İkiye Bölme Yöntemi $\frac{a+b}{2} = m_x$

$f(x) = e^x - 3x \quad [0,1] \quad |b-a| = 0,03 = 0$

	a	b	c
0.	0	1	0,5
1.	0,5	1	0,75
2.	0,5	0,75	0,625
3.	0,5	0,625	0,5625
4.	0,5625	0,625	0,59375
5.	0,59375	0,625	

$f(0) = e^0 - 3(0) = 1 - 0 = 1$

$f(1) = e^1 - 3(1) = 2,718 - 3 = -0,282$

$f(0,5) = e^{0,5} - 3(0,5) = 1,649 - 1,5 = 0,149$

$f(0,75) = e^{0,75} - 3(0,75) = 2,117 - 2,25 = -0,133$

$f(0,625) = e^{0,625} - 3(0,625) = 1,868 - 1,875 = -0,007$

$f(0,5625) = e^{0,5625} - 3(0,5625) = 1,755 - 1,6875 = 0,0675$

$f(0,59375) = e^{0,59375} - 3(0,59375) = 1,811 - 1,781 = 0,03$

$0,609 \approx 0,61 \approx x$

Yer Değiştirme Yöntemi $x_r = b - \frac{f(b)(b-a)}{f(b)-f(a)}$

$f(x) = x^3 - 4x - 9 \quad [2,3]$

	a	b
0.	2	3
1.	2,6	3
2.	2,693	3
3.	2,705	3
4.		
5.		

$f(2) = 8 - 8 - 9 = -9$

$f(3) = 27 - 12 - 9 = 6$

$x_r = 3 - \frac{6(3-2)}{6-(-9)}$

$f(2,6) = 2,6^3 - 4(2,6) - 9$

$= 17,576 - 10,4 - 9$

$= -1,824$

$3 - \frac{6}{15} = 3 - 0,4 = 2,6$

$x_r = 3 - \frac{6(3-2,6)}{6-(-1,824)} = 3 - \frac{2,4 \cdot 6(0,4)}{7,824} = 3 - 0,307 = 2,693$

$f(2,693) = 2,693^3 - 4(2,693) - 9$

$= 19,530 - 10,772 - 9$

$= -0,242$

$x_r = 3 - \frac{6(3-2,693)}{6-(-0,242)} = 3 - \frac{1,842 \cdot 0,307}{6,242} = 3 - 0,295 = 2,705$

$f(2,705) = 2,705^3 - 4(2,705) - 9$

$x_r = 3 - \frac{6(3-2,705)}{6-(-0,027)} = 3 - \frac{1,77 \cdot 0,295}{6,027} = 3 - 0,294 = 2,706$

$19,793 - 10,82 - 9 = 0,027$

$f(2,706) = 2,706^3 - 4(2,706) - 9 = 19,814 - 10,824 - 9 = -0,01$

Newton Raphson Yöntemi $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$

$f(x) = x^2 - 2 \quad f'(x) = 2x$

$x_1 = 1 - \frac{-1}{2} = \frac{3}{2} = 1,5$

$f(1) = 1 - 2 = -1 \quad f(1,5) = 0,25 \quad f(1,417) = 0,008$

$f'(1) = 2 \quad f'(1,5) = 3 \quad f(1,417) = 2,834$

$x_2 = 1,5 - \frac{0,25}{3} = 1,5 - 0,083 = 1,417$

$x_3 = 1,417 - \frac{0,008}{2,834} = 1,417 - 0,003 = 1,414$

$f(1,414) = 1,414^2 - 2 = 1,999 - 2 = 0,001$

$f(1,414) = 2 \cdot 1,414 = 2,828$

$x_4 = 1,414 - \frac{0,001}{2,828} = 1,414 - 0,0004 = 1,4136$

	$f(x)$	$f'(x)$
$x_0 = 1$	-1	2
$x_1 = 1,5$	$0,25$	3
$x_2 = 1,417$	$0,008$	$2,834$
$x_3 = 1,414$	$0,001$	$2,828$
$x_4 = 1,4136$		

$x_4 - x_3 = 1,414 - 1,4136$

$= 0,0004$

$x \approx 1,414$

Sekant Yöntemi $x_{k+1} = x_k - f(x_k) \frac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

$f(x) = e^{-x} - x \quad x_0 = 0 \quad x_1 = 1$

$f(0) = e^{-0} - 0 = 1$

$f(1) = e^{-1} - 1 = 0,368 - 1 = -0,632$

$x_2 = 1 - (-0,632) \frac{1 - 0}{-0,632 - 1} = 1 + 0,632 \frac{1}{-1,632}$

$= 1 + \frac{0,632}{-1,632} = 1 - 0,387$

$= 0,613$

$f(0,613) = e^{-0,613} - 0,613 = 0,542 - 0,613$

$= -0,071$

$x_3 = 0,613 - (-0,071) \frac{0,613 - 1}{-0,071 - (-0,632)} = 0,613 + 0,071 \frac{-0,387}{0,561}$

$= 0,613 + 0,071 \cdot -0,690$

$0,613 - 0,049 = 0,564$

$f(0,564) = e^{-0,564} - 0,564 = 0,569 - 0,564$

$0,005$

$x_4 = 0,564 - (0,005) \frac{0,564 - 0,613}{0,005 - (-0,071)}$

$= 0,564 - 0,005 \cdot \frac{-0,049}{0,076}$

$= 0,564 - 0,005 \cdot -0,645$

$= 0,564 + 0,003 = 0,567$

$f(0,567) = e^{-0,567} - 0,567$

$0,567 - 0,567 = 0$

$x \approx 0,567$

	x	$f(x)$
$x_0 =$	0	1
$x_1 =$	1	$-0,632$
$x_2 =$	$0,613$	$-0,071$
$x_3 =$	$0,564$	$0,005$
$x_4 =$	$0,567$	0

Düzeltilmiş Newton Raphson $x_{k+1} = x_k - \frac{f(x_k)f'(x_k)}{[f'(x_k)]^2 - f(x_k)f''(x_k)}$

$f(x) = x^3 - x - 2$ $\quad f(1,5) = 1,5^3 - 1,5 - 2 = 3,375$

$f'(x) = 3x^2 - 1$ $\quad = -0,125$

$f''(x) = 6x$ $\quad f'(1,5) = (1,5)^2 \cdot 3 - 1$

$= 6,75 - 1 = 5,75$

	f(x)	f'(x)	f''(x)
$x_0 = 1,5$	-0,125	5,75	9
$x_1 = 1,521$	-0,002	5,940	9,126
$x_2 = 1,5213$			
x_3			
x_4			

$x_1 = 1,5 - \frac{-0,125 \cdot 5,75}{(5,75)^2 - (0,125 \cdot 9)} = 1,5 - \frac{-0,719}{33,063 + 1,125} = 1,5 - \frac{-0,719}{34,188}$

$= 1,5 + 0,021 = 1,521$

$f(1,521) = 1,521^3 - 1,521 - 2 = 3,519 - 1,521 - 2$

$= -0,002$

$f'(1,521) = 3(1,521)^2 - 1 = 5,940$ $\qquad x \simeq 1,5213$

$6,940$

$f''(1,521) = 6 \cdot 1,521 = 9,126$

$x_2 = 1,521 - \frac{-0,002 \cdot 5,94}{(5,94)^2 - (0,002) \cdot 9,126} = 1,521 - \frac{-0,012}{35,284 + 0,018} = 1,521 + \frac{0,012}{35,302}$

$= 1,521 + 0,0003 = 1,5213$

Düzeltilmiş Sekant yöntemi $x_{k+1} = x_k - \frac{\delta x_k f(x_k)}{f(x_k + \delta x_k) - f(x_k)}$

$f(x) = x^3 - 2x - 5$ $\quad \delta = 0,01$

$f(0) = 8 - 4 - 5 = -1$

$x_1 = 2 - \frac{0,02 \cdot (-1)}{-0,798 - (-1)}$ $\quad f(2,02) = (2,02)^3 - 2(2,02) - 5$

$= 8,242 - 4,04 - 5 = -0,798$

$2 - \frac{-0,02}{0,202} = 2 + 0,099 = 2,099$

	f(x)	δx_i
$x_0 = 2$	-1	0,02
$x_1 = 2,099$		
x_2		

fc

Değiştirilmiş Secant Yöntemi

$$x_{k+1} = x_k - \frac{\delta x_k f(x_k)}{f(x_k + \delta x_k) - f(x_k)}$$

	x	f(x)	δx	$f(x+\delta x)$
x_0	2	-1	0,02	-0,798
x_1	2,099	0,05	0,021	0,288
x_2	2,095	0,005		
x_3				

$f(x) = x^3 - 2x - 5$ $\quad \delta = 0{,}01$

$f(2) = 8 - 4 - 5 = -1$ $\quad \varepsilon = 0{,}005$

$f(2{,}02) = 2{,}02^3 - 2(2{,}02) - 5$

$= 8{,}242 - 4{,}04 - 5 = -0{,}798$

$$x_1 = 2 - \frac{0{,}02 \cdot -1}{-0{,}798 - (-1)}$$

$$= 2 - \frac{-0{,}02}{0{,}202} = 2 + 0{,}99$$

$= 2{,}099$

$f(2{,}099) = 2{,}099^3 - 2(2{,}099) - 5 = 9{,}248 - 4{,}198 - 5 = 0{,}05$

$x_k + \delta x_k$

$2{,}099 + 0{,}021 = 2{,}12$

$f(2{,}12) = 2{,}12^3 - 2(2{,}12) - 5 = 9{,}528 - 4{,}24 - 5 = 0{,}288$

$$x_2 = 2{,}099 - \frac{0{,}021 \cdot 0{,}05}{0{,}288 - 0{,}05} = 2{,}099 - \frac{0{,}00105}{0{,}238} = 2{,}099 - 0{,}004$$

$2{,}095$

$f(2{,}095) = 2{,}095^3 - 2(2{,}095) - 5 = 9{,}195 - 4{,}19 - 5 = 0{,}005$

$2{,}099 - 2{,}095 = 0{,}004$

$f(x) = (x-2)^2$ $f(2) = 0$ $f'(x) = 2(x-2) \Rightarrow 2x-4$ $f'(2) = 0$

İkiye Bölme $[1, 3]$

$f(1) = (1-2)^2 = 1$
$f(3) = (3-2)^2 = 1$ işaretler aynı olmaz -

Yer Değiştirme

$$x_r = b - \frac{f(b)(b-a)}{f(b) - f(a)}$$ $1-1=0$ bölen 0 olmaz

$[1,5, 3]$

$f(1,5) = (1,5-2)^2 = 0,25$
$f(3) = (3-2)^2 = 1$ işaretler aynı olmaz

Newton Raphson

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$ $f(x) = (x-2)^2$ $f'(x) = 2(x-2)$

$f(3) = 1$ $f'(3) = 2$

$x_1 = 3 - \frac{1}{2} = 2,5$ $f(2,5) = 0,25$ $f'(2,5) = 1$

$x_2 = 2,5 - \frac{0,25}{2} = 2,25$ $f(2,25) = \frac{0,25^2}{0,0625}$ $f'(2,25) = 0,5$

$x_3 = 2,25 - \frac{0,0625}{0,5} = 2,25 - 0,125 = 2,125$

$f(2,125) = (2,125-2)^2 = 0,0156$ $f'(2,125) = 2(2,125-2) = 0,25$

$x_4 = 2,125 - \frac{0,0156}{0,25} = 2,125 + 0,0625 = 2,0625$

	$f(x)$	$f'(x)$
$x_0 = 3$	1	2
$x_1 = 2,5$	0,25	0,5
$x_2 = 2,25$	0,0625	0,5
$x_3 = 2,125$	0,0156	0,25
$x_4 = 2,0625$	0,0039	0,125
$x_5 = 2,01$		

⋮

linear

$f(x) = (x-2)^2$ $x_0 = 3$ $x_1 = 1,5$ $x_{k+1} = x_k - f(x_k) \dfrac{x_k - x_{k-1}}{f(x_k) - f(x_{k-1})}$

$\varepsilon = 0,05$

$f(3) = (3-2)^2 = 1$

$f(1,5) = (1,5-2)^2 = 0,25$

	x	f(x)
x_0	3	1
x_1	1,5	0,25
x_2	2	1
x_3	1,667	0,111
x_4	1,75	0,0625
x_5	1,857	0,02

$x_2 = 1,5 - 0,25 \dfrac{1,5-3}{0,25-1}$

$= 1,5 - \dfrac{0,25 \cdot -1,5}{-0,75} = 1,5 - \dfrac{-0,375}{-0,75}$

$1,5 - 0,5$

$= 1$

$f(1) = (1-2)^2 = 1$

$x_3 = 1 - 1 \dfrac{1-1,5}{1-0,25} = 1 + \dfrac{+0,5}{0,75} = 1 + 0,666 = 1,666$ $x \approx 1,857$

$f(1,667) = (1,667-2)^2 = 0,111$

$x_4 = 1,667 - 0,111 \dfrac{1,667-1}{0,111-1} = 1,667 + \dfrac{0,111 \cdot 0,667}{0,889} = 1,667 + \dfrac{0,074}{0,889}$

$= 1,667 + 0,083$

$= 1,75$

$f(1,75) = (1,75-2)^2 = 0,0625$

$x_5 = 1,75 - 0,0625 \dfrac{1,75-1,667}{0,0625-0,111} = 1,75 - \dfrac{0,0625 \cdot 0,083}{-0,0485} = 1,75 + \dfrac{0,0052}{0,0485} = 1,75 + 0,107$

$= 1,857$

$f(1,857) = (1,857-2)^2$

Düzeltilmiş Newton Raphson $x_{k+1} = x_k - \frac{f(x_k)\, f'(x_k)}{[f'(x_k)]^2 - f(x_k)\, f''(x_k)}$

$f(x) = (x-2)^2$ $\quad f(3) = (3-2)^2 = 1$

$f'(x) = 2(x-2)$ $\quad f'(3) = 2(3-2) = 2$

$f''(x) = 2$ $\quad f''(3) =$

	$f(x)$	$f'(x)$	$f''(x)$
$x_0 = 3$	1	2	2

$x_1 = 2$

$$x_1 = 3 - \frac{1 \cdot 2}{4 - 1 \cdot 2} = 3 - \frac{2}{2} = 2,$$

Düzeltilmiş Secant $x_{k+1} = x_k - \frac{\delta x_k f(x_k)}{f(x_k + \delta x_k) - f(x_k)}$

$f(x) = (x-2)^2$ $\quad \delta = 0,01$ $\quad \varepsilon = 0,08$

$f(3) = (3-2)^2 = 1$ $\quad f(3,03) = (3,03-2)^2 = 1,061$

$$x_1 = 3 - \frac{0,03 \cdot 1}{1,061 - 1} = 3 - \frac{0,03}{0,0609} = 3 - 0,493 = 2,507$$

$f(2,507) = (2,507-2)^2 = 0,257$

$f(2,507 + 0,025) = f(2,532) = (2,532-2)^2 = 0,283$

	x	f(x)	δx	$f(x_k, \delta x)$
0	3	1	0,03	1,061
1	2,507	0,257	0,025	0,283
2	2,261	0,068	0,023	0,081
3	2,138	0,019	0,001	0,025
4	2,0715	-		

$$x_2 = 2,507 - \frac{0,025 \cdot 0,257}{0,283 - 0,257} = 2,507 - \frac{0,0064}{0,026} = 2,507 - 0,246 = 2,261$$

$f(2,261) = (0,261)^2 = 0,068$

$f(2,261 + 0,023) = f(2,284) = (0,284)^2 = 0,081$

$$x_3 = 2,261 - \frac{0,023 \cdot 0,068}{0,081 - 0,068} = 2,261 - \frac{0,0016}{0,013} = 2,261 - 0,123 = 2,138$$

$f(2,138) = 0,019$

$f(2,138 + 0,001) = f(2,139) = 0,025$

$$x_4 = 2,138 - \frac{0,001 \cdot 0,019}{0,025 - 0,019} = 2,138 - \frac{0,000399}{0,006} = 2,138 - 0,0665 = 2,0715$$