Exercises

(p.368) In Ex. 1-5, find the projection matrix for the given subspace, and find the projection of the indicated vector on the subspace.

1. $[1,2,1]$ on $sp([1,2,1])$ in $\mathbb{R}^3$
3. $[2,1,3]$ on $sp([2,1,1],[1,1,1])$ in $\mathbb{R}^3$
5. $[1,3,1]$ on the plane $3x+2y+z=0$ in $\mathbb{R}^3$

(p.368) Answer the followings.

11. Find the projection matrix for the X_1, X_2, X_4-coordinate subspace of $\mathbb{R}^4$.
13. Give a geometric argument indicating that every projection matrix is idempotent.
14. Let $\mathbf{a}$ be a unit column vector in $\mathbb{R}^n$. Show that $\mathbf{a}\mathbf{a}^T$ is the projection matrix for the subspace $sp(\mathbf{a})$
15. Mark each of the following True or False.
    a. A subspace W of dimension k in $\mathbb{R}^n$ has associated with it a $K \times k$ projection matrix.
    b. Every subspace W of $\mathbb{R}^n$ has associated with it an $n \times n$ projection matrix.
    c. Projection of $\mathbb{R}^n$ on a subspace W is a linear transformation of $\mathbb{R}^n$ into itself.
    d. Two different subspaces of $\mathbb{R}^n$ may have the same projection matrix.
    e. Two different matrices may be projection matrices for the same subspace of $\mathbb{R}^n$.
    f. Every projection matrix is symmetric.
    g. Every symmetric matrix is a projection matrix.
    h. An $n \times n$ symmetric matrix A is a projection matrix if and only if $A^2 = I$.
    i. Every symmetric idempotent matrix is the projection matrix for its column space.
    j. Every symmetric idempotent matrix is the projection matrix for its row space.
17. What is the projection matrix for the subspace $\mathbb{R}^n$ of $\mathbb{R}^n$?
19. Let P be the projection matrix for a k-dimensional subspace of $\mathbb{R}^n$.
    a. Find all eigenvalues of P.
    b. Find the algebraic multiplicity and the geometric multiplicity of each eigenvalue found in part(a).
    c. Explain how we can deduce that P is diagonalizable, without using the fact that P is a symmetric matrix.
21. Find all invertible projection matrices.

(p.385) Find the least-squares fit to the given data by a linear function $f(x) = r_0 + r_1 x$. Graph the linear function and the data points.

5. $(0,1),(2,6),(3,11),(4,12)$
7. $(0,0),(1,1),(2,3),(3,8)$

(p.386) Answer the followings.

14. Let $(a_1, b_1), (a_2, b_2), \cdots, (a_m, b_m)$ be data points. If $\sum_{i=1}^{m} a_i = 0$ show that the line that best fits the data in the least-squares sense is given by $r_0 + r_1 x$ where $r_0 = (\sum_{i=1}^{m} b_i/m)$ and $r_1 = (\sum_{i=1}^{m} a_i b_i / \sum_{i=1}^{m} a_i^2)$.

(p.358) Verify that the given matrix is orthogonal, and find its inverse.

1. $(1\sqrt{2}) \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$

4. $\frac{1}{2} \begin{pmatrix} 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$

9. Supply a third column vector so that the matrix $\begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{3} & 0 \\ 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$ is orthogonal.

(p.358) Find a matrix C such that $D = C^{-1}AC$ is an orthogonal diagonalization of the given symmetric matrix A.

13. $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

15. $\begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

(p.358) Answer the followings.

19. Mark each of the following True or False.

   a. A square matrix is orthogonal if its column vectors are orthogonal.
   b. Every orthogonal matrix has null space $\{0\}$.
   c. If A^T is orthogonal then A is orthogonal.
   d. If A is an $n \times n$ symmetric orthogonal matrix then $A^2 = I$.
   e. If A is an $n \times n$ symmetric matrix such that $A^2 = I$ then A is orthogonal.
   f. If A and B are orthogonal $n \times n$ matrices then AB is orthogonal.
   g. Every orthogonal linear transformation carries every unit vector into a unit vector.
   h. Every linear transformation that carries each unit vector into a unit vector is orthogonal.
   i. Every map of the plane into itself that is an isometry (that is,preserves distance between points) is given by an orthogonal linear transformation.
   j. Every map of the plane into itself that is an isometry and that leaves the origin fixed is given by an orthogonal linear transformation.

(p.359) Answer the followings.

21. Let A be an orthogonal matrix. Show that A^2 is an orthogonal matrix too.

(p.359) Answer the followings.

27. Show that the real eigenvalues of an orthogonal matrix must be equal to 1 or -1 [Hint: Think in terms of linear transformations.]

(p.451) Find the spectral decomposition of the given symmetric matrix.

5. $\begin{pmatrix} 2 & 3 \\ 3 & 2 \end{pmatrix}$

8. $\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$