Calculus (II) – Final Exam

100%

(4%) 1. Prove that $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$

Sol：

Let $y = \cot^{-1} x$. Then $\cot y = x \Rightarrow -\csc^2 y \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = -\frac{1}{\csc^2 y} = -\frac{1}{1+\cot^2 y} = -\frac{1}{1+x^2}$

(4%) 2. Prove the reduction formula

$\int sin^n x dx = -\frac{1}{n} cosx \cdot sin^{n-1}x + \frac{n-1}{n}\int sin^{n-2}x dx$, where $n \geq 2$ is an integer.

Sol：

SOLUTION Let $u = \sin^{n-1}x \qquad dv = \sin x\, dx$

Then $du = (n-1)\sin^{n-2}x \cos x\, dx \qquad v = -\cos x$

so integration by parts gives

$$\int \sin^n x\, dx = -\cos x \sin^{n-1}x + (n-1)\int \sin^{n-2}x \cos^2 x\, dx$$

Since $\cos^2 x = 1 - \sin^2 x$, we have

$$\int \sin^n x\, dx = -\cos x \sin^{n-1}x + (n-1)\int \sin^{n-2}x\, dx - (n-1)\int \sin^n x\, dx$$

As in Example 4, we solve this equation for the desired integral by taking the last term on the right side to the left side. Thus we have

$$n\int \sin^n x\, dx = -\cos x \sin^{n-1}x + (n-1)\int \sin^{n-2}x\, dx$$

or

$$\int \sin^n x\, dx = -\frac{1}{n}\cos x \sin^{n-1}x + \frac{n-1}{n}\int \sin^{n-2}x\, dx \qquad \square$$

(6%) 3. Sketch $y = \frac{1}{1+e^{-x}}$ using the guideline of Sec. 3.5.

Sol：

$y = 1/(1+e^{-x})$ **A.** $D = \mathbb{R}$ **B.** No x-intercept; y-intercept $= f(0) = \frac{1}{2}$. **C.** No symmetry

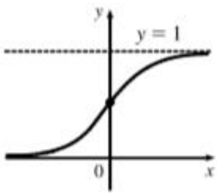
D. $\lim_{x\to\infty} 1/(1+e^{-x}) = \frac{1}{1+0} = 1$ and $\lim_{x\to-\infty} 1/(1+e^{-x}) = 0$ since $\lim_{x\to-\infty} e^{-x} = \infty$, so f has horizontal asymptotes

$y = 0$ and $y = 1$. **E.** $f'(x) = -(1+e^{-x})^{-2}(-e^{-x}) = e^{-x}/(1+e^{-x})^2$. This is positive for all x, so f is increasing on $\mathbb{R}$.

F. No extreme values **G.** $f''(x) = \dfrac{(1+e^{-x})^2(-e^{-x}) - e^{-x}(2)(1+e^{-x})(-e^{-x})}{(1+e^{-x})^4} = \dfrac{e^{-x}(e^{-x}-1)}{(1+e^{-x})^3}$

The second factor in the numerator is negative for $x > 0$ and positive for $x < 0$, and the other factors are always positive, so f is CU on $(-\infty, 0)$ and CD on $(0, \infty)$. IP at $(0, \frac{1}{2})$

H.



(4%) 4. Find the inverse function of $y = \frac{1-e^{-x}}{1+e^{-x}}$.

Sol：

$y = f(x) = \dfrac{1-e^{-x}}{1+e^{-x}} \Rightarrow y(1+e^{-x}) = 1 - e^{-x} \Rightarrow y + ye^{-x} = 1 - e^{-x} \Rightarrow ye^{x} + y = e^{x} - 1$ [multiply each term by e^x] $\Rightarrow ye^x - e^x = -y - 1 \Rightarrow e^x(y-1) = -y-1 \Rightarrow e^x = \dfrac{1+y}{1-y} \Rightarrow x = \ln\left(\dfrac{1+y}{1-y}\right)$.

Interchange x and y: $y = \ln\left(\dfrac{1+x}{1-x}\right)$. So $f^{-1}(x) = \ln\left(\dfrac{1+x}{1-x}\right)$.

(4%) 5. Solve each equation for x.

(a) $e - e^{-2x} = 1$.

(b) $\ln(2x + 1) = 2 - \ln x$.

Sol：

(a)

$$e - e^{-2x} = 1 \quad \Leftrightarrow \quad e - 1 = e^{-2x} \quad \Leftrightarrow \quad \ln(e-1) = \ln e^{-2x} \quad \Leftrightarrow \quad -2x = \ln(e-1) \quad \Leftrightarrow \quad x = -\tfrac{1}{2}\ln(e-1)$$

(b)

$$\ln(2x+1) = 2 - \ln x \quad \Rightarrow \quad \ln x + \ln(2x+1) = \ln e^2 \quad \Rightarrow \quad \ln[x(2x+1)] = \ln e^2 \quad \Rightarrow \quad 2x^2 + x = e^2 \quad \Rightarrow$$

$$2x^2 + x - e^2 = 0 \quad \Rightarrow \quad x = \frac{-1+\sqrt{1+8e^2}}{4} \quad [\text{since } x > 0].$$

(24%) 6. Find the limit.

(a) $\lim_{x\to\infty}[\ln(2+x) - \ln(1+x)] =$ ________.

(b) $\lim_{x\to(\frac{\pi}{2})^+} e^{\tan x} =$ ________.

(c) $\lim_{x\to 0}\frac{\sqrt{1+2x} - \sqrt{1-4x}}{x} =$ ________.

(d) $\lim_{x\to-\infty} x\cdot\ln(1-\frac{1}{x}) =$ ________.

(e) $\lim_{x\to 1}(2-x)^{\tan(\frac{\pi x}{2})} =$ ________.

(f) $\lim_{x\to 0^+}(\frac{1}{x} - \frac{1}{\tan^{-1}x}) =$ ________.

Sol：

(a)

$$\lim_{x\to\infty}[\ln(2+x) - \ln(1+x)] = \lim_{x\to\infty}\ln\left(\frac{2+x}{1+x}\right) = \lim_{x\to\infty}\ln\left(\frac{2/x+1}{1/x+1}\right) = \ln\frac{1}{1} = \ln 1 = 0$$

(b)

If we let $t = \tan x$, then as $x \to (\pi/2)^+$, $t \to -\infty$. Thus, $\lim_{x\to(\pi/2)^+} e^{\tan x} = \lim_{t\to-\infty} e^t = 0$ by (6).

(c)

This limit has the form $\frac{0}{0}$.

$$\lim_{x\to 0}\frac{\sqrt{1+2x}-\sqrt{1-4x}}{x} \overset{\text{H}}{=} \lim_{x\to 0}\frac{\frac{1}{2}(1+2x)^{-1/2}\cdot 2-\frac{1}{2}(1-4x)^{-1/2}(-4)}{1}$$
$$= \lim_{x\to 0}\left(\frac{1}{\sqrt{1+2x}}+\frac{2}{\sqrt{1-4x}}\right)=\frac{1}{\sqrt{1}}+\frac{2}{\sqrt{1}}=3$$

(d)

This limit has the form $(-\infty)\cdot 0$.

$$\lim_{x\to-\infty} x\ln\left(1-\frac{1}{x}\right)=\lim_{x\to-\infty}\frac{\ln\left(1-\frac{1}{x}\right)}{\frac{1}{x}} \overset{\text{H}}{=} \lim_{x\to-\infty}\frac{\frac{1}{1-1/x}\cdot\frac{1}{x^2}}{-\frac{1}{x^2}}=\lim_{x\to-\infty}\frac{-1}{1-\frac{1}{x}}=\frac{-1}{1}=-1$$

(e)

$$y=(2-x)^{\tan(\pi x/2)} \quad\Rightarrow\quad \ln y=\tan\left(\frac{\pi x}{2}\right)\ln(2-x) \quad\Rightarrow$$

$$\lim_{x\to 1}\ln y=\lim_{x\to 1}\left[\tan\left(\frac{\pi x}{2}\right)\ln(2-x)\right]=\lim_{x\to 1}\frac{\ln(2-x)}{\cot\left(\frac{\pi x}{2}\right)} \overset{\text{H}}{=} \lim_{x\to 1}\frac{\frac{1}{2-x}(-1)}{-\csc^2\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}}=\frac{2}{\pi}\lim_{x\to 1}\frac{\sin^2\left(\frac{\pi x}{2}\right)}{2-x}$$
$$=\frac{2}{\pi}\cdot\frac{1^2}{1}=\frac{2}{\pi} \quad\Rightarrow\quad \lim_{x\to 1}(2-x)^{\tan(\pi x/2)}=\lim_{x\to 1}e^{\ln y}=e^{(2/\pi)}$$

(f)

This limit has the form $\infty-\infty$.

$$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\tan^{-1}x}\right)=\lim_{x\to 0^+}\frac{\tan^{-1}x-x}{x\tan^{-1}x} \overset{\text{H}}{=} \lim_{x\to 0^+}\frac{1/(1+x^2)-1}{x/(1+x^2)+\tan^{-1}x}=\lim_{x\to 0^+}\frac{1-(1+x^2)}{x+(1+x^2)\tan^{-1}x}$$
$$=\lim_{x\to 0^+}\frac{-x^2}{x+(1+x^2)\tan^{-1}x} \overset{\text{H}}{=} \lim_{x\to 0^+}\frac{-2x}{1+(1+x^2)(1/(1+x^2))+(\tan^{-1}x)(2x)}$$
$$=\lim_{x\to 0^+}\frac{-2x}{2+2x\tan^{-1}x}=\frac{0}{2+0}=0$$

(20%) 7. Differentiate the function.

(a) $y = \sqrt{x} \cdot e^{x^2} \cdot (x^2+1)^{10}$.

(b) $f(x) = \frac{x}{1-\ln(x-1)}$.

(c) $y = (\sin x)^{\ln x}$.

(d) $y = x \cdot \log_4(\sin x)$.

(e) $y = \tan^{-1}(x - \sqrt{1+x^2})$.

Sol：

(a)

$$y = \sqrt{x}\, e^{x^2}(x^2+1)^{10} \quad\Rightarrow\quad \ln y = \ln\sqrt{x} + \ln e^{x^2} + \ln(x^2+1)^{10} \quad\Rightarrow\quad \ln y = \tfrac{1}{2}\ln x + x^2 + 10\ln(x^2+1) \quad\Rightarrow$$
$$\frac{1}{y}y' = \frac{1}{2}\cdot\frac{1}{x} + 2x + 10\cdot\frac{1}{x^2+1}\cdot 2x \quad\Rightarrow\quad y' = \sqrt{x}\, e^{x^2}(x^2+1)^{10}\left(\frac{1}{2x} + 2x + \frac{20x}{x^2+1}\right)$$

(b)

$$f(x) = \frac{x}{1-\ln(x-1)} \quad\Rightarrow$$

$$f'(x) = \frac{[1-\ln(x-1)]\cdot 1 - x\cdot\dfrac{-1}{x-1}}{[1-\ln(x-1)]^2} = \frac{\dfrac{(x-1)[1-\ln(x-1)]+x}{x-1}}{[1-\ln(x-1)]^2} = \frac{x-1-(x-1)\ln(x-1)+x}{(x-1)[1-\ln(x-1)]^2}$$
$$= \frac{2x-1-(x-1)\ln(x-1)}{(x-1)[1-\ln(x-1)]^2}$$

$$\text{Dom}(f) = \{x \mid x-1>0 \quad\text{and}\quad 1-\ln(x-1)\neq 0\} = \{x \mid x>1 \quad\text{and}\quad \ln(x-1)\neq 1\}$$
$$= \{x \mid x>1 \quad\text{and}\quad x-1\neq e^1\} = \{x \mid x>1 \quad\text{and}\quad x\neq 1+e\} = (1,1+e)\cup(1+e,\infty)$$

(c)

$$y = (\sin x)^{\ln x} \quad\Rightarrow\quad \ln y = \ln(\sin x)^{\ln x} \quad\Rightarrow\quad \ln y = \ln x\cdot\ln\sin x \quad\Rightarrow\quad \frac{1}{y}y' = \ln x\cdot\frac{1}{\sin x}\cdot\cos x + \ln\sin x\cdot\frac{1}{x} \quad\Rightarrow$$
$$y' = y\left(\ln x\cdot\frac{\cos x}{\sin x} + \frac{\ln\sin x}{x}\right) \quad\Rightarrow\quad y' = (\sin x)^{\ln x}\left(\ln x\cot x + \frac{\ln\sin x}{x}\right)$$

(d)

$$y = x\log_4\sin x \quad\Rightarrow\quad y' = x\cdot\frac{1}{\sin x\ln 4}\cdot\cos x + \log_4\sin x\cdot 1 = \frac{x\cot x}{\ln 4} + \log_4\sin x$$

(e)

$$y = \tan^{-1}\left(x - \sqrt{x^2+1}\right) \quad \Rightarrow$$

$$\begin{aligned}
y' &= \frac{1}{1+\left(x-\sqrt{x^2+1}\right)^2}\left(1-\frac{x}{\sqrt{x^2+1}}\right) = \frac{1}{1+x^2-2x\sqrt{x^2+1}+x^2+1}\left(\frac{\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\right) \\
&= \frac{\sqrt{x^2+1}-x}{2\left(1+x^2-x\sqrt{x^2+1}\right)\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}-x}{2\left[\sqrt{x^2+1}\,(1+x^2)-x(x^2+1)\right]} = \frac{\sqrt{x^2+1}-x}{2\left[(1+x^2)\left(\sqrt{x^2+1}-x\right)\right]} \\
&= \frac{1}{2(1+x^2)}
\end{aligned}$$

(44%) 8. Evaluate the integral

(a) $\int \frac{\sin 2x}{1+\cos^2 x} dx =$ ________.

(b) $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx =$ ________.

(c) $\int x \cdot 2^{x^2} dx =$ ________.

(d) $\int_0^{1/4} \frac{1}{\sqrt{1-4x^2}} dx =$ ________.

(e) $\int (\sin^{-1} x)^2 dx =$ ________.

(f) $\int \cos^{-1} x \, dx =$ ________.

(g) $\int \sec^3 x \, dx =$ ________.

(h) $\int_0^{\pi} \cos^4(2t) \, dt =$.

(i) $\int \tan^5 x \cdot \sec^3 x \, dx =$ ________.

(j) $\int \frac{x}{\sqrt{3-2x-x^2}} dx =$ ________.

(k) $\int \frac{\sqrt{9-x^2}}{x^2} dx =$ ________.

Sol：

(a)

$\displaystyle\int \frac{\sin 2x}{1+\cos^2 x}\,dx = 2\int \frac{\sin x \cos x}{1+\cos^2 x}\,dx = 2I$. Let $u = \cos x$. Then $du = -\sin x\,dx$, so

$\displaystyle 2I = -2\int \frac{u\,du}{1+u^2} = -2 \cdot \tfrac{1}{2}\ln(1+u^2) + C = -\ln(1+u^2) + C = -\ln(1+\cos^2 x) + C.$

Or: Let $u = 1 + \cos^2 x$.

(b)

Let $u = 1/x$, so $du = -1/x^2\,dx$. When $x = 1$, $u = 1$; when $x = 2$, $u = \frac{1}{2}$. Thus,

$$\int_1^2 \frac{e^{1/x}}{x^2}\,dx = \int_1^{1/2} e^u\,(-du) = -\left[e^u\right]_1^{1/2} = -(e^{1/2} - e) = e - \sqrt{e}.$$

(c)

Let $u = x^2$. Then $du = 2x\,dx$, so $\int x2^{x^2}\,dx = \frac{1}{2}\int 2^u\,du = \frac{1}{2}\frac{2^u}{\ln 2} + C = \frac{1}{2\ln 2}2^{x^2} + C$.

(d)

SOLUTION If we write

$$\int_0^{1/4} \frac{1}{\sqrt{1-4x^2}}\,dx = \int_0^{1/4} \frac{1}{\sqrt{1-(2x)^2}}\,dx$$

then the integral resembles Equation 12 and the substitution $u = 2x$ is suggested. This gives $du = 2\,dx$, so $dx = du/2$. When $x = 0$, $u = 0$; when $x = \frac{1}{4}$, $u = \frac{1}{2}$. So

$$\int_0^{1/4} \frac{1}{\sqrt{1-4x^2}}\,dx = \frac{1}{2}\int_0^{1/2} \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}u\Big]_0^{1/2}$$

$$= \frac{1}{2}\left[\sin^{-1}\left(\tfrac{1}{2}\right) - \sin^{-1}0\right] = \frac{1}{2}\cdot\frac{\pi}{6} = \frac{\pi}{12} \qquad \square$$

(e)

First let $u = (\arcsin x)^2$, $dv = dx \Rightarrow du = 2\arcsin x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$, $v = x$. Then

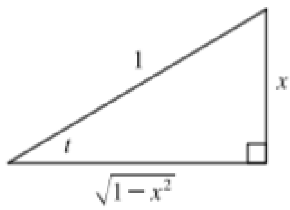
$I = \int (\arcsin x)^2 dx = x(\arcsin x)^2 - 2\int \frac{x\arcsin x}{\sqrt{1-x^2}}\,dx$. To simplify the last integral, let $t = \arcsin x$ $[x = \sin t]$, so

$dt = \frac{1}{\sqrt{1-x^2}}\,dx$, and $\int \frac{x\arcsin x}{\sqrt{1-x^2}}\,dx = \int t\sin t\,dt$. To evaluate just the last integral, now let $U = t$, $dV = \sin t\,dt \Rightarrow$

$dU = dt$, $V = -\cos t$. Thus,

$$\int t\sin t\,dt = -t\cos t + \int \cos t\,dt = -t\cos t + \sin t + C$$

$$= -\arcsin x \cdot \frac{\sqrt{1-x^2}}{1} + x + C_1 \quad \text{[refer to the figure]}$$



Returning to I, we get $I = x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$, where $C = -2C_1$.

(f)

Let $u = \cos^{-1} x$, $dv = dx \quad \Rightarrow \quad du = \dfrac{-1}{\sqrt{1-x^2}}\,dx$, $v = x$. Then by Equation 2,

$$\int \cos^{-1} x\,dx = x\cos^{-1} x - \int \frac{-x}{\sqrt{1-x^2}}\,dx = x\cos^{-1} x - \int \frac{1}{\sqrt{t}}\left(\frac{1}{2}\,dt\right) \qquad \begin{bmatrix} t = 1 - x^2, \\ dt = -2x\,dx \end{bmatrix}$$

$$= x\cos^{-1} x - \tfrac{1}{2}\cdot 2t^{1/2} + C = x\cos^{-1} x - \sqrt{1-x^2} + C$$

(g)

SOLUTION Here we integrate by parts with

$$u = \sec x \qquad dv = \sec^2 x\,dx$$

$$du = \sec x \tan x\,dx \qquad v = \tan x$$

Then

$$\int \sec^3 x\,dx = \sec x \tan x - \int \sec x \tan^2 x\,dx$$

$$= \sec x \tan x - \int \sec x\,(\sec^2 x - 1)\,dx$$

$$= \sec x \tan x - \int \sec^3 x\,dx + \int \sec x\,dx$$

Using Formula 1 and solving for the required integral, we get

$$\int \sec^3 x\,dx = \tfrac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C \qquad \square$$

(h)

$$\int_0^\pi \cos^4(2t)\,dt = \int_0^\pi [\cos^2(2t)]^2\,dt = \int_0^\pi \left[\tfrac{1}{2}(1+\cos(2\cdot 2t))\right]^2 dt \qquad \text{[half-angle identity]}$$

$$= \tfrac{1}{4}\int_0^\pi [1 + 2\cos 4t + \cos^2(4t)]\,dt = \tfrac{1}{4}\int_0^\pi [1 + 2\cos 4t + \tfrac{1}{2}(1+\cos 8t)]\,dt$$

$$= \tfrac{1}{4}\int_0^\pi \left(\tfrac{3}{2} + 2\cos 4t + \tfrac{1}{2}\cos 8t\right)\,dt = \tfrac{1}{4}\left[\tfrac{3}{2}t + \tfrac{1}{2}\sin 4t + \tfrac{1}{16}\sin 8t\right]_0^\pi = \tfrac{1}{4}\left[\left(\tfrac{3}{2}\pi + 0 + 0\right) - 0\right] = \tfrac{3}{8}\pi$$

(i)

Let $u = \sec x$, so $du = \sec x \tan x\,dx$. Thus,

$$\int \tan^5 x \sec^3 x\,dx = \int \tan^4 x \sec^2 x\,(\sec x \tan x)\,dx = \int (\sec^2 x - 1)^2 \sec^2 x\,(\sec x \tan x\,dx)$$

$$= \int (u^2-1)^2 u^2\,du = \int (u^6 - 2u^4 + u^2)\,du$$

$$= \tfrac{1}{7}u^7 - \tfrac{2}{5}u^5 + \tfrac{1}{3}u^3 + C = \tfrac{1}{7}\sec^7 x - \tfrac{2}{5}\sec^5 x + \tfrac{1}{3}\sec^3 x + C$$

(j)

SOLUTION We can transform the integrand into a function for which trigonometric substitution is appropriate by first completing the square under the root sign:

$$\begin{aligned} 3 - 2x - x^2 &= 3 - (x^2 + 2x) = 3 + 1 - (x^2 + 2x + 1) \\ &= 4 - (x + 1)^2 \end{aligned}$$

This suggests that we make the substitution $u = x + 1$. Then $du = dx$ and $x = u - 1$, so

$$\int \frac{x}{\sqrt{3 - 2x - x^2}}\,dx = \int \frac{u - 1}{\sqrt{4 - u^2}}\,du$$

We now substitute $u = 2\sin\theta$, giving $du = 2\cos\theta\,d\theta$ and $\sqrt{4 - u^2} = 2\cos\theta$, so

$$\begin{aligned} \int \frac{x}{\sqrt{3 - 2x - x^2}}\,dx &= \int \frac{2\sin\theta - 1}{2\cos\theta}\,2\cos\theta\,d\theta \\ &= \int (2\sin\theta - 1)\,d\theta \\ &= -2\cos\theta - \theta + C \\ &= -\sqrt{4 - u^2} - \sin^{-1}\left(\frac{u}{2}\right) + C \\ &= -\sqrt{3 - 2x - x^2} - \sin^{-1}\left(\frac{x + 1}{2}\right) + C \end{aligned}$$

$\square$

(k)

SOLUTION Let $x = 3\sin\theta$, where $-\pi/2 \le \theta \le \pi/2$. Then $dx = 3\cos\theta\, d\theta$ and

$$\sqrt{9 - x^2} = \sqrt{9 - 9\sin^2\theta} = \sqrt{9\cos^2\theta} = 3|\cos\theta| = 3\cos\theta$$

(Note that $\cos\theta \ge 0$ because $-\pi/2 \le \theta \le \pi/2$.) Thus the Inverse Substitution Rule gives

$$\begin{aligned}\int \frac{\sqrt{9 - x^2}}{x^2}\,dx &= \int \frac{3\cos\theta}{9\sin^2\theta}\,3\cos\theta\,d\theta \\ &= \int \frac{\cos^2\theta}{\sin^2\theta}\,d\theta = \int \cot^2\theta\,d\theta \\ &= \int (\csc^2\theta - 1)\,d\theta \\ &= -\cot\theta - \theta + C\end{aligned}$$

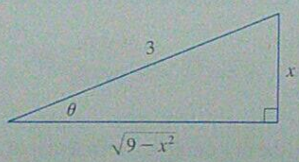


FIGURE 1

$\sin\theta = \frac{x}{3}$

Since this is an indefinite integral, we must return to the original variable x. This can be done either by using trigonometric identities to express $\cot\theta$ in terms of $\sin\theta = x/3$ or by drawing a diagram, as in Figure 1, where θ is interpreted as an angle of a right triangle. Since $\sin\theta = x/3$, we label the opposite side and the hypotenuse as having lengths x and 3. Then the Pythagorean Theorem gives the length of the adjacent side as $\sqrt{9 - x^2}$, so we can simply read the value of $\cot\theta$ from the figure:

$$\cot\theta = \frac{\sqrt{9 - x^2}}{x}$$

(Although $\theta > 0$ in the diagram, this expression for $\cot\theta$ is valid even when $\theta < 0$.) Since $\sin\theta = x/3$, we have $\theta = \sin^{-1}(x/3)$ and so

$$\int \frac{\sqrt{9 - x^2}}{x^2}\,dx = -\frac{\sqrt{9 - x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right) + C$$