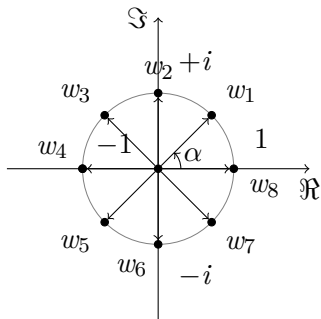
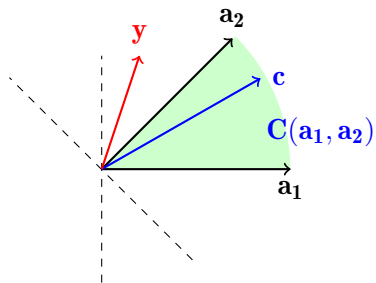
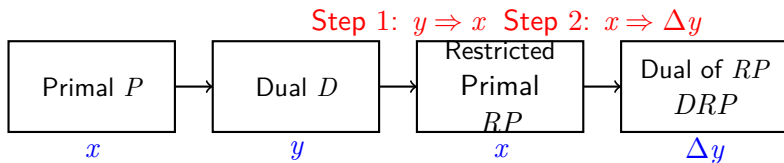


# Root of unity



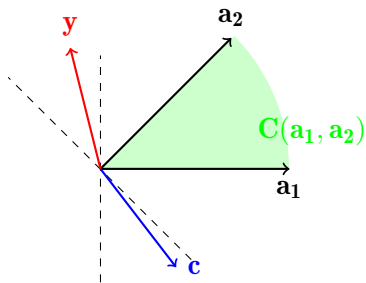
# Farkas lemma



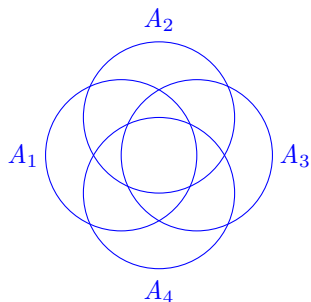


Step 3:  $y = y + \theta \times \Delta y$

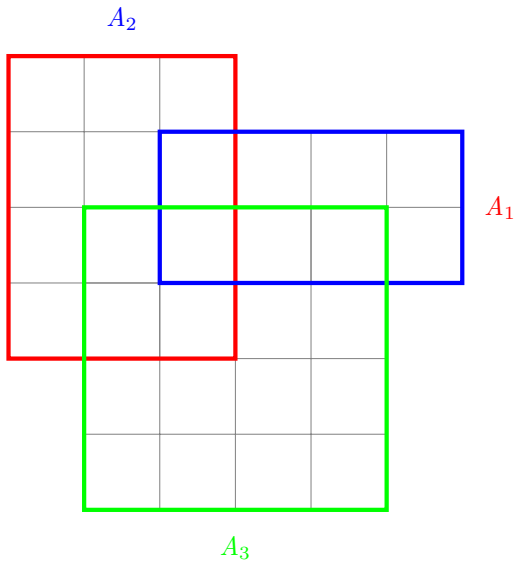
# Farkas lemma



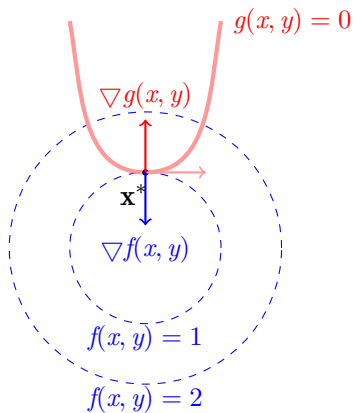
# Max Coverage Problem



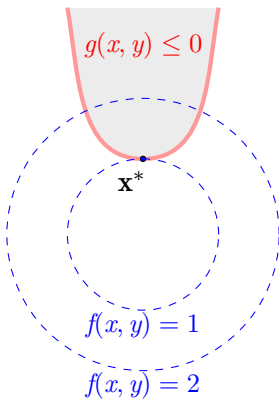
# Max Coverage Problem2



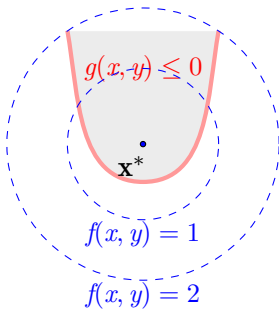
# Lagrangian

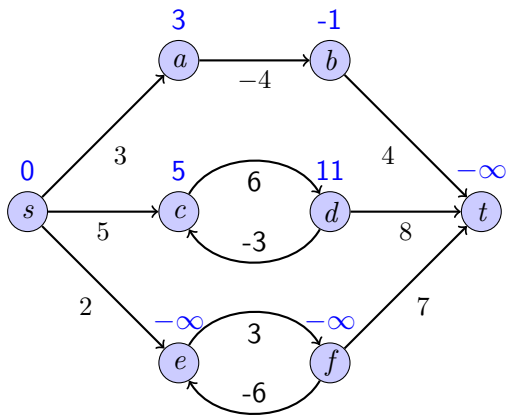


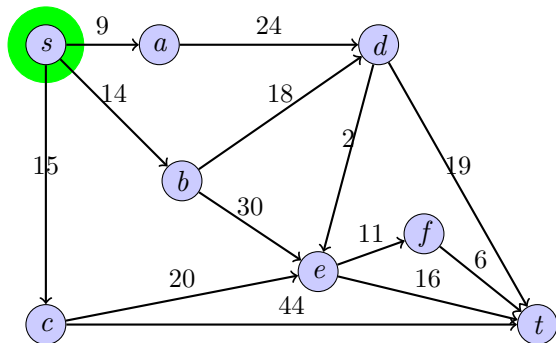
# Lagrangian

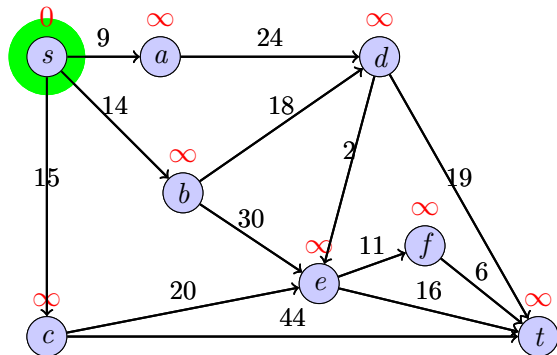


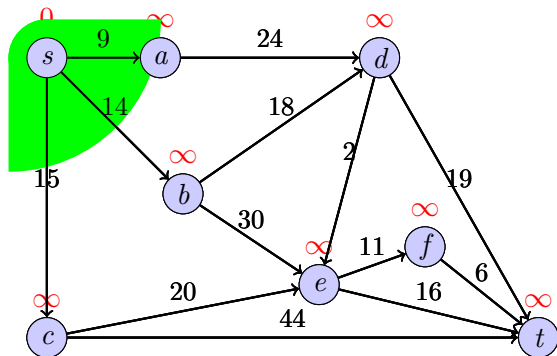


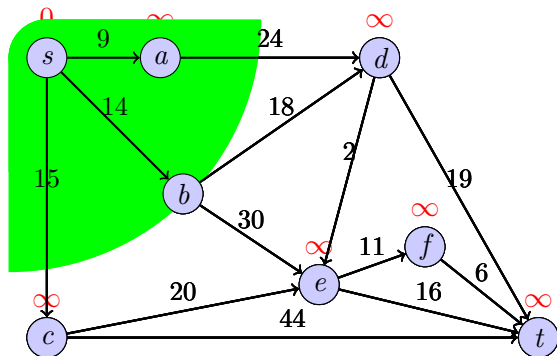


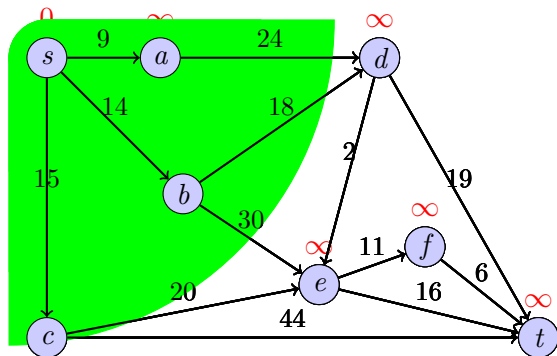


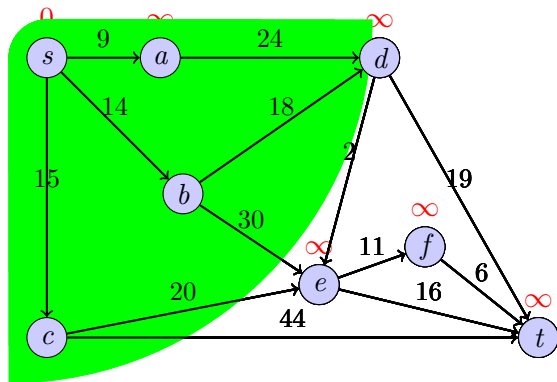




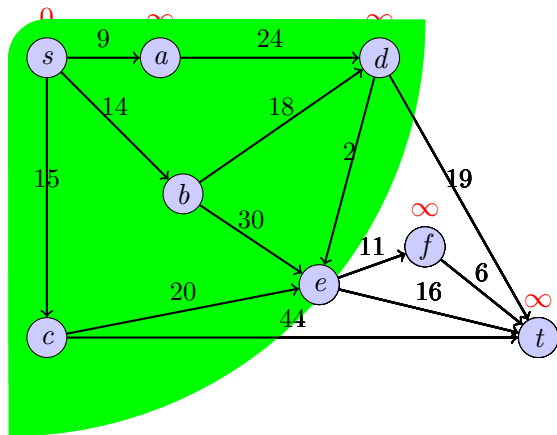


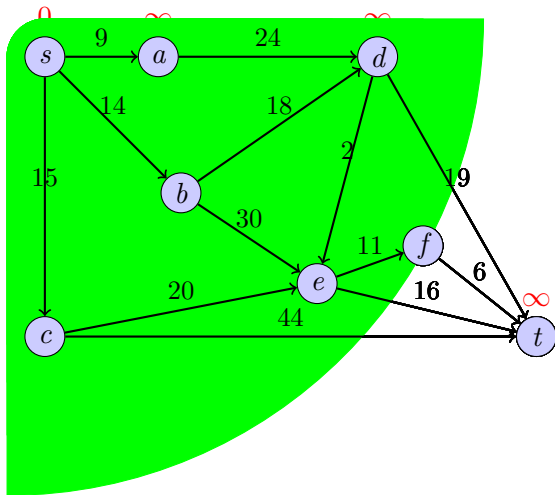


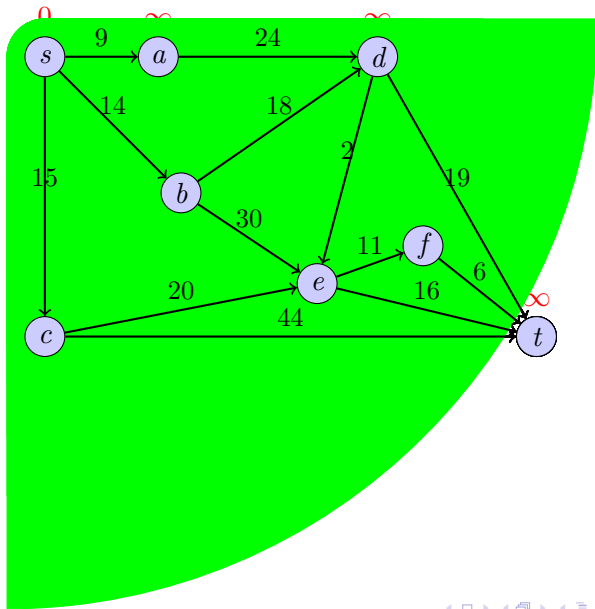


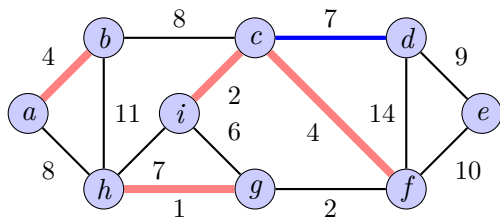




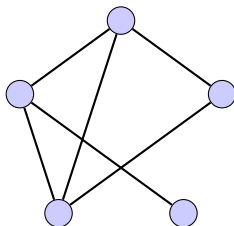


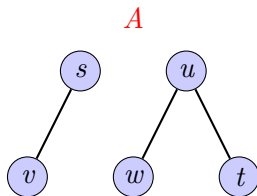
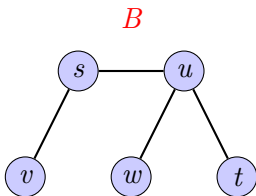
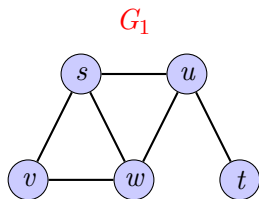


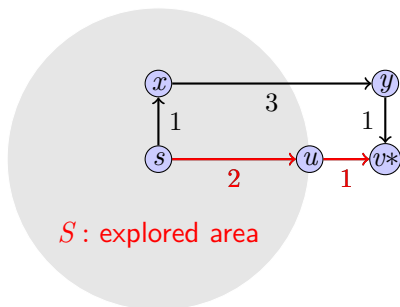


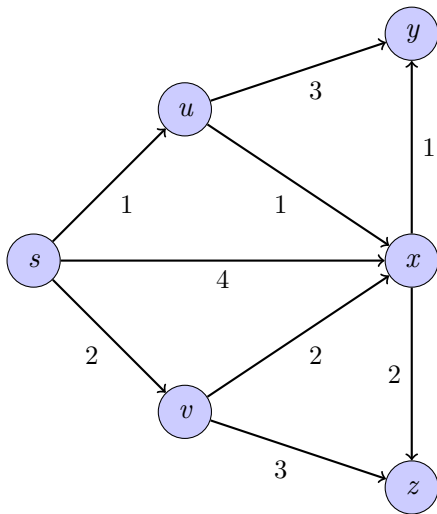


# Pentagon

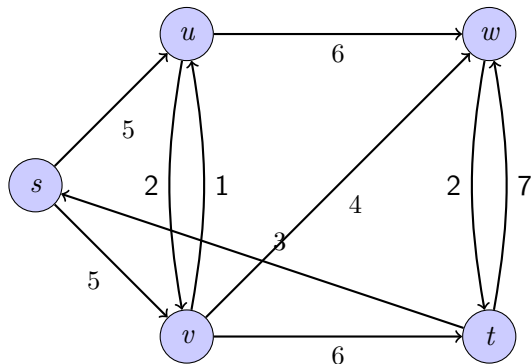












# Lec6 Alignment matrix Suffix FULL

4	0	-4	-10	-12	-16	-18	-22	-26	-30	E
5	3	-1	-7	-9	-13	-15	-19	-23	-27	C
3	6	2	-4	-6	-10	-12	-16	-20	-24	N
-1	2	5	-1	-3	-7	-9	-13	-17	-21	E
-5	-2	1	4	0	-4	-6	-10	-14	-18	R
-9	-6	-3	0	3	-1	-3	-7	-11	-15	R
-13	-10	-7	-4	-1	2	0	-4	-8	-12	U
-15	-12	-9	-6	-3	0	3	-1	-5	-9	C
-19	-16	-13	-10	-7	-4	-1	2	-2	-6	C
-23	-20	-17	-14	-11	-8	-5	-2	1	-3	O
-27	-24	-21	-18	-15	-12	-9	-6	-3	0	'
										'
										T
										S

# Lec6 Alignment matrix Prefix FULL

S:	'	'	O	C	U	R	R	A	N	C	E
T:	'	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
O		-3	1	-2	-5	-8	-11	-14	-17	-20	-23
C		-6	-2	2	-1	-4	-7	-10	-13	-16	-19
C		-9	-5	-1	1	-2	-5	-8	-11	-12	-15
U		-12	-8	-4	0	0	-3	-6	-9	-12	13
R		-15	-11	-7	-3	1	1	-2	-5	-8	-11
R		-18	-14	-10	-6	-2	2	-	-3	-6	-9
E		-21	-17	-13	-9	-5	-1	1	-1	-4	-5
N		-24	-20	-16	-12	-8	-4	-2	2	-1	-4
C		-27	-23	-19	-15	-11	-7	-5	-1	3	0
E		-30	-26	-22	-18	-14	-10	-8	-4	0	4

# Lec7 Shortest Path example FULL

	k=0	1	2	3	4	5
S	0	0	0	0	0	0
U	—	1	1	1	1	1
V	—	2	2	2	2	2
X	—	4	2	2	2	2
Y	—	—	4	3	3	3
Z	—	—	5	4	4	4

# Lec6 Alignment matrix Prefix

S:	'	'	O	C	U	R	R	A	N	C	E
T:	'	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
O		-3									
C		-6									
C		-9									
U		-12									
R		-15									
R		-18									
E		-21									
N		-24									
C		-27									
E		-30									

# Lec6 Step1

**OPT**

1	2	3	4	
0	6			1
	0	24		2
		0	60	3
			0	4

**SPLITTER**

1	2	3	4	
	1			1
		2		2
			3	3
				4

# Lec6 Step2

**OPT**

1	2	3	4	
0	6	18		1
	0	24	64	2
		0	60	3
			0	4

**SPLITTER**

1	2	3	4	
	1	2		1
		2	3	2
			3	3
				4

# Lec6 Step3

**OPT**

1	2	3	4	
0	6	18	38	1
	0	24	64	2
		0	60	3
			0	4

**SPLITTER**

1	2	3	4	
	1	2	3	1
		2	3	2
			3	3
				4



$$\frac{n}{2}$$

S: OCUR RANCE

T: OCCUR RENCE

$$1 \leq q \leq n$$

Pair

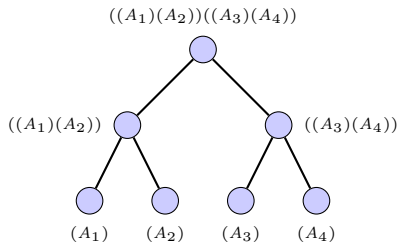
S: OCURRANC E  
 T: OCCURRENC E

Insertion

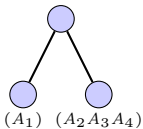
S: OCURRANC E  
 T: OCCURRENC E -

Deletion

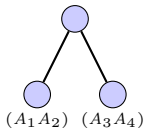
S: OCURRANCE -  
 T: OCCURRENC E



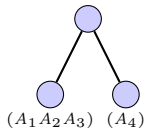
$k = 1$   
 $(A_1)(A_2 A_3 A_4)$

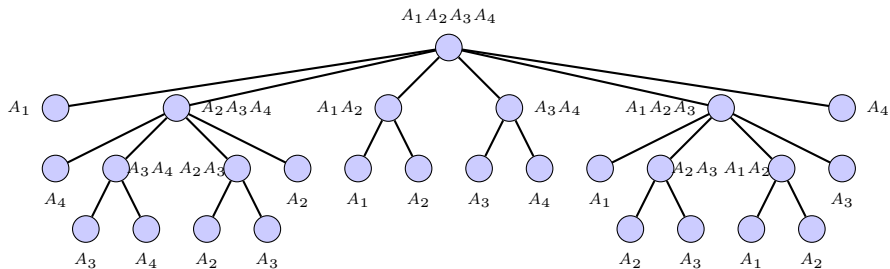


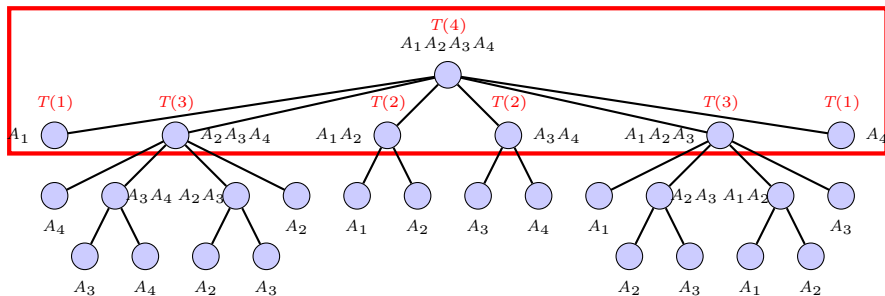
$k = 2$   
 $(A_1 A_2)(A_3 A_4)$

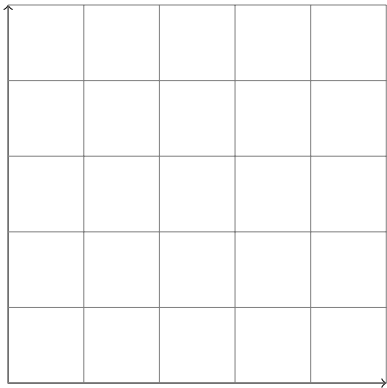


$k = 3$   
 $(A_1 A_2 A_3)(A_4)$



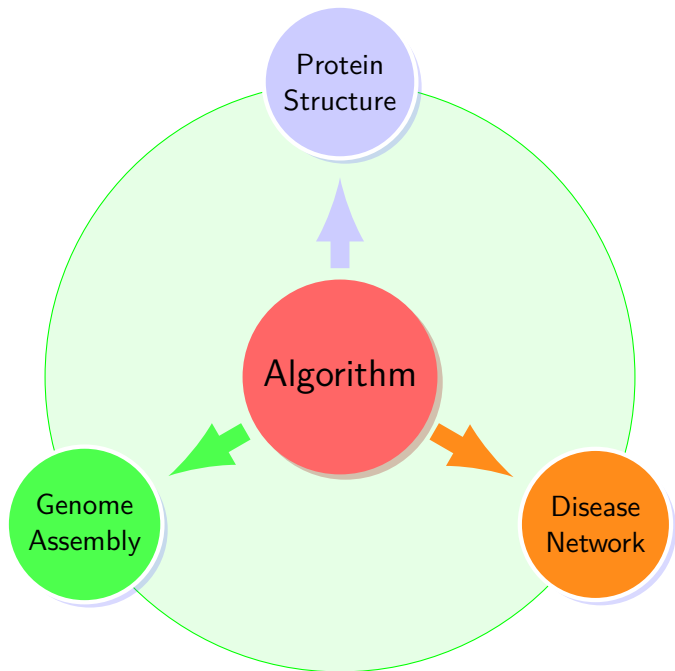


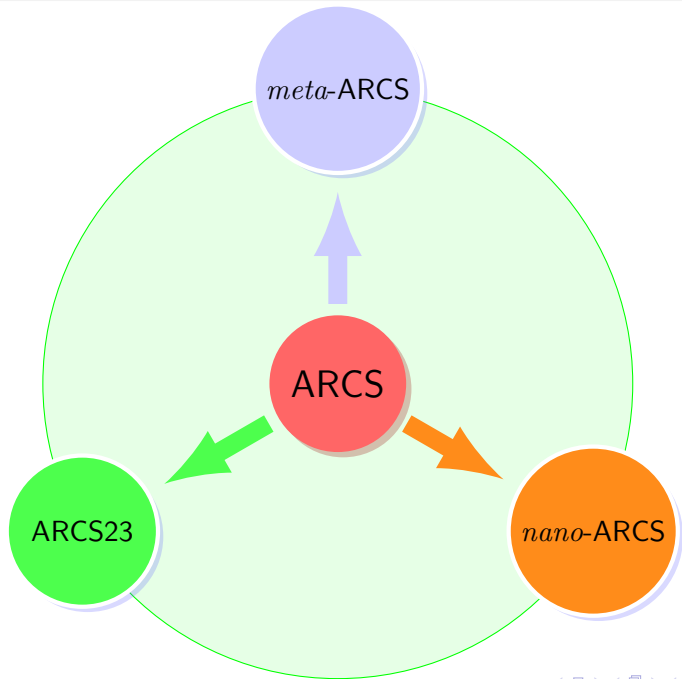


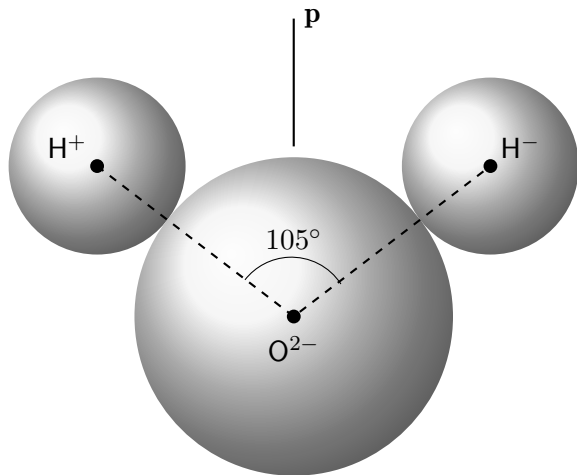


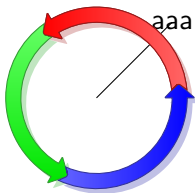
$$\begin{array}{cccc}
 & 1 \times 2 & 2 \times 3 & 3 \times 4 & 4 \times 5 \\
 A_1 = \begin{bmatrix} 1 & 2 \end{bmatrix} & A_2 = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{bmatrix} & A_3 = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix} & A_4 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}
 \end{array}$$



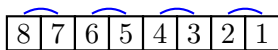




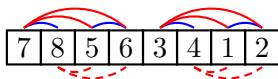




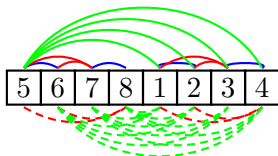
## Lec5. Where did we save? Merge sort



MERGE SORT step 1: 4 ops

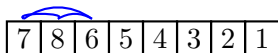
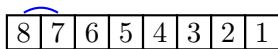


MERGE SORT step 2: 4 ops, save: 4 ops



MERGE SORT step 3: 4 ops, save: 12 ops

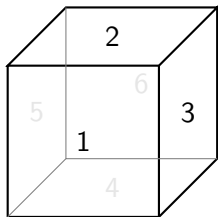
## Lec5. Where did we save?



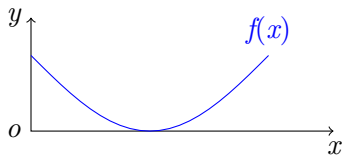
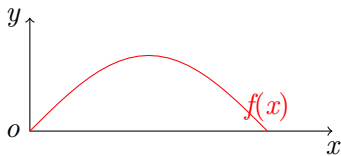
⋮



INSERTSORT: 28 ops

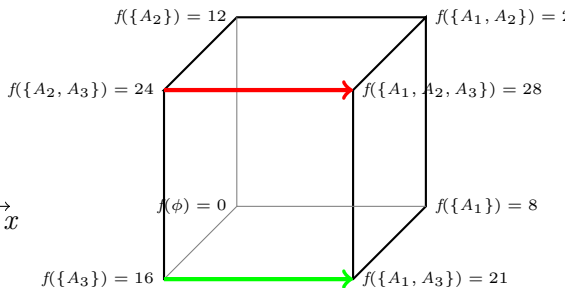
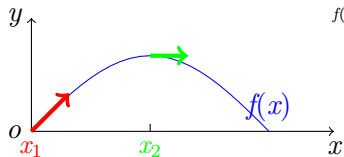


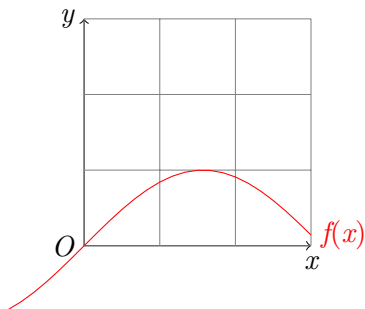
# $f(x)$ convex

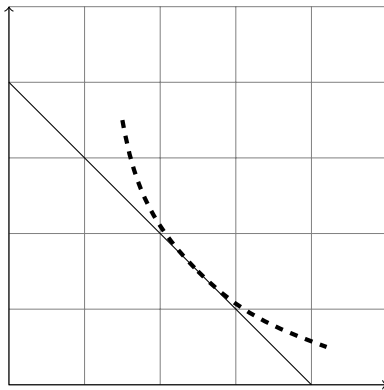




# $f(x)$ convex

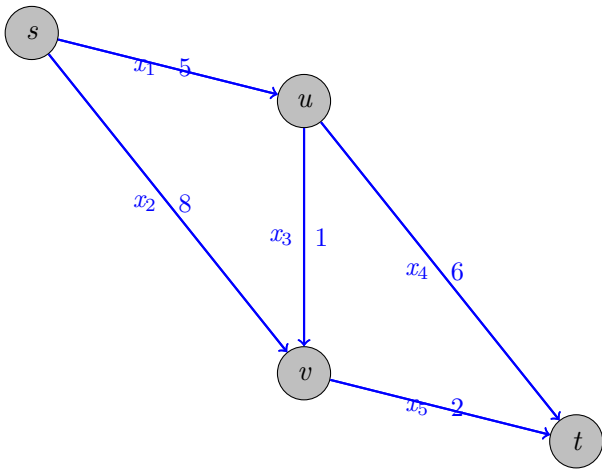




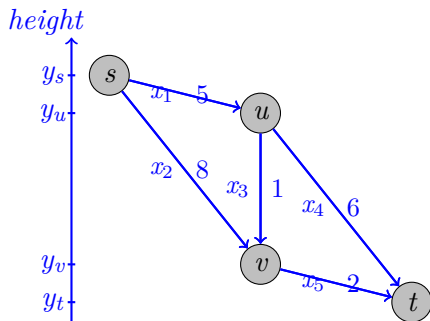


# Lec5 Closest Pair n points

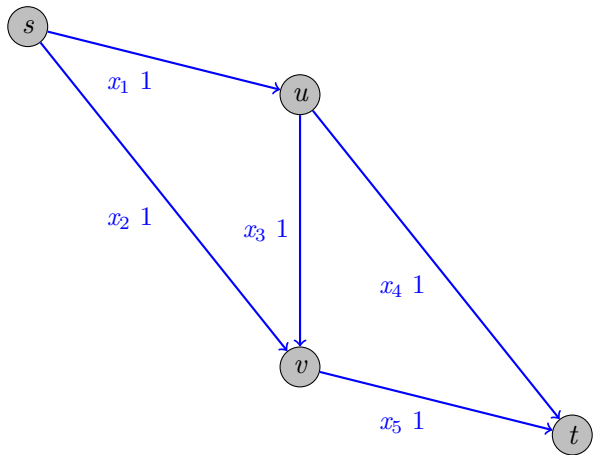




# Shortest path

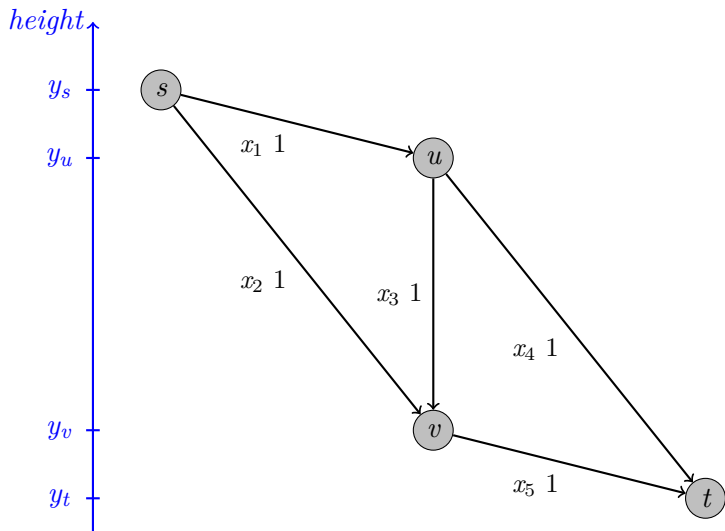


$$\begin{array}{ccccccc}
 & & & & x_n = 0 & & \\
 & & & & \uparrow & & \\
 & & & & \vee & & \\
 c_1 & c_2 & \dots & c_n & & \max & \\
 \parallel & \parallel & & & & & \\
 y_1 a_{11} & y_1 a_{12} & \dots & y_1 a_{1n} & y_1 b_1 & & \\
 + & + & & + & + & & \\
 y_2 a_{21} & y_2 a_{22} & \dots & y_2 a_{2n} & y_2 b_2 & & \\
 + & + & & + & + & & \\
 \vdots & \vdots & \dots & \vdots & \vdots & & \\
 + & + & & + & + & & \\
 y_m a_{m1} & y_m a_{m2} & \dots & y_m a_{mn} & y_m b_m & & 
 \end{array}$$

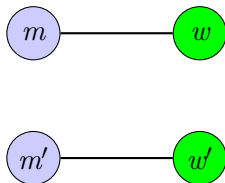




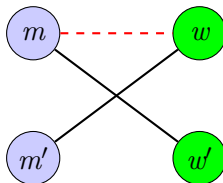
# dual of shortest path



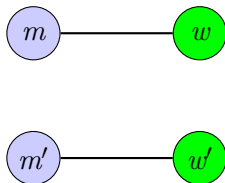
Match 1



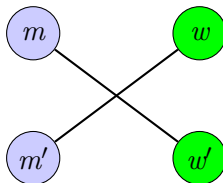
Match 2



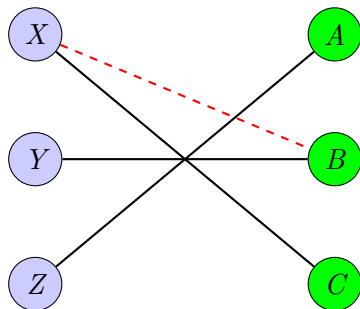
Match 1

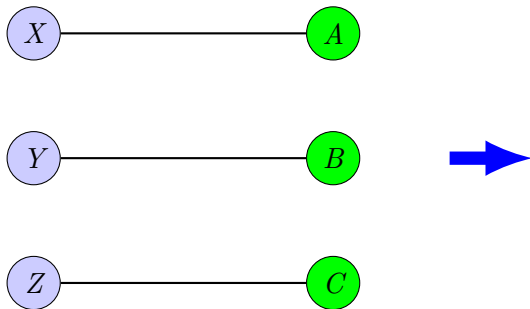


Match 2

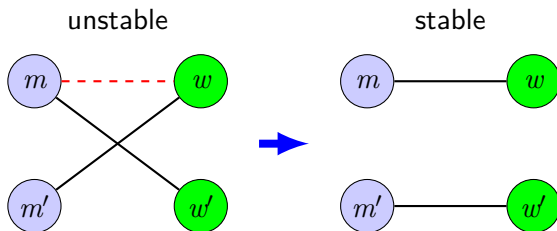


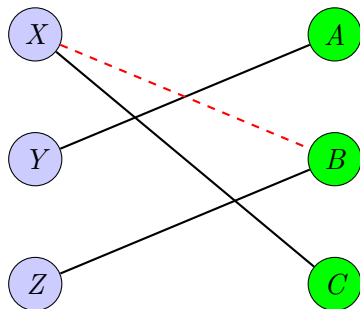
# Stable 4



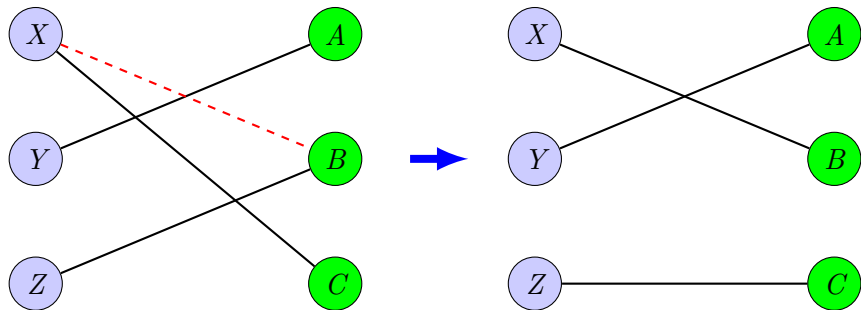


# Stable 6

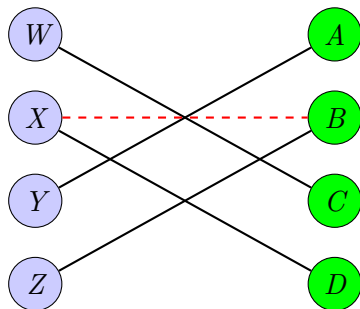




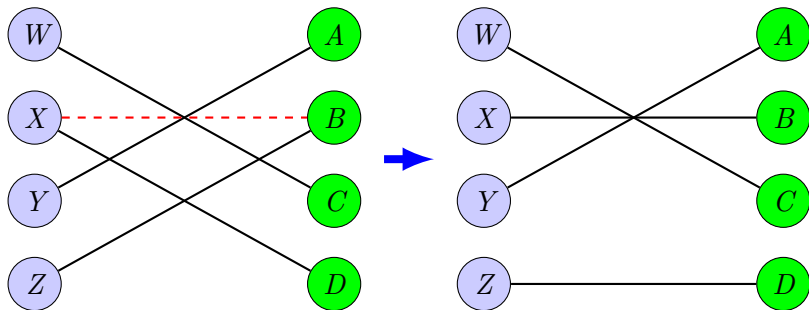
# Stable 8



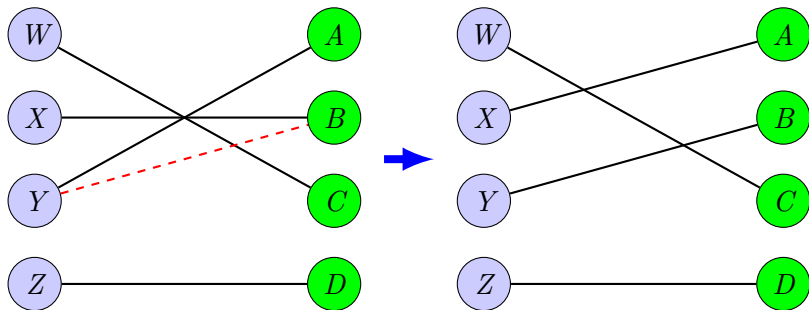




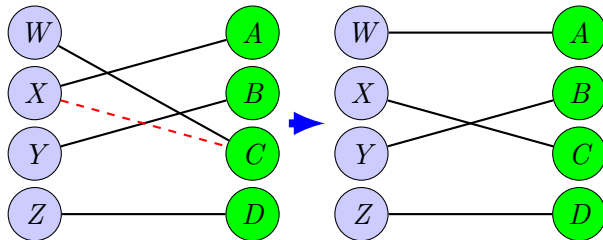
# Stable 10

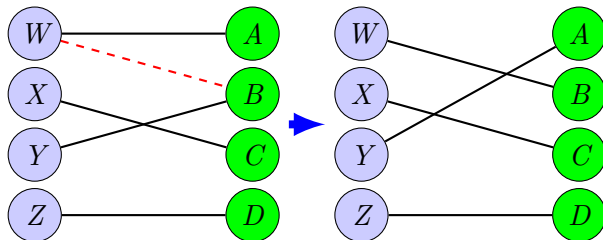


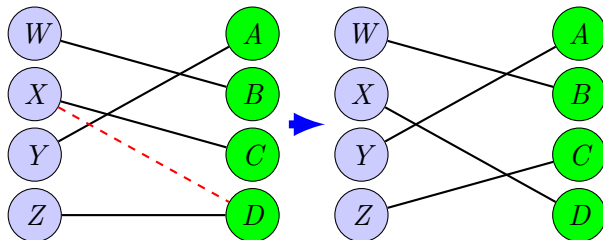
# Stable 11

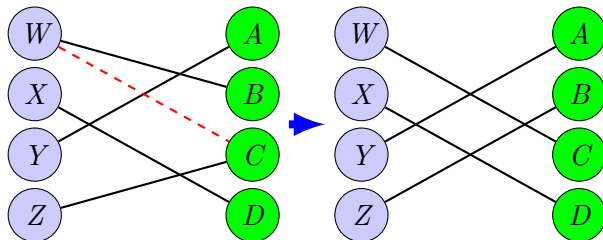


# Stable 12

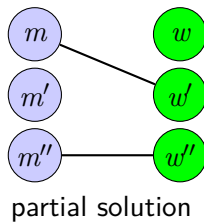
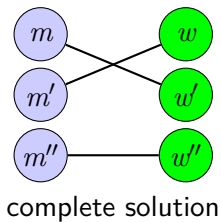






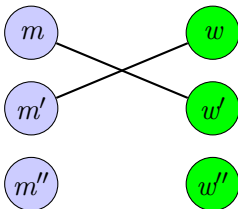


# Stable Proof 1

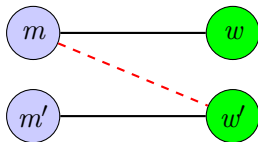




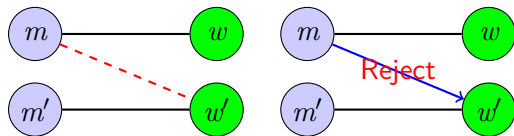
# Stable Proof 2



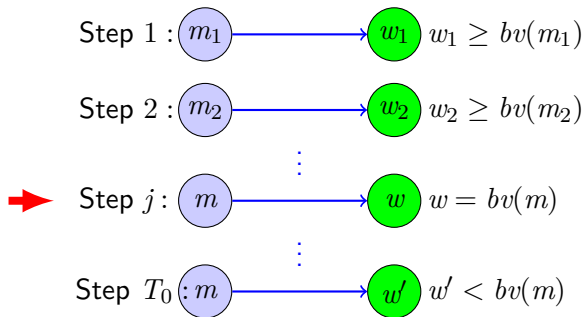
# Stable Proof 3



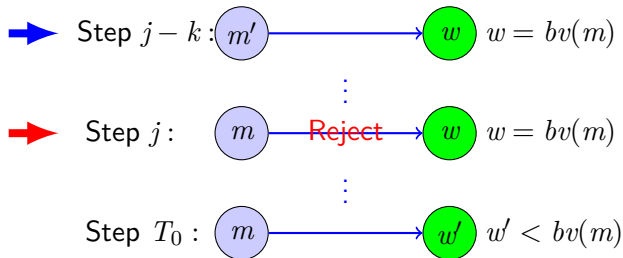
# Stable Proof 4



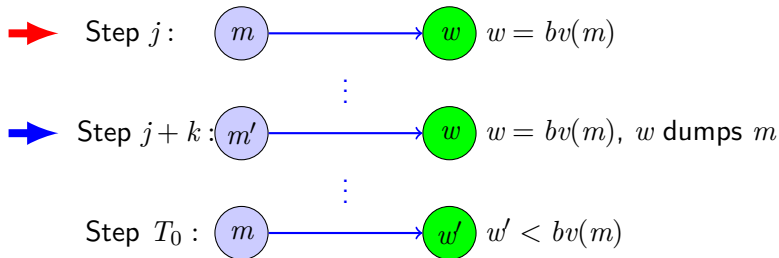
# Stable Proof 5



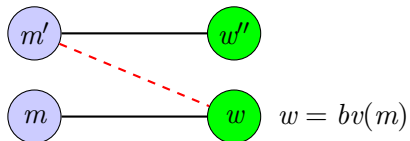
# Stable Proof 6

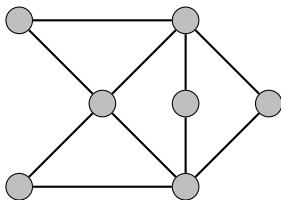


# Stable Proof 7

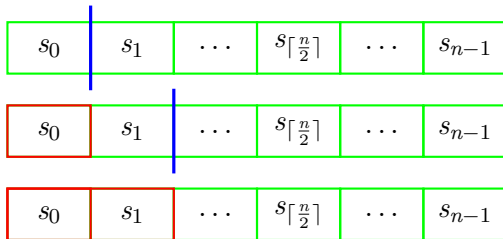


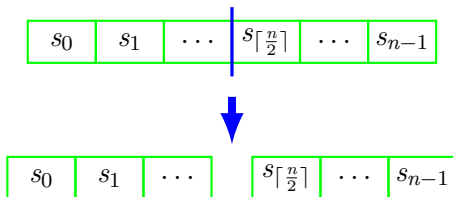
Stable match  $S'$



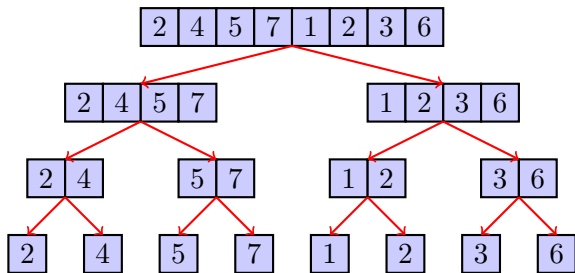




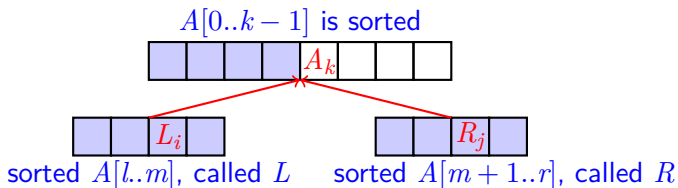




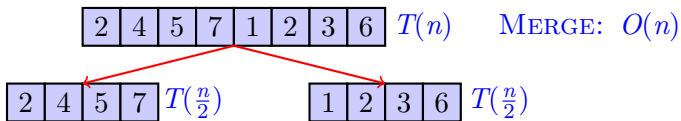
## Lec5 how to divide?



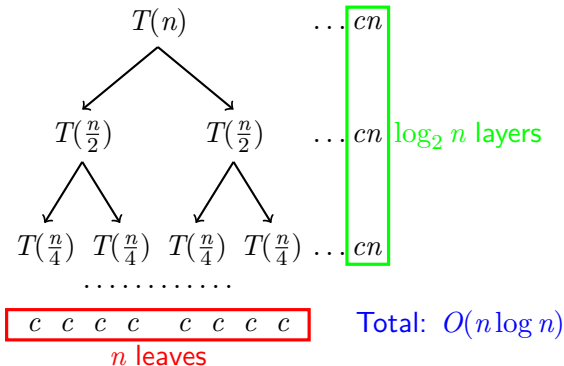
## Lec5 how to combine?



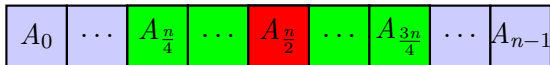
# Lec5 time complexity?



# Lec5 tree

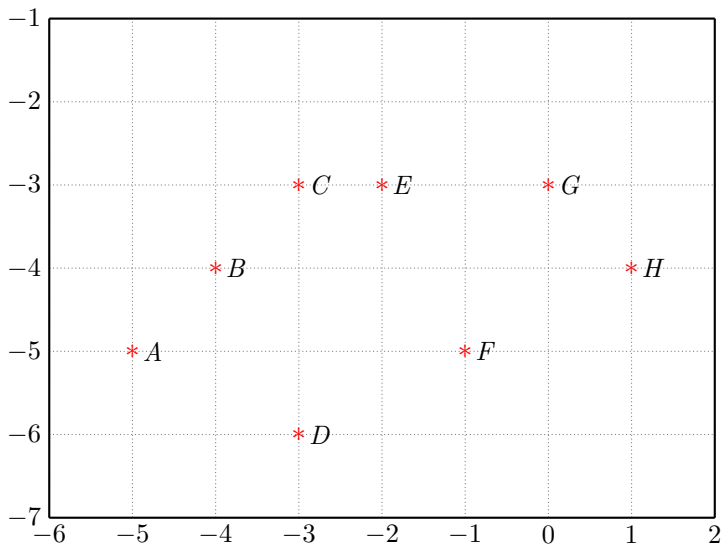


best splitter



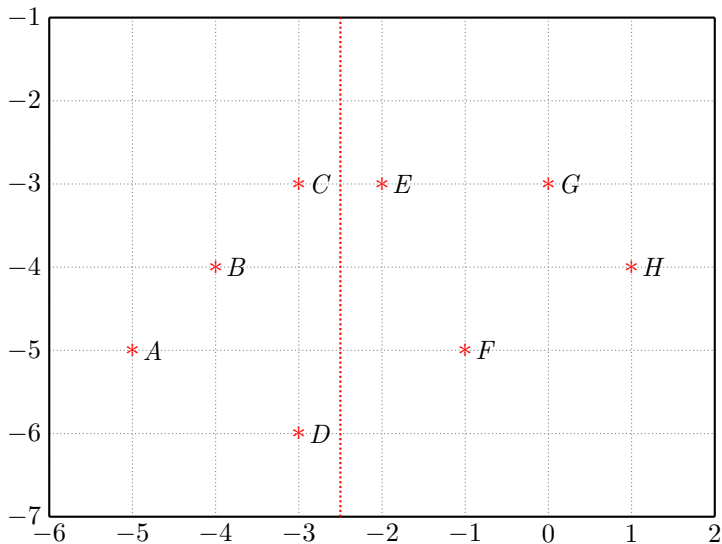
good splitters

# Lec5 Closest Pairs

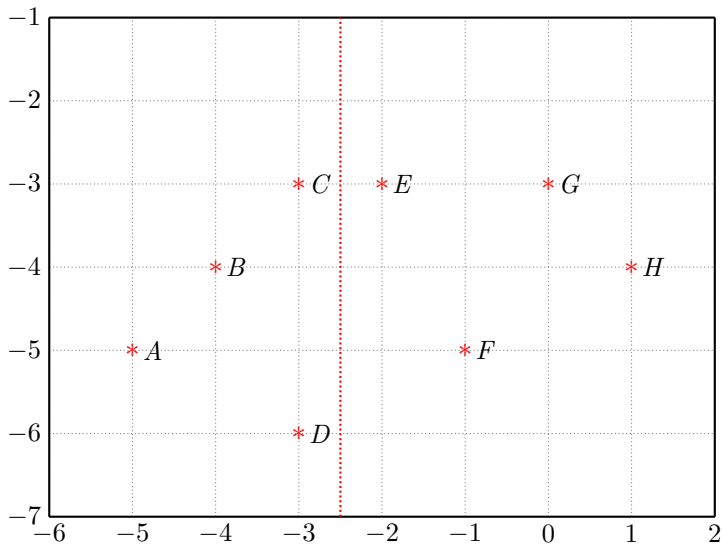




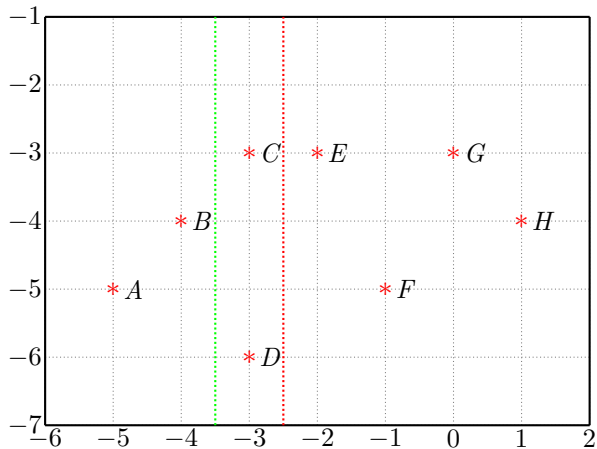
# Lec5 Closest Pairs



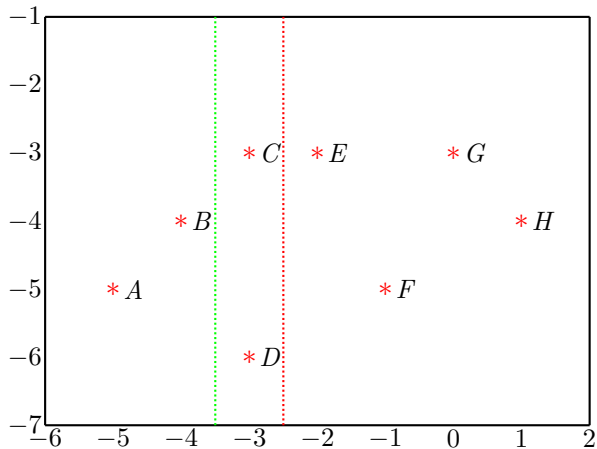
## Lec5 Closest Pairs Middle Red Line



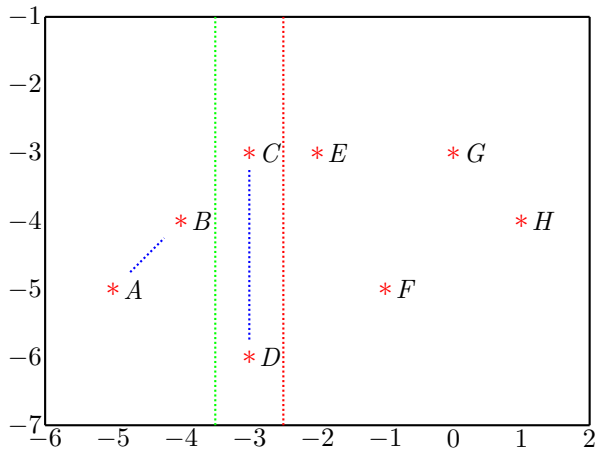
# Leech Closest Pairs Left



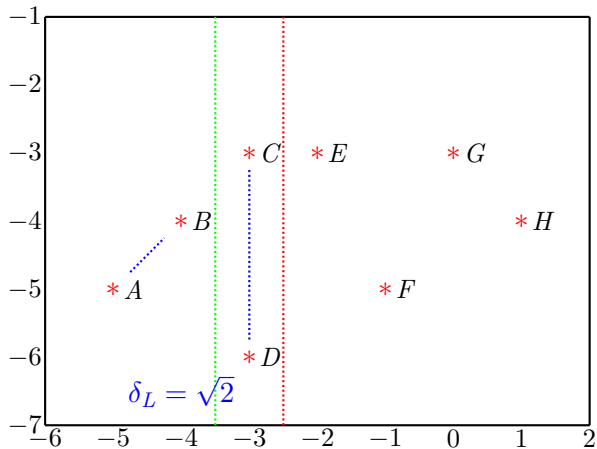
## Lec5 Closest Pairs Left Grids



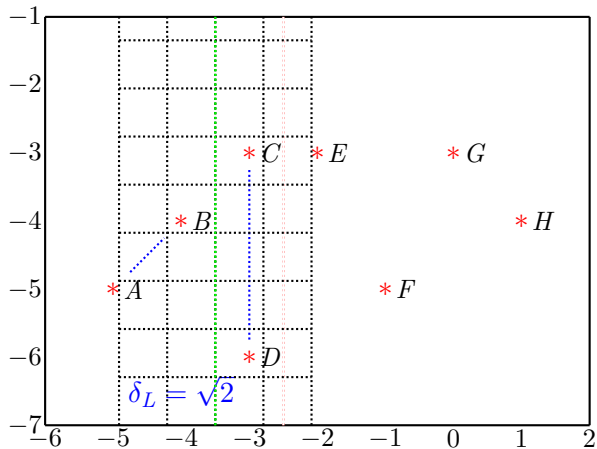
## Lec5 Closest Pairs Left Grids



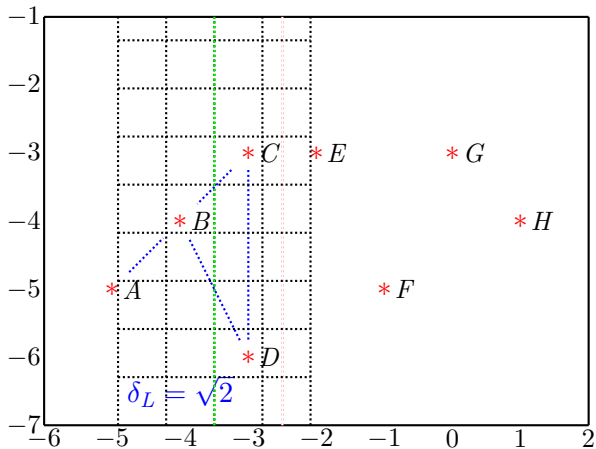
## Lec5 Closest Pairs Left Grids



# Lec5 Closest Pairs Left Grids

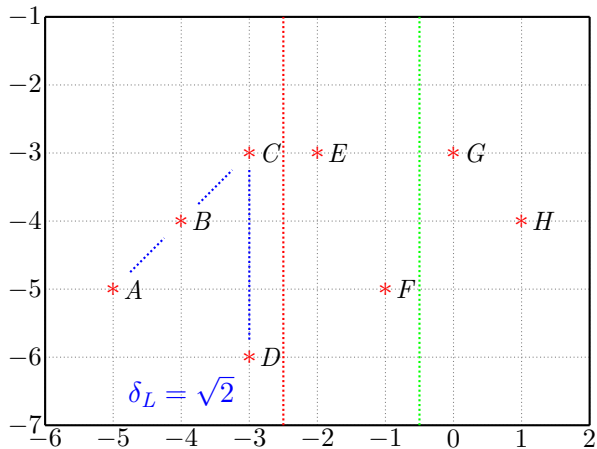


# Lec5 Closest Pairs Left Grids

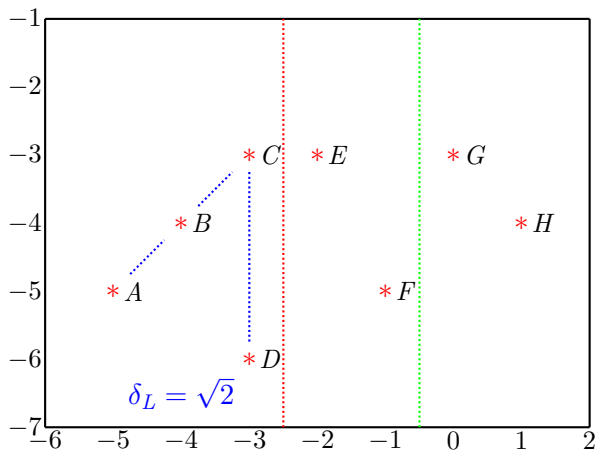




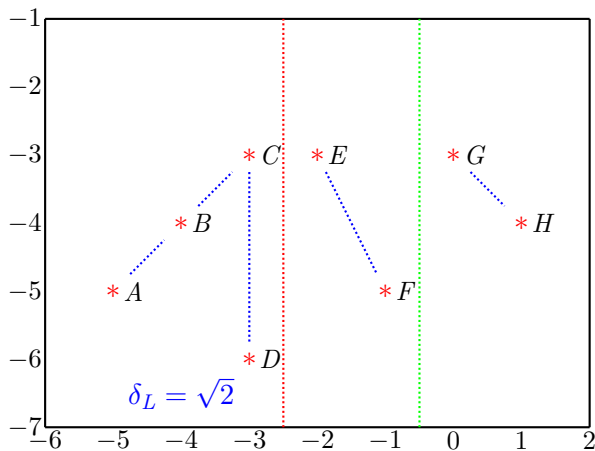
# Lec5 Closest Pair Right



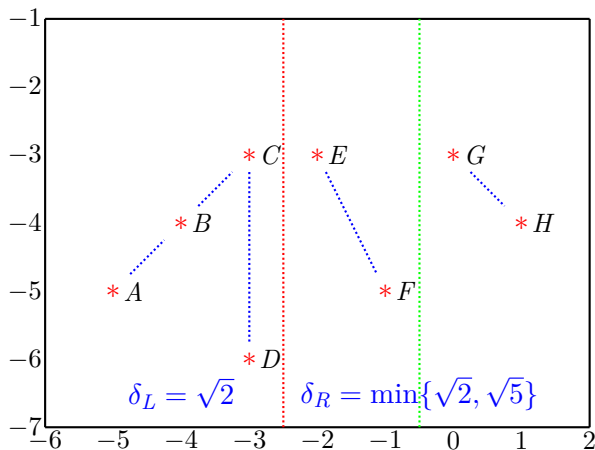
# Lec5 Closest Pairs Right Grids



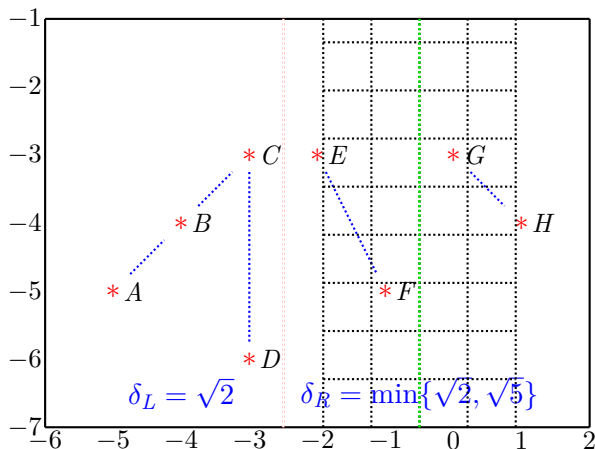
# Lec5 Closest Pairs Right Grids



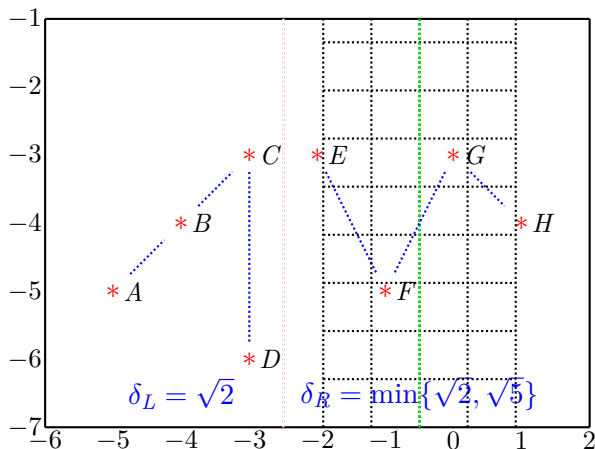
# Lec5 Closest Pairs Right Grids



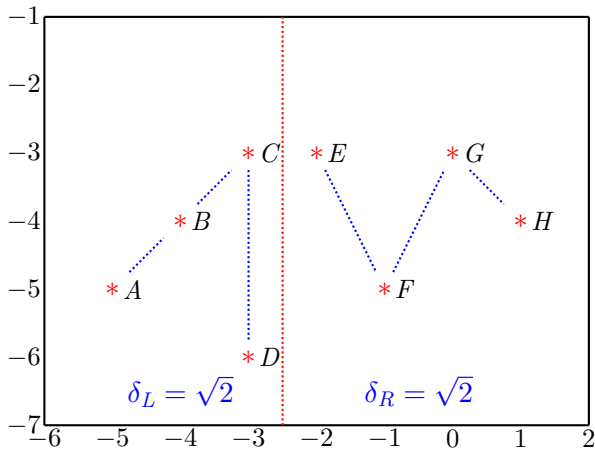
# Lec5 Closest Pairs Right Grids



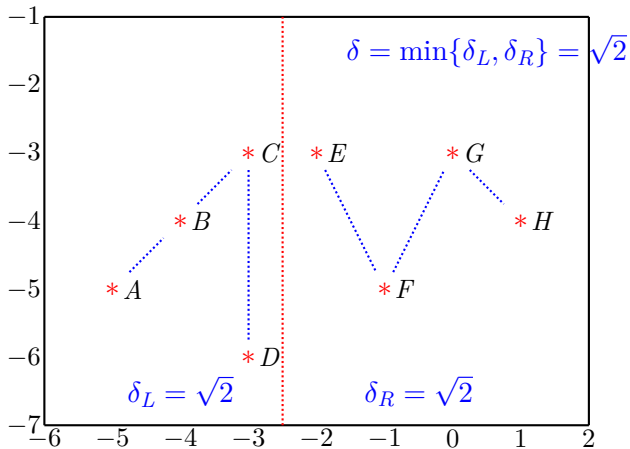
# Lec5 Closest Pairs Right Grids



# Lec5 Closest Pairs Final

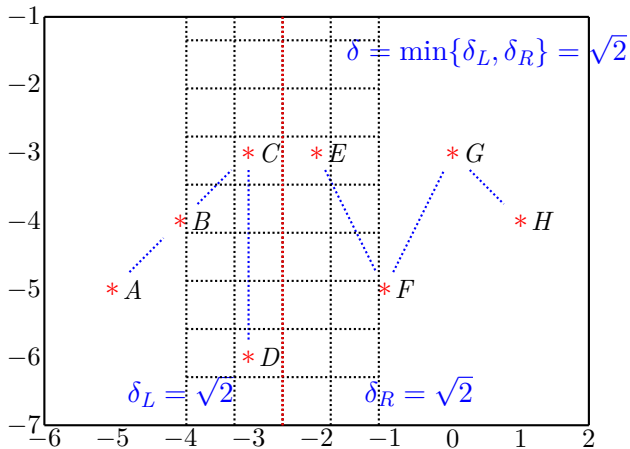


# Lec5 Closest Pairs Final

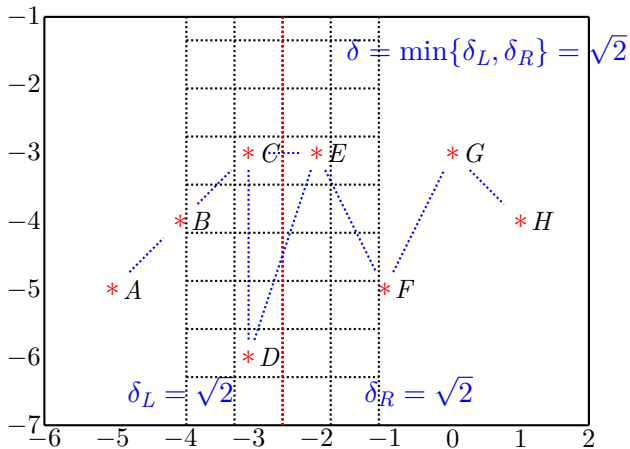




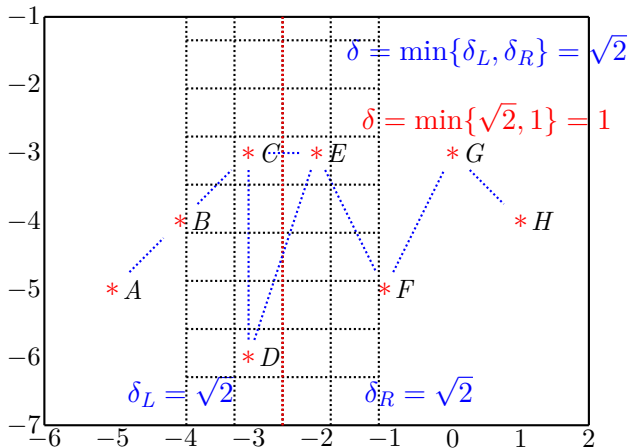
# Lec5 Closest Pairs Final



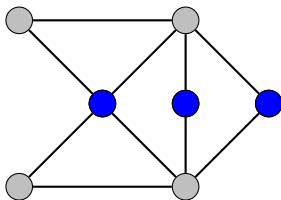
# Lec5 Closest Pairs Final

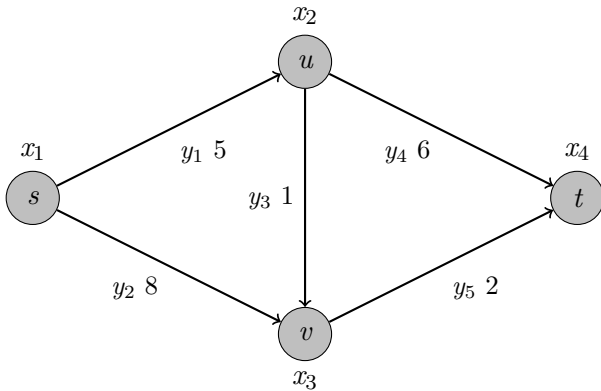


# Lec5 Closest Pairs Final

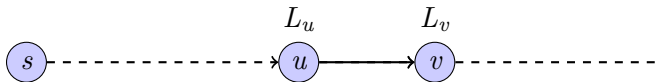


# Examples

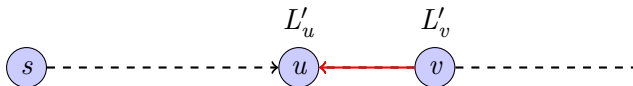




Step  $k$ :



Step  $k + 1$  :

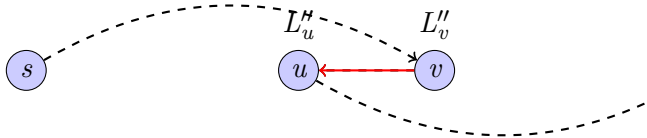


# Primal and dual

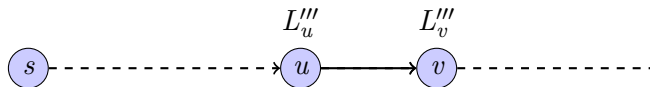


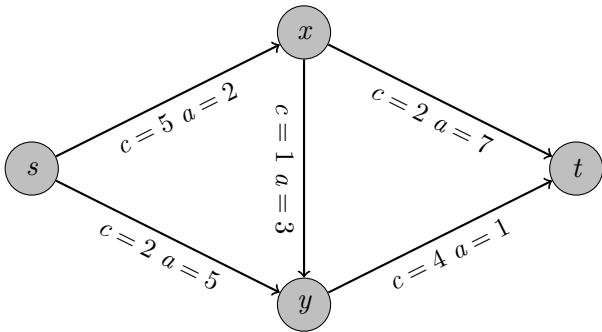


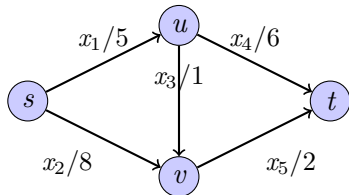
Step  $k''$  :

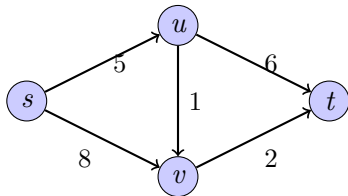


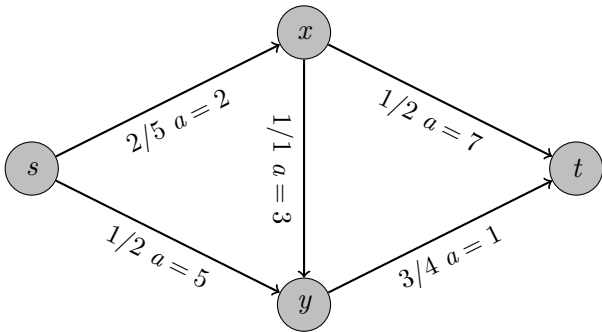
Step  $k'''$  :

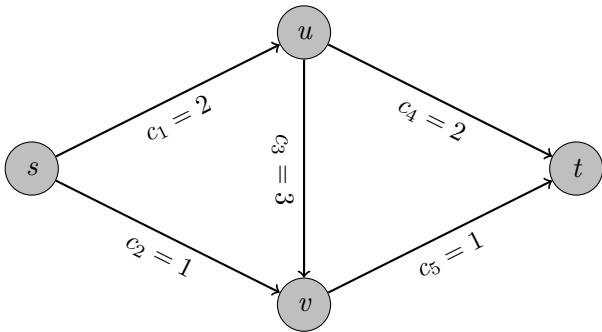


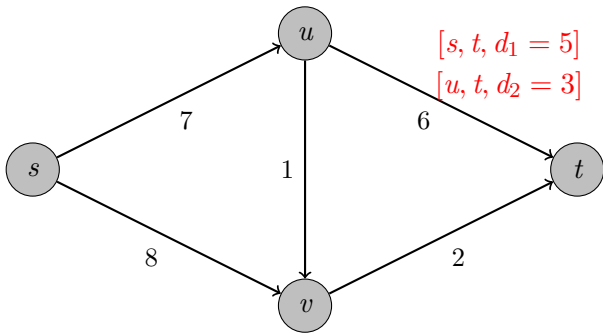




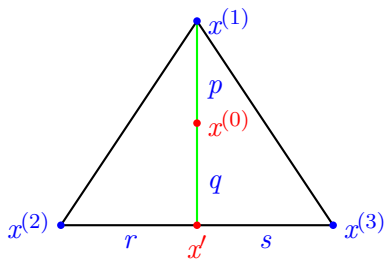




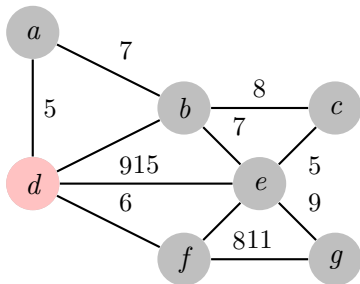




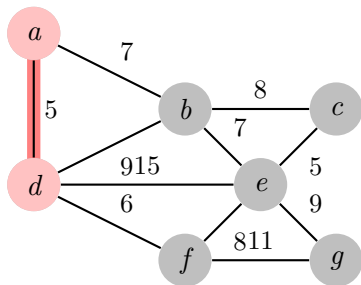




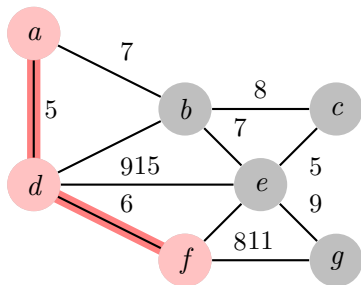
# Prim's algorithm



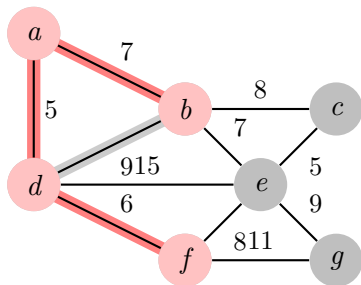
# Prim's algorithm



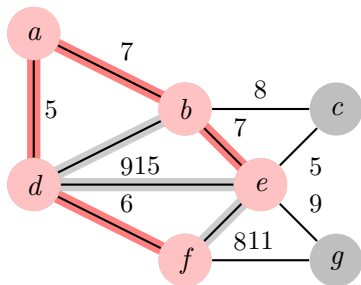
# Prim's algorithm



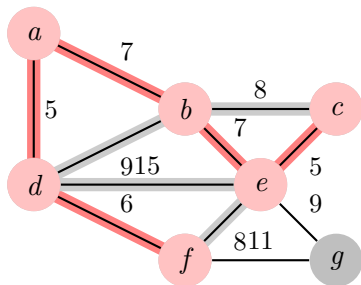
# Prim's algorithm



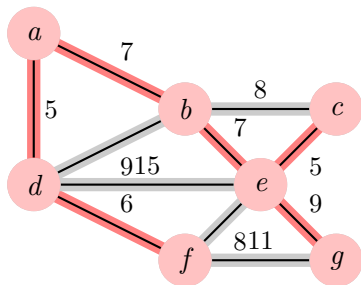
# Prim's algorithm



# Prim's algorithm

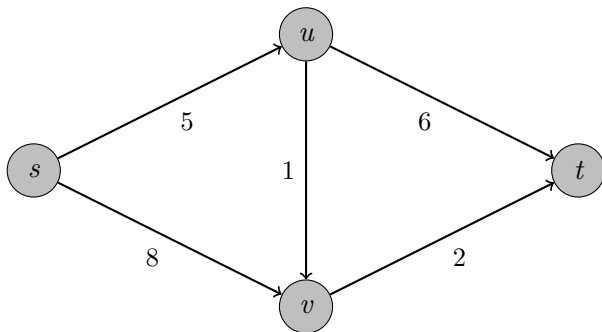


# Prim's algorithm

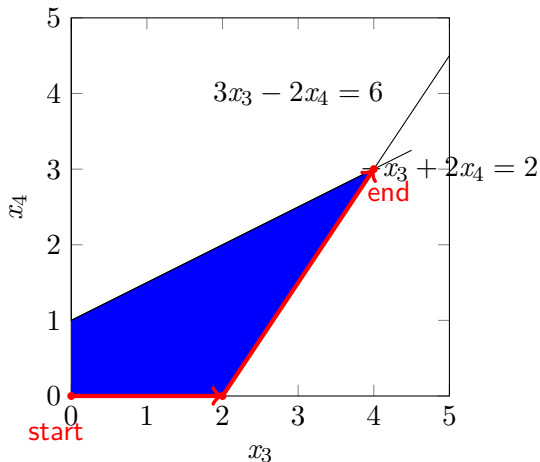


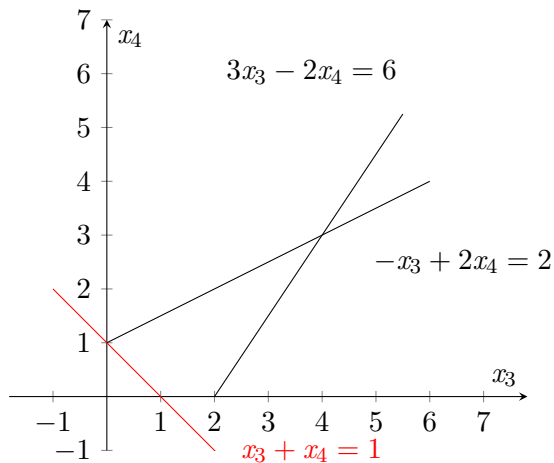


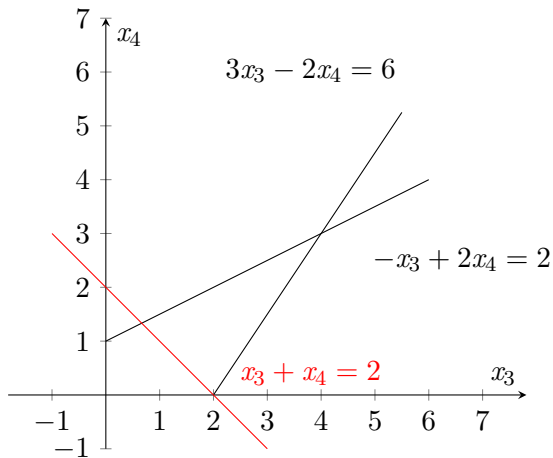
# Max Flow

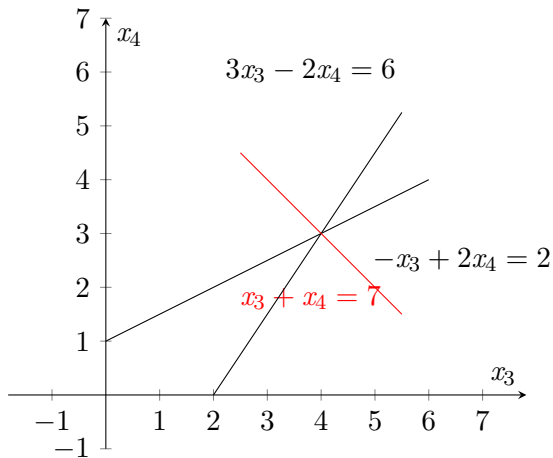


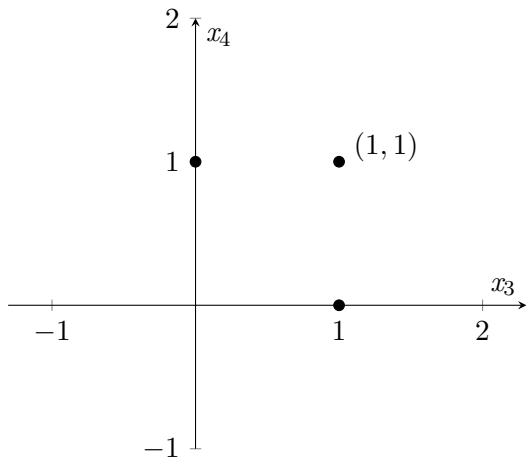
# LP example

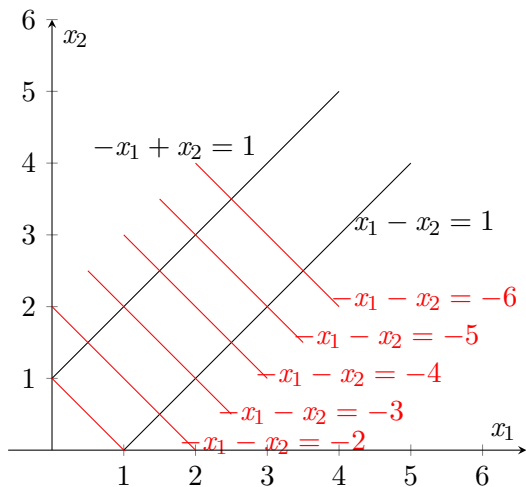


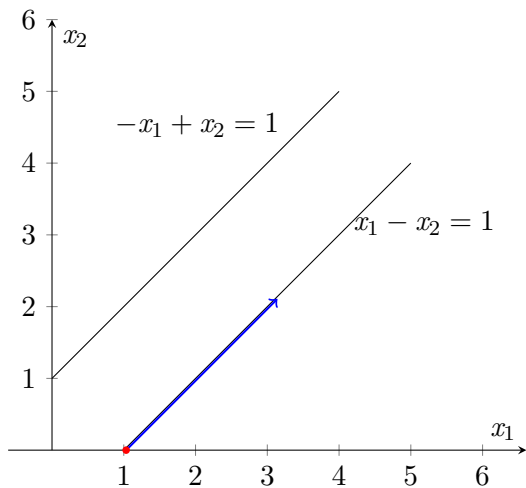




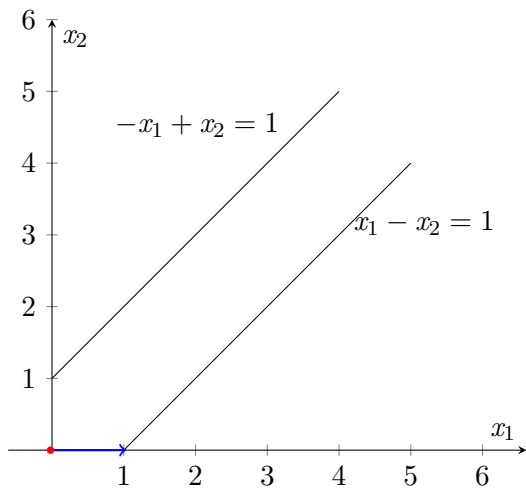












$$\begin{array}{cccccc}
 c_1 & c_2 & \dots & c_n & & \\
 a_{11} & a_{12} & \dots & a_{1n} & b_1 & \\
 a_{21} & a_{22} & \dots & a_{2n} & b_2 & \\
 & & & & & \\
 & & \dots & & & \\
 a_{m1} & a_{m2} & \dots & a_{mn} & b_m & 
 \end{array}$$

$$\min \quad c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \geq b_1$$

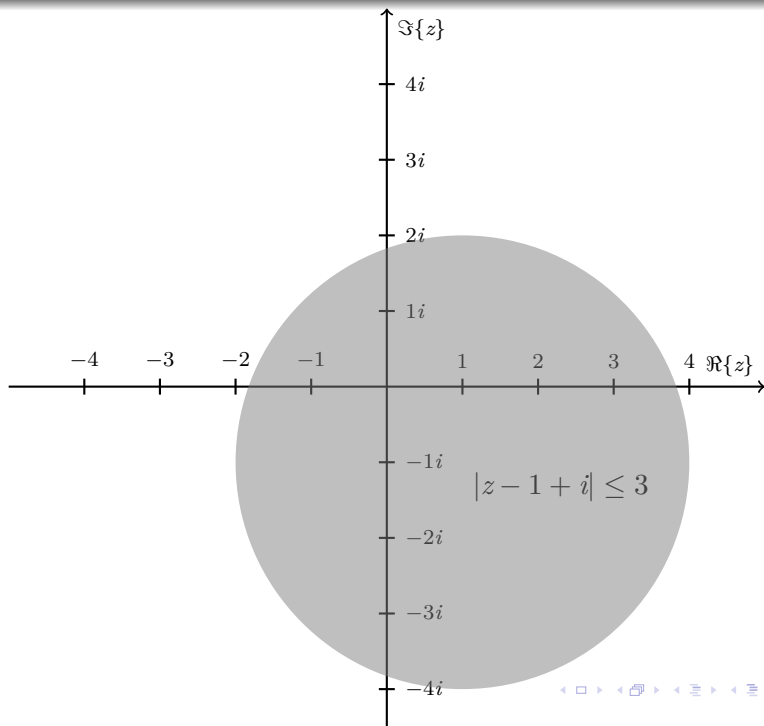
$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n \geq b_2$$

...

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n \geq b_m$$

$$x_i \geq 0 \quad \text{for each } i$$

$$\begin{array}{cccccc}
 c_1 & c_2 & \dots & c_n & & \\
 \vee & \vee & & \vee & \max & \\
 y_1 a_{11} & y_1 a_{12} & \dots & y_1 a_{1n} & y_1 b_1 & \\
 + & + & & + & + & \\
 y_2 a_{21} & y_2 a_{22} & \dots & y_2 a_{2n} & y_2 b_2 & \\
 + & + & & + & + & \\
 & & \dots & & & \\
 + & + & & + & + & \\
 y_m a_{m1} & y_m a_{m2} & \dots & y_m a_{mn} & y_m b_m & \\
 & & & y_j & \geq & 0 \text{ for each } j
 \end{array}$$



$$\begin{pmatrix} 0 & c_1 & c_2 & \cdots & c_m & \cdots & c_n \\ b_1 & a_{11} & a_{12} & \cdots & a_{1m} & \cdots & a_{1n} \\ b_2 & a_{21} & a_{22} & \cdots & a_{2m} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ b_m & a_{m1} & a_{m2} & \cdots & a_{mm} & \cdots & a_{mn} \end{pmatrix}$$

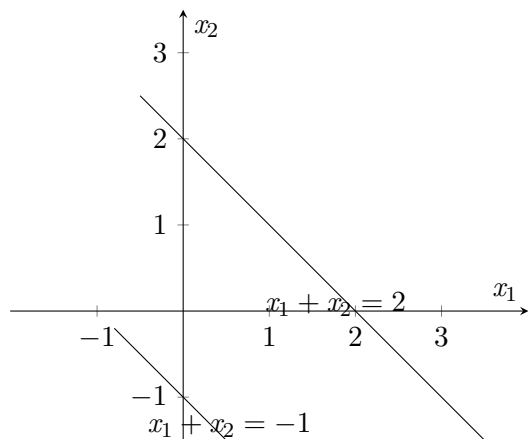
$$\Rightarrow \times \mathbf{B}^{-1}$$

$$\begin{pmatrix} \mathbf{c}_B^T \mathbf{B}^{-1} \mathbf{b} & 0 & 0 & \cdots & 0 & \mathbf{c}_N^T - \mathbf{c}_B^T \mathbf{B}^{-1} \mathbf{N} \\ & 1 & 0 & \cdots & 0 & \\ \mathbf{B}^{-1} \mathbf{b} & 0 & 1 & \cdots & 0 & \mathbf{B}^{-1} \mathbf{N} \\ & \vdots & \vdots & \ddots & \vdots & \\ & 0 & 0 & \cdots & 1 & \end{pmatrix}$$

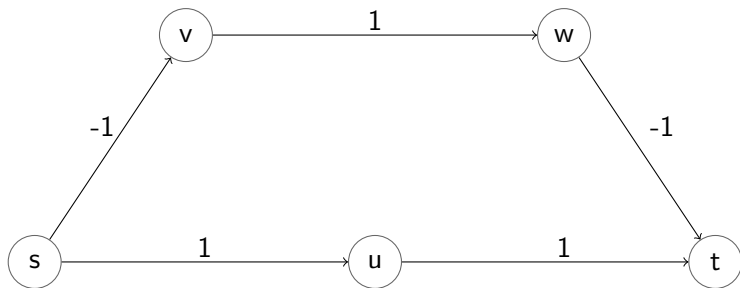
$$\begin{pmatrix} 0 & \dots & 0 & \dots & c_e & \dots \\ b_1 & \dots & 0 & \dots & a_{1e} & \dots \\ b_2 & \dots & 0 & \dots & a_{2e} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_l & \dots & 1 & \dots & a_{le} & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_m & \dots & 0 & \dots & a_{me} & \dots \end{pmatrix}$$

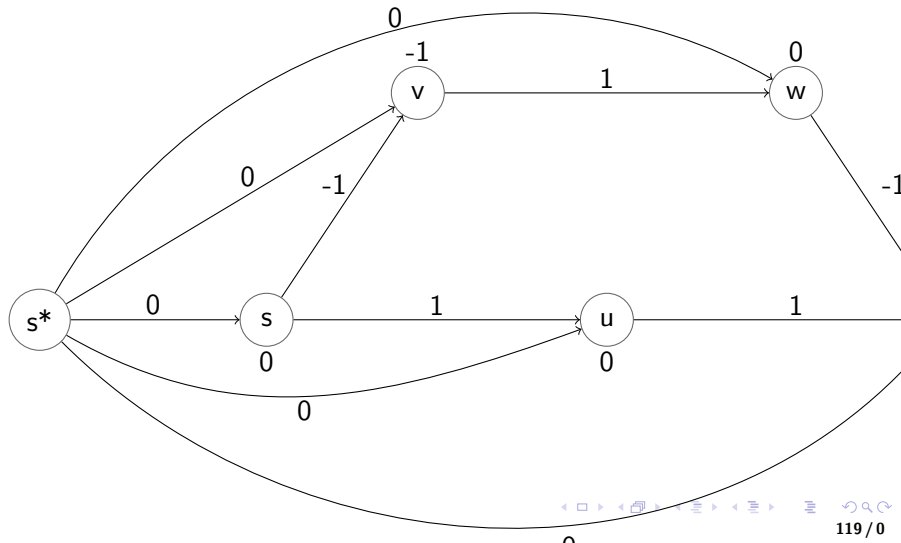
$\Rightarrow$

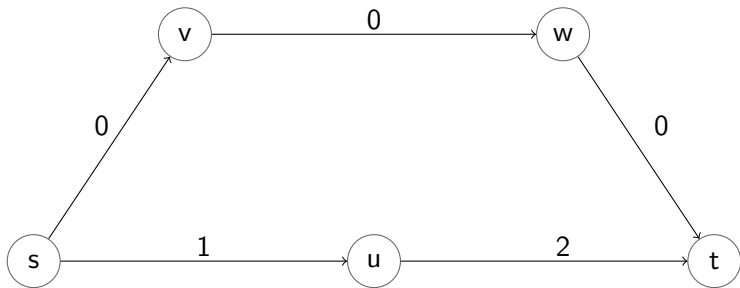
$$\begin{pmatrix} -\frac{a_{me}}{a_{le}} b_l & \dots & -\frac{c_e}{a_{le}} & \dots & 0 & \dots \\ b_1 - \frac{a_{1e}}{a_{le}} b_l & \dots & -\frac{a_{1e}}{a_{le}} & \dots & 0 & \dots \\ b_2 - \frac{a_{2e}}{a_{le}} b_l & \dots & -\frac{a_{2e}}{a_{le}} & \dots & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \frac{1}{a_{le}} b_l & \dots & \frac{1}{a_{le}} & \dots & 1 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_m - \frac{a_{me}}{a_{le}} b_l & \dots & -\frac{a_{me}}{a_{le}} & \dots & 0 & \dots \end{pmatrix}$$

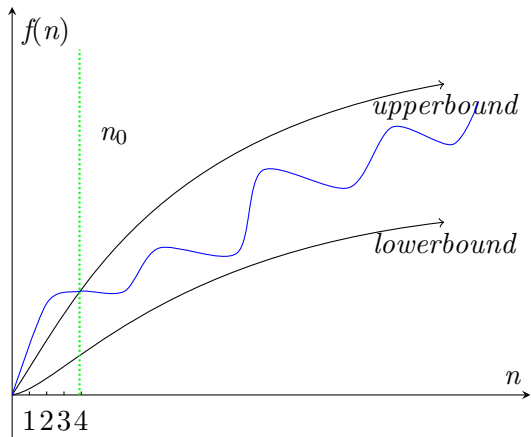


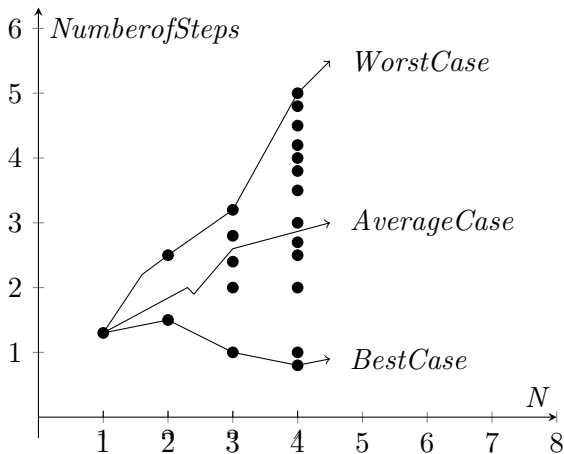


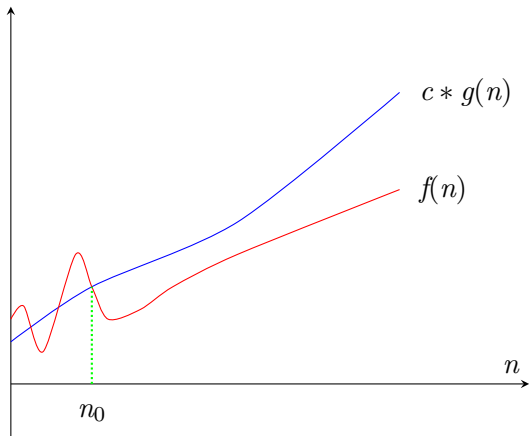


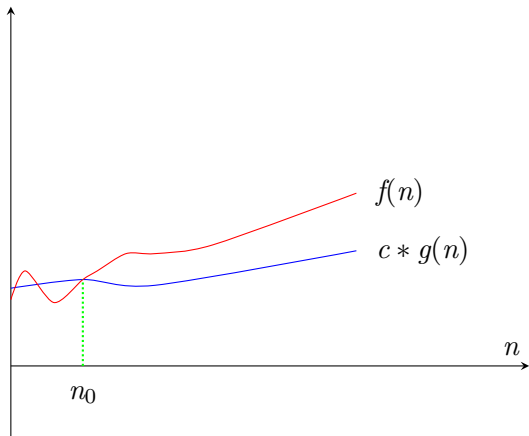


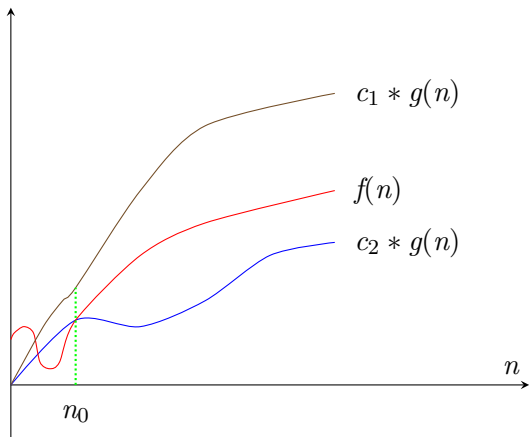














# Sequence alignment

S: ' ' O C U R R A N C E

T: ' ' O C U R R E N C E

	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
O	-3	1	-2	-5	-8	-11	-14	-17	-20	-23
C	-6	-2	2	-1	-4	-7	-10	-13	-16	-19
C	-9	-5	-1	1	-2	-5	-8	-11	-12	-15
U	-12	-8	-4	0	0	-3	-6	-9	-12	-13
R	-15	-11	-7	-3	1	1	-2	-5	-8	-11
R	-18	-14	-10	-6	-2	2	0	-3	-6	-9
E	-21	-17	-13	-9	-5	-1	1	-1	-4	-5
N	-24	-20	-16	-12	-8	-4	-2	2	-1	-4
C	-27	-23	-19	-15	-11	-7	-5	-1	3	0
E	-30	-26	-22	-18	-14	-10	-8	-4	0	4

# Backtrack path

S:	'	O	C	U	R	R	A	N	C	E	
T:	'	0	-3	-6	-9	-12	-15	-18	-21	-24	-27
O	-3	1	-2	-5	-8	-11	-14	-17	-20	-23	
C	-6	-2	2	-1	-4	-7	-10	-13	-16	-19	
C	-9	-5	-1	1	-2	-5	-8	-11	-12	-15	
U	-12	-8	-4	0	0	-3	-6	-9	-12	-13	
R	-15	-11	-7	-3	1	1	-2	-5	-8	-11	
R	-18	-14	-10	-6	-2	2	0	-3	-6	-9	
E	-21	-17	-13	-9	-5	-1	1	-1	-4	-5	
N	-24	-20	-16	-12	-8	-4	-2	2	-1	-4	
C	-27	-23	-19	-15	-11	-7	-5	-1	3	0	
E	-30	-26	-22	-18	-14	-10	-8	-4	0	4	