ADDIS ABABA UNIVERSITY

Conjectures

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1 CONJECTURE

Conjecture 1.1. [1]

$$\binom{n}{3} = \begin{cases} odd, & \textit{where } n \textit{ is prime and } n-2 \textit{ is its twin} \\ even, & \textit{otherwise} \end{cases}$$

Proof. (Intuition)

$$\binom{n}{3} = \frac{n!}{(n-3)!3!} = \frac{n(n-1)(n-2)}{6}$$

Now we have

$$\binom{n}{3} = \begin{cases} even & \text{if } 12|n(n-1)(n-2) \\ odd, & \text{otherwise} \end{cases}$$

For a trivial reason let $n > 3$.

Case 1: If n is even, then is $n-2$. For an obvious reason, from n, $n-1$ and $n-2$ one of them is a multiple of 3. Hence $12|n(n-1)(n-2)$.

Case 2: If n is odd.

□

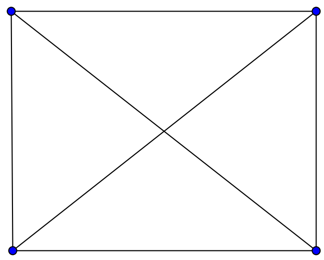
[1] Except for $n-1 = 12k$, where $k \in \mathbb{N}$.

Conjecture 1.2. *The number of possible triangles in a complete graph of order n (K_n) is*

$$\begin{cases} \frac{n^2(n-2)^2(n-1)^2)}{6^2}, & \textit{where } \binom{n}{3} \textit{ is odd.} \\ \frac{(n^2-n)(n^2-4)(n^2-5n+12)}{12^2}, & \textit{where } \binom{n}{3} \textit{is even.} \end{cases}$$

Proof. (Intuition) □

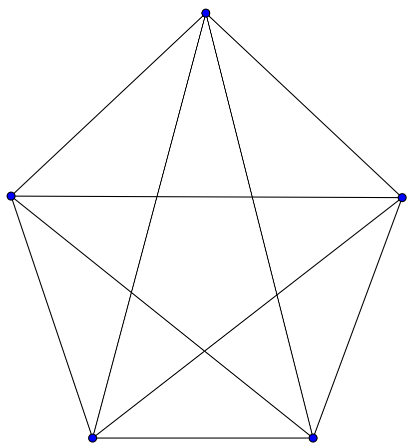
Let's look at some lower cases
There are 8 possible triangles in K_4(Exercise).



Now, let us compute the # of possible triangle in K_4 using (1.2). So $n = 4$ and $\binom{4}{3} = 4$ which is even. Thus, our formula is $\frac{(n^2-n)(n^2-4)(n^2-5n+12)}{12^2}$. Plugging 4 in place of n gives

$$\frac{(4^2-4)(4^2-4)(4^2-5(4)+12)}{12^2} = 8$$

There are 35 possible triangles in K_5(Exercise).



To compute the # of possible triangle in K_5 set $n = 5$. Then $\binom{5}{3} = 10$ which is even

$$\begin{aligned} \frac{(n^2-n)(n^2-4)(n^2-5n+12)}{12^2} &= \frac{(5^2-5)(5^2-4)(5^2-5(5)+12)}{12^2} \\ &= 35 \end{aligned}$$

Using (1.2) we predict the # of possible triangle in K_6 to be 120.

REFERENCES

[1] [Boris A. Kordemsky] The Moscaow Puzzles, 359 Mathematical recreations.