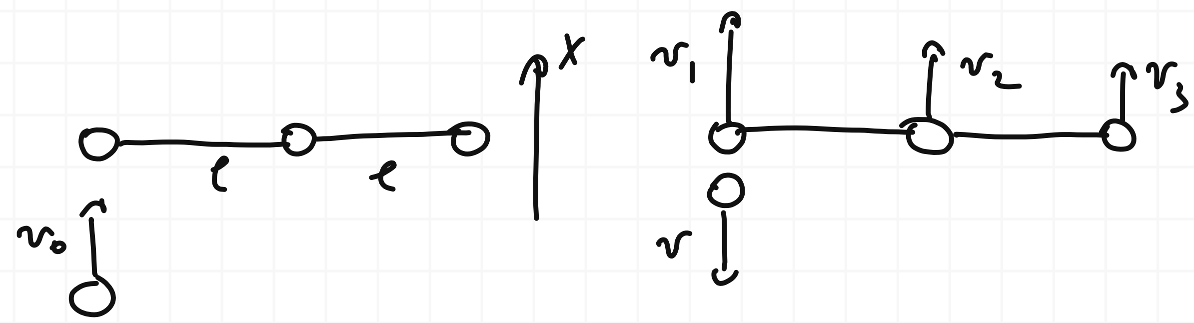
Неделя 7.

Плоское движение твердого тела. Качение.

6.15

v_0, l



ЗСИ$_x$: $v_0 = v_x + v_{1x} + v_{2x} + v_{3x}$

$v_0 = v_x + \underbrace{3v_c}_{\text{центра масс, т.е. II шарика}}$; $v_x = v_0 - 3v_c$

ЗСЭ: $v_0^2 = v^2 + v_1^2 + v_2^2 + v_3^2$

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + \frac{3}{2}mv_c^2 + \frac{1}{2}J\omega^2$; $J = 2\cdot ml^2$

$\frac{1}{2}\cancel{m}v_0^2 = \frac{1}{2}\cancel{m}v^2 + \frac{3}{2}\cancel{m}v_c^2 + \cancel{m}l^2\omega^2$

$v^2 = v_0^2 - 3v_c^2 - 2\omega^2 l^2$

ЗСМИ: $\Delta L = \Delta p \cdot l = J\omega$

$\cancel{m}(v_0 + v)\cancel{l} = 2\cancel{m}l^{\cancel{2}}\omega$; $v = 2\omega l - v_0$

$$\begin{cases} v = -v_x = 3v_c - v_0 \\ v^2 = v_0^2 - 3v_c^2 - 2\omega^2 l^2 \\ v = 2\omega l - v_0 \end{cases}$$

$3v_c - \cancel{v_0} = 2\omega l - \cancel{v_0}$

$v_c = \frac{2}{3}\omega l$

$(2\omega l - v_0)^2 = 4\omega^2 l^2 + v_0^2 - 4v_0\omega l =$

$= v_0^2 - \frac{4}{3}\omega^2 l^2 - 2\omega^2 l^2 = v_0^2 - (3 + \frac{1}{3})\omega^2 l^2$

$4\omega^{\cancel{2}} l^{\cancel{2}} - 4v_0 \cancel{\omega l} = (-3 - \frac{1}{3})\omega^{\cancel{2}} l^{\cancel{2}}$

$(7 + \frac{1}{3})\omega l = 4v_0$; $\frac{\overset{11}{\cancel{22}}}{3}\omega l = \overset{2}{\cancel{4}}v_0$; $\underline{\omega = \frac{6}{11}\frac{v_0}{l}}$

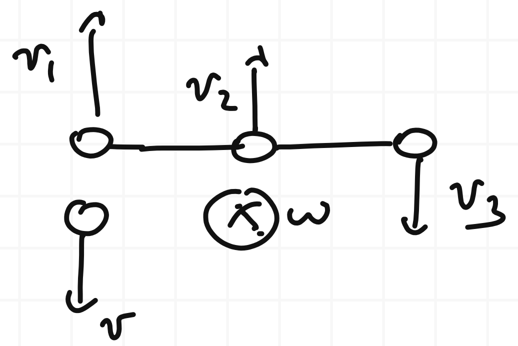
$\underline{v_c} = \frac{2}{3}\omega l = \frac{2}{\cancel{3}}\cdot\frac{\overset{2}{\cancel{6}}}{11}v_0 = \underline{\frac{4}{11}v_0}$

$$\underline{v_{1x}} = v_c + \omega l = \frac{4}{11}v_0 + \frac{6}{11}v_0 = \underline{\frac{10}{11}v_0}$$

$$\underline{v_{3x}} = v_c - \omega l = \frac{4}{11}v_0 - \frac{6}{11}v_0 = \underline{-\frac{2}{11}v_0}$$

$$\underline{v_{2x}} = v_c = \underline{\frac{4}{11}v_0}$$

$$\underline{v_{4x}} = v_x = v_0 - 2\omega l = v_0 - \frac{12}{11}v_0 = \underline{-\frac{1}{11}v_0}$$

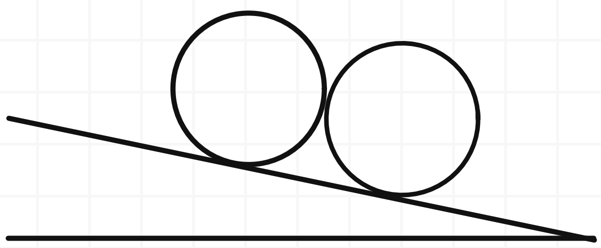


$$\underbrace{\begin{aligned} v_1 &= \frac{10}{11}v_0 \\ v_2 &= \frac{4}{11}v_0 \\ v_3 &= \frac{2}{11}v_0 \\ v &= \frac{1}{11}v_0 \end{aligned} \qquad \omega = \frac{6}{11}\frac{v_0}{l}}_{\text{ответ}}$$

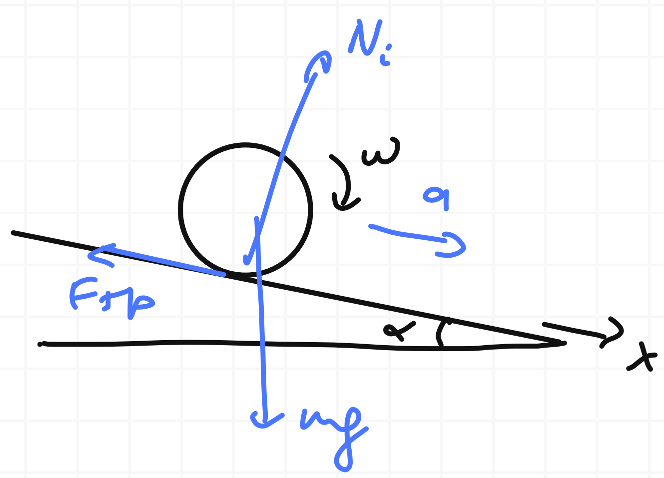
<u>9.71</u>

$$J_n = mr^2 = 2J_0$$

$$J_c = \frac{1}{2}mr^2 = J_0$$



нужно расположить цилиндр, который сам по себе скатывался быстрее друга



$$N = mg\cos\alpha\ ;\ F_{тр} = kN = kmg\cos\alpha$$

$$ma = mg\sin\alpha - F_{тр} = mg(\sin\alpha - k\cos\alpha)$$

$$a = g(\sin\alpha - k\cos\alpha)$$

$$M = F_{тр}\cdot R = kmg\cos\alpha \cdot R = J\beta = \frac{d\omega}{dt}J$$

$$\begin{cases} \frac{dv}{dt} = g(\sin\alpha - k\cos\alpha) \\ kmg\cos\alpha R^2 = J\frac{R\,d\omega}{dt} = J\frac{dv}{dt} \end{cases} \qquad \begin{cases} \frac{dv}{dt} = g(\sin\alpha - k\cos\alpha) \\ \frac{dv}{dt} = \frac{1}{J}kmg\cos\alpha R^2 \end{cases}$$

без проск.

$$Jg(\sin\alpha - k\cos\alpha) = kmg\cos\alpha R^2 \qquad |:\cos\alpha$$

$$Jg(\operatorname{tg}\alpha - k) = kmgR^2$$

$$\operatorname{tg}\alpha - k = \frac{kmgR^2}{gJ}; \qquad \operatorname{tg}\alpha = k\left(1+\frac{mR^2}{J}\right)$$

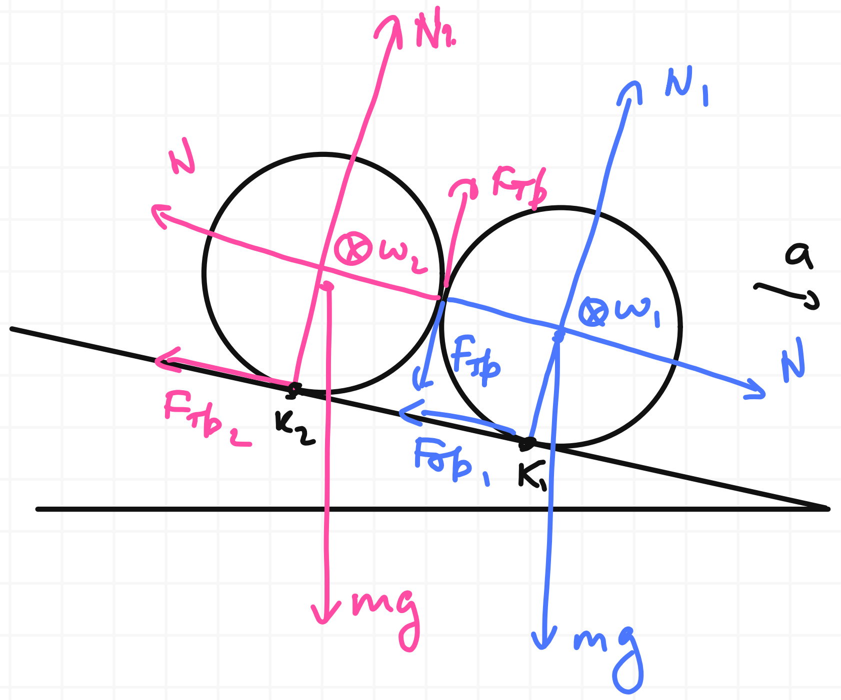
$$k_n = \frac{\operatorname{tg}\alpha}{1+\frac{mR^2}{2J_0}} = \frac{\operatorname{tg}\alpha}{1+\frac{mR^2}{mR^2}} = \frac{1}{2}\operatorname{tg}\alpha$$

$k_n > k_c$

$$k_c = \frac{\operatorname{tg}\alpha}{1+\frac{mR^2}{J_0}} = \frac{\operatorname{tg}\alpha}{1+\frac{mR^2}{\frac{1}{2}mR^2}} = \frac{1}{3}\operatorname{tg}\alpha$$

$a_n < a_c$

сплошной должен располагаться за полым



$$N_1 = N_2 = mg\cos\alpha$$

$$F_{тр_i} = k_i N_i = k_i mg\cos\alpha$$

$$k_1 = k_n = \frac{1}{2}\operatorname{tg}\alpha$$

$$k_2 = k_c = \frac{1}{3}\operatorname{tg}\alpha$$

$$F_{тр} = kN$$

$$M = J\beta = J\frac{d\omega}{dt}; \quad M = F_i \cdot R; \quad J\frac{d\omega}{dt} = F_i R$$

(k_1) $$J_1^{k_1}\frac{d\omega_1}{dt} = (mg\sin\alpha + N - kN)R$$

$$(mR^2 + mR^2)\frac{d\omega_1}{dt} = (N(1-k) + mg\sin\alpha)R$$

$$2m\frac{R\,d\omega_1}{dt} = 2m\frac{dv_1}{dt} = mg\sin\alpha + N(1-k)$$

(k_2) $$J_2^{k_2}\frac{d\omega_2}{dt} = (mg\sin\alpha - N - kN)R$$

$$\frac{3}{2}m\frac{dv_2}{dt} = mg\sin\alpha - N(1+k)$$

$$\frac{dv_1}{dt} = \frac{dv_2}{dt}$$

$$\frac{1}{2m}(mg\sin\alpha + N - kN) = \frac{2}{3m}(mg\sin\alpha - N - kN)$$

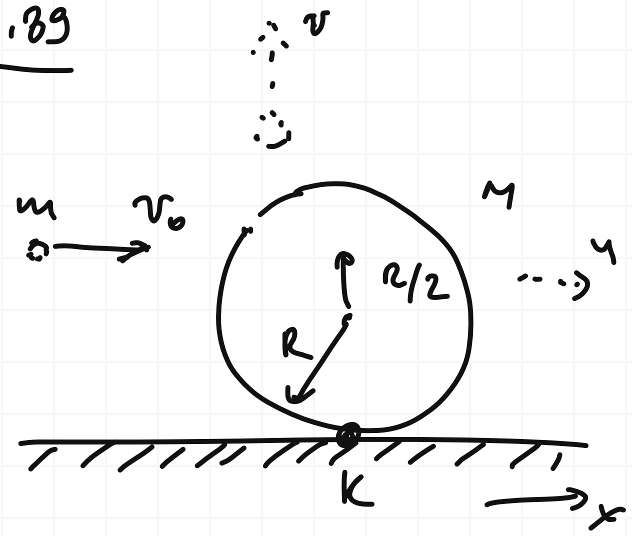
$$3mg\sin\alpha + 3N - 3Nk = 4mg\sin\alpha - 4N - 4Nk$$

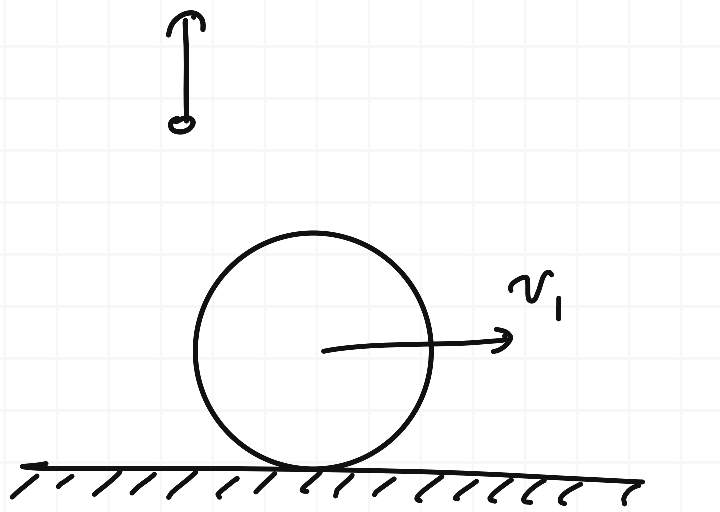
$$mg\sin\alpha = 7N + Nk = N(k+7)$$

$$N = \frac{mg\sin\alpha}{k+7}$$

$$a = \frac{dv}{dt} = \frac{mg\sin\alpha + N(1-k)}{2m} = \frac{1}{2}g\sin\alpha\left(1 + \frac{1-k}{k+7}\right) = \frac{4g\sin\alpha}{k+7}$$

9.89





равномерное качение

ЗСМИ отн (·)K:

$$m v_0 \cdot \frac{3}{2}R = MuR + mv\frac{\sqrt{3}}{2}R + I_k\omega_k$$

$$MuR + I_k\omega_k = Mv_1R + I_k\omega_1$$

$$\frac{3}{2}mv_0R = \frac{\sqrt{3}}{2}mvR + Mv_1R + I\frac{v_1}{R} = \frac{\sqrt{3}}{2}mvR + Mv_1R + \frac{2}{5}Mv_1R$$

$$\frac{3}{2}mv_0\cancel{R} = \frac{\sqrt{3}}{2}mv\cancel{R} + \frac{7}{5}Mv_1\cancel{R}; \quad \frac{\sqrt{3}}{2}\cancel{m}v = \frac{3}{2}\cancel{m}v_0 - \frac{7}{5}\frac{M}{m}v_1;$$

$$v = \sqrt{3}v_0 - \frac{14}{5\sqrt{3}}\frac{M}{m}v_1$$

9.187

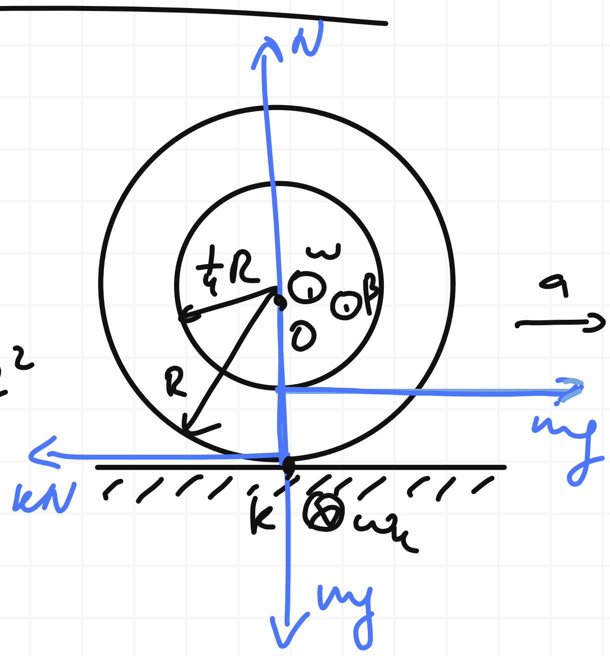
$s = 1м$

$R,\ \frac{1}{4}R$

$J_0 = \frac{1}{8}mR^2$

$k = \frac{3}{10}$

S_x − ?



$$(\cdot)K:\ \left(\frac{1}{8}\cancel{m}R^2 + \cancel{m}R^2\right)\frac{d\omega_k}{dt} = \cancel{m}g\cdot\frac{3}{4}\cancel{R}$$

$$\frac{9}{8}\frac{R\,d\omega_k}{dt} = \frac{9}{8}\frac{dv}{dt} = \frac{3}{4}g$$

Предположим, что катушка катится без проскальз.

$$a = \frac{dv}{dt} = \frac{3}{4}\cdot\frac{8}{9}g = \frac{2}{3}g$$

с другой стороны (II з.н.): $ma = mg - kN = mg - \frac{3}{10} mg = \frac{7}{10} g$

$a \neq a$, противоречие. Значит, катушка проскальз.

из II з.н: $a = \frac{dv}{dt} = \frac{7}{10} g = (1-k)g$

$S = \frac{1}{2} a t^2 = \frac{1}{2} \cdot \frac{7}{10} g t^2 = \frac{7}{20} g t^2$; $t^2 = \frac{20}{7} \frac{S}{g}$

(·) O: $M = J\beta$; $\frac{1}{4} \cancel{m} g \cancel{R} - k \cancel{m} g \cancel{R} = \frac{1}{8} \cancel{m} R^{\cancel{2}} \frac{d\omega}{dt}$

$\left(\frac{1}{4} - k\right) g = \frac{1}{8} R \beta$; $\beta = 8 \frac{g}{R} \left(\frac{1}{4} - k\right) = \frac{g}{R}(2 - 8k) =$

$= \frac{g}{R}\left(2 - \frac{\cancel{8}^{4} \cdot 3}{\cancel{10}_{5}}\right) = \frac{g}{R}\left(\frac{10-12}{5}\right) = -\frac{2}{5}\frac{g}{R}$

$S_x = \Delta\varphi \cdot R = \frac{1}{2}|\beta| t^2 R = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{5} \frac{\cancel{g}}{\cancel{R}} \cdot \frac{\cancel{20}^{4}}{7} \frac{S}{\cancel{g}} \cdot \cancel{R} = \frac{4}{7} S = \frac{4}{7}$ м

11.7

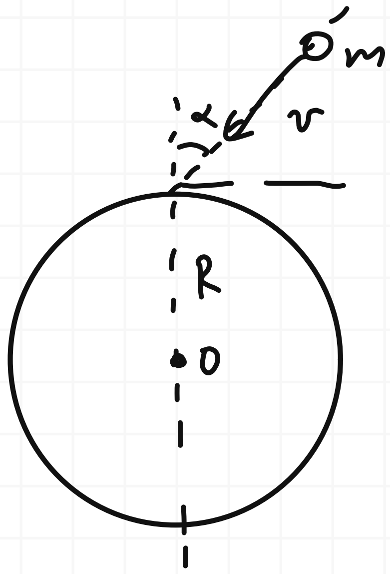
$\alpha = 45°$

$m = 1000$ т $= 1 \cdot 10^6$ кг

$v = 20$ км/с

$M = 6 \cdot 10^{24}$ кг

$R = 6400$ км



$J = J$ шара отн. точки $= \frac{3}{5} MR^2$

$L_1 = \frac{1}{\sqrt{2}} mvR + \frac{3}{5} MR^2 \omega_0$

$L_2 =$

11.8

u = 300 км/ч

R = 100 м

J = 7 кг·м^2

N = 1000 об/(мин)