Final Examination (Computer Algorithm)

2022.1.14

(總分 100，必須用中文(專有名詞可外加英文)作答，不可只用英文或符號作答)

1. (10%) (1)Define the following terminologies: P problem, NP problem, NP-complete problem (NPC), NP-hard problem; and (2) draw the relationship between them under current situation.

P: 輸入長度為 n 的問題 可在多項式時間內解決
NP: 可在多項式時間內驗證

2. (10%) 請說明(1)為何要了解 NPC 問題？主要原因？(2)如何證明一個問題是 NPC？

3. (10 %) Mathematically the 0-1-knapsack problem can be formulated as:

Let there be n items, z_1 to z_n where z_i has a value v_i and weight w_i. The maximum weight that we can carry in the bag is W.

$n \log n$
$n \log n$
n^2
$E \lg E = E \lg V^2$

$$\text{Maximize} \sum_{i=1}^{n} v_i x_i \quad \text{subject to} \quad \sum_{i=1}^{n} w_i x_i \leqslant W, \qquad x_i \in \{0,1\}$$

Solve by DP, the time complexity = O(nW) is a P problem?

(答案必須中文說明原因才給分)

4. (10 %) 現在請證明 3-CNF SAT is NP-complete(必須使用下例 SAT：

$\emptyset = (x_1 \land x_2) \lor (x_3 \rightarrow x_4)$ 證明。(全對才給分)

Then, 3CNF is satisfiable iff SAT is s

5(10%) 試證 **linear programming** 可為 ***minimum-weight vertex-cover problem*** 的 **2-approximation algorithm** (用中文說明)

逼近

(註：We define a variable x(v) for each vertex v, and x(v) =1 as **v** being in the vertex cover, and x(v) = 0 as v not being in the vertex cover. For any **edge (u, v)**, at least one of u and v must be in the vertex cover as x(u) + x(v) ≥ 1. We can formula this problem into ***0-1 integer program***:

$x(v)=1$ vertex-cover
$x(v)=0$ not
$x(v)+x(u)\geq 1$
edge(u.v)

$$\text{minimize} \quad \sum_{v \in V} w(v)x(v)$$

subject to

$$\begin{aligned} x(u) + x(v) &\geq 1 & \text{for each } (u, v) \in E \\ x(v) &\in \{0, 1\} & \text{for each } v \in V \,. \end{aligned}$$

再將他轉為 linear programming problem **(0 ≤ x(v) ≤ 1)**。再依下列演算法求解。

```
APPROX-MIN-WEIGHT-VC(G, w)
C ← Ø
compute x̄, an optimal solution to the linear program
for each v ∈ V
    do if x̄(v) ≥ 1/2
        then C ← C ∪ {v}
return C
```

$x(u)+x(v)\geq 1$ 必有一個大於 $\frac{1}{2}$ 涵蓋所有點

6. (10%) Suppose we have a set S ={a1, a2, .., an} of n proposed activities that wish to use a resource which can be used by only one activity at a time. Each activity ai has a start time si and a finish time fi, where 0 ≤ si ≤ fi ≤ ∞. If selected activity ai takes place during time interval [si, fi). Two activities ai and aj are compatible (相容) if the intervals [si, fi) and [sj, fj) do not overlap.

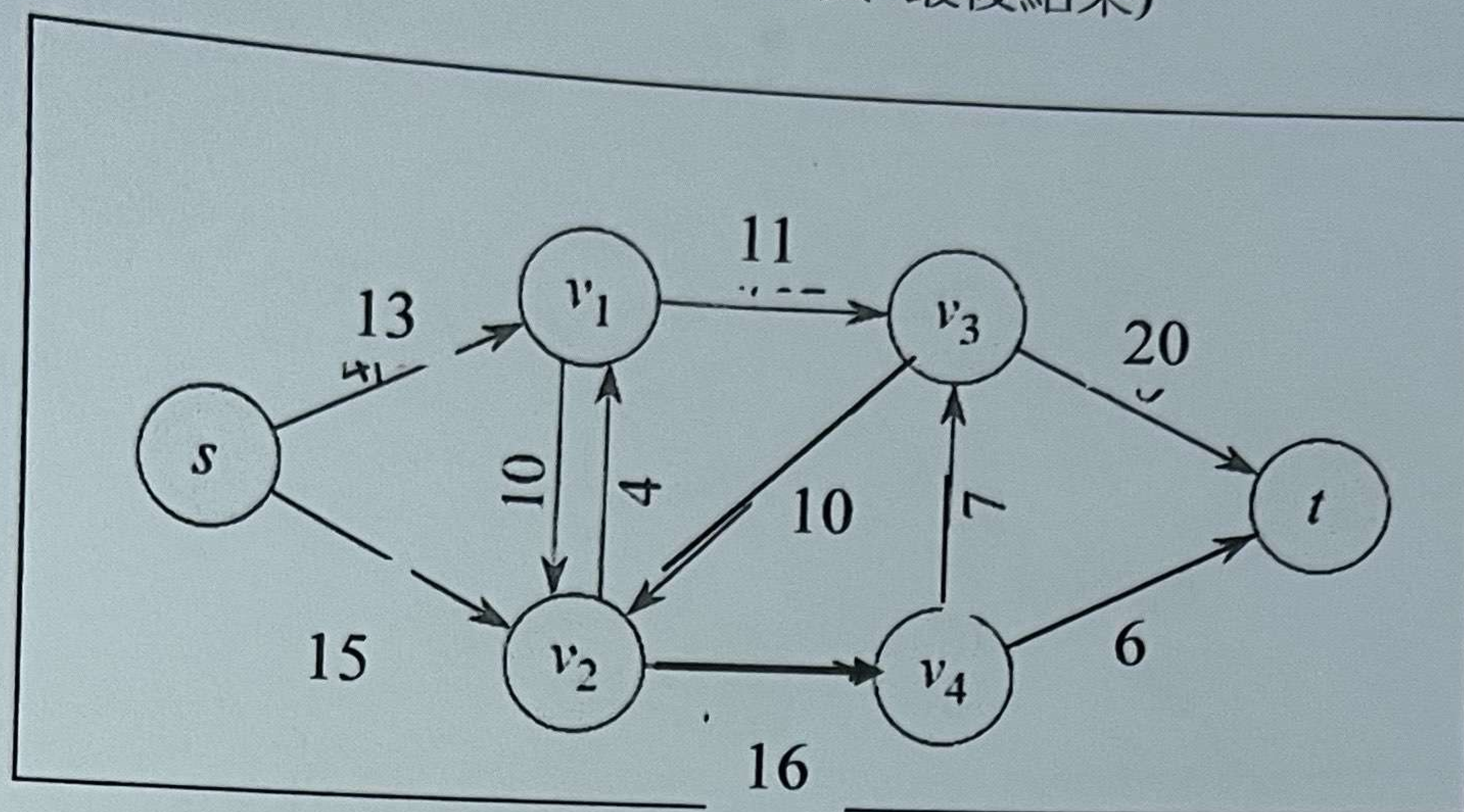
The activity-selection problem is to select a maximum-size subset of mutually compatible activities.

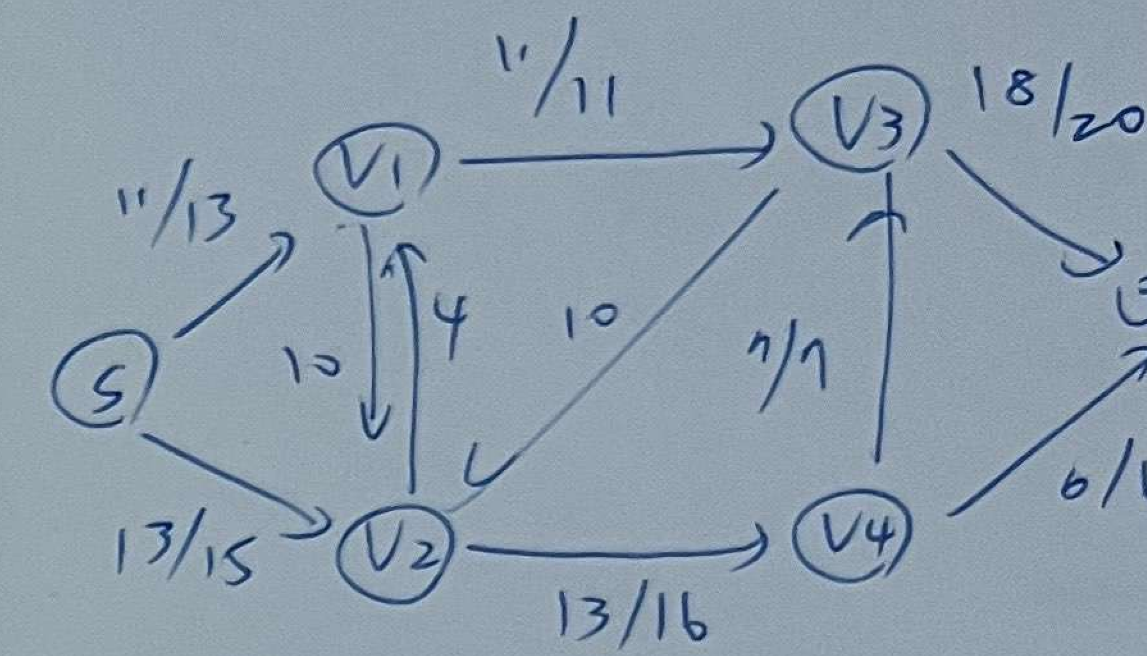
An activity selection problem is shown as followed:

i	1	2	3	4	5	6	7	8	9
Si	1	2	8	1	5	4	3	2	7
fi	3	4	10	2	7	9	6	5	8

用 Greedy algorithm 求解(每過程要說明)

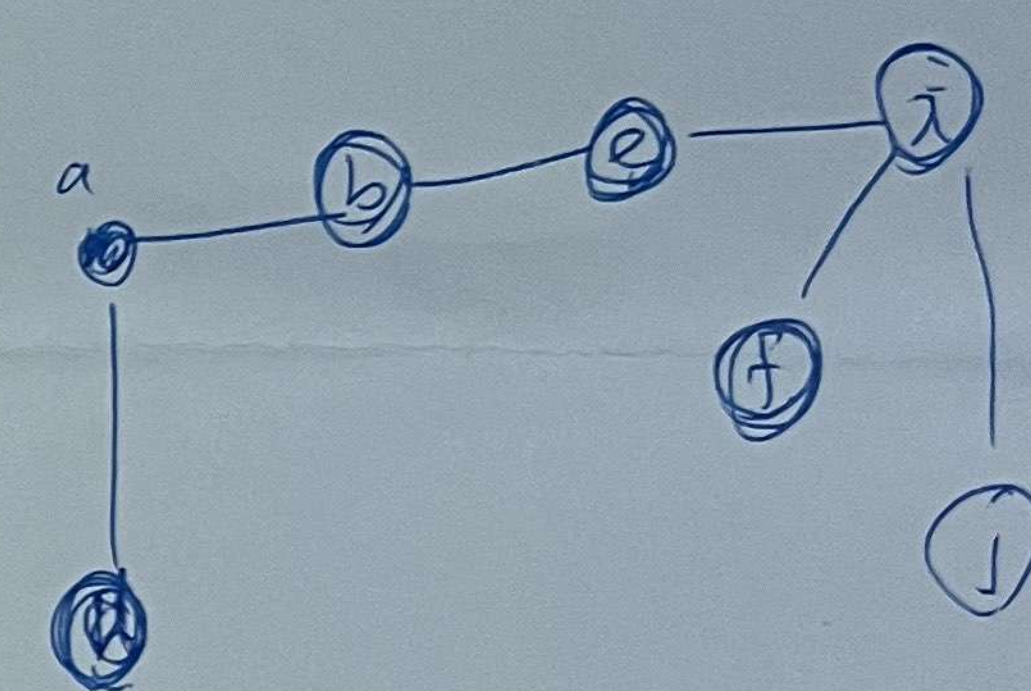
7. (10%) The Ford-Fulkerson algorithm is shown as follows. Use the following graph to find the max flow from s to t and show the state of each iteration.(畫出前兩步驟 flow network & residual network 與 最後結果)

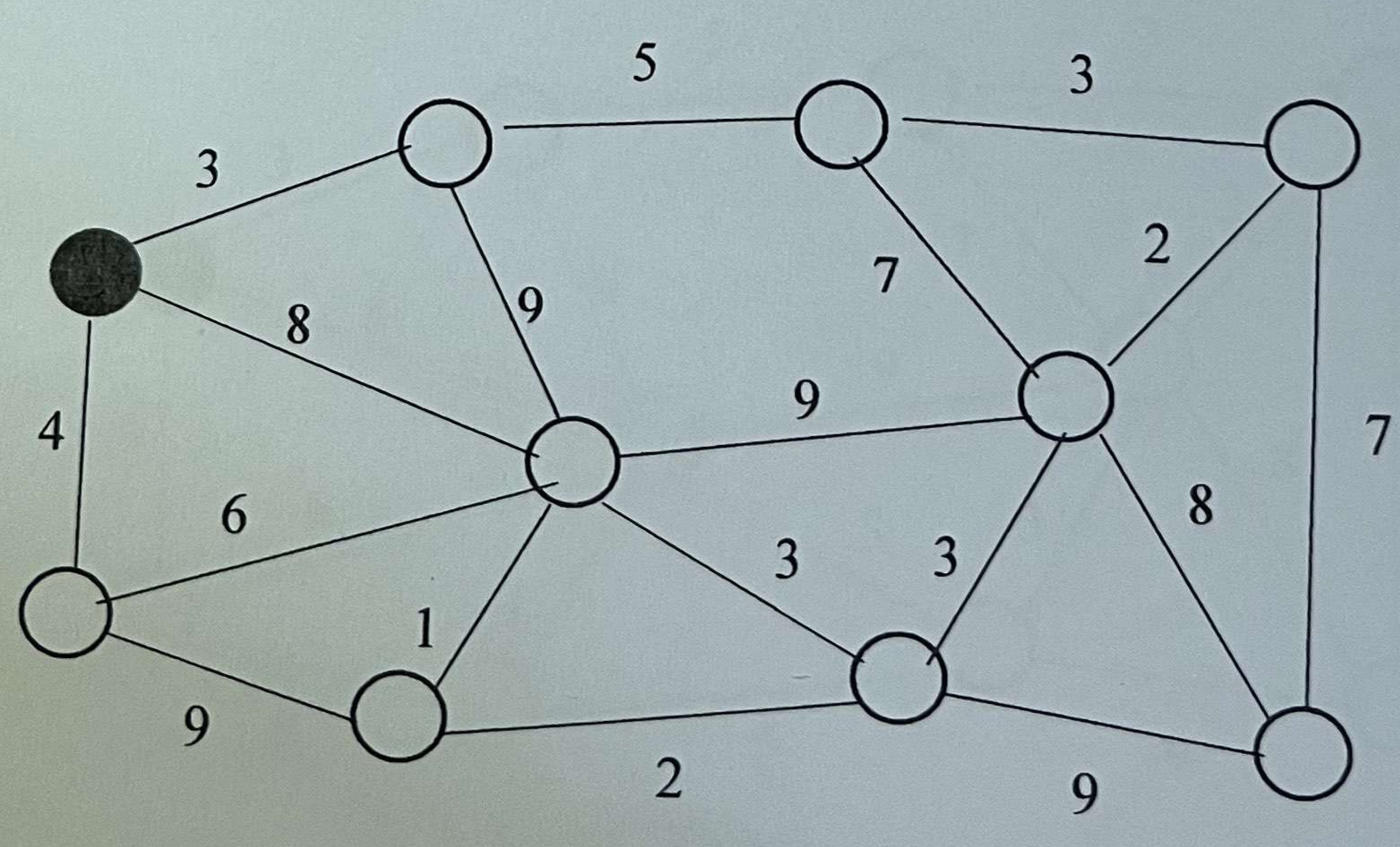




8. (10%) 使用 Prim's algorithm 求下圖 minimum spanning tree(由左上角黑點開始，選取次序標在答案紙圖上，每一步驟畫同一張圖)

```
MST-PRIM(G, w, r)
  for each u ∈ V[G]
      do key[u] ← ∞
         π[u] ← NIL
  key[r] ← 0                          u = r
  Q ← V[G]
  while Q ≠ Ø          →  v-1 ≒ V
      do u ← EXTRACT-MIN(Q)   — lg v
         for each v ∈ Adj[u]   —  v-1 ≒ V
             do if v ∈ Q and w(u, v) < key[v]
                   then π[v] ← u
                        key[v] ← w(u, v)
```

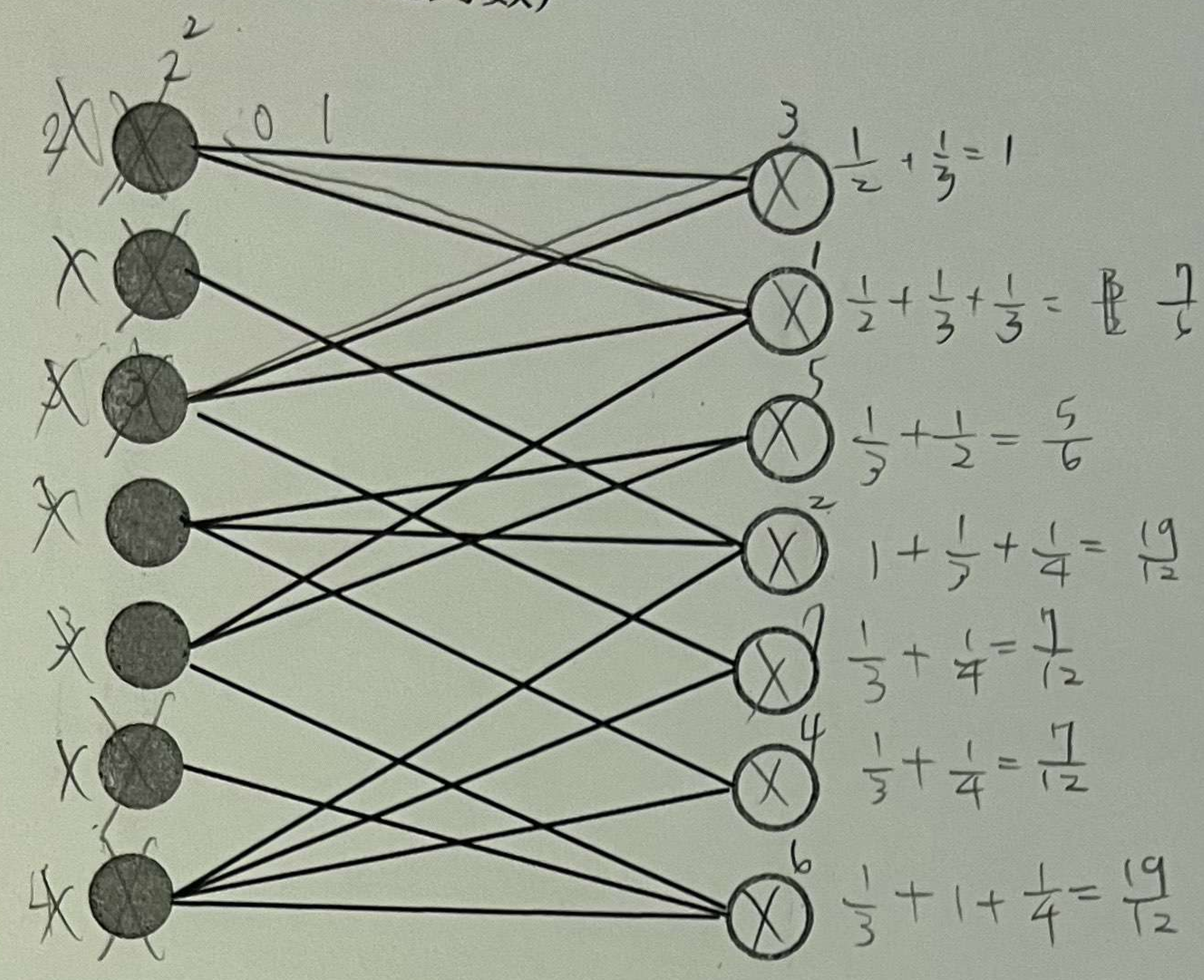




$\frac{8}{7}$

9. (10%) 下列圖為 7 位男生與 7 位女生在進行電腦擇友遊戲(連線為符合條件)，請找出最大配對？(1)說明方法？(不能用嘗試錯誤法) (2) 畫出前兩步驟 與 最後結果圖及 答案(最大配對數)

$\frac{3}{6}+\frac{2}{6}+\frac{2}{6}=\frac{7}{6}$



男　　　　　　　　女

10. (10%). 求解下列 線性規劃 答案(前兩步驟必須寫出)　　40

Max. $X_1 - X_2 + 2X_3$

sub.

$$X_1 + X_2 + X_3 \leqq 20$$

$$X_1 + 2X_2 + 3X_3 \leqq 60$$

$$2X_1 + X_2 + 3X_3 \leqq 72$$

$\frac{3}{6}+\frac{4}{6}$