Data Analytics

Assignment -TOPSIS Method-

윤장혁 교수님

산업공학과

201811527

이영은

Week3

- **Civic, Saturn, Ford, Mazda 중 최선의 선택과 최악의 선택을 다음 데이터를 기반으로 판단**

Weight	0.5	0.3	0.6	0.1
Impact	Positive	Positive	Positive	Negative
	Style	Reliability	Fuel Eco.	Cost
Civic	8	9	5	9
Saturn	8	5	7	8
Ford	9	9	7	7
Mazda	8	8	6	5

1. Normalized a given decision matrix.

1-1. Decision matrix X

Weight	0.5	0.3	0.6	0.1
Impact	Positive	Positive	Positive	Negative
	Style	Reliability	Fuel Eco.	Cost
Civic	8	9	5	9
Saturn	8	5	7	8
Ford	9	9	7	7
Mazda	8	8	6	5

1-2. Normalized decision matrix R

$\boldsymbol{R} = \{\boldsymbol{r_{ij}}\}_{\boldsymbol{m\times n}}$, $\boldsymbol{r_{ij}} = \frac{\boldsymbol{x_{kj}}}{\sqrt{\sum_{\boldsymbol{k=1}}^{\boldsymbol{m}} \boldsymbol{x_{kj}}^2}}$ 의 식을 사용하였고,

각 Criteria에 따라서 normalized decision matrix R를 구하는 풀이과정은 다음과 같습니다.

- 편의를 위해 풀이과정을 열 별로 분리하여 정규화 하였습니다.
- 모든 normalized 풀이과정에서 추후의 계산과정에서 오차를 줄이기 위하여 일곱째 자리에서 반올림하여 소수점 여섯째자리까지 구하였습니다.

■ Style

Weight	0.5
Impact	Positive
	Style
Civic	8
Saturn	8
Ford	9
Mazda	8

Normalized →

Weight	0.5
Impact	Positive
	Style
Civic	0.484182
Saturn	0.484182
Ford	0.544705
Mazda	0.484182

- **Civic** $8 / \sqrt{8^2 + 8^2 + 9^2 + 8^2} = 0.4841820.. \cong 0.484182$
- **Saturn** $8 / \sqrt{8^2 + 8^2 + 9^2 + 8^2} = 0.4841820.. \cong 0.484182$
- **Ford** $9 / \sqrt{8^2 + 8^2 + 9^2 + 8^2} = 0.5447047.. \cong 0.544705$
- **Mazda** $8 / \sqrt{8^2 + 8^2 + 9^2 + 8^2} = 0.4841820.. \cong 0.484182$

■ Reliability

Weight	0.3
Impact	Positive
	Reliability
Civic	9
Saturn	5
Ford	9
Mazda	8

Normalized →

Weight	0.3
Impact	Positive
	Reliability
Civic	0.568075
Saturn	0.315597
Ford	0.568075
Mazda	0.504956

- **Civic** $9 / \sqrt{9^2 + 5^2 + 9^2 + 8^2} = 0.5680749.. \cong 0.568075$
- **Saturn** $5 / \sqrt{9^2 + 5^2 + 9^2 + 8^2} = 0.3155972.. \cong 0.315597$
- **Ford** $9 / \sqrt{9^2 + 5^2 + 9^2 + 8^2} = 0.5680749.. \cong 0.568075$
- **Mazda** $8 / \sqrt{9^2 + 5^2 + 9^2 + 8^2} = 0.5049555.. \cong 0.504956$

■ Fuel Eco.

Weight	0.6
Impact	Positive
	Fuel Eco.
Civic	5
Saturn	7
Ford	7
Mazda	6

Normalized

Weight	0.6
Impact	Positive
	Fuel Eco.
Civic	0.396526
Saturn	0.555136
Ford	0.555136
Mazda	0.475831

- **Civic** $5 / \sqrt{5^2 + 7^2 + 7^2 + 6^2} = 0.3965257.. \cong 0.396526$
- **Saturn** $7 / \sqrt{5^2 + 7^2 + 7^2 + 6^2} = 0.5551361.. \cong 0.555136$
- **Ford** $7 / \sqrt{5^2 + 7^2 + 7^2 + 6^2} = 0.5551361.. \cong 0.555136$
- **Mazda** $6 / \sqrt{5^2 + 7^2 + 7^2 + 6^2} = 0.4758309.. \cong 0.475831$

■ Cost

Weight	0.1
Impact	Negative
	Cost
Civic	9
Saturn	8
Ford	7
Mazda	5

Normalized

Weight	0.1
Impact	Negative
	Cost
Civic	0.608164
Saturn	0.540590
Ford	0.473016
Mazda	0.337869

- **Civic** $9 / \sqrt{9^2 + 8^2 + 7^2 + 5^2} = 0.6081636.. \cong 0.608164$
- **Saturn** $8 / \sqrt{9^2 + 8^2 + 7^2 + 5^2} = 0.5405899.. \cong 0.540590$
- **Ford** $7 / \sqrt{9^2 + 8^2 + 7^2 + 5^2} = 0.4730161.. \cong 0.473016$
- **Mazda** $5 / \sqrt{9^2 + 8^2 + 7^2 + 5^2} = 0.3378686.. \cong 0.337869$

■ **Normalized decision matrix R**

Weight	0.5	0.3	0.6	0.1
Impact	Positive	Positive	Positive	Negative
	Style	Reliability	Fuel Eco.	Cost
Civic	0.484182	0.568075	0.396526	0.608164
Saturn	0.484182	0.315597	0.555136	0.540590
Ford	0.544705	0.568075	0.555136	0.473016
Mazda	0.484182	0.504956	0.475831	0.337869

2. Calculate the weighted normalized decision matrix.

■ w를 구하는 풀이는 다음과 같습니다.

■ **Style w :** $0.5 \div (0.5 + 0.3 + 0.6 + 0.1) = 0.333333$

■ **Reliability w :** $0.3 \div (0.5 + 0.3 + 0.6 + 0.1) = 0.2$

■ **Fuel Eco. w :** $0.6 \div (0.5 + 0.3 + 0.6 + 0.1) = 0.4$

■ **Cost w :** $0.1 \div (0.5 + 0.3 + 0.6 + 0.1) = 0.066667$

■ normalized matrix에 각각의 w를 곱한 결과는 다음과 같습니다.

- Weighted normalized decision matrix또한 추후 계산과정의 오차를 줄이기 위해 소수점 일곱째 자리에서 반올림하여 소수점 여섯째 자리까지 구하였습니다.

■ 모든 풀이과정은 다음과 같습니다.

위의 풀이과정과 동일하게 소수점 일곱째 자리에서 반올림하여 소수점 여섯째자리까지 표시하였습니다.

- **Style w : 0.333333**
  - Civic – $0.428182 \times 0.333333 = 0.1613938.. \cong 0.161394$
  - Saturn – $0.428182 \times 0.333333 = 0.1613938.. \cong 0.161394$
  - Ford – $0.544705 \times 0.333333 = 0.1815681.. \cong 0.181568$
  - Mazda – $0.428182 \times 0.333333 = 0.1613938.. \cong 0.161394$

- **Reliability w : 0.2**
  - Civic – $0.568075 \times 0.2 = 0.113615$
  - Saturn – $0.315597 \times 0.2 = 0.0631194.. \cong 0.063119$
  - Ford – $0.568075 \times 0.2 = 0.113615$
  - Mazda – $0.504956 \times 0.2 = 0.1009912.. \cong 0.100991$

- **Fuel Eco. w : 0.4**
  - Civic – $0.396526 \times 0.4 = 0.1586104 \cong 0.158610$
  - Saturn – $0.555136 \times 0.4 = 0.2220544 \cong 0.222054$
  - Ford – $0.555136 \times 0.4 = 0.2220544 \cong 0.0222054$
  - Mazda – $0.475831 \times 0.4 = 0.1903324 \cong 0.190332$

- **Cost w : 0.066667**
  - Civic – $0.608164 \times 0.066667 = 0.0405444.. \cong 0.040544$
  - Saturn – $0.540590 \times 0.066667 = 0.0360394.. \cong 0.036039$
  - Ford – $0.473016 \times 0.066667 = 0.0315345.. \cong 0.031534$
  - Mazda – $0.337869 \times 0.066667 = 0.0225247.. \cong 0.022525$

2-1. Weighted normalized decision matrix

Weighted normalized decision matrix T				
Weight	0.5	0.3	0.6	0.1
w	0.333333	0.2	0.4	0.06667
Impact	Positive	Positive	Positive	Negative
	Style (J+)	Reliability (J+)	Fuel Eco. (J+)	Cost (J-)
Civic	0.161394	0.113615	0.158610	0.040544
Saturn	0.161394	0.063119	0.222054	0.036039
Ford	0.181568	0.113615	0.222054	0.031534
Mazda	0.161394	0.100991	0.190332	0.022525

3. Identify positive/negative ideal solutions.

Weighted normalized decision matrix T				
Weight	0.5	0.3	0.6	0.1
w	0.333333	0.2	0.4	0.06667
Impact	Positive	Positive	Positive	Negative
	Style (J+)	Reliability (J+)	Fuel Eco. (J+)	Cost (J-)
Civic	0.161394	0.113615	0.158610	0.040544
Saturn	0.161394	0.063119	0.222054	0.036039
Ford	0.181568	0.113615	0.222054	0.031534
Mazda	0.161394	0.100991	0.190332	0.022525

	Style	Reliability	Fuel Eco.	Cost
PIS A*	0.181568	0.113615	0.222054	0.022525
NIS A′	0.161394	0.063119	0.158610	0.040544

■ 각각의 Criteria에서 positive와 negative ideal solution을 구하였습니다.

◆ Positive Impact (Style, Reliability, Fuel Eco)

- PIS A* : 최댓값 (빨간색)
- NIS A′ : 최솟값 (파란색)

(같은 값을 갖는 Alternatives모두 동일하게 표시하였습니다.)

◆ Negative Impact (Cost)

- PIS A* : 최솟값 (빨간색)
- NIS A′ : 최댓값 (파란색)

4. Calculate the relative closeness of each alternatives to the ideal solution.

■ 풀이 과정의 편의를 위해 Civic, Saturn, Ford, Mazda의 Alternatives로 나누어 각각을 계산한 풀이 과정을 첨부하였습니다.

■ 또한 위와 같이 풀이과정에서 소수점 여섯째자리까지 구하였습니다.

■ 각각의 풀이과정은 다음과 같습니다.

■ PIS $A* = \sqrt{\sum_{j=1}^{m}(t_j^* - t_{ij})^2}$ 의 식을 사용하여 구하였고,

NIS $A' = \sqrt{\sum_{j=1}^{m}(t_j' - t_{ij})^2}$ 의 식을 사용하였습니다.

또한 closeness $C*_i = A'_i/(A'_i + A*_i)$를 사용하였습니다.

■ Civic

	Style	Reliability	Fuel Eco.	Cost
PIS A*	0.181568	0.113615	0.222054	0.022525
NIS A'	0.161394	0.063119	0.158610	0.040544

	Style (J+)	Reliability (J+)	Fuel Eco. (J+)	Cost (J-)
Civic	0.161394	0.113615	0.158610	0.040544

Relative closeness each Alternative to the ideal solution			
	From PIS A*	From NIS A'	Closeness
Civic	0.06897	0.050496	0.422682

- 계산 과정은 다음과 같습니다.

- **Civic의 PIS A*, NIS A', Closeness 풀이과정**

- **PIS A***

$$\sqrt{(0.181568-0.161394)^2+(0.113615-0.113615)^2+(0.222054-0.158610)^2+(0.022525-0.040544)^2} = 0.0689696.. \cong 0.06897$$

- **NIS A'**

$$\sqrt{(0.161394-0.161394)^2+(0.063119-0.113615)^2+(0.158610-0.158610)^2+(0.040544-0.040544)^2} = 0.050496$$

- **Closeness**

$$\frac{0.050496}{(0.050496+0.0689696)} = 0.4226823.. \cong 0.422682$$

■ Saturn

	Style	Reliability	Fuel Eco.	Cost
PIS A*	0.181568	0.113615	0.222054	0.022525
NIS A'	0.161394	0.063119	0.158610	0.040544

	Style (J+)	Reliability (J+)	Fuel Eco. (J+)	Cost (J-)
Saturn	0.161394	0.063119	0.222054	0.036039

Relative closeness each Alternative to the ideal solution			
	From PIS A*	From NIS A'	Closeness
Saturn	0.056031	0.063604	0.531650

- 계산 과정은 다음과 같습니다.

- **Saturn의 PIS A*, NIS A', Closeness 풀이과정**

- **PIS A***

$$\sqrt{(0.181568-0.161394)^2+(0.113615-0.063119)^2+(0.222054-0.222054)^2+(0.022525-0.036039)^2} = 0.0560309.. \cong 0.056031$$

- **NIS A'**

$$\sqrt{(0.161394-0.161394)^2+(0.063119-0.063119)^2+(0.158610-0.222054)^2+(0.040544-0.036039)^2} = 0.0636037.. \cong 0.063604$$

- **Closeness**

$$\frac{0.063604}{(0.063604+0.056031)} = 0.5316504.. \cong 0.531650$$

■ Ford

	Style	Reliability	Fuel Eco.	Cost
PIS A*	0.181568	0.113615	0.222054	0.022525
NIS A'	0.161394	0.063119	0.158610	0.040544

	Style (J+)	Reliability (J+)	Fuel Eco. (J+)	Cost (J-)
Ford	0.181568	0.113615	0.222054	0.031534

Relative closeness each Alternative to the ideal solution			
	From PIS A*	From NIS A'	Closeness
Ford	0.00901	0.084043	0.903173

- 계산 과정은 다음과 같습니다.

- **Ford의 PIS A*, NIS A', Closeness 풀이과정**

- **PIS A***

$$\sqrt{(0.181568-0.181568)^2+(0.113615-0.113615)^2+(0.222054-0.222054)^2+(0.022525-0.031534)^2} = 0.009009.. \cong 0.00901$$

- **NIS A'**

$$\sqrt{(0.161394-0.181568)^2+(0.063119-0.113615)^2+(0.158610-0.222054)^2+(0.040544-0.031534)^2} = 0.0840425.. \cong 0.084043$$

- **Closeness**

$$\frac{0.084043}{(0.084043+0.00901)} = 0.9031734.. \cong 0.903173$$

■ Mazda

	Style	Reliability	Fuel Eco.	Cost
PIS A*	0.181568	0.113615	0.222054	0.022525
NIS A′	0.161394	0.063119	0.158610	0.040544

	Style (J+)	Reliability (J+)	Fuel Eco. (J+)	Cost (J-)
Mazda	0.161394	0.100991	0.190332	0.022525

Relative closeness each Alternative to the ideal solution			
	From PIS A*	From NIS A′	Closeness
Mazda	0.039657	0.052586	0.570083

- 계산 과정은 다음과 같습니다.

- **Ford의 PIS A*, NIS A′, Closeness 풀이과정**

- **PIS A***

$$\sqrt{(0.181568-0.161394)^2+(0.113615-0.100991)^2+(0.222054-0.190332)^2+(0.022525-0.022525)^2} = 0.0396565.. \cong 0.039657$$

- **NIS A′**

$$\sqrt{(0.161394-0.161394)^2+(0.063119-0.100991)^2+(0.158610-0.190332)^2+(0.040544-0.022525)^2} = 0.0525857.. \cong 0.052586$$

- **Closeness**

$$\frac{0.052586}{(0.052586+0.039657)} = 0.5700828.. \cong 0.570083$$

5. Relative closeness each Alternative to the ideal solution

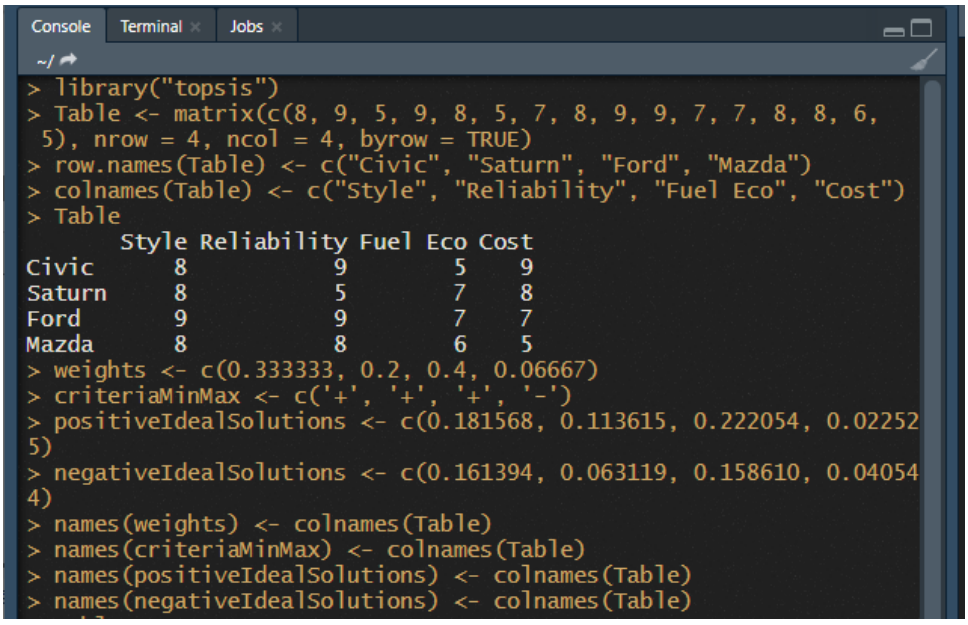
- 4에서 구한 풀이과정을 모두 합하여 나타내면 다음 표와 같습니다.

Relative closeness each Alternative to the ideal solution				
	From PIS A*	From NIS A'	Closeness	Priority
Civic	0.06897	0.050496	0.422682	4
Saturn	0.056031	0.063604	0.531650	3
Ford	0.00901	0.084043	0.903173	1
Mazda	0.039657	0.052586	0.570083	2

6. Conclusion : Civic, Saturn, Ford, Mazda 중 최선의 선택과 최악의 선택을 위의 데이터를 기반으로 판단

- 최선의 선택 : Ford
- 최악의 선택 : Civic

7. R Studio package 사용



```
> overall1 <- topsis(Table, weights, criteriaMinMax)
> overall1
  alt.row     score rank
1       1 0.4226778    4
2       2 0.5316502    3
3       3 0.9031702    1
4       4 0.5700835    2
>
```

R Studio topsis library를 사용하여 오차범위 안의 같은 결과를 구하였습니다.

■ 사용코드

```
Table <- matrix(c(8, 9, 5, 9, 8, 5, 7, 8, 9, 9, 7, 7, 8, 8, 6, 5), nrow = 4, ncol = 4, byrow = TRUE)

row.names(Table) <- c("Civic", "Saturn", "Ford", "Mazda")

colnames(Table) <- c("Style", "Reliability", "Fuel Eco", "Cost")

weights <- c(0.333333, 0.2, 0.4, 0.06667)

criteriaMinMax <- c('+', '+', '+', '-')

positiveIdealSolutions <- c(0.181568, 0.113615, 0.222054, 0.022525)

negativeIdealSolutions <- c(0.161394, 0.063119, 0.158610, 0.040544)


names(weights) <- colnames(Table)

names(criteriaMinMax) <- colnames(Table)

names(positiveIdealSolutions) <- colnames(Table)

names(negativeIdealSolutions) <- colnames(Table)


overall1 <- topsis(Table, weights, criteriaMinMax)
```

8. 참고 자료

- Package 'MCDA'
- http://github.com/paterijk/MCDA

Package 'MCDA'

May 29, 2019

Version 0.0.20

Date 2019-05-29

Title Support for the Multicriteria Decision Aiding Process

Author Patrick Meyer, Sébastien Bigaret, Richard Hodgett, Alexandru-Liviu Olteanu

Maintainer Patrick Meyer <patrick.meyer@imt-atlantique.fr>

Description
Support for the analyst in a Multicriteria Decision Aiding (MCDA) process with algorithms, preference elicitation and data visualisation functions. Sébastien Bigaret, Richard Hodgett, Patrick Meyer,
Tatyana Mironova, Alexandru Olteanu (2017) Supporting the multi-criteria decision aiding process :
R and the MCDA package, Euro Journal On Decision Processes, Volume 5, Issue 1 - 4, pages 169 - 194 <doi:10.1007/s40070-017-0064-1>.

Imports Rglpk, glpkAPI, methods, RColorBrewer, combinat

Suggests Rgraphviz, cplexAPI

License EUPL (== 1.1)

Encoding UTF-8

URL https://github.com/paterijk/MCDA

NeedsCompilation no

Repository CRAN

Date/Publication 2019-05-29 13:30:10 UTC

- P 73-74를 참고하여 R Studio 코드를 작성하였습니다.