PIGEONHOLE PRINCIPLE

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1 Introduction

Definition 1.1. [1] If $n+1$ objects(pigeons) are placed into n boxes (pigeonholes), then at least one box contains more than one object.

Definition 1.2 (Generalized pigeonhole principle). If m objects are placed into k boxes, then some box contains at least $\lceil \frac{m}{k} \rceil$ objects.

2 Application

Example 2.1. Given m integers $a_1, a_2, \ldots, a_m$ there exists k and l with $l \leq k < l \leq m$, such that

$$a_{k+1} + a_{k+2} + \cdots + a_l$$

is divisible by m.

Solution. Consider the following sums

$$a_1, a_1 + a_2, a_1 + a_2 + a_3, \ldots, a_1 + a_2 + \cdots + a_m$$

If one of the above sum is divisible by m, then we are done. But if we got a non-zero remainder when we divide the above sums by m, then we will get

$$1, 2, \ldots, m-1 \text{ as remainder.}$$

Since there are m sums, and $m-1$ possible values for remainder. Therefore there are integer k and l with $k < l$ such that $a_1 + a_2 + \cdots + a_k$ and $a_1 + a_2 + \cdots + a_l$ have the same remainder r when divided by m. Thus,

$$a_1 + a_2 + \cdots + a_k = am + r \tag{1}$$

$$a_1 + a_2 + \cdots + a_l = bm + r \tag{2}$$

Now subtract (2) from (1)

$$\Rightarrow a_{k+1} + a_{k+2} + \cdots + a_l = (b-a)m$$

Hence

$$m | (a_{k+1} + a_{k+2} + \cdots + a_l)$$

□

[1] Peter Gustav Lejeune Dirichlet, $(1805 - 1859)$.