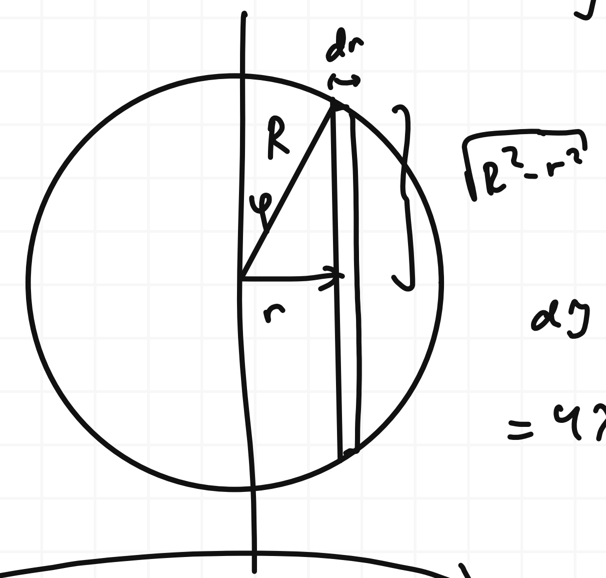
Неделя 5.

010

J - ?

m, R



$\exists\, \lambda$ – плотность ед. площади

$$dJ = r^2 dm$$

$$dm = 4\lambda \sqrt{R^2-r^2}\, dr$$

$$dJ = 4\lambda R \sqrt{1-\left(\frac{r}{R}\right)^2}\, r^2 dr =$$

$$= 4\lambda R^4 \sqrt{1-\left(\frac{r}{R}\right)^2}\left(\frac{r}{R}\right)^2 d\left(\frac{r}{R}\right) =$$

$$\frac{r}{R} = \sin\varphi$$

$$\lambda = \frac{m}{\pi R^2}$$

$$= 4\frac{m}{\pi R^2}\cdot R^4 \cos\varphi \cdot \sin^2\varphi\, d\sin\varphi =$$

$$= \frac{4mR^2}{\pi} \sin^2\varphi \cos^2\varphi\, d\varphi$$

$$// \sin^2\varphi\cos^2\varphi\, d\varphi = \frac{1}{4}\sin^2 2\varphi\, d\varphi = \frac{1}{8}\sin^2 2\varphi\, d(2\varphi)$$

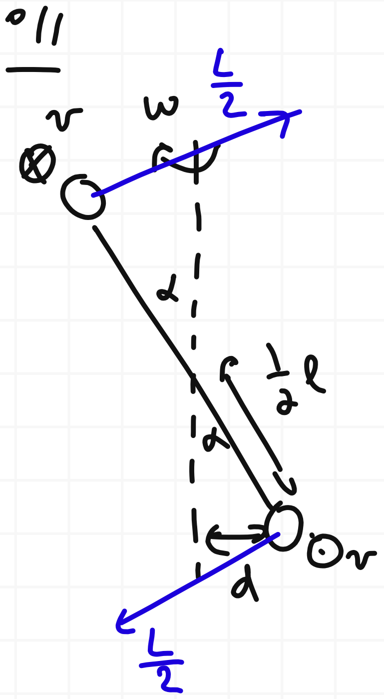
$$\frac{1}{8}\int_0^{\pi/2} \sin^2(2\varphi)\, d(2\varphi) = \frac{1}{8}\int_0^{\pi} \sin^2\alpha\, d\alpha = \frac{1}{32}\int_0^{2\pi}(1-\cos\beta)\, d\beta$$

$$= \frac{1}{32}(2\pi - 0) = \frac{\pi}{16} //$$

$$= \frac{\cancel{4}mR^2}{\cancel{\pi}}\cdot\frac{\cancel{\pi}}{\cancel{16}_{\,4}} = \frac{mR^2}{4}$$

Ответ: $\frac{mR^2}{4}$

011



$d = \frac{1}{2}l\sin\alpha$; $r = \frac{1}{2}l$

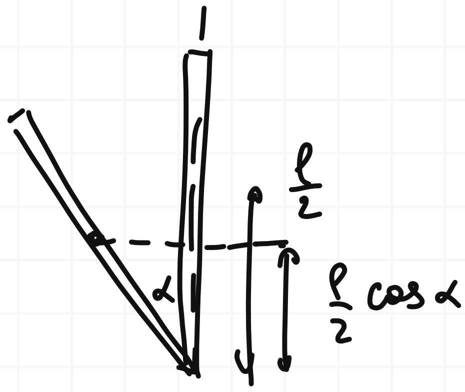
$$v = \omega d = \frac{1}{2}\omega l \sin\alpha$$

$$\vec{L} = [\vec{r}, \vec{p}] = m[\vec{r}, \vec{v}]$$

$$|\vec{L}| = L = 2mrv\sin 90^\circ = 2mrv = \cancel{2}m\,\frac{1}{\cancel{2}}l\cdot\frac{1}{2}\omega l\sin\alpha =$$

$$= \frac{1}{2}m\omega l^2\sin\alpha$$

№12



$$\frac{1}{2}J\omega^2 = \frac{1}{2}\frac{m\ell^2}{3}\cdot\omega^2 = mg\frac{\ell}{2}(1-\cos\alpha)$$

$$\frac{\cancel{m}\ell^2\omega^2}{\cancel{6}\,3} = \frac{\cancel{m}g\ell}{\cancel{2}}(1-\cos\alpha)$$

$$\omega^2 = \frac{3g}{\ell}(1-\cos\alpha)$$

$$\omega = \sqrt{3\frac{g}{\ell}(1-\cos\alpha)}$$

$$\varepsilon = \frac{d\omega}{dt} = \frac{d\omega}{d\alpha}\cdot\frac{d\alpha}{dt} = \frac{\omega\, d\omega}{d\alpha} \ominus$$

$$\left[\frac{d\omega}{d\alpha} = \sqrt{3\frac{g}{\ell}}\cdot\frac{\sin\alpha}{2\sqrt{1-\cos\alpha}} = \frac{1}{2}\sqrt{3\frac{g}{\ell}}\frac{\sin\alpha}{\sqrt{1-\cos\alpha}}\right]$$

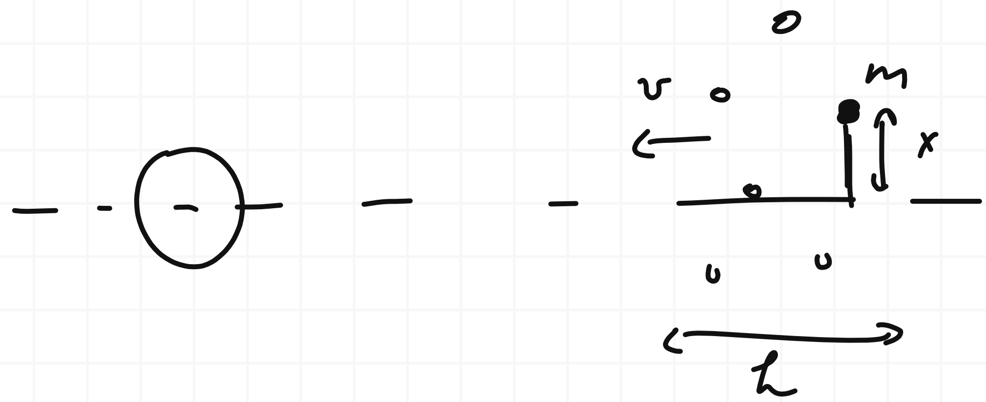
$$\ominus \sqrt{3\frac{g}{\ell}\cancel{(1-\cos\alpha)}}\cdot\frac{1}{2}\sqrt{3\frac{g}{\ell}}\frac{\sin\alpha}{\cancel{\sqrt{1-\cos\alpha}}} = \frac{3g}{2\ell}\sin\alpha$$

7.85.

$v = 5$ км/с

$h = 1000$

$n = 0{,}1$ км$^{-3}$



ЗСМИ: $\cancel{m}vx = \cancel{m}v_3R_3$; $x = \frac{v_3}{v}R_3$

ЗСЭ: $\frac{\cancel{m}v^2}{\cancel{2}} = \frac{\cancel{m}v_3^2}{\cancel{2}} - \frac{\cancel{2}\gamma M_3\cancel{m}}{\cancel{2}R_3}$; $v_3^2 = v^2 + \frac{2\gamma M_3}{R_3}$

$$x = \frac{\sqrt{v^2+\frac{2\gamma M_3}{R_3}}}{v}R_3$$

10 м/с $= \frac{0{,}01 \text{ км}}{c}$

$$N = \pi x^2\cdot h\cdot n = \frac{\pi n h R_3^2}{v^2}\left(v^2 + \underbrace{\frac{2\gamma M_3}{R_3}}_{2gR_3}\right) =$$

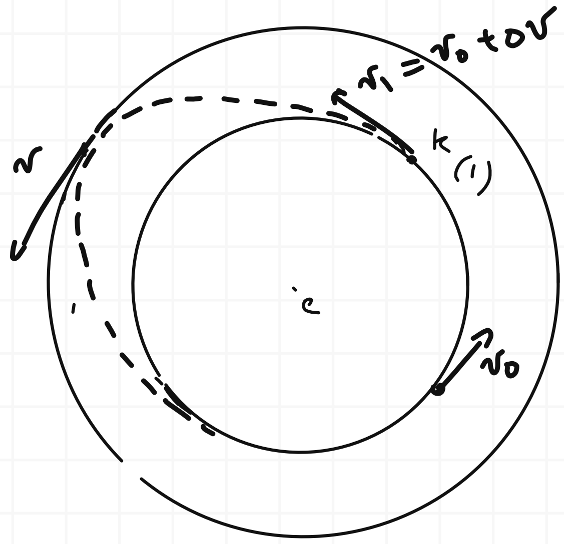
$$= \frac{3.14\cdot 0{,}1\text{ км}^{-3}\cdot 1000\text{ км}\cdot(6400\text{ км})^2}{(5\text{ км/с})^2}\left((5\text{ км/с})^2 + 2\cdot 0{,}01\frac{\text{км}}{c^2}\cdot 6400\text{ км}\right)$$

$$= 7{,}8\cdot 10^{10} \text{ (метеоритов)}$$

7.61 $R_З = 1{,}5\cdot 10^8$ км

$t - ?$ $R_М = 2{,}28\cdot 10^8$ км

$\gamma M_c = 1325\cdot 10^8\ \frac{км^3}{c^2}$



$$\frac{\cancel{m} v_0^2}{\cancel{R_З}} = \frac{\gamma \cancel{m} M_c}{R_З^{\cancel{2}}} \ ; \quad v_0 = \sqrt{\frac{\gamma M_c}{R_З}}$$

ЗСМИ (1), (2), отн. Солнца:

$$\cancel{m}(v_0 + \Delta v) R_З = \cancel{m} v_2 R_М \ ; \quad v_2 = \frac{R_З}{R_М}(v_0 + \Delta v)$$

ЗСЭ (1), (2):

$$\frac{\cancel{m}(v_0+\Delta v)^2}{2} - \frac{2\gamma \cancel{m} M_c}{2R_З} = \frac{\cancel{m} v_2^2}{2} - \frac{2\gamma \cancel{m} M_c}{2R_М} \quad |\cdot 2$$

] $v_1 = v_0 + \Delta v$

$$v_1^2 = v_2^2 + \frac{2\gamma M_c}{R_З} - \frac{2\gamma M_c}{R_М} = \left(\frac{R_З}{R_М}\right)^2 v_1^2 + 2\gamma M_c\left(\frac{1}{R_З} - \frac{1}{R_М}\right)$$

$$\left(1 - \left(\frac{R_З}{R_М}\right)^2\right) v_1^2 = 2\gamma M_c\left(\frac{1}{R_З} - \frac{1}{R_М}\right)$$

$$v_1 = \sqrt{\frac{2\gamma M_c\left(\frac{1}{R_З} - \frac{1}{R_М}\right)}{1 - \left(\frac{R_З}{R_М}\right)^2}} - \sqrt{\frac{\gamma M_c}{R_З}} =$$

$$= \sqrt{\frac{2\cdot 1325\cdot 10^8\ км^3/c^2\left(\frac{1}{1{,}5\cdot 10^8\ км} - \frac{1}{2{,}28\cdot 10^8\ км}\right)}{1 - \left(\frac{1{,}5}{2{,}28}\right)^2}} - \sqrt{\frac{1325\cdot 10^8\ км^3/c^2}{1{,}5\cdot 10^8\ км}} =$$

37,675 ≈

$$= 2{,}92\ \frac{км}{c}$$

t найдем из закона Кеплера

??

Т.2

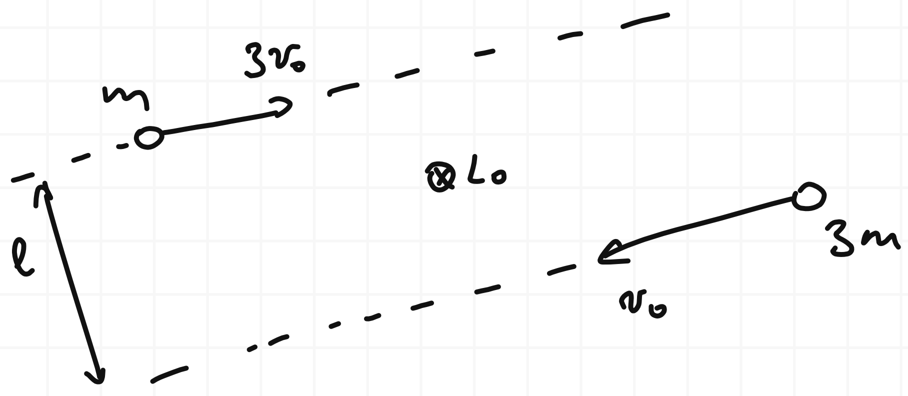
ЗСЭ + ЗСМИ

m, 3m

l

$3v_0$, v_0

$l_0 = l_{min}$ – ?



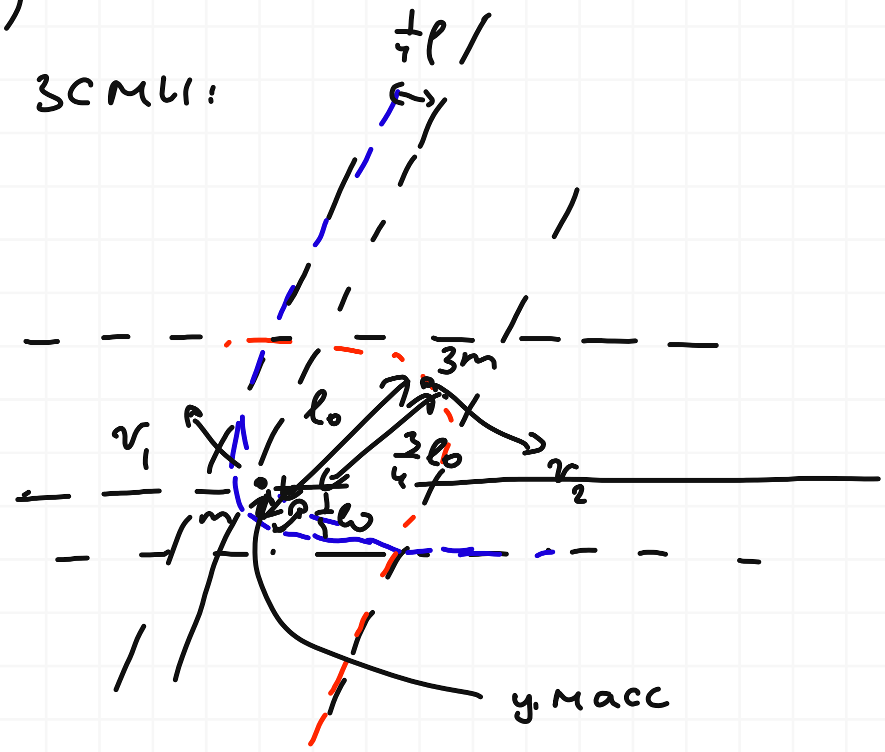
ЗСЭ: $\frac{\cancel{m}(3v_0)^2}{2} + \frac{3\cancel{m}v_0^2}{2} = \frac{\cancel{m}v_1^2}{2} + \frac{3\cancel{m}v_2^2}{2} - \frac{\gamma \cancel{m}(3m)}{l_0^2}$

$$12v_0^2 = v_1^2 + 3v_2^2 - \frac{6\gamma m}{l_0^2} \quad (2)$$

ЗСИ: $\sum \vec{p} = 0$ в $\forall$ момент времени

$$\cancel{m}v_1 = 3\cancel{m}v_2; \quad v_1 = 3v_2 \quad (1)$$

ц. масс неподвижен, запишем ЗСМИ:



$$L_0 = 3mv_0\frac{3}{4}l + 3mv_0\frac{1}{4}l = 3mv_0 l$$

$$L = 3mv_2\frac{3}{4}l_0 + mv_1\frac{1}{4}l_0 \overset{(1)}{=}$$

$$= \frac{9}{4}mv_2 l_0 + \frac{3}{4}mv_2 l_0 = 3mv_2 l_0$$

$$v_2 = \frac{l}{l_0}v_0$$

(2): $12v_0^2 = 9v_2^2 + 3v_2^2 - \frac{6\gamma m}{l_0}$

$$2v_0^2 = 2v_2^2 - \frac{\gamma m}{l_0} = 2\cdot\frac{l^2}{l_0^2}v_0^2 - \frac{\gamma m}{l_0} \quad \Big| \cdot l_0^2 \neq 0$$

$$2v_0^2 l_0^2 = 2l^2 v_0^2 - \gamma m l_0 \quad \Big| : 2v_0^2 \neq 0$$

$$l_0^2 + \frac{\gamma m}{2v_0^2}l_0 - l^2 = 0$$

$$D = \frac{\gamma^2 m^2}{4v_0^4} + 4l^2$$

$l_0 > 0$

$$\underline{l_0} = \frac{1}{2}\left(-\frac{\gamma m}{2v_0^2} + \sqrt{\frac{\gamma^2 m^2}{4v_0^4} + 4l^2}\right) = \sqrt{\frac{\gamma^2 m^2}{16v_0^4} + l^2} - \frac{\gamma m}{4v_0^2} =$$

$$= l\left(\sqrt{\left(\frac{\gamma m}{4lv_0^2}\right)^2 + 1} - \frac{\gamma m}{4lv_0^2}\right) = \left[v^* = \sqrt{\frac{\gamma m}{4l}}\right] =$$

$$= \underline{l\left(\sqrt{\frac{v^{*4}}{v_0^2} + 1} - \frac{v^{*2}}{v_0^2}\right)}$$

T.3

$r_0 = 1{,}5 \cdot 10^8$ км

$\Delta v = 15$ км/с

$T - ?$

$$\frac{\cancel{m} v_0^2}{\cancel{r_0}} = \frac{\gamma \cancel{m} M}{r_0^{\cancel{2}}} \; ; \quad v_0^2 = \frac{\gamma M}{r_0} \; ;$$

аналогично:

$$v^2 = \frac{\gamma M}{r}$$

$$\frac{\cancel{m}(v_0^2 + \Delta v^2)}{\cancel{2}} - \frac{2\gamma \cancel{m} M}{\cancel{2} r_0} = \frac{\cancel{m} v^2}{\cancel{2}} - \frac{2\gamma \cancel{m} M}{\cancel{2} r}$$

$$v_0^2 + \Delta v^2 - 2\frac{\gamma M}{r_0} = v^2 - \frac{2\gamma M}{r} = -\frac{\gamma M}{r}$$

$$\Delta v^2 - v_0^2 = -v^2$$

$$\frac{\gamma M}{r} + \Delta v^2 = \frac{\gamma M}{r_0} \quad \Big| : \gamma M$$

$$\frac{1}{r} = \frac{1}{r_0}\left(1 - \frac{\Delta v^2 r_0}{\gamma M}\right)$$

$$\frac{1}{\alpha} = \frac{r_0}{r} = 1 - \frac{\Delta v^2 r_0}{\gamma M} = 1 - \left(\frac{15 \text{ км}}{29{,}8 \text{ км}}\right)^2$$

$$\alpha = 1{,}34$$

$$\frac{T^2}{T_0^2} = \frac{r^3}{r_0^3} = \alpha^3 \; ; \quad \frac{T}{T_0} = \alpha^{3/2} = 1{,}55 \; ; \quad \underline{T = 1{,}55 \text{ лет} = \approx 565 \text{ суток}}$$

T.4

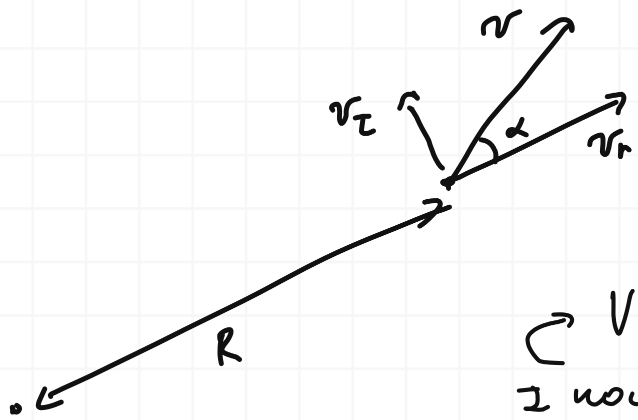
$R = 1$ а.е.

$v = 50 \frac{\text{км}}{\text{с}}$

$v_r = 40 \frac{\text{км}}{\text{с}}$

$r_p - ?$

$V = 30 \frac{\text{км}}{\text{с}}$



$$\cos\alpha = \frac{v_r}{v} = \frac{4}{5}$$

$$\sin\alpha = \frac{3}{5}$$

$$V = \sqrt{\frac{\gamma M}{R}} \; ; \quad \frac{\gamma M}{R} = V^2$$

I косм. скорость

ЗСМИ: $\quad m v R \sin\alpha = \frac{3}{5} \cancel{m} v R = \cancel{m} v^* r_p \; ; \quad 3vR = 5v^* r_p$

ЗСЭ: $\quad \frac{\cancel{m} v^2}{\cancel{2}} - \frac{2\gamma \cancel{m} M}{\cancel{2} R} = \frac{\cancel{m} v^{*2}}{\cancel{2}} - \frac{2\gamma \cancel{m} M \cdot R}{2 r_p \cdot R} \qquad v^* = \frac{3R}{5r_p} v$

$$v^2 - 2V^2 = v^{*2} - 2V^2 \frac{R}{r_p}$$

$$2500 - 2 \cdot 900 = v^{*2} - 2 \cdot \cancel{900}^{30} \cdot \frac{v^*}{\cancel{30}} = 0$$

$$v^{*2} - 60 v^* - 700 = 0$$

$$\frac{D}{4} = 1600 = 40^2$$

$$\underline{v^*} = 30 \oplus 40 = \underline{70 \frac{\text{км}}{\text{с}}}$$

$$\underline{r_p} = \frac{3R v_r}{5 v^*} = \underline{0{,}43 \text{ а.е.}}$$

7.139

$\rho_1 = 12 \frac{г}{см^3}$

$\rho_2 = 3 \frac{г}{см^3}$

$R_3 = 6400 км$

$\frac{g_{max}}{g} - ?$

$$\rho(r) = \rho_1 - \frac{\rho_1 - \rho_2}{R_3} r$$

$$g = \frac{\gamma M_3}{R_3^2}$$

$$dM = 4\pi r^2 \rho \, dr = 4\pi \rho r^2 dr = 4\pi \left(\rho_1 - \frac{\rho_1 - \rho_2}{R_3} r\right) r^2 dr$$

$$M = \frac{4}{3}\pi \rho_1 r^3 - 4\pi \frac{\rho_1 - \rho_2}{R_3} \frac{r^4}{4} = 4\pi r^3 \left(\frac{1}{3}\rho_1 - \frac{1}{4}\frac{r}{R_3}(\rho_1 - \rho_2)\right)$$

$$g(r) = \gamma \frac{M}{r^2} = \frac{\gamma}{\cancel{r^2}} \cdot 4\pi r^3 \left(\frac{1}{3}\rho_1 - \frac{1}{4}\frac{r}{R_3}(\rho_1 - \rho_2)\right) =$$

$$= 4\pi \gamma r \left(\frac{1}{3}\rho_1 - \frac{1}{4}\frac{r}{R_3}(\rho_1 - \rho_2)\right)$$

g_{max} $\quad g'(r) = 0$

$$g'(r) = 4\pi\gamma \cdot \frac{1}{3}\rho_1 - \frac{4\pi\gamma(\rho_1 - \rho_2)}{4R_3} \cdot 2r =$$

$$= 4\pi\gamma \left(\frac{1}{3}\rho_1 - \frac{1}{2}\frac{\rho_1 - \rho_2}{R_3} r\right) = 0$$

$$\frac{1}{3}\rho_1 = \frac{1}{2}\frac{\rho_1 - \rho_2}{R_3} r_m; \quad r_m = \frac{2}{3}\frac{\rho_1 R_3}{\rho_1 - \rho_2}$$

$$\frac{g_{max}}{g} = \frac{\cancel{4\pi\gamma} \cdot \frac{2}{3}\frac{\rho_1 \cancel{R_3}}{\rho_1 - \rho_2}\left(\frac{\cancel{2}}{\cancel{6}}\rho_1 - \frac{1}{\cancel{4}_2} \cdot \frac{\cancel{2}\rho_1 \cancel{R_3}}{3(\cancel{\rho_1 - \rho_2})} \cdot \frac{\cancel{\rho_1 - \rho_2}}{\cancel{R_3}}\right)}{\cancel{4\pi\gamma} \cancel{R_3}\left(\frac{1}{3}\rho_1 - \frac{1}{4}(\rho_1 - \rho_2)\right)} =$$

$$= \frac{\frac{2}{3}\frac{\rho_1}{\rho_1 - \rho_2} \cdot \frac{1}{3\cancel{6}}\rho_1}{\frac{1}{12}\rho_1 + \frac{1}{4}\rho_2} = \frac{\frac{2 \cdot 12}{3 \cdot 9} \cdot \frac{\cancel{12}^{2}}{\cancel{6}}}{1 + \frac{3}{4}} = \frac{2 \cdot 2 \cdot 12 \cdot 4}{3 \cdot 9 \cdot 7} = \underline{1,016}$$