Epsilon-Delta Proof

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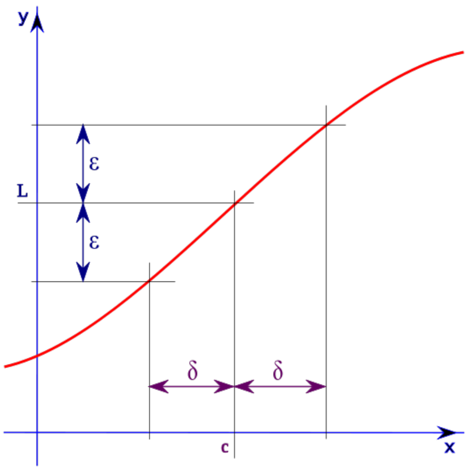
Single Variable



Precise Definition of Limit

Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. We say that the limit of $f(x)$ as x approaches x_0 is the number L, written as

$$\lim_{x \to x_0} f(x) = L$$

For every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$. Such that for all x.

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$$

Historical Note

Figure 1: Cauchy

The foundations of the rigorous study of *analysis* were laid in the nineteenth century, notably by the mathematicians Cauchy and Weierstrass.

Augustine Louis Cauchy

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Carl Weierstrass

Figure 2: Weierstrass

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Let D be a subset of $\mathbf{R}$ and let $f\colon D \to \mathbf{R}$ be a real-valued function on D. The function f is said to be *continuous* on D if, for all $\epsilon > 0$ and for all $x \in D$, there exists some $\delta > 0$ (which may depend on x) such that if $y \in D$ satisfies

One may readily verify that if f and g are continuous functions on D then the functions $f + g$, $f - g$ and $f.g$ are continuous. If in addition g is everywhere non-zero then f/g is continuous.

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Multi-variable

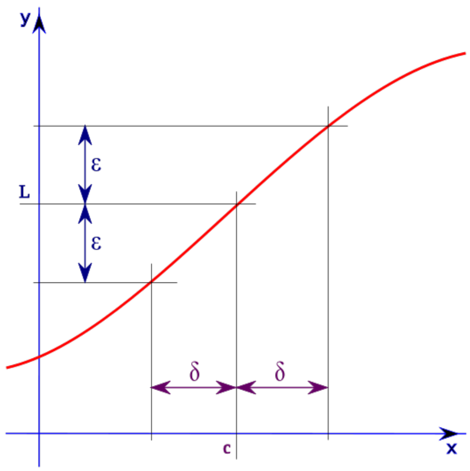
Precise Definition

Let $f(x)$ be defined on an open interval about x_0, except possibly at x_0 itself. We say that the limit of $f(x)$ as x approaches x_0 is the number L, written as

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

If ,for every number $\epsilon > 0$, there exists a corresponding number $\delta > 0$. Such that for all x.

$$0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$$



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$$|y - x| < \delta$$

then

$$|f(y) - f(x)| < \epsilon.$$

One may readily verify that if f and g are continuous functions on D then the functions $f + g$, $f - g$ and $f.g$ are continuous. If in addition g is everywhere non-zero then f/g is continuous.

References

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[2] [Kolman §Hill] Introduction to Linear Algebra with Applications.2000

[3] [Demissu Gemeda] An Introduction to Linear Algebra, Addis Ababa University Press.2000