$\exists x(\sim p \vee q)$

Middle Exam (Discrete Mathematics)

108.4.19

(滿分 100)

// 所有問題不能只寫答案(例如：Yes/ No)，必須加予說明，否則不計分

// 答案卷 請寫上姓名、學號

行ok ✓ 1. (4%) 下列命題(proposition)何者正確？ (錯一項，扣一分)

a) Toronto is the capital of Canada.

b) 1 + 2 = 2

c) x + 1 = 2

d) What time is it?

e) Read this carefully.

呃 ok ✓ 2. (4%) Determine whether $\exists x(P(x) \rightarrow Q(x))$ and $\exists x P(x) \rightarrow \exists x\, Q(x)$ are logically equivalent. Justify your answer.

行ok ✓ 3. (4%) Determine whether ∀x ∀y ∃z Q(x, y, z) and ∃z ∀x ∀y Q(x, y, z) are logically equivalent. (Q(x, y, z) = "x + y = z.", x, y, z ∈ R) Justify your answer.

行 ok ✓ 4. (4%) 推導 the logic equivalence $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (不行用真值表)

$(\sim p \vee q) \wedge (\sim q \vee p)$

行ok ✓ 5. (4%) 下列為 Prolog (Programming in Logic): facts 與 rules (全對才給分)

Fact: instructor(chan, EE001), instructor(wang, cs001),

instructor(lee, EE001), ~~instructor(sun, cs001)~~, instructor(lin, EE002),

enrolled(lee, EE001), enrolled(lee, cs001), ...

Rule: teachers(P, S) :- instructor(P, C), enrolled(S, C).

列出 Query:

? teachers(X, lee) 所有結果

$\sim p \wedge \sim q$

行ok ✓ 6. (4%) A 說: 有外國護照者，則比較有國際觀。

B 說: 那沒有外國護照者，就沒國際觀。

試問，B 說法正確否？

行ok ✓ 7. (4%) Show that if n is an integer and 3n + 2 is odd, then n is odd.

行ok ✓ 8. (4%) Two players, taking turns removing one, two, three, or four stones at a time from a pile (一堆) with 22 stones. The person who removes the last stone wins the game. Show that the first player can win the game no matter what the second player does. 第一位玩家第一次需拿幾個？每次要如何拿？

$\forall (P(x) \rightarrow (q(x))$

行 ok ✓ 9. (4%) Determine whether each of these statements is true or false.

a) $\phi \subset \{0\}$ b) $\phi \in \{0\}$ c) $\phi \in \{\phi\}$ d) $0 \in \{0\}$

行 ok ✓ 10. (4%) The domain is the set of integers, and P(x) is "$|x| = 2$"; Q(x) is "$x^2 = 8$," and R(x) is "$|x| = -x$." Find the truth set of P(x), Q(x), R(x) =?

行 ok ✓ 11. (4%) Determine function $f(x) = x^2$ from the set of integers to integers is one-to-one.

行 ok ✓ 12. (4%) 證明 $\sum_{j=0}^{n} ar^j = \frac{ar^{n+1}-a}{r-1}$ for $r \neq 1$.

$\frac{a(1-r^{n+1})}{}$

行 ok ✓ 13. (4%) How many functions are there for a Boolean function with degree 5?

行 ok ✓ 14. (4%) Sum both sides of the identity $k^2-(k-1)^2 = 2k - 1$ from k=1 to n and use the formula $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_0$ to find $\sum_{k=1}^{n}(8k-4)$ = ?

$8\frac{(1+n)n}{4 \; 2}$

行 ok ✓ 15. (4%) Let $g(x) = \lfloor x \rfloor$. Find a) $g^{-1}(\{1, 2, 3\})$. b) $g^{-1}(\{x \mid -2 < x < -1\})$.

ok ✓ 16. (4%) Find a minimal (最簡) sum-of-products expansion with *don't care* conditions indicated with *d*s.

行

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx	d	d	d	d
$w\bar{x}$	d	d	1	1
$\bar{w}\bar{x}$	d		d	d
$\bar{w}x$	1	1		1

$xyz = \bar{x}\bar{y}z$

111 011 = 100

000 100 = 011

行

ok ✓ 17. (4%) How many different Boolean functions F(x, y, z) are there such that

$F(\bar{x}, y, z) = F(x, \bar{y}, \bar{z})$ for all values of the Boolean variables x, y, and z. (需說明原因)

ok ✓ 18. (4%) Design a four switches control (四個開關一起控制同一燈光，全開時燈亮)，畫出函數表與函數即可，不需畫出線路。

行

ok 19. (4%) Determine whether each of these functions is a bijection (one-to-one correspondence) from R to R.

行

a) $f(x) = 2x + 1$

b) $f(x) = x / 2$

c) $f(x) = x^4$

d) $f(x) = |x| + 1$

ok 20. (4%) Determine whether f is a function from the set of all bit strings to the set of integers if

行

a) $f(S)$ is the number of 0-bits in S.

b) $f(S)$ is the position of a 0 bit in S.

21. (4%) 使用 | (NAND) 函數，將下列函數都改為 NAND.

行 ok

10100

5050 7+

(公式：$\bar{x} = x \mid x$.；$xy = (x \mid y) \mid (x \mid y)$.；$x + y = (x \mid x) \mid (y \mid y)$)

F(x,y,z) = x+y+z = ?

101·49

50

ok 22. (8%) 使用 Quine-McCluskey Method 化簡最簡函數 (寫出每一步驟)

行

$$xyz + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$$

1. Express each term in n variables by a n-bit string.(以 bit 表示)
2. Group the bit strings according to the number of 1s in them.(以 1 的數目分組)
3. Determine all products in n-1 variables (合併為 n-1variables).
4. Determine all products in n-2 variables from n-1 variables (合併為 n-2variables).
5. Continue combining as long as possible (重複合併步驟)
6. Find all Boolean products (找出所有合併項)

   例如：-11 and -01 => - - 1 則 取 z 代替 yz 與 $\bar{y}z$.

7. Find the solution from the products and original terms that can cover all original terms. (找最小 product 集合，包含所有項目)

ok 23. (8%) 若證明：The set of positive rational numbers (正有理數) is countable 其方式如下：

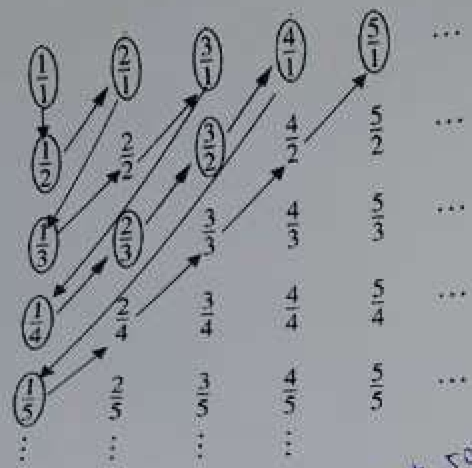
行

證明：正有理數: p / q : first row q =1 (first column p =1)

Second row q =2 (second column p =2), … 可對應所有 正有理數

自然數對應正有理數方式(下圖)若為：1->1/1, 2 ->1/2, 3->2/1 , 4-> 1/3, 5 ->2/2, 6->3/1 , 7->1/4, 8->2/3, 9->3/2, 10->4/1 , 11->1/5, … ，所以，

(1) 求自然數 5100 對應到何數？ 反之，(2) 正有理數 31/43 被哪一自然數對應？



$\frac{(1+414)}{2}+1$

11

5050 + 50

5100

2659

101
49

11
52

$\frac{10000}{2}$

5050

150

50
51
2550

49
50
2513
2450

$\frac{(1+101)72}{2}$

36
13
108
252
2626

101
102
202
1010
10302

50

$\frac{1}{5}$

102

63

.50

.010

10100

5052

173
36
438
219
2628
31
2659

36
73
108
252
2628
31
2659

173
×34
292
219
2482
31
2513

5026

50
51
2550

2628

101 +30
73 -30
42

31
43

國立臺南大學 學年度 學期 考試 答案卷

總分 93

考試科目：離散數學 班別：數位一

學號：[redacted] 姓名：[redacted]

1. (a) T

(b) F $1+2=3$

(c) F 不確定

(d) 非命題

(e) 非命題

-1

2, 第2題見最後一頁

2,

3. no $\forall x \forall y \exists z\, Q(x,y,z)$ 對於所有的 x,y 存在一個 z 與其對應 T

ex: $x=1$ $y=1$ $z=2$

$\exists z \forall x \forall y\, Q(x,y,z)$ 存在一個 z 使所有 x,y 與其對應 F

ex: $z=5$ $x+y$ 不一定 5

> not logically equivalent.

4. $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p) \equiv [(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$

$\equiv (\sim p \wedge \sim q) \vee (q \wedge \sim q) \vee (\sim p \wedge p) \vee (q \wedge p)$

$\equiv (\sim p \wedge \sim q) \vee F \vee F \vee (q \wedge p)$

$\equiv (\sim p \wedge \sim q) \vee (q \wedge p)$

$\equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

5.

teachers(X, lee) :- instructor(X, C), enrolled(lee, C)

X = sun, wang, (instructor(sun, cs001), enrolled(lee, cs001)
instructor(wang, cs001), enrolled(lee, cs001))

chan, lee (instructor(chan, EE001), enrolled(lee, EE001)
instructor(lee, EE001),

6. 否 設 p：有外國護照 q：有國際觀

A: $p \to q \equiv \sim p \vee q$

B: $\sim p \to \sim q \equiv \sim(\sim p) \vee \sim q \equiv p \vee \sim q$ 不等於 $\sim p \vee q$

7. 設 $3n+2 = 2k+1 \Rightarrow n = \cancel{\frac{2k-1}{3}} = \frac{2k+2-3}{3} = 2\left(\cancel{\frac{k+1}{3}}\right) - 1$ 為奇數

8. 第一位玩家拿2個 剩20個

之後每次拿完剩於5的倍數即可必贏

9.

(a) T ∅是任何子集

(b) F ∅不是元素是集合

(c) T 此時空集合為右邊集合中之元素

(d) T 0為集合中元素

10. P(x): $x = 2, -2$

Q(x): ∅ 沒有整數平方是8

R(x): $0 \geq x$ 0跟所有負整數

11. $f(x)=x^2$ $\quad f(1)=f(-1)=1$ not (one to one)

12. $\sum_{j=0}^{n} ar^j = ar^0 + ar^1 + ar^2 + \cdots + ar^n$

$r\left[\sum_{j=0}^{n} ar^j\right] = \quad ar^1 + ar^2 + \cdots + ar^n + ar^{n+1}$

2式相減 $(1-r)\left(\sum_{j=0}^{n} ar^j\right) = ar^0 - ar^{n+1} \Rightarrow \sum_{j=0}^{n} ar^j = \frac{a-ar^{n+1}}{1-r} = \frac{a(r^{n+1}-1)}{r-1}$ *

13. $2^{2^5} = 2^{32} = 1073741824 \times 4$

$= 4294967296$

14. $\sum_{k=1}^{n}(8k-4) = \sum_{k=1}^{n} 4(2k-1) = 4\sum_{k=1}^{n}\left[k^2-(k-1)^2\right]$ 令 $a_k = k^2$

$= 4\sum_{k=1}^{n}(a_k - a_{k-1}) = 4(a_n - a_0)$

$= 4(n^2 - 0^2) = 4n^2$

15. (a)

$1 = \lfloor x \rfloor \qquad 1 \le x < 2$

$2 = \lfloor x \rfloor \qquad 2 \le x < 3$

$3 = \lfloor x \rfloor \qquad 3 \le x < 4$

$1 \le g^{-1}(\{1,2,3\}) < 4$

(b) $-2 < g(x) < -1$ 但 $g(x)$ 一定是整數，$-2,-1$ 間無整數

$g^{-1}(\{x \mid -2 < x < -1\})$ 不存在

16.

d	d	d	d
d	d	1	1
d		d	d
1	1		1

$= W + Z + xy$

17.

$x\,y\,z$ / $1\,1\,1$ — $F(0,1,1) = F(1,0,0)$ 1

$x\,y\,z$ / $0\,1\,1$ — $F(1,1,1) = F(0,0,0)$ 2

$x\,y\,z$ / $0\,1\,0$ — $F(1,1,0) = F(0,0,1)$ 3

$x\,y\,z$ / $0\,0\,1$ — $F(1,0,1) = F(0,1,0)$ 4

$2^4 = 16$

W	X	Y	Z	亮否
1	1	1	1	1
0	1	1	1	0
1	1	1	0	1
0	1	1	0	0
1	1	0	1	1
0	1	0	1	0
1	1	0	0	0
0	1	0	0	1
1	0	1	1	1
0	0	1	1	0
1	0	1	0	0
0	0	1	0	1
1	0	0	1	0
0	0	0	1	1
1	0	0	0	1
0	0	0	0	0

$f(w,x,y,z)$

$= wxyz + \cancel{wxy\bar{z}} + wx\bar{y}z + \bar{w}x\bar{y}\bar{z}$

$+ \cancel{w\bar{x}}yz + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z}$

18.

w	x	y	z	亮否
1	1	1	1	1
1	1	1	0	0
1	1	0	1	0
1	1	0	0	1
1	0	1	1	0
1	0	1	0	1
1	0	0	1	1
1	0	0	0	0
0	1	1	1	0
0	1	1	0	1
0	1	0	1	1
0	1	0	0	0
0	0	1	1	1
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

$f(w,x,y,z)$

$= wxyz + wx\bar{y}\bar{z} + w\bar{x}y\bar{z} + w\bar{x}\bar{y}z + \bar{w}xy\bar{z}$

$+ \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}\bar{z}$

19 (a) 是 $f(x)=2x+1$ $\quad x = \frac{f(x)-1}{2}$

(b) 是 $f(x) = \frac{x}{2}$ $\quad x = 2f(x)$

(c) 否 ex: $f(1) = f(-1) = 1$

(d) 否 ex: $f(1) = f(-1) = 2$

20

(a) 是 0-bits 個數 多對一函數

(b) 否 0位置 一對多 非函數

21,

$F(x,y,z) = x+y+z$

$= x+(y+z) = (x|x)|((y+z)|(y+z))$

$= (x|x)|[[(y|y)|(z|z)]|[(y|y)|(z|z)]]$

22,

xyz	111
$x\bar{y}z$	101
$\bar{x}yz$	010
$\bar{x}\bar{y}z$	001
$\bar{x}y\bar{z}$	010
$\bar{x}\bar{y}\bar{z}$	000

xyz	111	— 1
$x\bar{y}z$	101	— 2
$\bar{x}yz$	011	— 3
$\bar{x}\bar{y}z$	001	— 4
$\bar{x}y\bar{z}$	010	— 5
$\bar{x}\bar{y}\bar{z}$	000	— 6

xz	1-1	— 1 2
yz	-11	— 1 3
$\bar{y}z$	-01	— 2 4
$\bar{x}y$	01-	— 3 5
$\bar{x}\bar{y}$	00-	— 4 6
$\bar{x}\bar{z}$	0-0	— 5 6

z	--1	1 2 3 4
$\bar{x}$	0--	3 4 5 6

$\boxed{z+\bar{x}}$

b | 0 1 0 0 1v

23 (1) 5100

$n = 100 \quad \frac{(1+n)n}{2} = 5050$

$5100 = 5050 + 50$

對應 $\frac{1+49}{101-49} = \frac{50}{52}$ #

(2) 31/43

$\frac{(1+72)72}{2} + 31$

$= 73 \times 36 + 31$

$= 2659$ #

ex

$P(x) = x > 2$ $Q(x) = x > 30$

2

2. $\exists x (P(x) \rightarrow Q(x))$

~~有時T有時F~~

$\exists x P(x) \rightarrow \exists x Q(x)$

$T \rightarrow T \rightarrow T$

not logically equivalent