Efficient Output-Side Singular Subspace Rotation for Downstream Task Adaptation

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BACKGROUND & OBJECTIVE

Parameter–Efficient Fine–Tuning (PEFT) is becoming more important as a way to train large–scale models. Recently, orthogonal rotation–based and SVD–based approaches have been actively studied.

In orthogonal rotation–based approaches, there is a limitation in that the computational cost for constructing the rotation matrix is high.

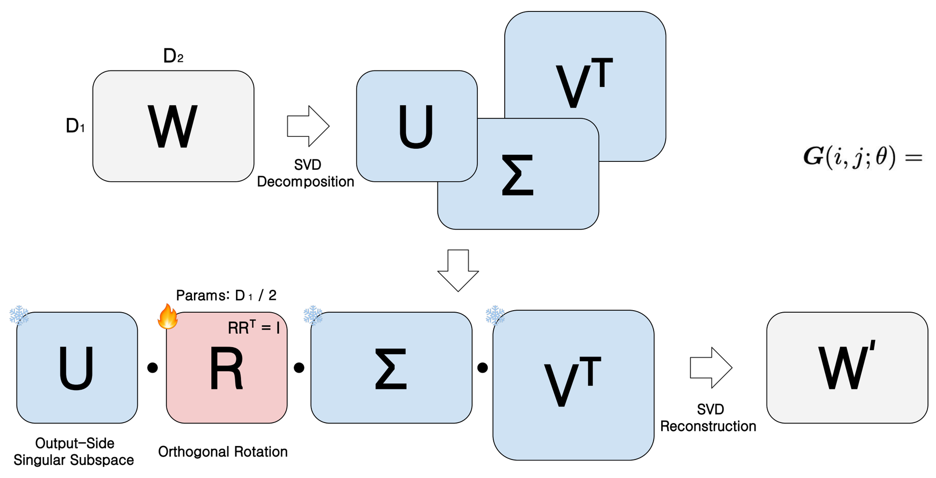
Objective

To adapt output–side representations of downstream task by **rotating the principal singular axes using SVD.**

To construct **the effective rotation matrix, while avoiding heavy computation and large parameter overhead.**

METHODOLOGY

Output-Side Singular Subspace Rotation



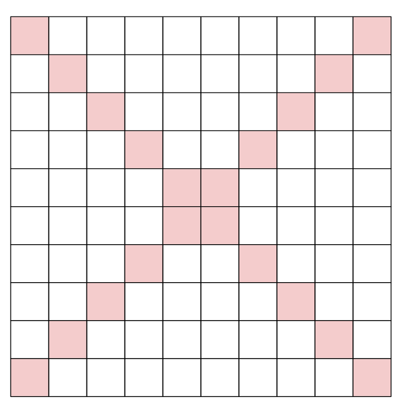
$$G(i,j;\theta) = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & \cos\theta & \cdots & -\sin\theta & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & \sin\theta & \cdots & \cos\theta & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

$$W^{'} = (UR)\Sigma V^T$$

We apply **SVD** to the pretrained weights and fine–tune the model by **orthogonally rotating the output–side singular subspace.**

To reduce the number of trainable parameters required for orthogonal rotation, we employ **Givens Rotation.** In particular, by forming **non–overlapping paired axes** , we can efficiently construct the rotation matrix in a single step without complex computations..

Variance-Aware Mirror-Paired Rotation Matrix



Mirror–Paired Givens Rotation

$$R^{(k)}_{ij}(\theta_k) = \begin{cases} \cos\theta_k, & \text{if } i = j = k \text{ or } i = j = d-k+1, \\ -\sin\theta_k, & \text{if } i = k \text{ and } j = d-k+1, \\ \sin\theta_k, & \text{if } i = d-k+1 \text{ and } j = k, \\ 1, & \text{if } i = j \text{ and } i \notin \{k, d-k+1\}, \\ 0, & \text{otherwise.} \end{cases}$$

$$m = \left\lfloor \frac{D}{2} \right\rfloor, \qquad \boldsymbol{\theta} = (\theta_1, \ldots, \theta_m).$$

The singular values obtained from SVD represent the variance along each principal axis, ordered from high to low importance. Axes with larger variance play a more critical role in shaping the pretrained weight representations.

By pairing each high–variance axis with a low–variance axis in a mirror– symmetric manner , we enable stable and effective training of important directions without being constrained by less informative axes.

EXPERIMENTAL RESULTS

Vision & Language Benchmark

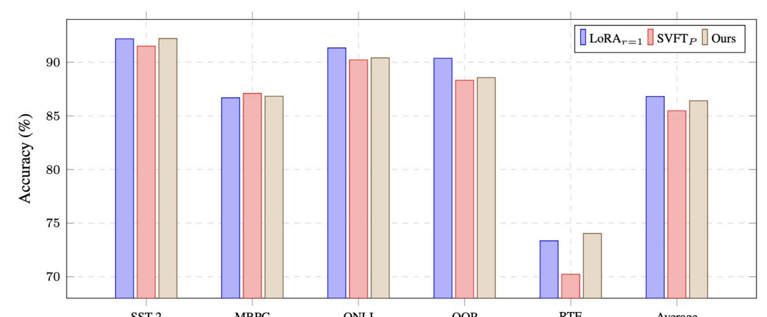


Figure 1: Comparison of GLUE benchmark performance across fine-tuning methods.

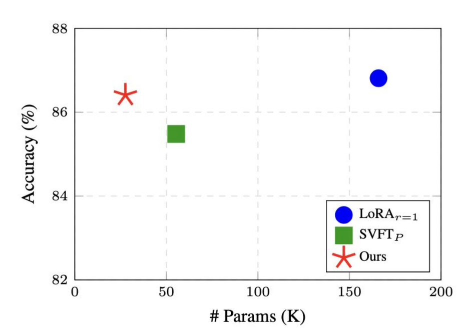


Figure 2: Average GLUE benchmark performance and the number of trainable parameters across fine-tuning methods.

GLUE Benchmark (SST–2, MRPC, QNLI, QQP, RTE)

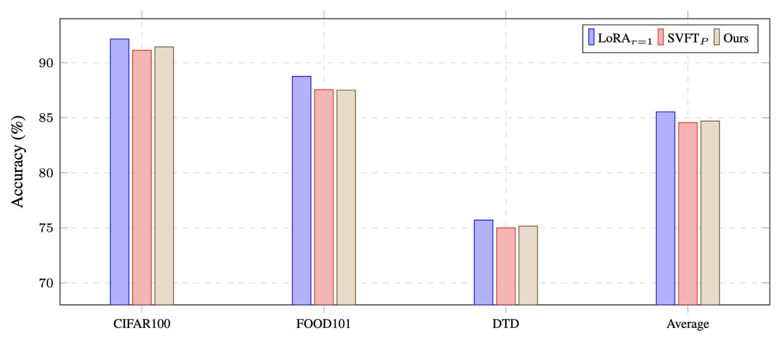


Figure 3: Comparison of image classification performance across fine-tuning methods.

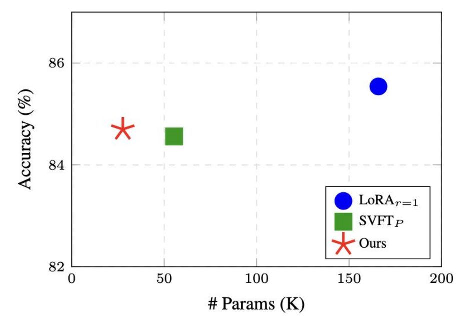


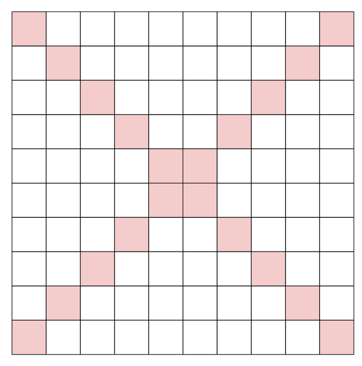
Figure 4: Average image classification performance and the number of trainable parameters across fine-tuning methods.

Various Image Classification Tasks (CIFAR100, FOOD101, DTD)

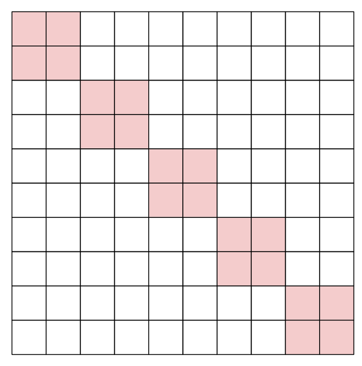
In GLUE benchmark and image classification tasks, **our method showed high performance on average with fewer parameters than SVFT** which is SVD–based approach.

In some tasks, our method showed comparable or better performance than LoRA with about 5 times more parameters.

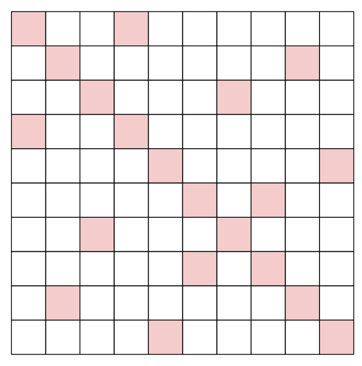
Comparison to Other Rotation Matrices



Mirror–Paired



Neighbor–Paired



Random–Paired

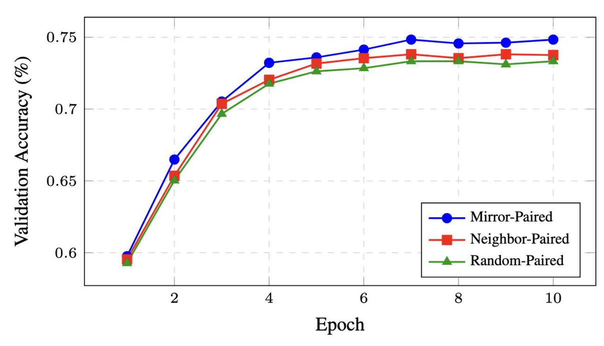


Figure 5: DTD validation accuracy per epoch across rotation matrices.

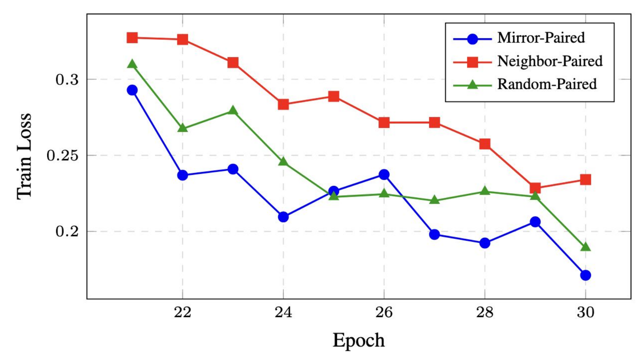


Figure 6: RTE training loss per epoch across rotation matrices.

We construct and train three types of paired rotation matrices: **Mirror–Pair, Neighbor–Pair, and Random–Pair** , and then extensively evaluate their performance.

Among them, **the proposed Mirror–Pair strategy consistently achieves superior performance across both vision and natural language tasks.**

CONCLUSION & FUTURE WORKS

Our method constructs an efficient orthogonal rotation matrix and adapts the output–side singular subspace, achieving competitive or superior performance with significantly fewer trainable parameters compared to existing PEFT approaches.

While the proposed Mirror–Pair rotation demonstrates robust performance across both vision and language tasks, it may not always be optimal for all downstream settings. So, it needs to explore methods for selecting task–adaptive pairing strategies.