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Cayley - Hamilton Theorem

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1 CAYLEY-HAMILTON

To prove Cayley-Hamilton Theorem we need the following lemma

Lemma 1.1. *Let f and g polynomials in x with matrix coefficients. Let A be a square matrix of order n.*
If $g(x) = f(x)(xI - A)$, then $g(A) = 0$.

Proof. □

Theorem 1.2. *Every square matrix of order n is a root of its characteristic polynomial. i.e. If A is a square matrix of order n, then $\chi_A(A) = 0$.*

Proof. Let $B = Adj(xI - A) = (p_{ij}(x))_{n\times n}$, where p_{ij}'s are polynomials.
B can be written as $B = B_0 + B_1x + \cdots + B_kx^k$.
$B_0, B_1, ..., B_k$ are matrices whose entries are scalars.
Let

$$\chi_A(x) = |xI - A| = a_0 + a_1x + \cdots + a_nx^n \tag{1}$$

But

$$\begin{aligned} B(xI - A) &= |xI - A|I \\ &= \chi_A(x)I \end{aligned}$$

□

REFERENCES

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