Неделя 4.

Преобразование т/д функций
Поверхностное натяжение

5.63

Ag
адиабат. сж

$\frac{\Delta V}{V} = 0{,}01$

$\frac{\Delta T}{T} = 0{,}028$

$\beta_T - ?$

$\alpha = 5{,}7 \cdot 10^{-5}\ K^{-1}$

$c_v = 0{,}23$ Дж/г·К

$\rho = 10{,}5$ г/см³

$$dS = \frac{m c_v dT}{T} + \left(\frac{\partial P}{\partial T}\right)_V dV = 0 \qquad (\delta Q = 0)$$

$$c_v m \int_T^{T+\Delta T} \frac{dT}{T} = -\left(\frac{\partial P}{\partial T}\right)_V \int_V^{V-\Delta V} dV$$

$$c_v m \ln\frac{T+\Delta T}{T} = \underbrace{\left(\frac{\partial P}{\partial T}\right)_V} \Delta V$$

$$\left.\begin{aligned} \beta_T &= -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \\ \alpha &= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P \end{aligned}\right\} : \quad \frac{\alpha}{\beta_T} = -\left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial P}{\partial V}\right)_T = \underline{\left(\frac{\partial P}{\partial T}\right)_V}$$

$$c_v \cancel{m} \ln\left(1 + \frac{\Delta T}{T}\right) = \frac{\alpha}{\beta_T} \Delta V = \frac{\alpha}{\beta_T} \frac{\Delta V}{V} \cdot \frac{\cancel{m}}{\rho}$$

$$c_v \ln\left(1 + \frac{\Delta T}{T}\right) = \frac{\alpha}{\beta_T} \cdot \frac{\Delta V}{V} \cdot \frac{1}{\rho}$$

$$\underline{\underline{\beta_T}} = \frac{\alpha}{\rho c_v} \cdot \frac{\Delta V}{V} \cdot \frac{1}{\ln\left(1 + \frac{\Delta T}{T}\right)} = \underline{\underline{8{,}55 \cdot 10^{-12}\ 1/\text{Па}}}$$

12.9

$\nu = 0{,}1$ моль

$r = 5$ см

$T = 290$ K

$\sigma = 70$ дин/см

$\frac{d\sigma}{dT} = -0{,}15 \frac{\text{дин}}{\text{см} \cdot \text{К}}$

$\Delta T - ?$

$$\left(\left(\begin{aligned} &Q = U^{\nearrow} + U^{\swarrow} = 2\sigma \cdot 4\pi r^2 - 2T\frac{d\sigma}{dT} \cdot 4\pi r^2 \\ &\Delta U = \frac{5}{2}\nu R \Delta T \end{aligned}\right.\right.$$

$$8\pi r^2 \left(\sigma - T\frac{d\sigma}{dT}\right) = \frac{5}{2}\nu R \Delta T$$

$$\underline{\underline{\Delta T}} = \frac{16}{5} \frac{\pi r^2}{\nu R}\left(\sigma - T\frac{d\sigma}{dT}\right) \approx \underline{\underline{3{,}14 \cdot 10^{-3}\ K}}$$

12.38

$r_0 = 5$ см

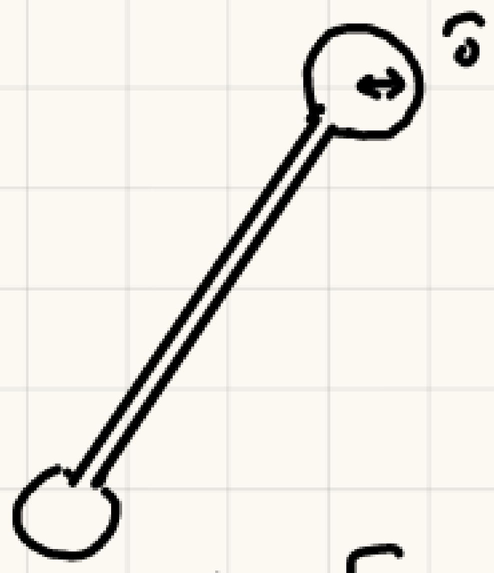
$\sigma = 30$ дин/см

$T = 300$ K

$\Delta S - ?$

Лаплас: $P_0 = \frac{4\sigma}{r_0}$ (вакуум)

$P = \frac{4\sigma}{r}$ (после открытия крана)



$$\left. \begin{aligned} &P_0 \cdot \frac{4}{3}\pi r_0^3 = \nu R T \\ &P \cdot \frac{4}{3}\pi r^3 = 2\nu R T \end{aligned} \right| \to \frac{P}{P_0} \cdot \frac{r^3}{r_0^3} = \frac{r^2}{r_0^2} = 2 \ ; \quad \begin{aligned} r &= \sqrt{2}\, r_0 \\ P_0 &= \sqrt{2}\, P_1 \end{aligned}$$

$$TdS = \cancel{dU}^{\,0} + pdV \;\; (\text{усл.}) ; \qquad dS = 2\nu R \frac{dV}{V}$$

$$\underline{\underline{\Delta S}} = 2\nu R \ln\frac{V}{V_0} = 2\nu R \ln\sqrt{2} = \nu R \ln 2 = \frac{4}{3}\frac{p_0 \pi r_0^3}{T}\ln 2 = \underline{\underline{\frac{16\sigma\pi r_0^2}{3T}\ln 2 \approx 28 \text{ эрг}/K}}$$

1.3

$\alpha = 1{,}8\cdot 10^{-4}\ K^{-1}$

$\beta = 3{,}9\cdot 10^{-6}$ атм

$\lambda - ?$

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V$$

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T P_{атм}\cdot\frac{1}{P_{атм}}\left(\frac{\partial P}{\partial T}\right)_V = \beta\lambda$$

$$\underline{\underline{\alpha = \beta\lambda P_{атм}}} ; \quad \lambda = \frac{\alpha}{\beta P_{атм}} = \underline{\underline{46 K^{-1}}}$$

10°

$f = \alpha T\left(\frac{l}{l_0} - \left(\frac{l_0}{l}\right)^2\right)$

$\alpha = 1{,}3\cdot 10^{-2}$ Н/К

$l_0 = 1$ м

$l = 2$ м

$T = 300 K$

$\Delta U - ?$

$\Delta F - ?$

(изотерм. растяж.)

$$dU = TdS - \delta A = TdS + \delta A^{ext} = TdS + f dl$$

$$dF = d(U - TS) = -SdT + f dl$$

$$dF_T = f dl = aT\left(\frac{l}{l_0} - \left(\frac{l_0}{l}\right)^2\right) dl \;\frac{l_0}{l_0} \;\underset{=}{\overset{x = \frac{l}{l_0}}{=}}\; aTl_0\left(x - \frac{1}{x^2}\right)dx$$

$$\underline{\underline{\Delta F_T}} = aTl_0 \int_1^2 \left(x - \frac{1}{x^2}\right)dx = aTl_0\left(\frac{x^2}{2} + \frac{1}{x}\right)\Big|_1^2 = \underline{\underline{aTl_0 \approx 3.9 \text{ Дж}}}$$

$$dS_T = \left(\frac{\partial S}{\partial l}\right)_T dl + \underbrace{\cancel{\left(\frac{\partial S}{\partial T}\right)_l dT}}_{\to 0 \; (T = const)} = -\left(\frac{\partial f}{\partial T}\right)_l dl = a\left[\left(\frac{l_0}{l}\right)^2 - \frac{l}{l_0}\right] dl$$

$$\Delta S_T = a l_0 \int_1^2 \left(\frac{1}{x^2} - x\right) dx = -a l_0$$

$$\underline{\underline{\Delta U_T}} = \Delta F_T + T\Delta S_T = aTl_0 - aTl_0 = \underline{\underline{0}}$$

11°

$m = 1$ г

$d = 2\cdot 10^{-4}$ см

изотерм

$\sigma = 25$ дин/см

$\rho = 0{,}9$ г/см³

r_1 – радиус в начале, r_2 – после дробления $\left(r_2 = \frac{d}{2}\right)$

$$m_1 = \frac{4}{3}\pi\rho r_1^3 ; \quad r_1 = \left(\frac{3m}{4\pi\rho}\right)^{1/3} \approx 0{,}64 \text{ см} \quad \rightsquigarrow W_1 = \frac{3m}{r_1 \rho}$$

$$m = \frac{4\pi r_2^2}{3}\cdot N \cdot r_2 \rho \quad \rightsquigarrow W_2 = \frac{3m}{r_2 \rho}$$

$$\underline{\underline{A_T^{ext}}} = \sigma(W_2 - W_1) = \frac{3m\sigma}{\rho}\left(\frac{1}{r_2} - \frac{1}{r_1}\right) \approx \underline{\underline{8{,}7\cdot 10^5 \text{ эрг}}}$$

$\underline{12^\circ}$

$h = 0{,}1$ мм

$\theta = 60^\circ$

$\sigma = 73 \cdot 10^{-3}$ Н/м

$H - ?$

$$P_0 = P(H) + \frac{\sigma}{r} = P(H) + \rho g H$$

$$H = \frac{\sigma}{\rho g r} = \frac{2\sigma \cos\theta}{\rho g h} \simeq \underline{\underline{7{,}5 \text{ см}}}$$