CS10 Assignment 2

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Notice:

1. Due Nov. 15, 2010.
2. Please hand in your answer in hard copy.
3. You can arbitrarily choose 1 problem.

The following problems can be used for your reduction to prove NP-completeness:
SAT, 3SAT, INDEPENDENT-SET, VERTEX-COVER
3-COLORING, SET-COVER, SUBSET-SUM, HAMILTON-CYCLE

1 NP-completeness (10 marks)

The SUBGRAPH-ISOMORPHISM problem takes two graphs G_1 and G_2 and asks whether G_1 is isomorphic to a subgraph of G_2. Show that the subgragh-isomorphism problem is NP-complete.

2 NP-completeness (10 marks)

Given an integer $m \times n$ matrix A and an integer m-vector b, the 0-1 INTEGER-PROGRAMMING problem asks whether there is an integer n-vector x with elements in the set $\{0, 1\}$ such that $Ax \geq b$. Prove that 0-1 integer programming is NP-complete.

3 NP-completeness (10 marks)

The SET-PARTITION problem takes as input a set S of numbers. The question is whether the numbers can be partitioned into two sets A and B and $B = S - A$ such that $\Sigma_{x \in A} x = \Sigma_{x \in B} x$. Show that the set-partition problem is NP-complete.

4 NP-completeness (10 marks)

In the HALF-3SAT problem, we are given a 3SAT formula ϕ with n variables and m clauses, where m is even. We wish to determine whether there exists a truth assignment to the variables of ϕ such that exactly half the clauses evaluate to 0 and exactly half the clauses evaluate to 1. Prove that the $half - 3SAT$ problem is NP-complete.

5 NP-completeness (10 marks)

Show that UNDIRECTED-HAMILTON-CYCLE problem is NP-complete.
UNDIRECTED-HAMILTON-CYCLE(DECISION PROBLEM):
Input: an undirected gragh G
Output: 1 if G has a Hamilton cycle which visits each vertex exactly once and also returns to the starting vertex, 0 for others