Predictive Extended Frobenius Finite Memory Estimation with Application to Mobile Robot Localization

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***Abstract*—In this paper, we present a new approach for estimating the state of nonlinear systems. Kalman filter (KF) and particle filter (PF), which are widely used in practical applications, may show poor performance or divergence phenomenon due to existence of disturbance or missing measurement. This paper proposes a new predictive extended Frobenius finite memory estimation (PEFFME) algorithm to overcome the performance degradation problem from the measurement missing. To ensure robustness against disturbance and model uncertainty, the PEFFME is designed with finite impulse response (FIR) structure based on the minimization of Frobenius norm and the prediction of measurements. It shows excellent estimation performance and successfully prevents divergence even when measurement observation fails temporarily. The accuracy and robustness of the new estimation method are verified by mobile robot localization experiments in which kidnapped robot problem and multiple missing measurements occur.**

***Index Terms*—Finite memory estimation, predictive estimation, mobile robot localization**

I. Introduction

Since Kalman filter (KF) has been shown to successfully estimate the state of linear systems, there have been attempts to improve it and to apply it to nonlinear systems. One of the results, extended KF (EKF), is based on linearization of state and measurement equations [1], [2]. Unscented KF (UKF), on the other hand, adopts unscented transformation to approximate multidimensional integrals [3]. In addition, particle filter (PF) has been developed for state estimation to handle high nonlinearity of the system [4], [5]. However, KF and PF can suffer from divergence problems due to severe disturbance, model uncertainty or incorrect noise information because they have infinite impulse response (IIR) structure which uses all measurements from the initial time to estimate the state. Furthermore, UKF and PF increase the amount of computation as the number of samples or particles increases for better performance.

In this respect, finite impulse response (FIR)-type estimators have been presented as alternative solutions. Because of their bounded-input bounded-output (BIBO) characteristics, they have intrinsic robustness property [6], [7]. Researches have also been conducted to estimate state of nonlinear system using FIR-type estimators. In [8], a hybrid particle/FIR filtering algorithm was proposed to solve the sample impoverishment problem of PF. A fuzzy horizon group shift FIR filtering using Takagi-Sugeno model was proposed in [9] for nonlinear state estimation. Extended FIR filters for nonlinear systems were discussed in [10], which is accurate in ideal situations where there is no missing measurement, but is vulnerable enough to diverge immediately if a missing measurement occurs at a certain timing. In recent years, a variety of systems are often operated in real time, and accordingly the need to estimate the state of the system in real time has also increased. However, in reality, it is difficult to always receive the measurements correctly, especially in real-time systems with high complexity such as wireless sensor networks (WSNs). Therefore, missing measurement is a frequent problem, and finding a way to cope with it is urgent and important.

In this paper, we propose a new state estimation method, which is called predictive extended Frobenius finite memory estimation (PEFFME), for nonlinear state-space discrete-time systems. It overcomes the limitations of existing methods by measurement prediction in case of measurement missing. It is derived as an unbiased estimation of FIR structure that guarantees superior robustness while considering predicted measurement. The gain values for the PEFFME are set to minimize degradation of performance due to unexpected observational error. Via the predictive unbiased estimation, it shows a more robust performance in harsh environments than the existing methods. It also has important features in terms of design. No process or observation noise statistics are required at design, suggesting that it can be used effectively in systems with uncertain noise information. While adopting noise covariances that are difficult to apply correctly to existing methods as design parameters, it adopts horizon size, which is convenient to implement. Since its computational burden is not much large, it is reasonable to solve the real-time state estimation problems in the real world. The accuracy, robustness and real-time performance of the proposed PEFFME will be verified through mobile robot localization experiments, where kidnapped robot problem or multiple missing measurements occur.

The remainder of this paper is organized as follows. In section II, we discuss how to proceed with measurement prediction in situations where missing measurements occur and how to derive the PEFFME to have accurate and robust performance. Section III provides problem formulation for mobile robot localization and verifies the proposed algorithm through experiments for mobile robot localization and perfor-

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mance analysis. Conclusions are drawn in section IV.

II. PEFFME Algorithm

Consider a nonlinear system which has following discrete time state space representation:

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k, \tag{1}$$
$$\mathbf{z}_k = h(\mathbf{x}_k) + \mathbf{v}_k, \tag{2}$$

where $\mathbf{x}_k \in \mathbb{R}^n$, $\mathbf{u}_k \in \mathbb{R}^r$, $\mathbf{z}_k \in \mathbb{R}^m$, $\mathbf{w}_k \in \mathbb{R}^n$ and $\mathbf{v}_k \in \mathbb{R}^m$ are state, control input, measurement, process noise and observation noise at time k, respectively. $\mathbf{w}_k$ and $\mathbf{v}_k$ are assumed to be zero mean white Gaussian with covariances Q and R, respectively. The functions $f\colon \mathbb{R}^n \to \mathbb{R}^n$ and $h\colon \mathbb{R}^n \to \mathbb{R}^m$ are supposed to be continuously differentiable on an open connected set $D \subset \mathbb{R}^n$. Ideally, $\mathbf{z}_k$ exists for all k but if observation fails, it does not. So, we consider a modified measurement equation as follows:

$$\mathbf{z}_k = \alpha_k h(\mathbf{x}_k) + (1-\alpha_k)\ h(f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})) + \mathbf{v}_k, \tag{3}$$

where α_k has a value of zero or one, indicating whether observation has been successful. If observation is successful, it has a value of one, otherwise it is zero. Through (3), measurement can be predicted when missing measurement occurs. To simplify the analysis of (1) and (3), we use a linearization technique here. By defining Jacobian matrices as

$$F_k = \frac{\partial f}{\partial \mathbf{x}}|_{\hat{\mathbf{x}}_k},\ H_k = \frac{\partial h}{\partial \mathbf{x}}|_{\hat{\mathbf{x}}_k},\ H_k^- = \frac{\partial h}{\partial \mathbf{x}}|_{\hat{\mathbf{x}}_k^-},$$

where $\hat{\mathbf{x}}_k^- = f(\hat{\mathbf{x}}_{k-1}, \mathbf{u}_{k-1})$, (1) can be approximated with the first-order Taylor series expansion as below:

$$\begin{aligned}\mathbf{x}_{k+1} &\simeq f(\hat{\mathbf{x}}_k, \mathbf{u}_k) + \frac{\partial f}{\partial \mathbf{x}}|_{\hat{\mathbf{x}}_k}(\mathbf{x}_k - \hat{\mathbf{x}}_k) + \mathbf{w}_k \\ &= F_k\mathbf{x}_k + [f(\hat{\mathbf{x}}_k, \mathbf{u}_k) - F_k\hat{\mathbf{x}}_k] + \mathbf{w}_k \\ &= F_k\mathbf{x}_k + \tilde{\mathbf{u}}_k + \mathbf{w}_k, \end{aligned}\tag{4}$$

where $\tilde{\mathbf{u}}_k = f(\hat{\mathbf{x}}_k, \mathbf{u}_k) - F_k\hat{\mathbf{x}}_k$ is an auxiliary control input. In the same manner, (3) can be approximated as below:

$$\begin{aligned}\mathbf{z}_k \simeq\ & \alpha_k[h(\hat{\mathbf{x}}_k) + \frac{\partial h}{\partial \mathbf{x}}|_{\hat{\mathbf{x}}_k}(\mathbf{x}_k - \hat{\mathbf{x}}_k)] \\ & + (1-\alpha_k)\,[h(\hat{\mathbf{x}}_k^-) + \frac{\partial h}{\partial \mathbf{x}}|_{\hat{\mathbf{x}}_k^-}(\mathbf{x}_k - \hat{\mathbf{x}}_k^-)] + \mathbf{v}_k \\ =\ & [\alpha_k H_k + (1-\alpha_k)H_k^-]\mathbf{x}_k + \alpha_k(h(\hat{\mathbf{x}}_k) - H_k\hat{\mathbf{x}}_k) \\ & + (1-\alpha_k)(h(\hat{\mathbf{x}}_k^-) - H_k^-\hat{\mathbf{x}}_k^-) + \mathbf{v}_k. \end{aligned}\tag{5}$$

Introduce an auxiliary measurement:

$$\begin{aligned}\tilde{\mathbf{z}}_k = &\ \mathbf{z}_k - \alpha_k(h(\hat{\mathbf{x}}_k) - H_k\hat{\mathbf{x}}_k) \\ & - (1-\alpha_k)(h(\hat{\mathbf{x}}_k^-) - H_k^-\hat{\mathbf{x}}_k^-). \end{aligned}\tag{6}$$

Now we can present linearized state space representation using (4), (5) and (6) as follows:

$$\mathbf{x}_{k+1} = F_k\mathbf{x}_k + \tilde{\mathbf{u}}_k + \mathbf{w}_k, \tag{7}$$
$$\tilde{\mathbf{z}}_k = \bar{H}_k\mathbf{x}_k + \mathbf{v}_k, \tag{8}$$

where $\bar{H}_k = \alpha_k H_k + (1-\alpha_k)H_k^-$. Here we define a predetermined natural number $N \geq n$ as horizon size for finite memory estimation. For integer $s \in [1, N-1]$, the auxiliary measurement at time $k-s$ can be represented as follows by (7) and (8):

$$\begin{aligned}\tilde{\mathbf{z}}_{k-s} = &\ \bar{H}_{k-s}\underbrace{F_{k-s-1}\cdots F_{k-N}}_{N-s}\mathbf{x}_{k-N} \\ & + \bar{H}_{k-s}\underbrace{F_{k-s-1}\cdots F_{k-N+1}}_{N-s-1}(\tilde{\mathbf{u}}_{k-N+1} + \mathbf{w}_{k-N+1}) \\ & + \cdots \\ & + \bar{H}_{k-s}F_{k-s-1}(\tilde{\mathbf{u}}_{k-s-2} + \mathbf{w}_{k-s-2}) \\ & + \bar{H}_{k-s}(\tilde{\mathbf{u}}_{k-s-1} + \mathbf{w}_{k-s-1}) + \mathbf{v}_{k-s}. \end{aligned}\tag{9}$$

Let $\Gamma_k\colon \mathbb{I}\times\mathbb{I} \to \mathbb{R}^{m\times n}$ be a function which satisfies

$$\Gamma_k(a,b) = \begin{cases} \bar{H}_{k-N+a}\prod_{i=1}^{a-b}F_{k-N+a-i}, & \text{if } a > b \\ \bar{H}_{k-N+a}, & \text{if } a = b \\ \mathbf{0}_{m\times n}, & \text{otherwise.} \end{cases}\tag{10}$$

Combining all the results of (9) for $s \in [1, N-1]$ and using (8), an augmented auxiliary measurement vector $\tilde{Z}_{k-1} = \left[\tilde{\mathbf{z}}_{k-N}^T\tilde{\mathbf{z}}_{k-N+1}^T\cdots\tilde{\mathbf{z}}_{k-1}^T\right]^T$ can be represented as

$$\tilde{Z}_{k-1} = \mathcal{A}_{k-1}\mathbf{x}_{k-N} + \mathcal{B}_{k-1}\tilde{U}_{k-1} + \mathcal{B}_{k-1}\tilde{W}_{k-1} + \tilde{V}_{k-1}, \tag{11}$$

where

$$\mathcal{A}_{k-1} = \begin{bmatrix}\Gamma_{k-1}(1,1) \\ \Gamma_{k-1}(2,1) \\ \vdots \\ \Gamma_{k-1}(N,1)\end{bmatrix},$$

$$\mathcal{B}_{k-1} = \begin{bmatrix}\mathbf{0}_{m\times n} & \cdots & \mathbf{0}_{m\times n} & \mathbf{0}_{m\times n} \\ \Gamma_k(1,1) & \cdots & \Gamma_k(1,N-1) & \mathbf{0}_{m\times n} \\ \vdots & \ddots & \vdots & \vdots \\ \Gamma_k(N-1,1) & \cdots & \Gamma_k(N-1,N-1) & \mathbf{0}_{m\times n}\end{bmatrix},$$

$$\tilde{U}_{k-1} = \left[\tilde{\mathbf{u}}_{k-N}^T\tilde{\mathbf{u}}_{k-N+1}^T\cdots\tilde{\mathbf{u}}_{k-1}^T\right]^T,$$
$$\tilde{W}_{k-1} = \left[\mathbf{w}_{k-N}^T\mathbf{w}_{k-N+1}^T\cdots\mathbf{w}_{k-1}^T\right]^T,$$
$$\tilde{V}_{k-1} = \left[\mathbf{v}_{k-N}^T\mathbf{v}_{k-N+1}^T\cdots\mathbf{v}_{k-1}^T\right]^T.$$

Using (7), $\mathbf{x}_k$ can be represented in terms of $\mathbf{x}_{k-N}$ as follows:

$$\mathbf{x}_k = \mathcal{F}_{k-1}(0)\mathbf{x}_{k-N} + \mathcal{C}_{k-1}\tilde{U}_{k-1} + \mathcal{C}_{k-1}\tilde{W}_{k-1}, \tag{12}$$

where

$$\mathcal{F}_k(a) = \begin{cases}\underbrace{F_kF_{k-1}\cdots F_{k-N+a+1}}_{N-a}, & \text{if } a < N \\ I_{n\times n}, & \text{otherwise,}\end{cases}$$
$$\mathcal{C}_{k-1} = \begin{bmatrix}\mathcal{F}_{k-1}(1) & \mathcal{F}_{k-1}(2) & \cdots & \mathcal{F}_{k-1}(N)\end{bmatrix}.$$

We propose an unbiased estimation method in the following form:

$$\hat{\mathbf{x}}_{k|k-1} = L_{k-1}\tilde{Z}_{k-1} + M_{k-1}\tilde{U}_{k-1}. \tag{13}$$

By (11), (12) and (13), we obtain

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} = &\ (L_{k-1}\mathcal{A}_{k-1} - \mathcal{F}_{k-1}(0))\,\mathbf{x}_{k-N} + \mathbf{x}_k \\ & + (L_{k-1}\mathcal{B}_{k-1} - \mathcal{C}_{k-1})\left(\tilde{U}_{k-1} + \tilde{W}_{k-1}\right)\end{aligned}$$

$$+ L_{k-1}\tilde{V}_{k-1} + M_{k-1}\tilde{U}_{k-1}. \quad (14)$$

Taking expectation to (14) yields

$$E\left[\hat{\mathbf{x}}_{k|k-1}\right] = \left(L_{k-1}\mathcal{A}_{k-1} - \mathcal{F}_{k-1}(0)\right)E\left[\mathbf{x}_{k-N}\right] + E\left[\mathbf{x}_k\right] + \left(L_{k-1}\mathcal{B}_{k-1} - \mathcal{C}_{k-1} + M_{k-1}\right)\tilde{U}_{k-1}. \quad (15)$$

In view of unbiased estimation, $E\left[\hat{\mathbf{x}}_{k|k-1}\right]$ and $E\left[\mathbf{x}_k\right]$ should be the same. Consequently, we obtain the following unbiasedness conditions:

$$L_{k-1}\mathcal{A}_{k-1} = \mathcal{F}_{k-1}(0), \quad (16)$$

$$M_{k-1} = -L_{k-1}\mathcal{B}_{k-1} + \mathcal{C}_{k-1}. \quad (17)$$

On the other hand, to ensure that observational errors have a minimal effect on the estimate, we should find L_{k-1} that minimizes its own Frobenius norm. So, we consider a optimization problem to minimize the following cost function:

$$J = \frac{1}{2}\|L_{k-1}\|_F^2 \quad \text{subject to (16) and (17)}, \quad (18)$$

where $\|\cdot\|_F$ denotes Frobenius norm. To solve (18), we define Lagrange function as

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\|L_{k-1}\|_F^2 - \Lambda^T\left(L_{k-1}\mathcal{A}_{k-1} - \mathcal{F}_{k-1}(0)\right) \\ &= \frac{1}{2}\mathrm{tr}\left[L_{k-1}^T L_{k-1}\right] - \Lambda^T\left(L_{k-1}\mathcal{A}_{k-1} - \mathcal{F}_{k-1}(0)\right), \quad (19)\end{aligned}$$

where Λ is a Lagrange multiplier matrix. Partial derivative of $\mathcal{L}$ with respect to L_{k-1} should be zero as

$$\frac{\partial\mathcal{L}}{\partial L_{k-1}} = L_{k-1} - \Lambda^T\mathcal{A}_{k-1}^T = 0. \quad (20)$$

Substituting (20) into (16) yields

$$\Lambda^T = \mathcal{F}_{k-1}(0)\left(\mathcal{A}_{k-1}^T\mathcal{A}_{k-1}\right)^{-1}. \quad (21)$$

Therefore, we determine the gain matrices for the proposed estimator by (17), (20) and (21) as below:

$$L_{k-1} = \mathcal{F}_{k-1}(0)\left(\mathcal{A}_{k-1}^T\mathcal{A}_{k-1}\right)^{-1}\mathcal{A}_{k-1}^T, \quad (22)$$

$$M_{k-1} = -\mathcal{F}_{k-1}(0)\left(\mathcal{A}_{k-1}^T\mathcal{A}_{k-1}\right)^{-1}\mathcal{A}_{k-1}^T\mathcal{B}_{k-1} + \mathcal{C}_{k-1}. \quad (23)$$

The derived PEFFME is finally represented as follows:

$$\begin{aligned}&\hat{\mathbf{x}}_{k|k-1} \\ &= \mathcal{F}_{k-1}(0)\left(\mathcal{A}_{k-1}^T\mathcal{A}_{k-1}\right)^{-1}\mathcal{A}_{k-1}^T\tilde{Z}_{k-1} \\ &\quad + \left(-\mathcal{F}_{k-1}(0)\left(\mathcal{A}_{k-1}^T\mathcal{A}_{k-1}\right)^{-1}\mathcal{A}_{k-1}^T\mathcal{B}_{k-1} + \mathcal{C}_{k-1}\right)\tilde{U}_{k-1}. \quad (24)\end{aligned}$$

Remark 1. *To compensate for the missing measurement, the measurement equation (2) is replaced by (3). This implies that when a missing measurement occurs, a new measurement is taken into account using the estimate from the previous time step and the system model. The most dependable information when observation fails in a discrete-time system is the previous state estimate, which is the most recent available data. Here, accordingly to (1), (2) and the assumptions of system noises* $\boldsymbol{w}_k$ *and* $\boldsymbol{v}_k$, $h(f(\hat{\mathbf{x}}_{k-1}, \boldsymbol{u}_{k-1}))$ *becomes a maximum-likelihood estimate of* $\boldsymbol{z}_k$. *This justifies the modified measurement equation (3). By "predicting" a measurement from the previous state estimate, it is possible to determine the most reliable measurement and to prevent divergence, which has become a persistent problem in existing estimators.*

Remark 2. *Note that* $\|A\|_F = \sqrt{E\left[\frac{\|AX\|_2^2}{\|X\|_2^2}\right]}$, *where* $\|\cdot\|_2$ *denotes 2-norm and* X *is non-zero uniformly random vector, respectively. Since the square of the 2-norm of a vector physically means its energy, the Frobenius norm of matrix indicates the average energy transfer rate. If an augmented auxiliary measurement vector* $\tilde{Z}$ *is deformed by unexpected observational errors, this leads to an incorrect estimate according to (13). By presenting the cost function as in (18) and minimizing it, we can design a robust estimator which minimizes the energy transferred from the observational errors.*

III. Application to Mobile Robot Localization

This section applies the proposed PEFFME to mobile robot localization problem with experiments.

A. System Model Description

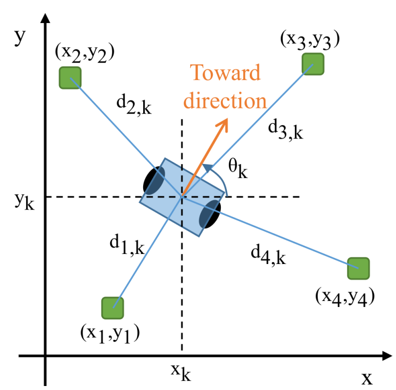

Fig. 1: Schematics of the localization system

Figure 1 illustrates a spatial distribution of a wheeled mobile robot (WMR) and real-time locating system (RTLS) anchors. The anchors are located at (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4), which are user-specified positions. The distances between mobile tag mounted on the WMR and the anchors are measured in real time. The WMR has an inertial measurement unit (IMU) that is able to measure its own heading angle. The purpose of the localization system to be designed is to estimate the two-dimensional position coordinates x_k and y_k of the WMR and its heading angle θ_k. Defining pose to be state vector so that $\mathbf{x}_k = [x_k\ y_k\ \theta_k]^T$, motion model of WMR can be described as follows [11]:

$$x_{k+1} = f_1 = x_k + u_{l,k}\cos(\theta_k + \frac{1}{2}u_{\theta,k}), \quad (25)$$

$$y_{k+1} = f_2 = y_k + u_{l,k}\sin(\theta_k + \frac{1}{2}u_{\theta,k}), \quad (26)$$

$$\theta_{k+1} = f_3 = \theta_k + u_{\theta,k}, \quad (27)$$

where $u_{l,k}$ and $u_{\theta,k}$ are the incremental changes in distance and heading angle, respectively. The measured tag-anchor distances and heading angle from the IMU are transmitted to an external computer via wireless communication. Then the computer generates distance difference values $d_{12,k}$, $d_{13,k}$ and $d_{14,k}$ as below:

$$d_{1,k} = \sqrt{(x_k - x_1)^2 + (y_k - y_1)^2}, \tag{28}$$

$$d_{12,k} = h_1 = d_{1,k} - \sqrt{(x_k - x_2)^2 + (y_k - y_2)^2}, \tag{29}$$

$$d_{13,k} = h_2 = d_{1,k} - \sqrt{(x_k - x_3)^2 + (y_k - y_3)^2}, \tag{30}$$

$$d_{14,k} = h_3 = d_{1,k} - \sqrt{(x_k - x_4)^2 + (y_k - y_4)^2}. \tag{31}$$

The computer does not apply any special treatment to the received heading angle value as

$$h_4 = \theta_k. \tag{32}$$

The equations (25), (26), (27), (29), (30), (31) and (32) are expressed in the form of (1) and (2) by assigning $f = [f_1\ f_2\ f_3]^T$, $h = [h_1\ h_2\ h_3\ h_4]^T$, $\mathbf{z}_k = [d_{12,k}\ d_{13,k}\ d_{14,k}\ \theta_k]^T$ and $\mathbf{u}_k = [u_{l,k}\ u_{\theta,k}]^T$.

B. Experimental Setup

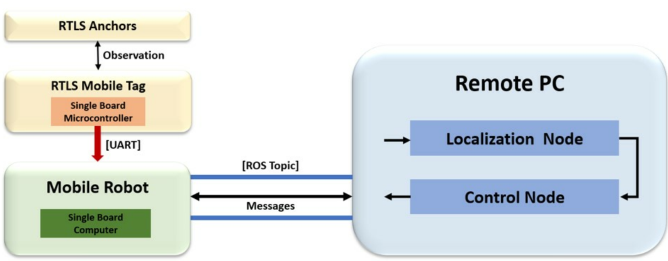

Fig. 2: Experimental setup for localization

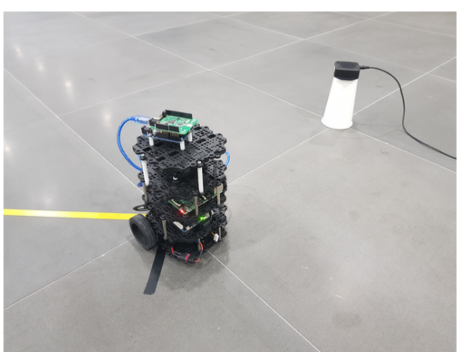

Fig. 3: WMR and an anchor used in the experiments

Figure 2 shows how the information is exchanged between the equipment used in the experiments. The Pozyx developer's kit was used for the RTLS tag and anchors. The tag is attached to a single board microcontroller, Arduino UNO, so that a single board computer of the WMR, Raspberry Pi 3, loads real-time tag-anchor distance values via serial communication. A Wi-Fi router was used for wireless connection of WMR and remote PC. The remote PC sends control inputs or receive sensor data.

C. Experimental Verification

The robustness of the proposed PEFFME will be demonstrated through the experimental results in this section. Its performance will be analyzed with EKF and regularized PF (RPF) for comparison. This section considers three situations: normal case in which no artificial disturbances are injected; kidnapped case; and measurement missing case. The numerical values used in the experiments are as follows:

1) Locations of anchors: $(0, 0)$m, $(0, 6)$m, $(6, 6)$m and $(6, 0)$m
2) Horizon size for PEFFME: $N = 10$
3) Covariance of process noise for EKF and RPF: $Q = \text{diag}\left\{0.01\text{m}^2, 0.01\text{m}^2, 0.01\text{rad}^2\right\}$
4) Covariance of observation noise for EKF and RPF: $R = \text{diag}\left\{0.2\text{m}^2, 0.2\text{m}^2, 0.2\text{m}^2, 0.1\text{rad}^2\right\}$
5) The number of particles for RPF: 50
6) Sampling time period: 0.2s

Remark 3. *RPF is a sequential Monte-Carlo method, and the larger the number of particles, the higher the computational burden. Since too many particles induce high computational complexity, there is a realistic limit to the number of particles in the real-time estimation environment. The number of particles presented above was chosen so that RPF can provide real-time computation results at every sampling time.*

1) Normal case: First, an experiment was conducted in a situation where external force or communication disturbance did not occur in the WMR. Figure 4 shows the results of position estimation when the WMR performs a straight motion. The WMR started at $(0.40, 0.10)$m at time 6.1s and reached $(4.40, 5.20)$m at time 66.1s. Figure 5 plots the pose estimation error of the three estimators: EKF, RPF and PEFFME. Sometimes, even in the normal case, some or all of tag-anchor distance values are not passed correctly to the WMR, which makes the estimators unable to take into account the correct measurements. This is why the EKF has very large peak values in the estimation error at the timings of this phenomenon (e.g. around 7s, 51s, 54s and 57s). The RPF shows inaccurate estimation due to the small number of particles. The proposed PEFFME provides superior performance than the aforementioned estimators in the same environment.

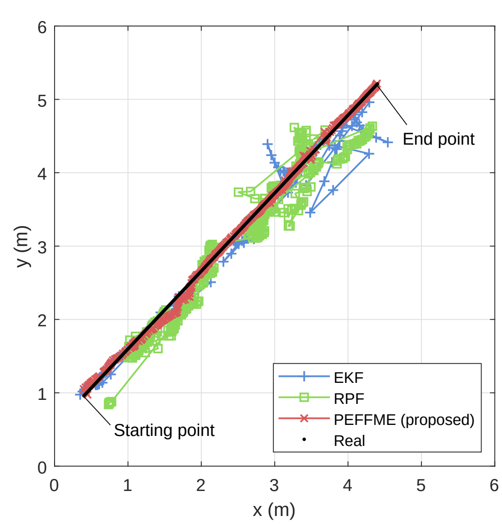

Fig. 4: Localization results in normal case.

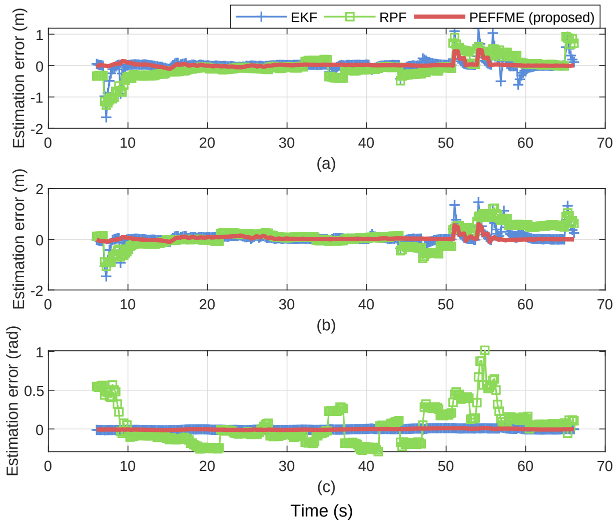

Fig. 5: Estimation errors in normal case. (a) Coordinate x. (b) Coordinate y. (c) Heading angle θ.

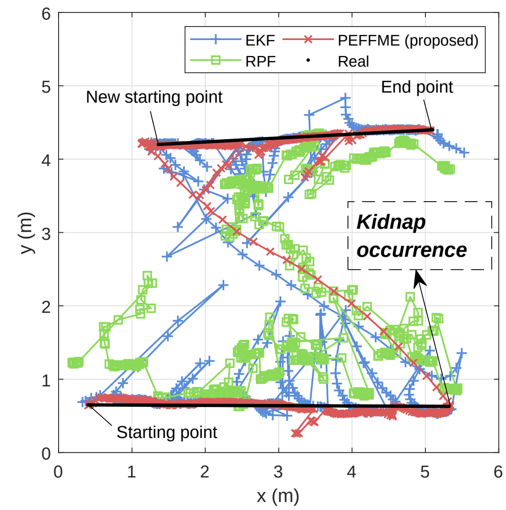

Fig. 6: Localization results in kidnapped case.

2) Kidnapped case: The kidnapped robot problem is challenging from an estimation point of view since the sudden change of the pose is unpredictable. Such a change drastically reduces the reliability of the system model and, in some cases, causes divergence problems. In order to observe the estimation performance in the kidnapped case, an experiment was carried out to artificially move the WMR in operation. As shown in Figure 6, the WMR moved along the x-axis starting at $(0.40, 0.65)$m at time 7.3s. When it reached $(5.30, 0.65)$m at time 57.7s, it was forcibly moved to position $(1.35, 4.20)$m and started normal travel in the x-axis direction again at time 65.7s. Then it reached $(5.10, 4.40)$m at time 103.1s and the experiment was terminated. As plotted in Figure 7, EKF, RPF, and PEFFME show similar performance to those in the normal case before the occurrence of the kidnap. However, during the kidnap process, particles in the RPF could not be updated with the correct values, which caused a large estimation error at the time when the WMR started again. Despite the kidnap, the proposed PEFFME exhibits the highest accuracy and convergence among three estimators.

3) Measurement missing case: We evaluated the estimation performance in situation where it is not possible to receive measurements over some arbitrary time intervals. We created the situation by turning off the all of anchors for some period of time. The experimental method is the same as the normal case except that the power was cut for 5 seconds twice in the middle and the results of position estimation are shown in Figure 8. The anchor power was cut off from the time of 16.5s to 21.9s and from the time of 49.1s to 54.2s, and the EKF and RPF diverge in these time periods, as shown in Figure 9. On the other hand, the proposed PEFFME successfully copes with measurement missing, so it shows accurate estimation without divergence.

To compare the performance of the estimators in the entire experiments, we introduce positioning error $e_{pos} = \sqrt{(x-\hat{x})^2 + (y-\hat{y})^2}$. For the three experiments, the root mean square (RMS) of positioning error e_{RMS} and the average

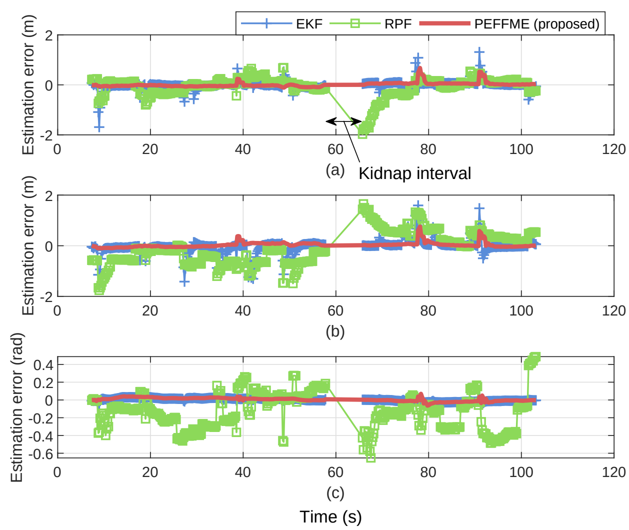

Fig. 7: Estimation errors in kidnapped case. (a) Coordinate x. (b) Coordinate y. (c) Heading angle θ.

computation time per step ΔT_{avg} are analyzed in Table I and Table II, respectively. PEFFME exhibits the smallest RMS of positioning error in the three experiments, demonstrating the highest accuracy and robustness of it. Notice that the average computation time per step of the RPF is long enough to be close to the sampling time, 0.2s. On the other hand, PEFFME takes much less time to produce estimation results, which verifies it is feasible to perform real-time computation.

IV. Conclusion

In this paper, we have proposed the PEFFME, a novel state estimation method for nonlinear systems with high robustness. The PEFFME is designed to solve situations where measurement is not normally received through measurement prediction. It has a FIR structure which is robust to disturbance and is designed to minimize the Frobenius norm of its gain matrices for minimizing the effect of observational error. The

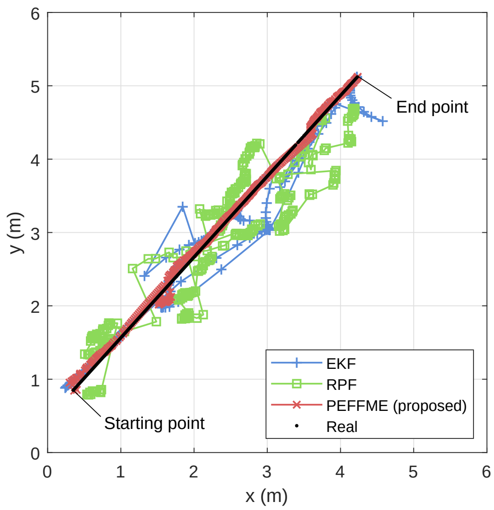

Fig. 8: Localization results in measurement missing case.

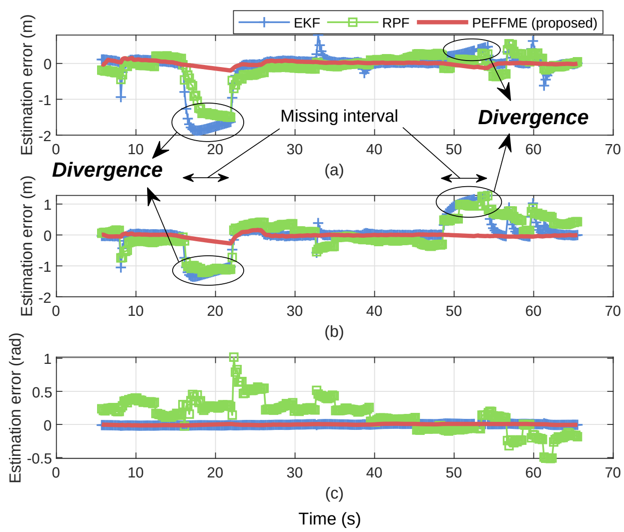

Fig. 9: Estimation errors in measurement missing case. (a) Coordinate x. (b) Coordinate y. (c) Heading angle θ.

TABLE I: Comparison of RMS of positioning error e_{RMS} between the estimators.

	Normal case	Kidnapped case	Measurement missing case
EKF	0.35m	0.37m	0.77m
RPF	0.53m	0.75m	0.69m
PEFFME	0.13m	0.15m	0.10m

TABLE II: Comparison of average computation time per step ΔT_{avg} between the estimators.

	Normal case	Kidnapped case	Measurement missing case
EKF	8.7×10^{-5}s	7.2×10^{-5}s	7.8×10^{-5}s
RPF	0.17s	0.18s	0.17s
PEFFME	4.2×10^{-4}s	3.8×10^{-4}s	4.0×10^{-4}s

performance of PEFFME in kidnapped robot problem and measurement missing, which are realistic in mobile robot localization, is compared with existing estimators. And it is shown to have superior accuracy and robustness and sufficient real-time performance. The proposed PEFFME can be widely used for estimation problems, such as object tracking, in which measurements are received conditionally. We expect PEFFME to help solve estimation problems in various fields.

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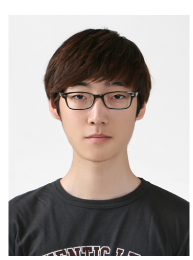

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