

# CAS10 Assignment 1

Institute of Computing Technology,  
Chinese Academy of Sciences, Beijing, China

October 17, 2010

- Due Nov. 5, 2010.
- Please send your answer to wangchao1987@ict.ac.cn, shaomingfu@gmail.com, huangchunlin@ict.ac.cn, zhanghaicang@ict.ac.cn
- You can choose one problem from Problem 1-4, and one problem from Problem 5-6.

## 1 True or False (10 marks)

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

*True or false?* In every instance of the Stable Matching Problem, there is a stable matching containing a pair  $(m, w)$  such that  $m$  is ranked first on the preference list of  $w$  and  $w$  is ranked first on the preference list of  $m$ .

## 2 True or False (10 marks)

Decide whether you think the following statement is true or false. If it is true, give a short explanation. If it is false, give a counterexample.

*True or false?* Consider an instance of the Stable Matching Problem in which there exists a man  $m$  and a woman  $w$  such that  $m$  is ranked first on the preference list of  $w$  and  $w$  is ranked first on the preference list of  $m$ . Then in every stable matching  $S$  for this instance, the pair  $(m, w)$  belongs to  $S$ .

## 3 Time Analysis (10 marks)

Can you give another analysis method to prove that the algorithm of Stable Matching Problem must be stopped in  $O(n^2)$  time?

## 4 Programming (5 marks)

Please write a program to implement the algorithm of the Stable Matching Problem in C/C++ or Java.

You can refer to the algorithm of Page 21 of the book **Algorithm Design**.

## 5 Potential Method (10 marks)

Suppose that a counter begins at a number with  $b$  1's in its binary representation, rather than at 0. Show that the cost of performing  $n$  **INCREMENT** operations is  $O(n)$  if  $n = \Omega(b)$ . (Do not assume that  $b$  is constant.)

## 6 Aggregate Analysis (10 marks)

If the set of stack operation included a **MULTIPUSH** operation, which pushes  $k$  items onto the stack, would the  $O(1)$  bound on the amortized cost of stack operations continue to hold?