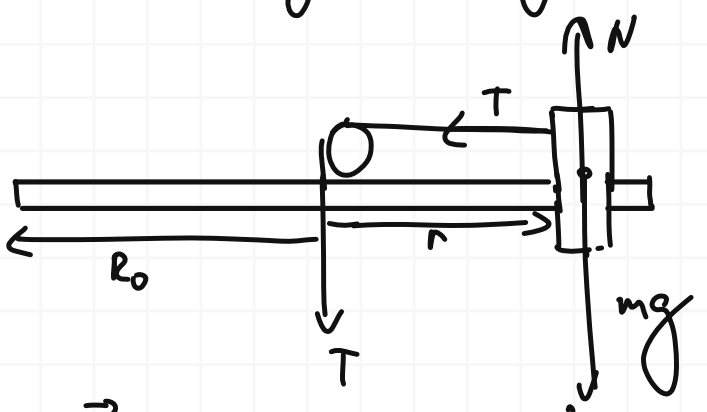
Неделя 6.

Момент импульса. Вращение твердого тела вокруг неподвижной оси.

6.9

ω_0



$J(r) = mr^2$

ЗСМИ, т.к. $\sum \vec{M} = 0$: $m\omega_0 R_0^2 = m\omega r^2$; $\omega = \frac{R_0^2}{r^2}\omega_0$

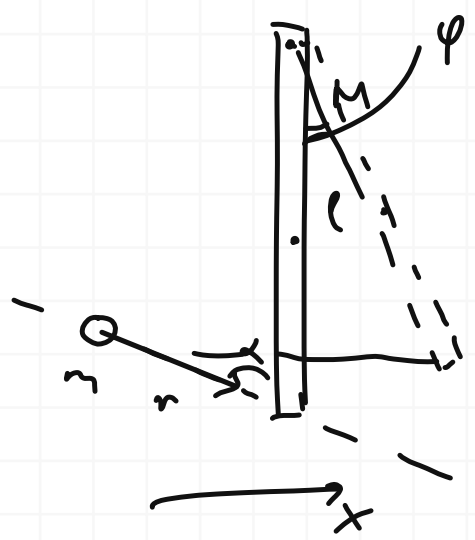
II З.М.: $T = m\omega^2 r$; $T(r) = m\cdot\left(\frac{R_0^2}{r^2}\omega_0\right)^2 r = \frac{R_0^4}{r^3} m\omega_0^2 R_0 = m\omega_0^2 R_0^4 \cdot \frac{1}{r^3}$

ЗИЭ: $A = \frac{1}{2} J\omega^2 - \frac{1}{2} J_0\omega_0^2 = 2m\omega_0^2 R_0^2 - \frac{1}{2} m_0\omega_0^2 R^2 = \frac{3}{2} m\omega_0^2 R_0^2$

// $\frac{1}{2} m\left(\frac{R_0}{2}\right)^2 \cdot \left(\frac{R_0^2}{(R_0/2)^2}\omega_0\right)^2 = \frac{1}{4} m\frac{R_0^2}{4} \cdot 16\omega_0^2 = 2m\omega_0^2 R_0^2$ //

9.105

M, ℓ, m, v, α



$mv\ell$

$L = J\omega$

ЗИМИ: $mv\sin\alpha\,\ell = J\omega = \left(\frac{1}{3}M\ell^2 + m\ell^2\right)\omega$

$\omega = \frac{mv\sin\alpha\,\ell}{\left(\frac{1}{3}M + m\right)\ell^2} = \frac{m\sin\alpha}{\frac{1}{3}M + m}\cdot\frac{v}{\ell}$

$\frac{1}{2} J\omega^2 = Mg\frac{\ell}{2}(1-\cos\varphi) + mg\ell(1-\cos\varphi) = \left(\frac{1}{2}M + m\right)g\ell(1-\cos\varphi)$

$1-\cos\varphi = \frac{\frac{1}{2}\left(\frac{1}{3}M+m\right)\ell^2 \cdot m^2\sin^2\alpha\, v^2}{\left(\frac{1}{3}M+m\right)^2 \ell^2 \cdot \left(\frac{1}{2}M+m\right)g\ell} = \frac{\frac{1}{2}m^2v^2\sin^2\alpha}{g\ell\left(\frac{1}{3}M+m\right)\left(\frac{1}{2}M+m\right)}$

$\cos\varphi = 1 - \frac{3(mv\sin\alpha)^2}{g\ell(M+3m)(M+2m)}$

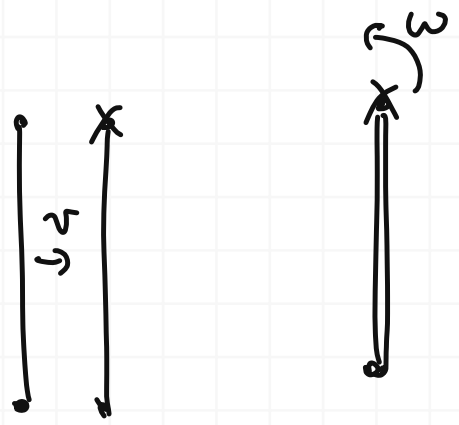
9.126

$\ell = 0{,}3$ м

$k = 0{,}1$

$v = 3$ м/с

$n - ?$



$$J = \frac{1}{3} m\ell^2 \cdot 2 = \frac{2}{3} m\ell^2$$

ЗСМИ: $\quad mv \cdot \frac{\ell}{2} = J\omega_0 = \frac{2}{3} m\ell^2 \omega_0 ; \quad \omega_0 = \frac{\frac{1}{2} mv\ell}{\frac{2}{3} m\ell^2} = \frac{3}{4} \frac{v}{\ell}$

$$A_{тр} = \Delta W_{кин} = -\frac{1}{2} J\omega_0^2 = -\frac{1}{2} \cdot \frac{2}{3} m\ell^2 \cdot \frac{3}{16} \frac{v^2}{\ell^2} = -\frac{3}{16} mv^2$$

$$dA_{тр_i} = F(r) \cdot r\, d\varphi = -k m_i g \cdot r_i\, d\varphi ; \quad A_{тр_i} = -k m_i g r_i \cdot \varphi =$$

$$= -k g m_i r_i \cdot \varphi$$

$$\int dA_{тр} = -\int_0^{\ell} k g \varphi \cdot \lambda r\, dr =$$

$$= -\frac{2m}{\ell} k g \varphi \int_0^{\ell} r\, dr = -2 k m g \varphi \cdot \frac{\ell^2}{2\ell} = -k m g \ell \varphi =$$

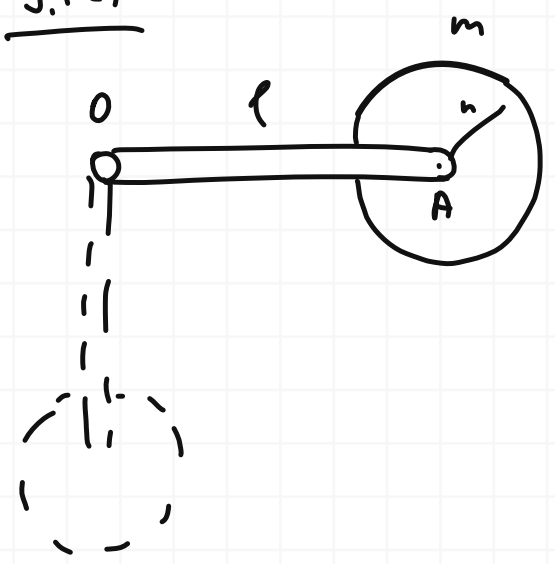
$$= -k(2m) g \cdot \underbrace{\frac{1}{2} \varphi}_{\text{Sy.m.}}$$

n – колич. оборотов

$$\varphi = 2\pi n$$

$$+\frac{3}{16} m v^2 = + k m g \ell \cdot 2\pi n ; \quad n = \frac{3v^2}{32\pi k g \ell}$$

9.121



ЗСЭ: $\quad mg\ell = \frac{1}{2} m v^2 ; \quad v = \sqrt{2g\ell}$

$$\omega_1 = \frac{v}{\ell} = \sqrt{\frac{2g}{\ell}}$$

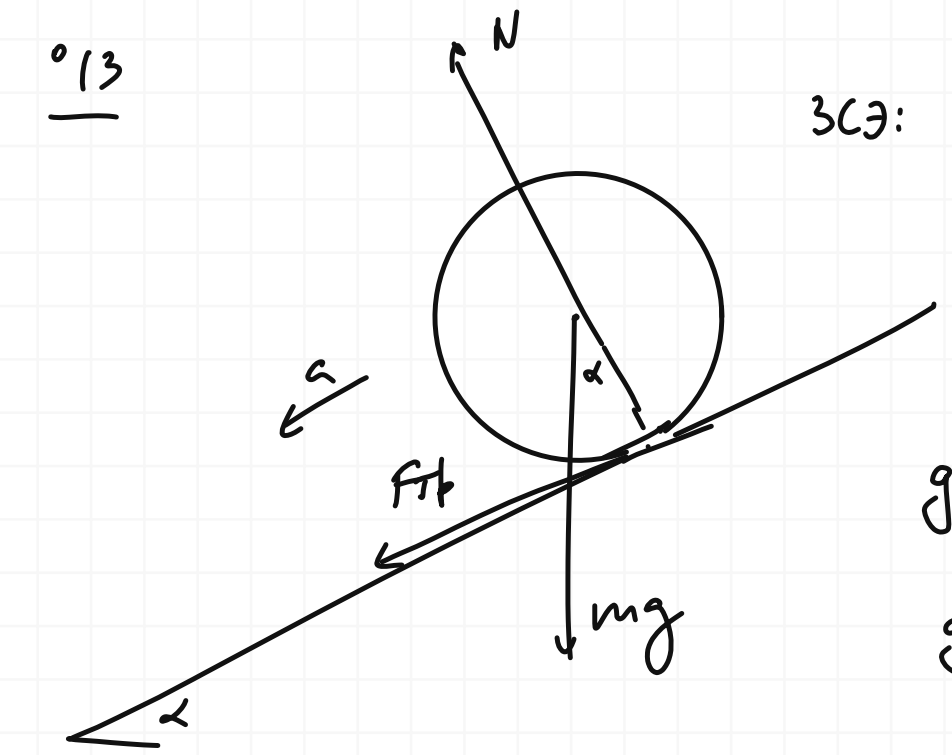
ЗСМИ: $\quad m\ell^2 \sqrt{\frac{2g}{\ell}} = m\left(\frac{1}{2} r^2 + \ell^2\right) \omega_2$

$$\omega_2 = \sqrt{\frac{2g}{\ell}} \cdot \frac{\ell^2}{\frac{1}{2} r^2 + \ell^2} ; \quad \omega_2^2 = \frac{2g \cdot \ell^3}{\left(\frac{1}{2} r^2 + \ell^2\right)^2}$$

ЗСЭ: $\quad mgx = \frac{1}{2}\left(\frac{1}{2} m r^2 + m\ell^2\right) \omega_2^2$

$$x = \frac{1}{2g}\left(\frac{1}{2} r^2 + \ell^2\right) \omega^2 = \frac{\left(\frac{1}{2} r^2 + \ell^2\right) \cdot 2g\ell^3}{2g\left(\frac{1}{2} r^2 + \ell^2\right)^2} = \frac{\ell^3}{\frac{1}{2} r^2 + \ell^2}$$

№13



ЗСЭ: $mgh = \frac{1}{2} J\omega^2 + \frac{1}{2} mv^2 =$

$= \frac{1}{2} \cdot \frac{2}{3} mv^2 + \frac{1}{2} mv^2 = \frac{5}{6} mv^2$

$h = l \sin\alpha$; $l = \frac{at^2}{2}$; $v = at$

$gl\sin\alpha = \frac{g a t^2 \sin\alpha}{2} = \frac{5}{6} a^2 t^2$

$g\sin\alpha = \frac{5}{3} a$; $a = \frac{3}{5} g\sin\alpha$

№14

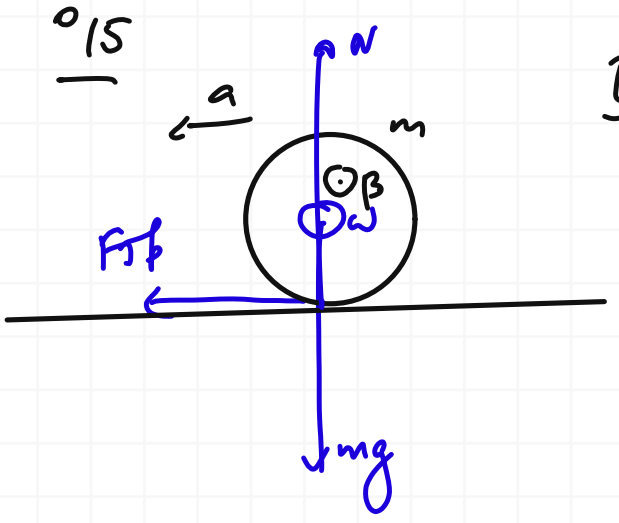
$K = \frac{1}{2} mv^2 + \frac{1}{2} J\omega^2$

$K_м = \frac{1}{2} mv^2 + \frac{1}{2} \cdot \frac{2}{5} mv^2 = \frac{7}{10} mv^2$

$K_у = \frac{1}{2} mv^2 + \frac{1}{2} \cdot \frac{1}{2} mv^2 = \frac{3}{4} mv^2$

$\frac{K_м}{K_у} = \frac{7 \cdot 4}{10 \cdot 3} = \frac{14}{15}$

№15



II З.Н.: $ma = F_{тр} = \mu mg$; $a = \mu g$; $v = \mu gt$

$M = J\beta$; $\beta = \frac{M}{J} = \frac{-\mu mgR}{\frac{2}{5} mR^2} = -\frac{5}{2} \frac{\mu g}{R}$

$\begin{cases} dv = \mu g\, dt \\ d\omega = -\frac{5}{2} \frac{\mu g}{R} dt \end{cases}$ $\begin{cases} v = \mu g\tau; \ \tau = \frac{v}{\mu g} \\ \frac{v}{R} - \omega = -\frac{5}{2} \frac{\mu g}{R} \tau \end{cases}$

$\omega - \frac{v}{R} = \frac{5}{2} \frac{\mu g}{R} \cdot \frac{v}{\mu g}$; $\omega = \frac{5}{2} \frac{v}{R} + \frac{v}{R} = \frac{7}{2} \frac{v}{R}$

$v = \frac{2}{7} \omega R$