Algebraic Algorithms

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October 2022

1 Theoretical part

1. Provide example of an integral domain which is:
   - Gaussian and Notherian;
   - Gaussian and non-Noetherian;
   - non-Gaussian and Noetherian;
   - non-Gaussian and non-Noetherian.
2. A (commutative) ring R is *Artinian* iff any descending chain of ideals from R eventually stabilizes; in other words there is no infinite chain of ideals $I_1 \supset I_2 \supset I_3 \supset ...$ Provide example of an integral domain which is:
   - Artinian and Notherian;
   - ** Artinian and non-Noetherian;
   - non-Artinian and Noetherian;
   - non-Artinian and non-Noetherian.
3. Is it true that:
   - every finite graph is terminating;
   - every terminating graph becomes a forest after forgetting its' orientation;
   - every (finite) non-oriented graph can be oriented to a terminating graph.
4. An element $nf(x)$ is called a *normal form* of an element x iff $nf(x)$ is a terminal and $x \xrightarrow{*} nf(x)$. Prove that a graph G is convergent iff any element of G has a unique normal form.
5. ** Show that the problem of deciding whether a polynomial $p(\overline{x})$ belongs to a given ideal $I \leq \mathbb{Q}[\overline{x}]$ is NP-hard.

2 Computational part

1. Study the first two sections of the sage tutorial.