<요약>

1.1 Vector in Euclidean spaces

- That two nonzero vectors in $\mathbb{R}^n$ is a scalar multiple by other is parallel.
- A Linear combination of vectors $v_1, v_2, \cdots, v_k$ in $\mathbb{R}^n$ is a vector of the form $r_1v_1 + \cdots + r_kv_k$, (r_i is Scalar)

  The set of all such linear combination is the span of the vectors $v_1, \cdots, v_k$ and is denoted by span $(v_1, \cdots, v_k)$

- $0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$, $e_n = \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix}$

1.2 The Norm and the dot product.

Let $v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ $w = \begin{pmatrix} w_1 \\ \vdots \\ w_n \end{pmatrix}$

- The norm or magnitude of v is $\|v\| = \sqrt{v_1^2 + \cdots + v_n^2}$
- $\|v\| \geq 0$ ($v = 0$ 일때 $\|v\| = 0$), $\|rv\| = |r|\,\|v\|$, $\|v+w\| \leq \|v\| + \|w\|$
- A unit vector is a vector of magnitude 1. (방향만 관계)
- A dot product of v and w is $v \cdot w = v_1w_1 + \cdots + v_nw_n$
- $v \cdot v = \|v\|^2 \geq 0$, $|v \cdot w| \leq \|v\|\,\|w\|$, $\|v+w\| \leq \|v\| + \|w\|$
- $v \cdot w = \|v\|\,\|w\| \cos\theta$
- the vector v and w are orthogonal if $v \cdot w = 0$

1.3 Matrices and their Algebra.

- $m \times n$ matrix is made of m rows and n columns.
- $m \times 1$ matrix is column vector with m components, and a $1 \times n$ matrix is a row vector with n components
- The product Ab of an $m \times n$ matrix A and a column vector b with components $b_1, b_2, \cdots, b_n$ is the column vector equal to the linear combination of column vectors of A where the scalar coefficient of the jth column vector of A is b_j.

  $A = (A_1, \cdots, A_n) \times \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = A_1b_1 + \cdots + A_nb_n$

- In general, $AB \neq BA$
- If $A = (a_{ij})$, $B = (b_{ij})$ are matrices of the same size, then $A+B$ is the matrix of that size with entry $a_{ij} + b_{ij}$ in the jth row and jth column.
- For any matrix A and scalar r, the matrix rA is found by multiplying each entry in A by r.
- The transpose of an $m \times n$ matrix A is the $n \times m$ matrix A^T, which has as its kth row vector the kth column vector.

* Properties of the matrix operations.

- $(A^T)^T = A$, $(A+B)^T = A^T + B^T$, $(AB)^T = B^T A^T$

1.4 Solving system of linear equations.

- A linear system has an associated augmented matrix, having the coefficient matrix of the system on the left of the partition and the column vector of constants on the right of the partition.

- The elementary row operations on a matrix are as follow
  - (Row interchange) Interchange of two rows
  - (Row scaling) Multiplication of a row by a nonzero scalar
  - (Row addition) Addition of a multiple of a row to a different row.

- Matrices A and B are row equivalent (written $A \sim B$) if A can be transformed into B by a sequence of elementary row operations.

- If $Ax = b$ and $Hx = c$ are systems such that the augmented matrices $(A|b)$ and $(H|c)$ are row equivalent, the systems $Ax = b$ and $Hx = c$ have the same solution set.

- Matrix is in row-echelon form if:
  a. All rows containing only zero entries are grouped together at the bottom of the matrix.
  b. The first nonzero element (the pivot) in any row appears in a column to the right of the first nonzero element in any preceding row.

- A matrix is in reduced row-echelon form if it is in row-echelon form and, in addition, each pivot is 1 and is the only nonzero element in its column.
  Every matrix is row equivalent to a unique matrix in reduced row-echelon form.

- In Gauss method with back substitution, we solve a linear system by reducing the augmented matrix so that portion to the left of the partition is in row-echelon form
  The solution is then found by back substitution.

- The Gauss-Jordan method is similar to the Gauss method, except that pivots are adjusted to be 1 and zeros are created above as well as below the pivots.

- A Linear system $Ax = b$ has no solutions if and only if, after $(A|b)$ is row-reduced so that A is transformed into row-echelon form, there exists a row with only zero entries to the left of the partition but with nonzero entry to the right of the partition.
  The linear system is then inconsistent.

- If $Ax = b$ is a consistent linear system and if a row-echelon form H of A has at least one column containing no (nonzero) pivot, the system has an infinite number of solutions. The free variables corresponding to the columns containing no pivots can be assigned any values, and the reduced linear system can then be solved for the remaining variable.

- An elementary matrix E is one obtained by applying a single elementary row operation to an identity matrix I. Multiplication of matrix A on the left by E effects the same elementary row operation on A.

1.5 Inverse of square matrices.

- Let A be a square matrix. A square matrix C such that $CA = AC = I$ is the inverse of A and is denoted by $C = A^{-1}$. If such an inverse of A exists, then A is said to be invertible. The inverse of invertible matrix A is unique. A square matrix that has no inverse is called singular.

- The inverse of a square matrix A exists if and only if A can be reduced to identity matrix I by means of elementary row operations, or (equivalently) if and only if A is a product of elementary matrices. In this case, A is equal to the product, in left-to-right order, of inverses of successive elementary matrices corresponding to the sequence of row operations used to reduce A to I

- To find A^{-1}, if it exists, form the augmented matrix $(A|I)$ and apply the Gauss Jordan method to reduce this matrix to $(I|C)$. If this can be done, then $A^{-1} = C$.
$C = A^{-1}I$
Otherwise, A is not invertible.

- The inverse of a product of invertible matrices is the product of the inverses in the reverse order.

1.6 Homogeneous systems, subspaces, and bases

1. A linear system $Ax=b$ is homogeneous if $b=0$

2. Every linear combination of solutions of a homogeneous system $Ax=0$ is again a solution of the system.

3. A subset W of $\mathbb{R}^n$ is closed under vector addition if the sum of two vectors in W is again in W. The subset W is closed under scalar multiplication if every scalar multiple of every vector in W is in W. If W is nonempty and closed under both operation, then W is subspace of $\mathbb{R}^n$.

4. The span of any k vectors in $\mathbb{R}^n$ is subspace of $\mathbb{R}^n$. If A is an $m\times n$ matrix, the row space of A is span in $\mathbb{R}^n$ of the row vectors of A, the column space of A is span in $\mathbb{R}^m$ of the column vectors, and the nullspace of A is the solution set of $Ax=0$ in $\mathbb{R}^n$.

5. A subset $\{w_1, \dots, w_k\}$ of a subspace W of $\mathbb{R}^n$ is a basis for W if every vector in W can be expressed uniquely as a linear combination of $w_1, \cdots, w_k$

? 6. The set $\{w_1, \cdots, w_k\}$ is a basis for span $(w_1, \cdots, w_k)$ if and only if $0w_1 + \cdots + 0w_k$ is the unique linear combination of the w_j that is equal to the zero vector.

7. A consistent linear system $Ax=b$ of m equations in n unknowns has a unique solution if and only if the reduced row-echelon form of A appears as the $n\times n$ identity matrix

8. A consistent linear system having fewer equations than unknowns is underdetermined — that is, it has an infinite number of solutions.

9. A square linear system has a unique solution if and only if its coefficient matrix is row equivalent.

10. The solutions of any consistent linear system $Ax=b$ are precisely the vectors $p+h$, where p is any one particular solution of $Ax=b$ and h varies through the solution set of the homogeneous system $Ax=0$.

<문제 정리>

A	$\times B$	$=$	C
역x	역x		역x
역o	역x		역x
역o	역o		역o