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On the contact domain method: A comparison of penalty and Lagrange multiplier implementations

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ABSTRACT

This work focuses on the assessment of the relative performance of the so-called contact domain method, using either the Lagrange multiplier or the penalty strategies. The mathematical formulation of the contact domain method and the imposition of the contact constraints using a stabilized Lagrange multiplier method are taken from the seminal work (as cited later), whereas the penalty based implementation is firstly described here. Although both methods result into equivalent formulations, except for the difference in the constraint imposition strategy, in the Lagrange multiplier method the constraints are enforced using a stabilized formulation based on an interior penalty method, which results into a different estimation of the contact forces compared to the penalty method. Several numerical examples are solved to assess certain numerical intricacies of the two implementations. The results show that both methods perform similarly as one increases the value of the penalty parameter or decreases the value of the stabilization factor (in case of the Lagrange multiplier method). However there seems to exist a clear advantage in using the Lagrange multiplier based strategy in a few critical situations, where the penalty method fails to produce convincing results due to excessive penetration.

1. Motivation

The computational modeling and analysis of structural contact problems have been important subjects of interest over the past several decades. Despite the significant progress achieved in the subject, it still poses a challenge in non-linear problems, especially when the aim is to develop accurate and efficient algorithms based on implicit methods.

While developing a contact formulation two main ingredients may be basically chosen:

- A scheme to discretize the contact surfaces or the interface between them.
- A technique to enforce the contact constraints.

Most of the existing contact formulations impose the contact constraints on the boundary of one of the contacting bodies, which necessitates the projection of certain quantities from one contacting surface onto the other. The most popular discretization strategy in the context of large deformation contact problems is the node-to-segment approach developed by Hallquist et al. [7] and further extended to more general cases by Bathe and Chaudhary [2], Simo et al. [19], Wriggers and Simo [23] and Papadopoulos and Taylor [16].

The non-penetration conditions are enforced by preventing the nodes on one of the contact surface (the 'slave' one) from penetrating on the counterpart contact surface (the 'master' one). The methodology inherits some drawbacks such as: (a) failure in passing the contact patch test for a single pass algorithm [8] and (b) being prone to lock when considered as a two-pass algorithm due to the overconstraining of the contact surface. Moreover in the classical formulation, the algorithm is unable to deal with some cases where the identification of the master segment is ambiguous with respect to the slave nodes. Recently, Zavarise and De Lorenzis [26] have facilitated new techniques to overcome such problems.

In recent years other discretization schemes were also developed, based on a continuous treatment of the contact constraints. The latest segment-to-segment discretization strategies are based on the so-called mortar method initially introduced in the context of domain decomposition methods [3]. In contrast to the node-to-segment discretization, the continuity constraints are not enforced at discrete nodal points but they are formulated along the entire coupling boundary in a weak integral sense. This method is particularly well suited to exchange information of two discretized domains along common, in general non-conforming, surface grids and it provides the optimal convergence rate of the finite element solution by imposing a weak coupling between degrees of freedom.

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Due to its obvious advantages, this method has found its application in many of the recently developed frictionless and frictional contact formulation focusing on large deformation problems [25,18,17,5].

Alike the discretization schemes, a variety of numerical methodologies have been proposed in the literature to deal with the contact constraints. Among them, the enforcement of the constraints using Lagrange multiplier methods, penalty methods, the augmented Lagrangian approach or the relatively new Nitsche method are the most common methodologies.

Lagrange multiplier methods introduce additional variables (the Lagrange multipliers) to enforce directly and exactly the contact constraint. Despite the obvious advantage of the exact enforcement of the constraint condition, the method poses some difficulties due to the additional effort required to solve the multipliers. In addition, the equations for the Lagrange multipliers introduce zeros in the diagonal of the system of equations, which pose additional difficulties if direct solution techniques are used. However applications of Lagrange multiplier based formulations are widespread till date and they can be found in [5,10,20], to name a few.

On the other hand, penalty methods avoid the need for additional variables by introducing an approximation of the constraint condition. An additional term enters in the weak form of the governing equations, which penalizes the dissatisfaction of the constraint condition by a large positive penalty parameter. Theoretically, as the penalty parameter tends to infinity, the contact constraint is enforced exactly. Unfortunately, the resulting system of equations may become ill-conditioned as the penalty parameter increases, so the choice of an appropriate penalty parameter becomes a balancing act between accuracy and stability. Way back in 1995 [4], a consistent penalty method had been used to model metal forming process using a visco-elastic formulation. Recently, Fischer and Wriggers [6] used penalty methods for solving 2D frictional contact for large deformation problems. Along with these conventional techniques, the augmented Lagrangian method is often chosen to cope with the contact inequality constraints inasmuch as it combines the regularizing effect of the penalty method and the exact satisfaction of the constraints ensured by the Lagrange multiplier method, without having the ill-conditioning problem inherent to the former [21,27]. In a recent work by Wriggers and Zavarise [24], a purely displacement based formulation has been presented based on a non-standard variational formulation introduced by Nitsche. It has been found that the new discretization scheme performs better than the standard penalty formulation for frictionless contact problems.

In the present paper the recently developed method for discretizing the contact interface named as *contact domain method* (CDM), originally proposed in [14], and its relative performance is assessed with respect to the schemes used for imposing the contact constraints i.e. the Lagrange multiplier and the penalty strategies. As mentioned in [14], in the CDM the contact domain can be interpreted as a fictitious intermediate region that connects the potential contact surfaces and has the same dimension as the deformable contacting bodies. The utilized contact domain is discretized with a non-overlapping set of patches that leads to a pairing of the contacting entities (nodes, segments and surfaces) in the contact boundaries. Based on this discretization scheme, the geometric normal and tangential contact constraints are formulated in terms of dimensionless, strain-like measures.

Although the detailed theoretical derivations and numerical implementations are described in [14], for the relevance of the present discussion the equations for describing the CDM are reconsidered in the following sections. Then the two strategies for imposing the contact constraints (Lagrange multipliers and a regularized penalty) are described and their equivalences are found. Finally, both schemes are compared by using the patch test and a number of relevant numerical examples, and their relative performance is evaluated in terms of accuracy and robustness and some relevant conclusions are presented.

2. Review of the contact domain method: stabilized Lagrange multiplier implementation

The contact domain method was originally proposed for 2D contacting bodies in [14] and validated with a number of numerical examples in [9]. An extension to the frictionless 3D case was given in [11]. The method poses some specific features for modeling contact between two largely deformable bodies, namely:

- The discretization scheme allowing the contact restrictions to be applied in a manifold (the contact domain) of the same dimension as the contacting bodies, which is in contrast with other (node-to-segment, mortar etc.) methods, where normally those restrictions are imposed in a domain whose dimension is one order less than the dimension of the body,
- That defined contact domain is then discretized, and completely covered by a set of non-overlapping contact patches, which implicitly determine all possible node-to-segment pairings, this having relevant consequences as for passing the patch.
- The contact domain, covering the inter-space between the contacting bodies, is endowed with a displacement field interpolated from the contacting boundaries. This allows defining the gaps in terms of stretch-like measures, which facilitates the derivation of the gap differentiation and its linearization.

As for the type of contact patches, it is worth mentioning that, for a 2D problem, triangular or quadrilateral patches can be used for the contacting domain irrespective of the meshing pattern used for the elements. However, linear triangular shape poses little advantages as mentioned in Section 2.2 of [14]. Hence, the entire formulation is restricted to linear triangular shaped contact patches in [14] and herein.

2.1. Contact patches and gap definitions

Let us consider two contacting bodies $\mathcal{B}^{(\alpha)},\ \alpha = 1,2$ undergoing large deformations (see Fig. 1); the associated deformation maps $\boldsymbol{\varphi}_t^{(\alpha)}$ connect the material points $\mathbf{X}^{(\alpha)}$ at the reference configuration onto points $\mathbf{x}^{(\alpha)}$ in the current configuration.

As mentioned in [14], at the time step $[t_n, t_{n+1}]$, the *contact domain* D_n, with boundary ∂D_n joining part of the boundaries of the contacting bodies, can be approximated by a domain D_n^l partitioned in n_p patches $D_n^{(p)}$ such that

$$D_n \approx D_n^l = \bigcup_{p=1}^{n_p} D_n^{(p)}, \tag{1}$$

the partition having the property that the patches do not overlap and D_n^l converges to the contact domain D_n as the number of patches $D_n^{(p)}$ and their vertices $V^{(i)}$ increase.

With reference to Fig. 2, we introduce $\mathbf{u}^D(\mathbf{x})$ as the *pseudo incremental displacement field*, at time step $[t_n, t_{n+1}]$ in the contact domain, conveniently interpolated from the incremental displacement at the nodes of the contacting boundaries using the linear shape functions at the contact patches; thus defining the extrapolated *pseudo incremental motion of the contact domain* $\phi : D_n \to D_{n+1}$, $\phi(\mathbf{x}_n) \equiv \phi^D(\mathbf{x}_n) = \mathbf{x}_n + \mathbf{u}^D(\mathbf{x}_n)$ as explained in [14].

We also introduce the entities $\mathbf{x}_n,\ \bar{\mathbf{x}}_n,\ \mathbf{x}_{n+1} = \phi(\mathbf{x}_n)$, respectively, as the position of a pseudo-material point in the contact patch at time t_n, its projection on the base-side of the contact patch