Euler Formula

Miliyon T.

March 24, 2015

1 Euler factorial formula

Theorem 1.1 (Euler formula)**.** *Let a and n be nonnegative integers with $a \geq n$. Then*

$$n! = \sum_{k=0}^{n} (-1)^k \binom{n}{k} (a-k)^n$$

Which is equivalent to

$$n! = a^n - \binom{n}{1}(a-1)^n + \binom{n}{2}(a-2)^n - \binom{n}{3}(a-3)^n + \cdots + (-1)^n \binom{n}{n}(a-n)^n$$

Proof. It is trivial For $n = 1$

$$1! = a^1 - \binom{1}{1}(a-1)^1$$
$$1 = a - (a-1) = 1$$

Then, we assume it is true for n

$$n! = \sum_{k=0}^{n} (-1)^k \binom{n}{k} (a-k)^n$$

Now let's proof that it is true for $n + 1$

$$\begin{aligned} \sum_{k=0}^{n+1} (-1)^k \binom{n+1}{k} (a-k)^{n+1} &= \sum_{k=0}^{n} (-1)^k \binom{n}{k} (a-k)^n \\ &= \sum_{k=0}^{n} (-1)^k \binom{n}{k} (a-k)^n \end{aligned}$$

$\square$

References

[1] [Demissu Gemeda] Topics in Linear Algebra, Addis Ababa University. 2005

[2] [Sheldon Axler] Linear Algebra Done Right, Springer Publishing. 1997

[3] [Caroline Laroche Turnage] Selected Proofs of Fermat's Little Theorem and Wilson's Theorem.