

Algebraic Algorithms

mykyta.narusevych@matfyz.cuni.cz

October 2022

1 Theoretical part

1. Provide example of an integral domain which is:
 - Gaussian and Noetherian;
 - Gaussian and non-Noetherian;
 - non-Gaussian and Noetherian;
 - non-Gaussian and non-Noetherian.
2. A (commutative) ring R is *Artinian* iff any descending chain of ideals from R eventually stabilizes; in other words there is no infinite chain of ideals $I_1 \supset I_2 \supset I_3 \supset \dots$. Provide example of an integral domain which is:
 - Artinian and Noetherian;
 - ** Artinian and non-Noetherian;
 - non-Artinian and Noetherian;
 - non-Artinian and non-Noetherian.
3. Is it true that:
 - every finite graph is terminating;
 - every terminating graph becomes a forest after forgetting its' orientation;
 - every (finite) non-oriented graph can be oriented to a terminating graph.
4. An element $nf(x)$ is called a *normal form* of an element x iff $nf(x)$ is a terminal and $x \xrightarrow{*} nf(x)$. Prove that a terminating graph G is convergent iff any element of G has a unique normal form. * Is termination condition really necessary for the equivalence to hold?
5. * Show that the problem of deciding whether a polynomial $p(\bar{x})$ belongs to a given ideal $I \leq \mathbb{Q}[\bar{x}]$ is NP-hard (and is co-NP-hard, as well).

2 Computational part

1. Study the first two sections of the sage tutorial.