Goldbach's proof of infinitude of primes

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Lemma 1. The Fermat numbers $F_n = 2^{2^n} + 1$ are pairwise relatively prime.

Proof. It's easy to show by induction that

$$F_m - 2 = F_0 F_1 \cdots F_{m-1}$$

This means that if d divides both $F_n \& F_m$ (with $n < m$), then d also divides $F_m - 2$; so d divides 2. But every Fermat number is odd. So d is 1. □

Theorem 1. There are infinitely many primes

Proof. Choose a prime divisor p_n of each Fermat number F_n. By () we know these primes are all distinct, showing that there are infinitely many primes.

□