at least two journals have had dedication issues on the occasion of his 60th birthday: *IEEE Transactions on Automatic Control*, October 1981, and the *Journal of Optimization Theory and Applications*, December 1980), his compassionate

personality is not well known except to those of us who were fortunate enough to have known him. I shall mention several examples to illustrate this.

Many of us know how bad the traffic is from Professor Bellman's office in USC to Santa Monica in the Los Angeles area. Somehow, I always underestimated the driving time and, thus, was always late. Dick was very concerned that I might be in too much of a hurry and, thus, cause an accident. He repeatedly told me in his earnest and sincere way to slow down. In fact, on the night of March 17, he gave me a long and sincere talk on the importance of driving carefully to avoid any accidents. One cannot imagine the shock received when told, barely 48 hours later, the news of Dick's death.

Dick was a humanitarian and constantly concerned about the contribution of scientific progress to the betterness of mankind. This is the principle reason for his interest and tremendous contribution in medicine, environment, and energy. The year after the Watts riots, Dick invited the Bootstrap Director to send students to the computer classes and followed up with a summer school program for teaching high school students computer programming and mathematics. This probably was the first computer literacy course ever given at the high school level.

Dick's breath of knowledge was tremendous. But, above all, he had the extraordinary capacity to give, to support, and to persevere. He gave me many new ideas during our weekly discussions on the phone. He could recall past work to the exact date, author, and title of the article.

Dick Bellman cast a giant shadow, not only in the scientific community, which is well known, but also as a person, a teacher, and a friend. Both in his contributions to science and as a person, he was the ultimate example in scientific creativity and success, personal courage and strength, friendly devotion and support.

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**ABSTRACT:** The development of DP (dynamic programming) is presented in a chronological order. Although it seems awkward, it does show that science never develops in a

correct and straightforward fashion. It is only afterwards that a scientific development can be made logical and rational. The emphasis is placed on the actual development of DP and its interconnections with other basic areas, such as invariant imbedding. Another area of emphasis is the possible future research directions. Most of the detailed derivations and equations are omitted.

### The Formulation Period

The concept of DP (dynamic programming) was gradually formulated during 1948 to 1952. The first paper on DP was published in 1952 [1].

In 1948, the main mathematical activity at the mathematics division at RAND was the theory of games, under the influence of von Neuman who visited RAND from time to time. There are many difficulties in applying this theory to the real world, and it formulates typical multistage decision processes. One of the problems of great interest at the time was the selection of enemy targets, or the optimal use of guided missiles against enemy targets. These targets can be stationary or moving, such as attacking missiles or planes. Consider the simple model of  $N$  targets. The values of these targets are  $V_1, V_2, \dots, V_N$ . There are  $S$  identical missiles, which we wish to use. The purpose is to allocate the missiles so as to reflect the maximum damage to the targets. Since this damage is stochastic, let us use the total expected damage as the measure of success.

A number of  $S_i$  of the  $S$  missiles are allocated to the destruction of the  $i$ th target. Let  $P_i(S_i)$  be the probability that the  $i$ th target will be destroyed by these  $S_i$  missiles, then the total expected damage is

$$E(D) = \sum_{i=1}^N p_i(S_i) V_i \quad (1)$$

Our problem is to maximize Eq. (1) subject to the restrictions

$$\sum_{i=1}^N S_i = S \quad (2)$$

$$0 \leq S_i \leq S, \quad i = 1, 2, \dots, N \quad (3)$$

In solving this simple stochastic allocation problem, many difficulties were encountered.

tered. Among the available techniques at the time, the classical calculus of variations is probably the most used one. But, this classical technique cannot be applied easily. Observe that  $S_i$  only assumes integer values, and  $P_i(S_i)$  may not be differentiable. Even if these two difficulties were overcome, we still face the difficulty that  $S_i$  may be zero.

There are many other practical problems that have similar difficulties and that have the stochastic multistage decision process structure. Some typical examples of these problems are:

(1) We are given a quantity  $x > 0$  that is to be utilized to accomplish a certain task. If an amount  $y$ , where  $0 \leq y \leq x$ , is used on any single attempt, the probability of success is  $a(y)$ . If the task is not accomplished on the first try, we continue with the new quantity  $x - y$ . How does one proceed in order to maximize the overall probability of success?

(2) We are informed that a particle is in either state 0 or 1, and we are given, initially, the probability  $x$  that is in state 1. Use of operation  $A$  will reduce this probability to  $ax$ , where  $a$  is some positive constant less than 1, and whereas operation  $L$ , which consists of observing the particle, will tell us, definitely, which state it is in. If it is desirable to transform the particle into state 0 in a minimum time, what is the optimal procedure?

(3) At each stage in the sequence of the actions, we are allowed a choice of one of the two actions. The first has associated a probability  $p_1$  of gaining one unit, a probability  $p_2$  of gaining two units, and a probability  $p_3$  of terminating the process. The second has a similar set of probabilities  $p'_1, p'_2, p'_3$ . What sequence of choices maximizes the probability of attaining at least  $n$  units before the process is terminated?

(4) We are fortunate enough to possess two gold mines,  $A$  and  $B$ , the first of which possesses an amount  $x$  of gold, while the second possesses an amount  $y$ . If the only gold-mining machine we have is used in  $A$ , there is a probability  $p_1$  that  $r$  percent of the gold there will be brought up safely, the machine still being usable, and a probability  $(1 - p_1)$  that the machine will be damaged, will mine no gold, and will be of no further use. Similarly, mine  $B$  has the probabilities  $q_1$  and  $(1 - q_1)$  associated with it. How does one proceed in order to maximize the total amount of gold obtained before the machine is defunct?

(5) Let us consider the above problem in the case in which we know only the expected amounts of gold in each mine and the expected amount mined each time, without being able to observe the results of individual operations.

(6) Two players,  $A$  and  $B$ , the first possessing  $x$  dollars and the second possessing  $y$  dollars, play a modified coin-tossing game described by the matrix

$$M = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$$

Assuming that each player is motivated by a desire to ruin the other, how does each play?

Another approach is to use enumeration. Since computers were beginning to be available at that time, a detailed enumeration to solve the problem may be possible. Consider Problem (3), listed above with  $N$  stages. This requires  $2 \cdot 4^{N-1}$  listings in order to enumerate all possible rules. We now know that the amount of computation required is prohibitive even for a fairly simple, practical allocation problem.

The stochastic nature of the problem makes the enumeration approach even more difficult. The outcome of any action is indeterminate, specified only by a distribution function. Problems of a second order of difficulty, overlapping the domain of sequential analysis, are those in which the distribution function is only partially known. Third-order problems would perhaps be those in which it is not known whether or not a distribution function exists. There existed no effective techniques to solve these large-class practical problems on a computer.

After trying enumeration and various other approaches, Dr. Bellman discovered a technique or approach that circumvents the dimensional explosion for multistage processes with enumeration. Furthermore, in trying to solve various multistage processes, it was discovered that the same technique was used over and over again to obtain the functional equation. This functional equation has basically the same character for all the different multistage stochastic decision processes, and the following common features are apparent:

- (1) The state of the system is described by a small set of parameters.
- (2) The effect of a decision is to transform this set of parameters into a similar set.
- (3) The past history of the system is of no importance in determining future actions, a Markovian property.

Dr. Bellman decided to call this technique "the principle of optimality," which states "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

It should be pointed out that "the principle of optimality" is not only not rigorous, it is

not even precise. However, it does guide the formulation of the functional equation.

Consider problem (1), let

$$f(x) = \text{overall probability of success using an optimal procedure} \quad (4)$$

If we use an amount  $y$  on the first try, our probability of success is  $a(y)$ . If we fail on the first try, an occurrence with probability  $[1 - a(y)]$ , we use an optimal policy starting with the residual amount  $x - y$ . Hence, applying the principle of optimality, we have

$$f(x) = \max_{0 \leq y \leq x} \{a(y) + [1 - a(y)]f(x - y)\} \quad (5)$$

As one can see, if we apply the principle to the above different problems, we are repeatedly obtaining equations similar to Eq. (5). The most important feature is that the principle function or optimal return depends only on the residual amount, or the amount remaining and the number of remaining stages.

### Why "Dynamic Programming"?

An interesting question is, "Where did the name *dynamic programming* come from?" The technique really is for solving multistage stochastic decision processes. The word *stochastic* is not used lightly. DP was developed for stochastic processes. It was realized fairly late that DP can also be used to solve deterministic problems.

The 1950s were not good years for mathematical research. We had a very interesting gentleman in Washington as the Secretary of Defense, and he actually had a pathological fear and hatred of the word *research*. His face would suffuse, he would turn red, and he would get violent if people used the term *research* in his presence. One can imagine how he felt, then, about the term *mathematical*. The RAND Corporation was employed by the Air Force, and the Air Force had the Secretary of Defense as its boss, essentially. Hence, Dr. Bellman felt that he had to do something to shield the Air Force from the fact that he was really doing mathematics inside the RAND Corporation. What title, what name, could one choose? In the first place, one was interested in planning, in decision making, in thinking. But *planning* is not a good word for various reasons. He decided, therefore, to use the word *programming*. He wanted to get across the idea that this was dynamic, this was multistage, this was time-varying; let's kill two birds with one stone. Let's take a word that has an absolutely precise meaning, namely *dynamic*, in the classical, physical sense. It also has a very interesting property as an adjective, that is, it's impossible to use the

word *dynamic* in the pejorative sense. Try thinking of some combination that will possibly give it a pejorative meaning. It's impossible. Thus, *dynamic programming* was a good name.

A RAND report on DP was published in 1953 [2], and a book on DP was published in 1957 [3].

## Deterministic Problems and Optimal Control

By this time, Dr. Bellman started to work on control theory. The tool used was calculus of variations. It was discovered that a very simple problem required great ingenuity to solve. A small change in the problem caused a great change in the solution. It was concluded that the calculus of variation is not an effective tool for obtaining numerical solutions. A monograph was published as a RAND report [4]. This monograph was translated into Russian, but it was never published in English.

However, there clearly has to be some connection between DP and calculus of variations. A number of the mathematical models of DP was analyzed using calculus of variations.

But, the problem remained, how does one obtain numerical solutions of optimization problems of the deterministic type? DP was developed for stochastic decision processes. One did not contemplate the application of DP to control processes of deterministic types. However, after contemplating for a long time, the thought finally occurred that the desired solution in a control process was a policy, "Do thus-and-thus if you find yourself in this portion of state space with this amount of time left." Conversely, once it was realized that the concept of policy was fundamental in control theory, the mathematization of the basic engineering concept of "feedback control," then the emphasis upon a state variable formulation became natural. We see, then, a very interesting interaction between DP and control theory. This reinforces the point that when working in the field of analysis, it is exceedingly helpful to have some underlying physical processes clearly in mind.

What is worth noting about the foregoing development is that one should have seen the application of DP to control theory several years before. One should have, but didn't. It is all very well to start a lecture by saying, "Clearly, a control process can be regarded as a multistage decision process in which..." but it is a bit misleading. Scientific developments can always be made logical and rational with sufficient hindsight. It is amazing, however, how clouded the

crystal ball looks beforehand. We all wear such intellectual blinders and make such inexplicable blunders that it is amazing that any progress is made at all.

By treating deterministic control process as a multistage process and applying DP, we have overcome the various difficulties encountered when calculus of variations is used. Some of these difficulties are: the two-point boundary value problem, the requirement of differentiable functions, and the various constraint difficulties.

## Numerical Solution

It is obvious that DP can be supplied to solve many practical problems. However, the effectiveness of the numerical algorithm of DP needs to be tested. A final goal of any scientific theory must be the derivation of numbers. Theories stand or fall, ultimately, depending upon numbers. It is usually, if not always, impossible to predict where a theoretical investigation will end once started. But what one can be certain of is that the investigation of a meaningful scientific area will lead to meaningful mathematics. Inevitably, as soon as one pursues the basic theory of obtaining numerical answers to numerical questions, one will be led to all kinds of interesting and significant problems in pure mathematics.

The computation was programmed in machine language. It was before FORTRAN and other simple languages. The Johnniac was the computer used. Time on the computer was preempted by the physics and engineering divisions. Consequently, Stuart Dreyfus slept during the day so that he could use the computer at night.

Many processes were investigated. The algorithm worked uniformly well. The results were published in 1962 [5].

## Invariant Imbedding

C. Chandrasekhar published his book on radiation transfer in 1956 [6]. It was obvious that the principle of invariants in the book is DP without optimization. It is known that DP could take problems in calculus of variations and, by introducing suitable variables, make them initial value problems. Apparently, Ambarzumian's method could do the same for many parts of mathematical analysis and mathematical physics.

We shall not go into details on the development of invariant imbedding. The reader is referred to the various publications [7-12]. However, the connection between DP and invariant imbedding needs to be explored. In DP, we have a type of approximation, approximation in policy space, which does not

exist in classical analysis. Every sequence of decisions corresponds to a function, but not every function corresponds to a sequence of decisions. What makes the method so powerful is that many questions can be considered to arise from a decision process. In invariant imbedding, we have approximation in process space, again an approximation that does not exist in classical analysis. Every process gives rise to a function, but not every function corresponds to a process.

It should be noted that the Riccati equation plays a basic role in both DP and invariant imbedding. Also, we discovered that the basic invariant-imbedding functional equation can be obtained in many different ways [11]. The derivation can start from a consideration of the original physical system, or it can start from the equations that represent the original physical system. The former has its origin of particle counting in physics, and the latter is essentially a typical mathematical derivation by the use of the Taylor series and by the observation that the desired function depends only on the resources remaining and the remaining duration of the process. These are the same observations that form the basis in DP. If the process is linear, the functional equation of invariant imbedding reduces to the Riccati equation.

Consider the nonlinear boundary-value problem

$$\frac{dx}{dt} = f(x, y, t) \quad \frac{dy}{dt} = g(x, y, t) \quad (6)$$

with boundary conditions

$$x(0) = c \quad y(t_f) = 0 \quad (7)$$

with  $0 \leq t \leq t_f$ . In order to avoid the various computational difficulties in solving the above boundary-value problem, we shall convert it into an initial-value problem. In other words, the missing initial condition  $y(0)$  will be obtained by using the invariant-imbedding concept. To do this, consider the problem with the more general boundary conditions

$$x(a) = c \quad y(t_f) = 0 \quad (8)$$

where  $a \leq t \leq t_f$  and  $a$  is the starting value of the independent variable  $t$ . However, it should be kept in mind that  $a$  also controls the duration of the process. If  $a$  assumes different values from zero to  $t_f$ , say  $a = 0, \Delta, 2\Delta, \dots$ , then there will be a family of problems. Each member of this family has a different starting value of  $a$  and is represented by Eqs. (6) and (8). Let us consider obtaining the missing initial conditions  $y(a)$  for this family of problems. The idea is that neighboring processes are related to each other. It may be possible to obtain the miss-

ing initial condition for the original problem  $y(0)$  by examining the relationships between the neighboring processes.

Notice that the missing initial condition  $y(a)$  for this family of problems is not only a function of the starting point of the process  $a$  but also a function of the starting state or the given initial condition  $c$ . Define

$r(c, a)$  = the missing initial condition for the system represented by Eqs. (6) and (8) where the process begins at  $t = a$  with  $x(a) = c$ . (9)

Obviously,

$$y(a) = r(c, a) \quad (10)$$

Notice that  $x(a)$  and  $y(a)$  represent the starting state of the process.

We shall consider  $r$  as the dependent variable, and  $c$  and  $a$  as the independent variables. An expression for  $r$  in terms of  $c$  and  $a$  will be obtained.

After some simple and typical manipulations by the use of the Taylor series, we finally obtain the functional equation of invariant imbedding

$$\begin{aligned} r(c, a) + g(c, \gamma(c, a), a)\Delta \\ = r(c + f(c, r(c, a), a)\Delta, a + \Delta) \end{aligned} \quad (11)$$

Notice that the function  $r(c, a)$  corresponds to the optimal return function in dynamic programming and the definition in Eq. (9) emphasizes the dependence of  $r$  on the remaining initial state and the remaining duration of the process. Equation (11) is exactly the functional equation of dynamic programming without optimization. If we take to the limits as  $\Delta$  tends to zero, we obtain the partial differential equation of DP without optimization

$$\begin{aligned} f(c, r(c, a), a) \frac{\partial r(c, a)}{\partial c} + \frac{\partial r(c, a)}{\partial a} \\ = g(c, r(c, a), a) \end{aligned} \quad (12)$$

with the initial condition

$$r(c, t_f) = 0 \quad (13)$$

For linear systems, let

$$\begin{aligned} f(x, y, t) &= q_{11}(t)x + q_{12}(t)y \\ g(x, y, t) &= q_{21}(t)x + q_{22}(t)y \end{aligned}$$

Obviously, because it is linear and homogeneous, we have

$$r(c, a) = R(a)c \quad (14)$$

Substituting Eq. (14) into Eq. (12), we have

$$\begin{aligned} \frac{dR(a)}{da} &= q_{21}(a) + [q_{22}(a) - q_{11}(a)]R(a) \\ &\quad - q_{12}(a)R^2(a) \end{aligned} \quad (15)$$

with the condition

$$R(t_f) = 0 \quad (16)$$

Equation (15) is the Riccati equation.

## The Dimensionality Difficulty

One of the principal difficulties in using DP is the "curse of dimensionality." As the number of state variables increase, not only the computer time, which we have if the problem is important enough, increases rapidly but also the required computer memory, of which we only have a very limited amount, increases exponentially. Because of this limitation on computer memory, we probably will never solve a very large general DP problem, as we know it today, without approximation.

The research on DP, after the basic development, is essentially how to overcome the dimensionality difficulties. Various techniques have been devised. We have had a certain amount of success. However, the basic dimensionality problem that we encounter in almost every applied mathematics endeavor to solve large problems is still and will always be with us. Enumeration, the various search techniques, combinatorial problems, and artificial intelligence are all limited by the exponential increase of the possible various combinations. One approach to solve this dimensionality problem seems to be by the use of approximation, selective computation, and heuristic judgment as being developed in artificial intelligence.

## Discussion and Conclusions

We have discussed the development of DP and its interconnections with other areas according to the actual chronological order. Although it seems awkward, science never develops in the correct and straightforward order. It is easy to see now that invariant imbedding or the semigroup theory is the most general concept. If we add optimization to the invariant-imbedding equation, we obtain the functional equation of DP. Optimal control is a special case of DP. The calculus of variations problems can be reformulated by using either the principle of optimality or the invariant-imbedding formulation to change the original boundary-value problem into initial value problems. Furthermore, the new formulation has the very unusual and attractive feature of approximation in policy space rather than the approximation in function space, which is the case in the original classical approach.

In our discussion, we have omitted most of the equations and the detailed developments. This is done for two reasons. In the first

place, DP is well known; most readers would have a familiarity with these equations. Secondly, it is obviously infeasible to go into any detail even only for the most recent developments in such a short paper. We have given a little more detail in invariant imbedding, because we feel that the concept of invariant imbedding has not been fully explored. Since this concept is completely different from the usual or classical approach, some of the computational features certainly should be further explored.

Many people are intimidated with the dimensionality difficulty of DP and, thus, misunderstand the concept of invariant imbedding. The dimensionality difficulty of invariant imbedding is quite different from that of DP. This is because of the absence of the maximum or minimum operation. Furthermore, it should be emphasized that invariant imbedding is only a concept. It can be applied and developed in many different directions.

*Large systems.* A new genre of mathematical problems has arisen in the recent years, namely, controlling large systems. Furthermore, it is not even a question of control; that is much too ambitious. It is a question of feasible operation. Those of us living in big cities, afflicted by air pollution, water pollution, traffic jams, noise, overcrowding, and all the other "blessings" of big-city civilization, can appreciate this point.

There is little difficulty nowadays in using any of several methods to obtain either an analytic or computational solution of the determination of optimal control of a system specified by four state variables. As computers become more powerful and as our analytic tools are sharpened, we can begin to feel confident in obtaining detailed solutions when faced with eight or sixteen state variables.

But if one examines some of the problems connected with the control of the American economy or, on a much more humble level, with the feasible operation of a chemical refinery, one begins to realize how far we are from understanding problems of this nature. The conventional descriptions require thousands upon thousands of equations, subject to the demands of "on-line" control.

The general problem of operating a large system with a limited amount of time available for observation, for data processing, and for implementation of control, generates a new kind of mathematical questions that have not yet been precisely formulated, and certainly not resolved.

*Artificial intelligence.* In order to solve these complicated problems, we are leading to another area of development, namely, artificial intelligence. This area was started

many years ago. However, due to the limitation of computer capability, this area did not make much progress until recently. It should be noted that the expert system is a knowledge-based multistage decision process. The decision may be heuristic or merely the use of rules of thumb. Thus, it is not in the strict sense of optimization as in DP. However, this can be made in the general sense of invariant imbedding, with the aid of approximations. Recently, we have had some success in the systematization of this area by the concept of invariant imbedding.

**Stochastic processes.** DP is ideally sited for stochastic decision processes. Various studies on stochastic decision processes by the use of DP have been carried out. These studies include both numerical and analytical solutions.

Let us look at optimal control of a stochastic process. The basic assumption of control is that we observe the system at each time, and on this basis choose the appropriate control or action. Consider what the calculus of variations prescribes. It chooses a function of time, the optimal control function, then never bothers to observe the system from the initial state on. According to this approach, once we have decided what to do, we do not look at the system again.

How can we interpret these concepts when we accept the existence of uncertainty? One way to interpret the phrase "never bothers to" is to say that we are not able to. Hence, we can say that in the face of uncertainty, the classical approach is invoked in situations where it is impossible to obtain any further information concerning the process, and DP is used when we do possess means of observing the system in operation and of utilizing the data obtained in this way. In other words, we use DP when we possess complete or partial information concerning the state of the system at each time, but we use the classical approach when we must make an a priori decision and subsequently have no chance to correct it as further information is obtained.

This difference is a very exciting thought. It means that when we introduce uncertainty

in decision making, we automatically introduce a spectrum of processes intermediate between the "locus of points" formulation and that of the "envelope of tangents."

In order to analyze mathematically these new kinds of control processes, we must introduce the basic concept of "information." What was completely neglected in a classical theory of control was the examination of the kinds and accuracy of the information available to the decision maker. Today, we have all the possibilities before us. At any given time, we can assume that no information is available, that all information is available, or that various kinds of partial information is ascertainable. Furthermore, we can introduce the idea of interrupted control processes where there are probabilities of certain kinds of data being unavailable at any time. The many categories of the possible kinds of uncertainties have not yet been systematically explored.

Once we have attained this plateau of sophistication, we can introduce the possibility of a choice of the kind of information we want for decision making at any time. Let us further realistically accept the condition that either we have only a specific time for decision making or we have a specific number of sensing devices with various capabilities. The problem then arises of deciding what parts of the system to observe, what kinds of accuracy are required, and when to observe.

Consequently, we can start with the simple idea of a control process, and by constant analysis of what is possible, what is available, what constraints exist, and so forth, obtain many new interesting kinds of mathematical problems. Very few such problems have so far been either formulated or analyzed [13].

## References

- [1] R. Bellman, "On the Theory of Dynamic Programming," *Proc. Nat. Acad. Sci.*, vol. 38, p. 716, 1952.
- [2] R. Bellman, *An Introduction to the Theory of Dynamic Programming*, R-245, The RAND Corp., June 1953.

- [3] R. Bellman, *Dynamic Programming*, Princeton, NJ: Princeton University Press, 1957.
- [4] R. Bellman, I. Glicksterg, and O. A. Gross, *Some Aspect of the Mathematical Theory of Control Processes*, R-313, The RAND Corp., Jan. 1958.
- [5] R. Bellman and S. Dreyfus, *Applied Dynamic Programming*, Princeton, NJ: Princeton University Press, 1962.
- [6] C. Chandrasekhar, *Radiation Transfer*, Dover, NY, 1960.
- [7] R. Bellman, R. Kalaba, and M. Prestrud, *Invariant Imbedding and Radiation Transfer in Slabs of Finite Thickness*, American Elsevier, NY, 1962.
- [8] R. Bellman, H. Kagiwada, R. Kalaba, and M. Prestrud, *Invariant Imbedding and Time-Dependent Processes*, American Elsevier, NY, 1964.
- [9] G. M. Wing, *An Introduction to Transport Theory*, NY: Wiley, 1962.
- [10] R. Bellman, and G. M. Wing, *An Introduction to Invariant Imbedding*, NY: Wiley, 1975.
- [11] E. S. Lee, *Quasilinearization and Invariant Imbedding*, NY: Academic Press, 1968.
- [12] M. R. Scott, *Invariant Imbedding and Its Applications to Ordinary Differential Equations*, Reading, MA: Addison-Wesley, 1973.
- [13] R. Bellman, and E. S. Lee, *Stochastic Invariant Imbedding*, to be published.



**E. Stanley Lee** holds the M.S. degree and Ph.D. degree in chemical engineering from North Carolina State University and Princeton University, respectively. He has been a member of the research group of Phillips Petroleum Company and Professor of Chemical and Electrical Engineering at the University of Southern

California, Los Angeles. He joined Kansas State University in 1966 and is currently a Professor of Industrial Engineering. In addition to teaching, Dr. Lee is the recipient of many research grants and is an editor for various mathematical journals. He has four books to his credit and has published more than 100 research papers.

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Mar. 26-29, 1985	1985 IEEE International Conference Acoustics, Speech, & Signal Processing	Hyatt Regency Hotel Tampa, FL	Dr. Vijay Jain, Dept. of Electrical Engineering Univ. of South Florida, Tampa, FL 33620	Past
June 19-21, 1985	1985 American Control Conference	Boston Marriott Hotel Copley Place Boston, MA	Prof. David Wormley, Dept. of Mechanical Engr. MIT, Cambridge, MA 02139	Past
June 24-28, 1985	IFAC Symp. Auto. Cont. in Space	Toulouse, France	Prof. M. Pelegrin, CERT, Complexe Aerospatial de Lespinet, 2, Avenue Edouard Belin, BP 4025 F-31055 Toulouse Cedex, France	N/A