



Logistics Scheduling with Due Date Assignment and Batch Delivery

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Abstract: This paper aims to study the logistics scheduling with due date assignment and batch delivery. A set of orders needs to be processed by the manufacturer and delivered to the customers upon completion. Each customer has a specific acceptable lead time. The manufacturer assigns a due date for one order by negotiating with the customers. The problem is to assign appropriate due date for the orders, and find a joint schedule of order processing at the manufacturer and order delivery from the manufacturer to the customers that minimize the total cost. The objective is to minimize the total tardiness penalty, due date assignment and delivery costs. For this problem, we analyze the complexity of the general case and study the solvability of multiple case by providing efficient algorithms.

Keywords: logistics scheduling, batch delivery, due date, algorithms

1 Introduction

Production and distribution operations are two key operational functions in a supply chain. To achieve optimal operational performance in a supply chain, it is critical to integrate these two functions and plan and schedule them jointly in a coordinated manner. There are various integrated models of production scheduling and product distribution in the literature. The objective of such models is typically to optimize both customer service level and distribution cost. Joint consideration of production and delivery schedules is also beneficial in making higher level decisions in the supply chain. To learn more about research results on this aspect, the reader is referred to see (Hall and Potts, 2003, 2005; Chen and Variraktarakis, 2005; Chen and Pundoor, 2006; Chen and Lee, 2008).

Meeting due dates has always been one of the most important objectives in scheduling and supply chain management. Customers demand that suppliers meet contracted delivery dates or face large penalties. While traditional scheduling models considered due dates as given by exogenous decisions, in an integrated system they are determined by taking into account the system's ability to meet the quoted delivery dates. In order to avoid tardiness penalties, including the possibility of losing customers, companies are under increasing pressure to quote attainable delivery dates. At the same time, promising delivery dates too far into the future may not be

acceptable to the customer or may force a company to offer price discounts in order to retain the business. Thus there is an important trade-off between assigning relatively short due dates to customer orders and avoiding tardiness penalties. This is the reason that increasingly large number of recent studies have viewed due date assignment as part of the scheduling process, and showed how the ability to control due dates can be a major factor in improving system performance.

Another line of research related to the problem under study focuses on scheduling problem with due date assignment. There are extensive research results on this problem in the literature. Early research in the area of due date assignment in scheduling was due to Seidmann et al. (1981) and Panwalkar et al. (1982). Panwalkar et al. (1982) studied the constrained version where the scheduler must decide on a common due date for all jobs (this method is usually referred to as the CON method), while Seidmann et al. (1981) dealt with the unrestricted case where each job can have a different due date (we refer to this method as the DIF method). These two papers started extensive research in the area of due date assignment, with most papers focusing on the common due date assignment problem (e.g. Bagchi et al., 1986; Kahlbacher and Cheng, 1993; Adamopoulos and Pappis, 1995; Cheng and Kovalyov, 1996; Mosheiov, 2001; Biskup and Jahnke, 2001; Birman and Mosheiov, 2004; Chang et al., 2009). A survey on common due date assignment problems was given by Gordon et al. (2002). Shabtay and Steiner (2006, 2007, 2008) study some due date assignment problem in single and parallel machines environment. They study many scheduling problems with due date assignment, analysis the complexity of the problems and give efficient algorithms for these problems.

In this paper we consider a make-to-order production-distribution system consisting of one manufacturer and multiple customers. At the beginning of a planning horizon, each customer places a set of orders with the manufacturer. Each customer has a specific acceptable lead time. The manufacturer assigns a due date for one order by negotiating with the customers. The problem is to assign appropriate due date for the orders, and find a joint schedule of order processing at the manufacturer and order delivery from the manufacturer to the customers that minimize the total cost. The objective is to minimize the total tardiness penalty, due date

assignment and delivery costs.

The remainder of this paper is organized as follows. In section 2 we formulate the integrated scheduling model. In section 3 we show that the general problem is strongly NP-hard, and discuss some important special cases which can be solved in polynomial time. In the last section we give some concluding remarks.

2 Problems formulation

In this section, we define our problems mathematically and introduce some optimality properties satisfied by all the problems that we will use in later sections.

There is a manufacturer obtains orders from m customers that are located at different sites at time 0. Let $J_i = \{J_{i1}, J_{i2}, \dots, J_{in_i}\}$ be the order set of customer i , for $i = 1, 2, \dots, m$, where $n_1 + n_2 + \dots + n_m = n$. There is a single machine available for processing all the orders. The processing time of order J_{ij} is p_{ij} and preemption is not permitted. Each customer has a specific acceptable lead time A_i . The manufacturer assigns a due date d_{ij} for order J_{ij} by negotiating with the customers. If $d_{ij} > A_i$, then the manufacturer should give α_i penalty per time for the customer i .

The problem is to assign appropriate due date for the orders, and find a joint schedule of order processing at the manufacturer and order delivery from the manufacturer to the customers that minimize the total cost. The orders are to be delivered in batches to the downstream customer in the supply chain. We assume that there is no capacity limitation on a batch delivery and that the cost per delivery is fixed; that is, the cost is independent of the orders delivered at this time. Hence, it may be cheaper to delay shipping of a order until the delivery time of the following order, since the delay saves a delivery charge.

Once completed on the machine, each order needs to be delivered to its customer. We assume that there are an unlimited number of transport vehicles available (e.g., they are provided by a third-party logistics service firm) to deliver the processed orders from the factory to customers at any time. Each vehicle can load at most $B \geq n$ jobs on each trip from the factory to a customer, so the capacity of the vehicle is unlimited. Let t_i and c_i denote the time and cost that a vehicle travels from the factory to customer i , for $i = 1, 2, \dots, m$. Note that the batch transportation time and cost are independent of the batch size. For a given schedule of the problem, we define:

C_{ij} = the completion time of order $J_{ij} \in N_i$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n_i$.

D_{ij} = the delivery time of order $J_{ij} \in N_i$, which means the time when order J_{ij} is delivered to the customer i , $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n_i$.

$L_{ij} = D_{ij} - d_{ij}$. The lateness of job $J_{ij} \in N_i$.

$U_{ij} = 1$ if $L_{ij} > 0$; else $U_{ij} = 0$.

$T_{ij} = \max\{D_{ij} - d_{ij}, 0\}$, which is the tardiness of order $J_{ij} \in N_i$.

The objective function can be represented as follows:

$$Z(\pi, d^*(\pi)) = \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha_i \max\{0, d_{ij} - A_i\} + \sum_{i=1}^m \sum_{j=1}^{n_i} \beta_i T_{ij} + \sum_{i=1}^m b_i c_i$$

Where α_i and β_i are nonnegative parameters representing the per unit lead-time and tardiness penalty for order i . The objective is to minimize the total tardiness penalty, due date assignment and delivery costs. We refer to this problem as total weighted tardiness with due date assignment problem.

In order to specify each problem, we use the three-field notation $\alpha|\beta|\gamma$ widely used in scheduling research to denote the problems under study. We study the following problem:

$$1|BD, DIF \left| \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha_i \max\{0, d_{ij} - A_i\} + \sum_{i=1}^m \sum_{j=1}^{n_i} \beta_i T_{ij} + \sum_{i=1}^m b_i c_i \right|$$

In the α field, 1 means that the jobs are processed on a single machine. In the β field, DIF means that the jobs may need to be assign different due dates. BD means that processed orders need to be delivered by batch. The γ field is the objective function to be minimized. The objective function is to minimize the total tardiness penalty, due date assignment and delivery costs.

When the due dates are given, the problem is NP hard. This, however, does not settle the complexity of our general problem with individually assignable due dates, as the common due dates constraint theoretically may make the problem harder. In the next subsection, we will prove that this is not the case, by showing that the problem is strongly NP-hard. We will also discuss some important special cases which are solvable in polynomial time.

3 Algorithm for the problem

When each customer has more than one orders, we need to assign the due dates for each order. It is easy to see that the orders delivery in the same batch will assign the same due date. If one order belong to the customer i belong to the delivery batch B_i , then define the completion time of this batch is C_i , and the due date assignment of this batch is as follows:

$$d_i^*(\pi) = \begin{cases} A_i, & \text{if } C_i + t_i > A_i, \alpha_i \geq \beta_i \\ C_i + t_i, & C_i + t_i \leq A_i, \text{ or } C_i + t_i > A_i, \alpha_i < \beta_i \end{cases}$$

First we analyze some properties of the optimal due date assignment strategy for a given schedule π . It is easy to see that for a given π , the due date assignment problem has a separable objective function and we can determine the optimal due date assignment for job by determining the that minimizes the following objective:

$$Z_i(d_i) = \alpha_i \max\{0, d_i - A_i\} + \beta_i T_i$$

Lemma 1. If $C_i + t_i \leq A_i$ for a given π , then set d_i to

any value from $[C_i + t_i, A_i]$.

Proof. It is easy to see that if $C_i + t_i \leq A_i$ for a given π , and we set $d_i = d \in [C_i + t_i, A_i]$, there will be no penalty for order i , this completes the proof.

Lemma 2. If $C_i + t_i > A_i$ for a given π , then the optimal due date for order i is greater than A_i , i.e., $d_i \geq A_i$.

Proof. Assume the lemma is false, and let π^0 be an optimal schedule in which there exists a order i with $C_i + t_i > A_i$, and $d_i < A_i$. Let $A_i - d_i = \Delta > 0$. We can define an alternative schedule π which is exactly the same as schedule π^0 except we set $d_i = A_i$. The difference between the objective values of the two schedules can be written as follows:

$$Z(\pi^0) - Z(\pi) = Z_i(d_i = A_i - \Delta) - Z_i(d_i = A_i) = \beta_i \cdot \Delta > 0$$

which contradicts the optimality of π^0 and completes the proof.

Lemma 3. If $C_i + t_i > A_i$ for a given π , when $\alpha_i \geq \beta_i$, set $d_i = A_i$, else then set $d_i = C_i + t_i$.

Proof. If $C_i + t_i > A_i$ for a given π , then we have $A_i \leq d_i \leq C_i + t_i$ for any schedule π , $Z_i(d_i = A_i + \Delta) - Z_i(d_i = A_i) = \Delta \times (\alpha_i - \beta_i)$, $0 \leq \Delta \leq C_i + t_i - A_i$.

So if $\alpha_i \geq \beta_i$, set $d_i = A_i$, else then set $d_i = C_i + t_i$, this completes the proof.

Then we discuss the complexity of the problem we study.

Theorem 1. Even when each customer has only one order, the problem is still NP-hard in the strong sense even for the case where $\alpha_i = \beta_i$ for $i = 1, 2, \dots, m$. Furthermore, the problem is NP-hard in the ordinary sense if $\alpha_i = \alpha$ and $\beta_i = \beta$ for $i = 1, 2, \dots, m$.

Proof. Observe that our reduced problem has the same format for its objective as the well-known single-machine scheduling problem with the weighted tardiness objective with implied due dates. When $t_i = 0$, $\alpha_i = \beta_i$, $i = 1, 2, \dots, m$, the problem can be reduced to find a schedule π to minimize the following objective function:

$$Z(\pi, d^*(\pi)) = \sum_{i=1}^m \alpha_i \max(0, C_i - A_i) + \sum_{i=1}^m c_i$$

Because the second term $\sum_{i=1}^m c_i$ is a constant, $\sum_{i=1}^m \alpha_i \max(0, C_i - A_i)$ is equivalent to

$$1 \left\| \sum_{i=1}^m \omega_i T_i \right\|, \text{ and the latter problem is strongly NP-Hard.}$$

When $\alpha_i = \alpha$ and $\beta_i = \beta$ for $i = 1, 2, \dots, m$, the

problem can be reduced to $1 \left\| \sum_{i=1}^m T_i \right\|$, and this problem is ordinary NP-Hard.

Because the general problem is strongly NP-Hard, we study some special cases.

3.1 Each customer's order sequence is given

When each customer's order sequence is given, then we know the processing sequence of every customer.

$A_i = 0$ means the specific acceptable lead time of each customer is zero. This assumption is used in due date assignment problems with no acceptable lead time. This is reasonable when the customer wants a delivery of an order as soon as possible and may even agree to pay for speedier delivery.

When $A_i = 0$, the objective function can be reduced as follows:

$$Z(\pi, d^*(\pi)) = \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha_i d_{ij} + \sum_{i=1}^m \sum_{j=1}^{n_i} \beta_i T_{ij} + \sum_{i=1}^m b_i c_i$$

the due date assignment of this batch is as follows:

$$d_i^*(\pi) = \begin{cases} 0, & \text{if } \alpha_i \geq \beta_i \\ C_i + t_i, & \text{if } \alpha_i < \beta_i \end{cases}, i = 1, 2, \dots, m$$

When each customer's job processing sequence is given, we develop the following dynamic programming algorithm.

Algorithm 1

Define the value function $V(l_1, \dots, l_m; b, k)$ as the minimum value of the objection function for scheduling that satisfy the following conditions:

(1) the total number of scheduled orders is $l_1 + \dots + l_m$, of which l_u orders are from the top of the given processing sequence for customer u , $u = 1, 2, \dots, m$;

(2) the number of orders in the last delivery batch is b , and the orders belong to customer k ($1 \leq k \leq m$).

Boundary condition: $V(0, \dots, 0; 0, 0) = 0$

Recurrence relation: $V(l_1, \dots, l_m; b, k) =$

$$\min \begin{cases} V(l_1, \dots, l_k - 1, \dots, l_m; b - 1, k) + \omega_k \cdot \left(\sum_{i=1}^m \sum_{j=1}^{l_i} p_{ij} + t_k + b \cdot p_{kl_k} \right), & \text{if } b > 1 \\ V(l_1, \dots, l_k - 1, \dots, l_m; b', h) + \omega_k \cdot \left(\sum_{i=1}^m \sum_{j=1}^{l_i} p_{ij} + t_k \right), & \text{if } b = 1 \end{cases}$$

Optimal solution value: $\min V(n_1, \dots, n_m; b, k)$ for all $1 \leq b \leq n$, $1 \leq k \leq m$.

Theorem 2. Algorithm 1 finds an optimal solution for this case in $O(mn^{m+1})$ time.

Proof. Algorithm 1 proceeds according to the forward state transition mode. The dynamic program exploits properties of an optimal schedule by the given sequence, so the algorithm eventually finds an optimal solution. The ranges of the values l_i , b and k are $1 \leq l_i \leq n_i$, $i = 1, 2, \dots, m$; $1 \leq b \leq n$; $1 \leq k \leq m$, respectively, therefore the algorithm requires an overall

computational time of $O(mn^{m+1})$.

From the algorithm 1, we can know that when m is a fixed number, this problem can be solved in polynomial time, so this special case is polynomial solvable.

3.2 Customer's delivery batch is determined

Each customer's delivery batch is determined means for customer i 's orders J_{i1}, \dots, J_{in_i} , it is divided into B_i batch, $i = 1, 2, \dots, m$, and each batch contains the certain jobs. Let l_{i1}, \dots, l_{iB_i} be the orders number for customer i , P_{i1}, \dots, P_{iB_i} be the processing time for each batch, $i = 1, 2, \dots, m$.

When $A_i = 0$, the objective function can be reduced as follows:

$$Z(\pi, d^*(\pi)) = \sum_{i=1}^m \sum_{j=1}^{n_i} \alpha_i d_{ij} + \sum_{i=1}^m \sum_{j=1}^{n_i} \beta_i T_{ij} + \sum_{i=1}^m b_i c_i$$

the due date assignment of this batch is as follows:

$$d_i^*(\pi) = \begin{cases} 0, & \text{if } \alpha_i \geq \beta_i \\ C_i + t_i, & \text{if } \alpha_i < \beta_i \end{cases}, i = 1, 2, \dots, m$$

Because the job delivery batch is determined, so the total delivery cost is $\sum_{i=1}^m B_i c_i$. We can consider every delivery batch for all the customers as a large job, and let $\omega_i = \min(\alpha_i, \beta_i)$, then the objective function can be reduced as follows:

$$Z(\pi, d^*(\pi)) = \sum_{i=1}^m \sum_{j=1}^{B_i} l_{ij} \omega_i (C_{ij} + t_i) + \sum_{i=1}^m B_i c_i = \sum_{i=1}^m \sum_{j=1}^{B_i} l_{ij} \omega_i C_{ij} + \sum_{i=1}^m B_i \omega_i t_i + \sum_{i=1}^m B_i c_i$$

The last two expressions are constant numbers, so the problem reduces to minimum $\sum_{i=1}^m \sum_{j=1}^{B_i} l_{ij} \cdot \omega_i \cdot C_{ij}$. We consider $l_{ij} \cdot \omega_i$ as the weight of job J_{ij} for i -th customer, and the problem can be turned into minimum the total weighted completion in a single machine. We can schedule all the delivery batch in the non decreasing order of $P_{ij} / l_{ij} \cdot \omega_i$, then we can get the optimal schedule of the problem, and the computational time is $O(n \log n)$. So this problem can be solved in polynomial time.

The algorithm for solving this problem is as follows:
Algorithm 2

Step 1: Renumber all the delivery batch in the non decreasing order of $P_{ij} / l_{ij} \cdot \omega_i$ sequence.

Step 2: Formulate each customer's delivery batch as the given sequence.

Step 3: Determine the optimal batch delivery time for each batch.

Step 4: Assign the due dates for each batch according to Lemma 3.

The computational time of the algorithm is $O(n \log n)$, so this problem can be solved in polynomial time, this special case is polynomial solvable.

$A_i = 0$, and $\alpha_i = \alpha, \beta_i = \beta$

When $A_i = 0$, and $\alpha_i = \alpha, \beta_i = \beta$, the objection function can be reduced as follows:

$$Z(\pi, d^*(\pi)) = \alpha \sum_{i=1}^m \sum_{j=1}^{n_i} d_{ij} + \beta \sum_{i=1}^m \sum_{j=1}^{n_i} T_{ij} + \sum_{i=1}^m b_i c_i$$

the due date assignment of this batch is as follows:

$$d_i^*(\pi) = \begin{cases} 0, & \text{if } \alpha_i \geq \beta_i \\ C_i + t_i, & \text{if } \alpha_i < \beta_i \end{cases}, i = 1, 2, \dots, m$$

Then we develop the following dynamic programming algorithm.

Algorithm 3

Define the value function $F(l_1, \dots, l_m; i, k)$ as the minimum value of the objection function for scheduling that satisfies the following conditions:

(1) The total number of scheduled orders is $l_1 + \dots + l_m$, of which l_u orders are from the top of the SPT sequence for customer u , $u = 1, 2, \dots, m$;

(2) The number of order in the last delivery batch is k , and the orders belong to customer i ($1 \leq i \leq m$).

Boundary condition: $F(0, \dots, 0; 0, 0) = 0$

Recurrence relation: $F(l_1, \dots, l_m; i, k) =$

$$\min \begin{cases} F(l_1, \dots, l_i - 1, \dots, l_m; i, k - 1) + \alpha \left[\sum_{u=1}^m \sum_{j=1}^{l_u} p_{uj} + (k - 1) \cdot p_{il_i} + t_i \right], & \text{if } k > 1; \\ \min_{1 \leq g \leq m} \{ F(l_1, \dots, l_i - 1, \dots, l_m; g, h) + \alpha \left[\sum_{u=1}^m \sum_{j=1}^{l_u} p_{uj} + t_i \right] + c_i \}, & \text{if } k = 1; \end{cases}$$

Optimal solution value:

$$\min \{ F(l_1^*, \dots, l_m^*; i, k) \mid 0 \leq l_i^* \leq n_i, 1 \leq i \leq m, 0 \leq k \leq l_i^* \}$$

Theorem 3. Algorithm 3 finds an optimal solution for this case in $O(n^{2m+1} m^2)$ time.

Proof. Algorithm 3 proceeds according to the forward state transition mode. The dynamic program exploits properties of an optimal schedule by the given sequence, so the algorithm eventually finds an optimal solution. The ranges of the values l_i , i and k are $1 \leq l_i \leq n_i$, $i = 1, 2, \dots, m$; $1 \leq k \leq n$; $1 \leq i \leq m$, respectively, therefore the algorithm requires an overall computational time of $O(n^{2m+1} m^2)$.

From the algorithm 3, we can know that when m is a fixed number, this problem can be solved in polynomial time, so this special case is polynomial solvable.

4 Conclusions

In this paper we study the logistics scheduling with due date assignment and batch delivery. The problem is to assign appropriate due date for the orders, and find a joint schedule of order processing at the manufacturer and

order delivery from the manufacturer to the customers that minimize the total cost. The objective is to minimize the total tardiness penalty, due date assignment and delivery costs. We show that the general problem is strongly NP-hard, and discuss some important special cases which can be solved in polynomial time.

There are some research topics remain open for future investigation. First, the problems with other customer-related objectives such as total earliness of the jobs may be studied. The second research topic is to consider the case of processing jobs on parallel machines or flowshop and delivering them to multiple customers.

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