1.8 Basic concepts of vector spaces.

T/F

1. The set consisting of the zero vector is a subspace for every vector space. T

2. Every vector space has at least two distinct subspaces. F

3. Every vector space with a nonzero vector has at least two distinct subspaces.

4. If $\{v_1, \cdots, v_n\}$ is subset of a vector space V, then v_i is in span $(v_1, \cdots, v_n)$ for $i = 1 \cdots n$ T

5. If $\{v_1, \cdots, v_n\}$ is a subset of a vector space V, then the sum $v_i + v_j$ is in span $(v_1, \cdots, v_n)$ for all choices of i and j from 1 to n. T sum space

6. If $u+v$ lies in subspace W of a vector space V, then both u and v lies in W F

7. Two subspaces of a vector space V may have empty intersection. F

8. If S is independent, each vector in V can be expressed uniquely as a linear combination of vectors in S F

9. If S is independent and generates V, each vector in V can be expressed uniquely as a linear combination of vectors in S T

10. If each vector in V can be expressed uniquely as a linear combination of vectors in S, then S is an independent set. T

- Let V be a vector space with basis $\{v_1, v_2, v_3\}$ $\longrightarrow$ $\{v_1, v_1+v_2, v_1+v_2+v_3\}$ also basis for V.

- Let V be a vector space with basis $\{v_1, \cdots, v_n\}$ and $W = \text{span}(v_3, v_4, \cdots, v_n)$
  If $w = r_1v_1 + r_2v_2$ is in W $\longrightarrow$ $w = 0$

- Let $\{v_1, v_2, v_3\}$ be basis for a vector space V. $\rightarrow$ vectors $w_1 = v_1 + v_2$, $w_2 = -v_2 + v_3$, $w_3 = v_1 - v_3$ do not generate V.

- Let $\{v_1, \cdots, v_n\}$ be a basis for a vector space V,
  Let $w = t_1v_1 + t_2v_2 + \cdots + t_kv_k$ with $t_k \neq 0$. $\rightarrow$ $\{v_1, v_2, \cdots, v_{k-1}, w, v_{k+1}, \cdots, v_n\}$ is basis for V

2.2 Rank and Nullity of a Matrix and Linear Transformation.

1. The number of independent row vectors in matrix is the same as the number of independent column vectors. T (pivot과 동일)

2. If H is a row-echelon form of a matrix A, then the nonzero column vectors in H form a basis for the column space of A F

3. If H is a row-echelon form of a matrix A, then the nonzero row vectors in H are a basis for the row space of A. T

4. If an $n \times n$ matrix A is invertible, then $\text{rank}(A) = n$. T

5. For every matrix A, we have $\text{rank}(A) > 0$. F 0은 rank 0

6. For all positive integers m and n, the rank of an $m \times n$ matrix might be any number from 0 to the maximum of m and n. F

7. For all positive integers m and n, the rank of an $m \times n$ matrix might be any number from 0 to the minimum of m and n. T

8. For all positive integers m and n, the nullity of an $m \times n$ matrix might be any number from 0 to n F positive 면 n이라면 다 0이어야 ' but positive

9. For all positive integers m and n, the nullity of an $m \times n$ matrix might be any number from 0 to m. F

10. For all postive integers m and n, with $m \geq n$, the nullity of an $m \times n$ matrix might be any number from 0 to n. T

· If A is a square matrix, the nullity of A is the same as the nullity of A^T.

3.2 Determinants of a square matrix

1. The diterminant det(A) is defined for any matrix A. F

2. The determinant det(A) is defined for each square matrix A. T

3. The determinant of a square matrix is a scalar T.

4. If a matrix A is multiplied by a scalar c, the determinant of the
   -를 곱하다.
   resulting matrix is $c \cdot \det(A)$. F

5. If an $n \times n$ matrix A is multiplied by a sclar c, the determinant of the resulting matrix is $c^n \cdot \det(A)$ T

6. For every square matrix A, we have $\det(AA^T) = \det(A^TA) = [\det(A)]^2$. T

7. If two rows and also two columns of a square matrix A are interchanged,
   2x2
   the determinant changes sign 부호 F (row 번)

8. The determinant of an elementary matrix is nonzero. T ($\det(A)=0$ 은 non invertible)

9. If $\det(A) = 2$ and $\det(B) = 3$, then $\det(A+B) = 5$. F ($\det(A+B)$?

10. If $\det(A) = 2$ and $\det(B) = 3$, then $\det(AB) = 6$. T

3.2 Determinants of a square matrix.

1. The determinants of a square matrix is the product of the entries on its main diagonals. T 주대각 성분의 곱!

2. The determinant of an upper-triangular square matrix is the product of the entries on its main diagonals F.

3. The determinant of a lower-triangular square matrix is the product of the entries on its main diagonal. T

4. A square matrix is non singular if and only if its determinant is positive. F $\leftarrow$ o $\rightarrow$ x
[illegible]

5. The column vectors of an $n \times n$ matrix are independent if and only if the determinant of the matrix is nonzero T

6. A homogeneous square linear system has a nontrivial solution if and only if the determinant of its coefficient matrix is zero F
계수행렬

7. the product of a square matrix and its adjoint is the identity matrix. T

8. the ~~dr~~ product of a square matrix and its adjoint is equal to some scalar times the identity matrix T

9. The transpose of the adjoint of A is the matrix of cofactor of A. F

10. The formular $A^{-1} = \frac{1}{\det(A)} \operatorname{adj} A$ is of the practical use in computing the inverse of a large nonsingular matrix. T

3.7 Eigenvalues and Eigenvectors.

1. Every square matrix has real eipen value. F

2. Every $n \times n$ matrix has n distinct eipen values. F

3. Every $n \times n$ matrix has n not necessarily distinct and possibly complex eipenvalues. T

4. There can be only one eipenvalue associated with an eipenvetor of a linear transformation. T

5. There can be only one eipenvector associated with an eipenvalue of a linear transformation. F

6. If v is an eipen vector of a matrix A, then v is an eipen vector of $A + cI$ for all scalar c. T

7. If λ is an eipen value of matrix A, then λ is an eipenvalue of $A + cI$ foll all scalar c. F

8. The v is an eipenvector of an invertible matrix A, then cv is an eipen vector of A^{-1} for all nonzero scalar c. T

9. Every vector in a vector space V is an eipenvector of the identity transformation of V into V. F

10. Every nonzero vector in a vector space V is an eipenvector of the identity transformation of V into V. T

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1. Every $n \times n$ matrix is diaponalizable. F
2. If an $n \times n$ matrix has n distinct real eipenvalues, it is diaponalizable. T
3. Every $n \times n$ real symmetric matrix is real diaponalizable. T
4. An $n \times n$ matrix is diaponalizable if and only if it has n distinct eipen values. F
5. A $n \times n$ matrix is diagonalizable if and only if the alpebraic multiplicity of each of its eipenvalues equals the peometric multiplicity. T
6. Every invertible matrix is diaponalizable. F
7. Every triangular matrix is diaponalizable. F
8. If A and B are similar square matrices and A is diaponalizable, then B is also diaponalizable. T
9. If an $n \times n$ matrix A is diaponalizable, there is a nuique diaponal matrix D that is similar to A F
10. If A and B are similar square matrixes, then $\det(A) = \det(B)$ T

4.1 projections

1. A subspace W of dimension k in $\mathbb{R}^n$ has associated with it a $k \times k$ projection matrix. F

2. Every subspace W of $\mathbb{R}^n$ has associated with it an $n \times n$ projection matrix. T

3. Projection of $\mathbb{R}^n$ on subspace W is a linear transformation ~~of $\mathbb{R}^n$~~ into itself. T

4. Two different subspaces of $\mathbb{R}^n$ may have the same projection matrix. F

5. Two different subspaces of $\mathbb{R}^n$ may be projection matrices for the same subspace of $\mathbb{R}^n$ F

6. Every projection matrix is symmetric T

7. Every symmetric matrix is a projection matrix. F

8. An $n \times n$ symmetric matrix A is a projection matrix if and only if $A^2 = I$. F

9. Every symmetric idempotent matrix is the projection matrix for its column space. T

10. Every symmetric idempotent matrix is the projection matrix for its row space. T

4.2 properties of symmetric matrices.

1. A square matrix is orthogonal if its column vectors are orthogonal. F
2. Every orthogonal matrix has null space $\{0\}$ T
3. If A^T is orthogonal, then A is orthogonal. T
4. If A is an $n \times n$ symmetric orthogonal matrix, then $A^2 = I$. T
5. If A is an $n \times n$ symmetric matrix such that $A^2 = I$, then A is orthogonal. T
6. If A and B are orthogonal $n \times n$ matrix, then AB is orthogonal. T
7. Every orthogonal linear transformation carries every unit vector into a unit vector T
8. Every linear transformation that carries every unit vector is orthogonal. T
9. Every map of the plane into itself that is an isometry. 같이 거리 T F
10. Every map of the plane into itself that is an isometry and that leaves the origin fixed is given by an orthogonal linear transformation. T