CS711008Z Algorithm Design and Analysis

Lecture 3. NP and intractability (Part I) [1]

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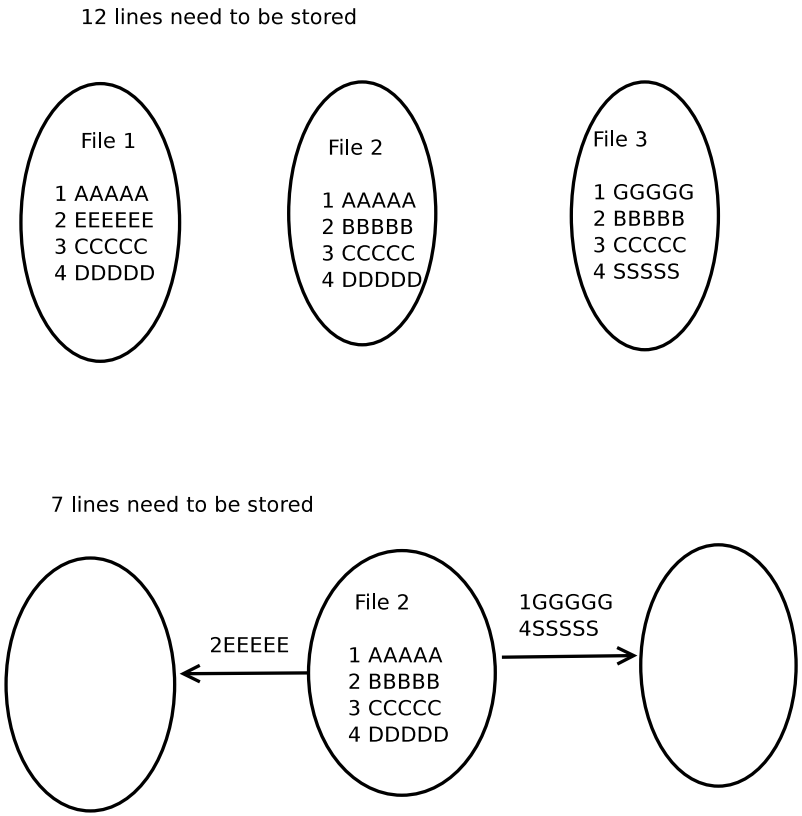
[1] The slides are prepared based on Introduction to algorithms, Algorithm design, and Computer and Intractability.

Storage Compression Problem

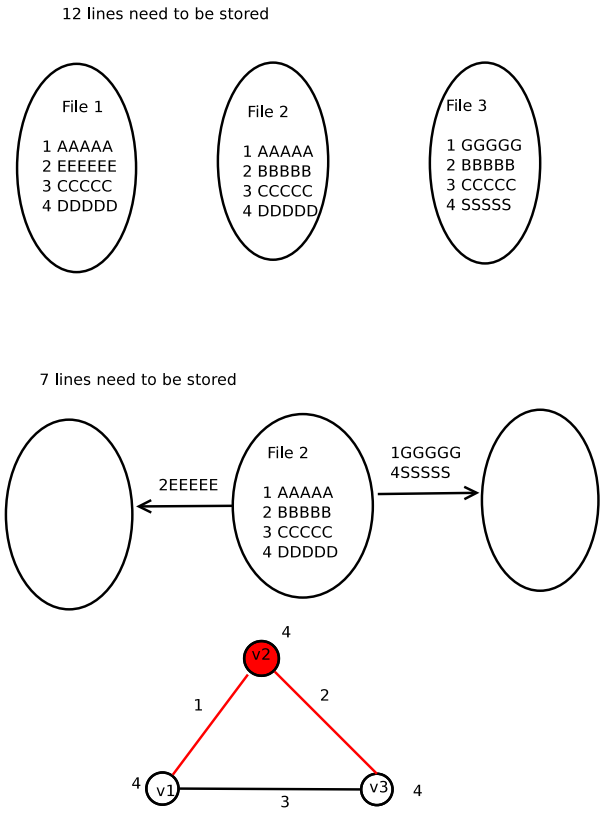
Practical Problem:
Data should be compressed as much as possible due to lots of redundancy. The complete data can be restored from the compressed data.



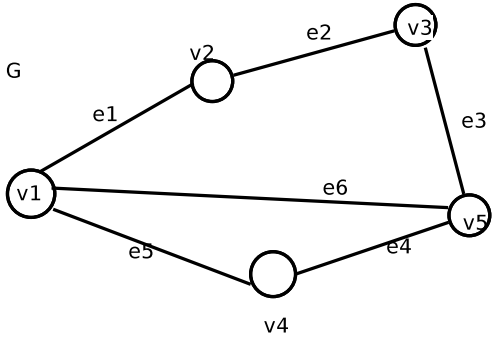
Formalized Definition:

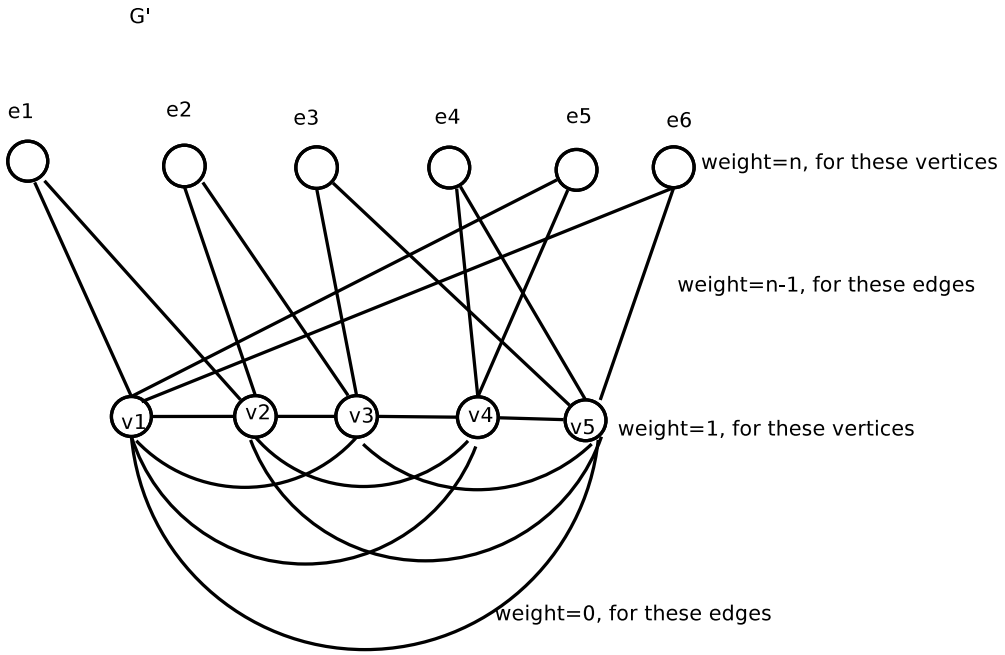
Input: Given a graph $G =< V, E >$ which has weighted vertices and edges , and a positive number W.
Let $w(v_i)$ denote the weight of vertex v_i and $d(e_k)$ denote the weight of edge e_k . For each $e_k = (v_i, v_j)$, $d(e_k) \geq |w(v_i) - w(v_j)|$ must be guranteed in order to restore complete data
Output: is there a set of vertices $S \subseteq V$,a set of edges $P \subseteq E$, such that each vertex is in S or is adjacent to one vertex in S through one edge in P and $\sum_{v_i \in S} w(v_i) + \sum_{e_k \in P} d(e_k) \leq W$?

Storage Compression Problem —Formalization



Vertex Cover $\leq_P$ Storage Compression Problem: Transformation

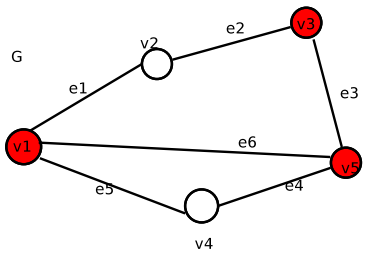




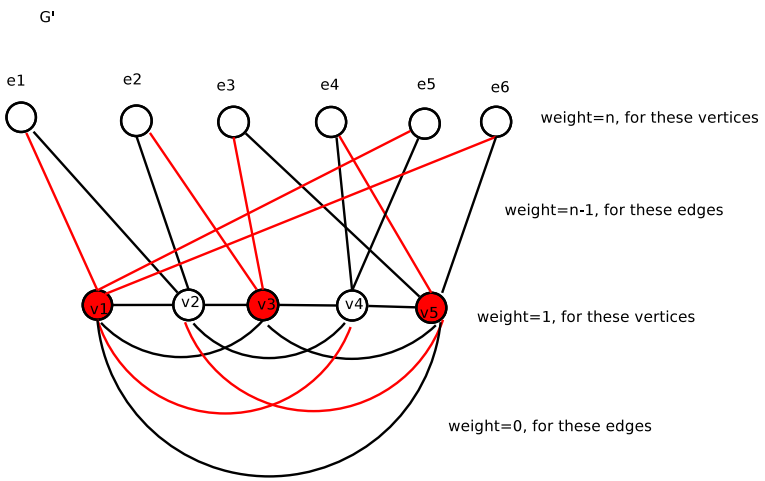
Vertex Cover $\leq_P$ Storage Compression Problem: Transformation

It can be easily verified that

Graph G has a vertex cover with size less than k iff the graph G' has a solution with size less than $(n-1)*n+k$



The size of vetex cover is 3. And the solution is {v1,v3, v5}.



the size of the solution is 3 + 6* 5 = 33

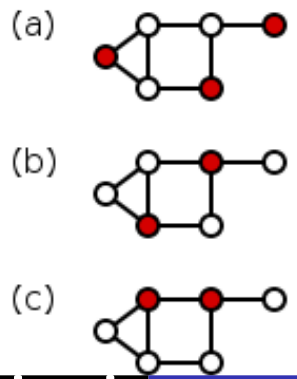
Dominating Set $\leq_P$ Storage Compression Problem: Transformation

In graph theory, a dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is joined to at least one member of D by some edge.

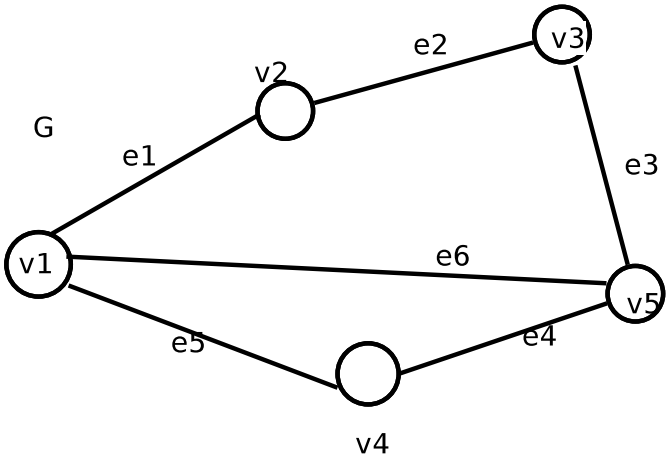
Dominateng Set Problem:

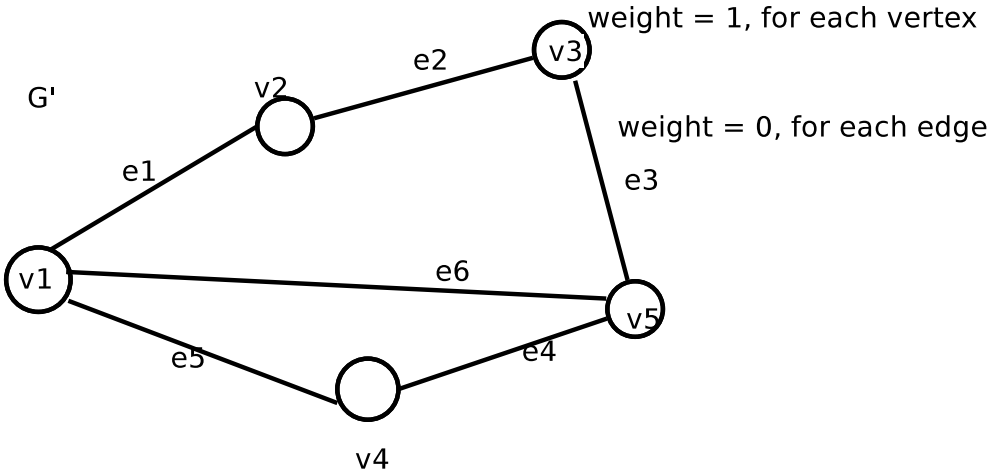
Input: Given a graph $G =< V, E >$, and an integer number k,
Output: is there a dominating set of size k?

It has been proved that Dominating Set Problem is NP-complete.Please refer to $http://en.wikipedia.org/wiki/Dominating\_set$ for details.



Dominating Set $\leq_P$ Storage Compression Problem: Transformation

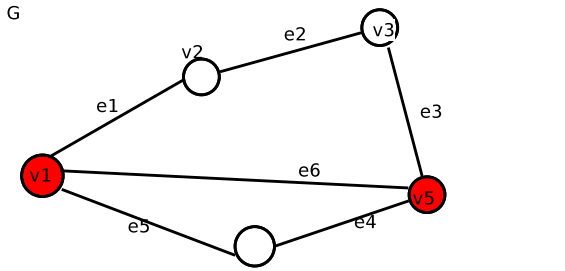




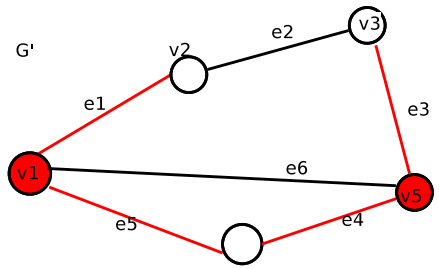
Dominating Set $\leq_P$ Storage Compression Problem: Transformation

It can be easily verified that

Graph G has a dominating set with size less than k iff the graph G' has a solution with size less than k



The size of dominating set is 2. The solution is {v1, v5}



The size of the solution is 2