Неделя 1. Основы кинематики

~~1.5~~
~~1.11~~
~~1.21~~
1.24
~~0.3~~
~~2.1~~
~~2.5~~

№3

$\dot{x} = x/t$

$\tau = 10c$

$t = \tau$

$x_0 = 1\,м$

$x(\tau) - ?$

$\dot{x}(\tau) - ?$

$$\dot{x} = \frac{x}{\tau} \; ; \quad \dot{x}_0 = \frac{x_0}{\tau} = \frac{1м}{10c} = 0{,}1\ м/с$$

$$\frac{dx}{dt} = \frac{x}{\tau} \; ; \int_{x_0}^{x(t)} \frac{dx}{x} = \frac{1}{\tau}\int_0^t dt \; ; \quad \ln\frac{x(t)}{x_0} = \frac{1}{\tau}t \; ;$$

$$x(t) = x_0 \exp\left(\frac{t}{\tau}\right) = [t=\tau] = x_0 \exp(1) = x_0 e \approx$$

$$\approx 1\,м \cdot 2{,}72 = 2{,}72\,м$$

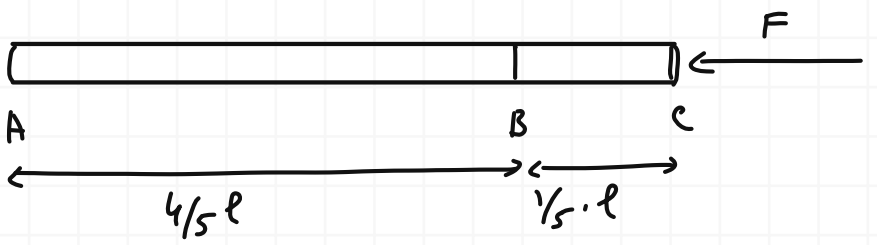
$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\left(\frac{x}{\tau}\right)}{dt} = \frac{1}{\tau}\frac{dx}{dt} = \frac{1}{\tau}\dot{x} = \frac{1}{\tau}\cdot\frac{x}{\tau} = \frac{x}{\tau^2}$$

$$\ddot{x}(\tau) = \frac{x(\tau)}{\tau^2} = \frac{x_0 e}{\tau^2} = \frac{1м \cdot 2{,}72}{(10c)^2} = 2{,}72 \cdot 10^{-2}\ м/с^2$$

№2.1

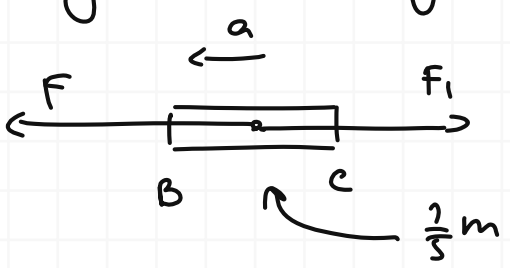
$AC, m, \ell, F,$
$AB = \frac{4}{5}\ell$

$F_1 - ?$



Пусть стержень (в частности, кусочек BC) движется с ускорением a.

I Масса всего стержня – m.



$$F = ma$$

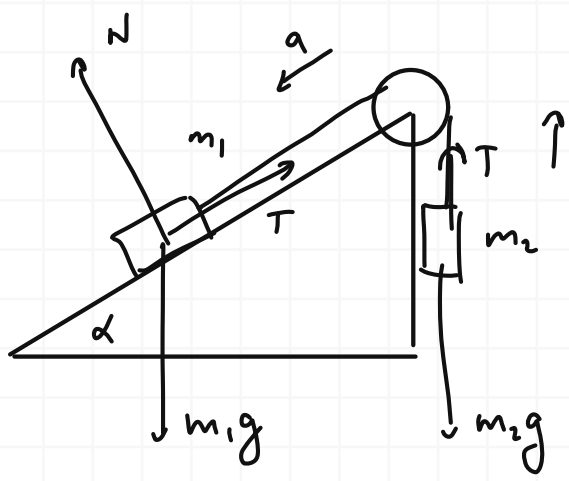
$$F - F_1 = \frac{1}{5}ma \quad + \Rightarrow \quad ma - F_1 = \frac{1}{5}ma$$

$$F_1 = \frac{4}{5}ma$$

№2.5

m_1, m_2, α

$a, T - ?$



II З.К.:

$$\uparrow a \quad m_2 a = T - m_2 g$$

$$+ \quad m_1 a = m_1 g \sin\alpha - T$$

$$(m_1 + m_2)a = m_1 g \sin\alpha - m_2 g$$

$$a = \frac{(m_1 \sin\alpha - m_2)g}{m_1 + m_2}$$

$$T = m_2(a+g) = \frac{m_2}{m_1+m_2}(m_1\sin\alpha - m_2)g + m_2 g =$$

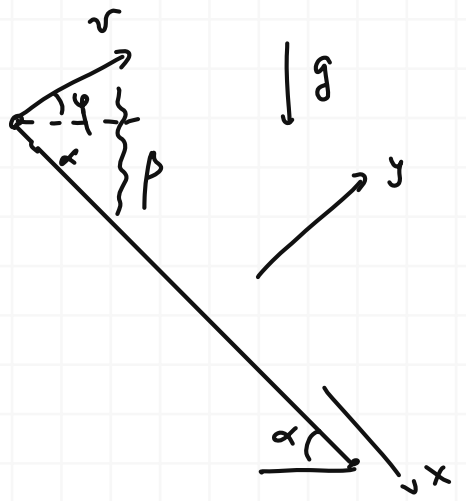
$$= m_2 g\left(\frac{m_1\sin\alpha - \cancel{m_2} + m_1 + \cancel{m_2}}{m_1+m_2}\right) = \frac{m_1 m_2 (1+\sin\alpha)g}{m_1+m_2}$$

№1.5

$\varphi - ?$

$\alpha = 45°$

L_{max}



$$\cos\alpha = \sin\alpha = \frac{1}{\sqrt{2}}$$

$$g_x = \frac{1}{\sqrt{2}}g\,;\ g_y = -\frac{1}{\sqrt{2}}g$$

$$x = v\cos(\alpha+\varphi)t + \frac{1}{2}\left(\frac{1}{\sqrt{2}}g\right)t^2 = \left[\alpha+\varphi := \beta\right] =$$

$$= v\cos\beta t + \frac{1}{2\sqrt{2}}gt^2$$

$$y = v\sin\beta t - \frac{1}{2\sqrt{2}}gt^2$$

момент падения: $x = L,\ y = 0$

$$t\left(v\sin\beta - \frac{1}{2\sqrt{2}}gt\right) = 0\ ,\ t \neq 0$$

$$v\sin\beta = \frac{1}{2\sqrt{2}}gt\,;\ t = 2\sqrt{2}\,\frac{v\sin\beta}{g}$$

$$L = v\cos\beta\cdot\frac{2\sqrt{2}\,v\sin\beta}{g} + \cancel{\frac{1}{2\sqrt{2}}g}\,\frac{(2\sqrt{2})^2 v^2\sin^2\beta}{g^2} =$$

$$= \frac{2\sqrt{2}\,v^2\sin\beta\cos\beta}{g} + \frac{2\sqrt{2}\,v^2\sin^2\beta}{g} = \frac{2\sqrt{2}\,v^2}{g}\sin\beta(\cos\beta+\sin\beta) \to \max$$

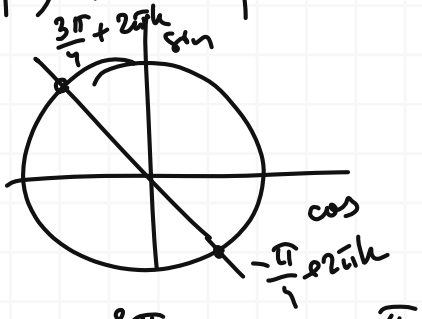
$$\left(\sin\beta(\cos\beta+\sin\beta)\right)'_\beta = 0$$

$$0 = \sin\beta(-\sin\beta+\cos\beta) + \cos\beta(\cos\beta+\sin\beta) =$$

$$= -\sin^2\beta + \sin\beta\cos\beta + \cos^2\beta + \sin\beta\cos\beta =$$

$$= \cos^2\beta - (1-\cos^2\beta) + \sin 2\beta = 2\cos^2\beta - 1 + \sin 2\beta = \cos 2\beta + \sin 2\beta$$

$\cos 2\beta = -\sin 2\beta$



$$2\beta = -\frac{\pi}{4} + \pi k;\ k \in \mathbb{Z}$$

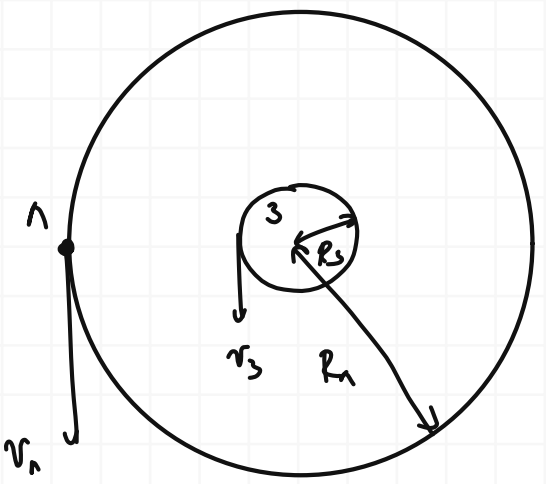
$$2\beta = \frac{3\pi}{4}\,;\ \beta = \frac{3\pi}{8} = \alpha+\varphi = \frac{2\pi}{8} + \varphi\,;\ \varphi = \frac{\pi}{8}$$

№1.11

$R_л = 3{,}8 \cdot 10^5$ км

$R_з = 6{,}4 \cdot 10^3$ км

$v_т$ – ?



T_i – период

$T_л = 28$ дней $= 28 \cdot 24 \cdot 60 \cdot 60$ с $= 2419200$ с

$T_з = 1$ день $= 24 \cdot 60 \cdot 60$ с $= 86400$ с

$\omega = \frac{2\pi}{T}$;

$\omega_л = \frac{2\pi}{T_л}$, $\omega_з = \frac{2\pi}{T_з}$

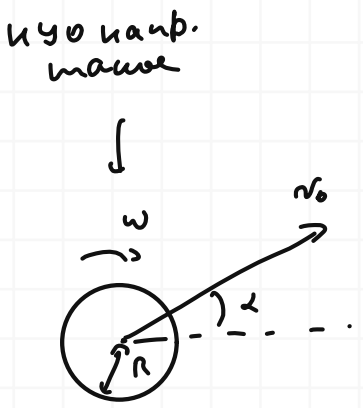
$v_л = \omega_л R_л = \frac{2\pi}{T_л} R_л$; $v_з = \omega_з R_з = \frac{2\pi}{T_з} R_з$

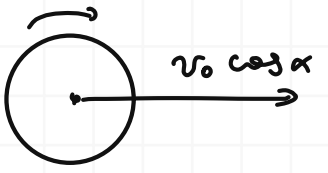
$v_т = v_л - v_з = 2\pi \left(\frac{R_л}{T_л} - \frac{R_з}{T_з} \right) = 2 \cdot 3{,}14 \left(\frac{3{,}8 \cdot 10^8}{2419200} - \frac{6{,}4 \cdot 10^6}{86400} \right) = 521{,}5$ м/с

№1.21

R, ω, α, v_0

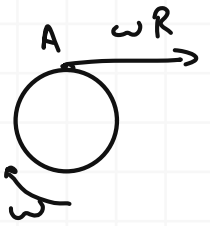
ρ – ?



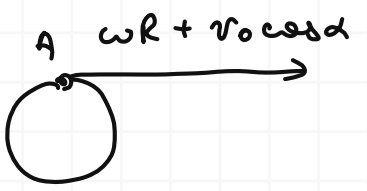


t момент наиб. подъёма

СО центра диска:



в ИСО:



Пусть m – масса маленького кусочка колеса рядом с A

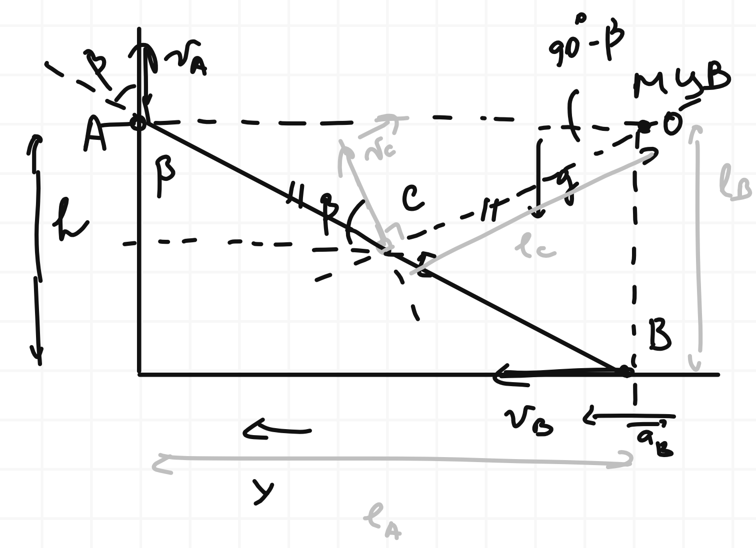
$m \frac{v_A^2}{R_{кр}} = m\omega^2 R + mg$; $R_{кр} = \frac{v_A^2}{\omega^2 R + g} = \frac{(\omega R + v_0 \cos\alpha)^2}{\omega^2 R + g}$

№1.24

l, v_A, β

$\vec{a}_c$ - ?

(велич, напр - ?)



стержень нерастяжим:

$v_A \cos\beta = v_B \sin\beta$; $v_B = v_A \operatorname{ctg}\beta$

восстановим ⊥ к $\vec{v}_A$, $\vec{v}_B$, их пересечение — МЦВ

соед. МЦВ и (·)C, построим $\vec{v}_C$ перп. этому отрезку и напр. по движению

$\cos\beta = \frac{h}{l}$

$$\frac{v_A}{l_A} = \frac{v_B}{l_B} = \frac{v_C}{l_C}$$

$$l_C = AC = \frac{1}{2}AB = \frac{1}{2}l$$

$$l_A = l\sin\beta$$

$$\frac{v_A}{l\sin\beta} = \frac{v_C}{\frac{1}{2}l}; \quad v_C = \frac{1}{2\sin\beta} v_A$$

$$y_c = \frac{y_a}{2}; \quad \ddot{y}_c = \frac{\ddot{y}_a}{2} = 0; \quad a_y = 0 : \quad \vec{a}_c = \begin{pmatrix} a_{cx} \\ 0 \end{pmatrix}$$

$$\dot{v}_c = \left(\frac{v_A}{2\sin\beta}\right)'_t = \frac{v_A}{2}\left(\frac{1}{\sin\beta}\right)'_t$$

$$\sin\beta(t) = \sqrt{1 - \left(\frac{y_A}{2l}\right)^2} \Rightarrow (\sin\beta)'_t = \frac{-2\frac{y_a}{2l}\cdot\frac{v_a}{2l}}{2\sqrt{1-\left(\frac{y_a}{2l}\right)^2}} = \frac{-y_a v_a}{2l\sqrt{4l^2 - y_a^2}}$$

$$\dot{v}_c = \frac{v_a}{2}\cdot\frac{-(\sin\beta)'_t}{\sin^2\beta} = \frac{v_a}{2}\,\frac{y_a v_a}{2l\sqrt{4l^2 - v_a^2}\,\sin^2\beta} = \frac{v_a^2}{4l\sin^2\beta}\cdot\frac{y_a}{\sqrt{4l^2 - y_a^2}} =$$

$$= \frac{v_a^2\cos\beta}{4l\sin^2\beta\,\sin\beta} = \frac{v_a^2\cos\beta}{4l\sin^3\beta}$$

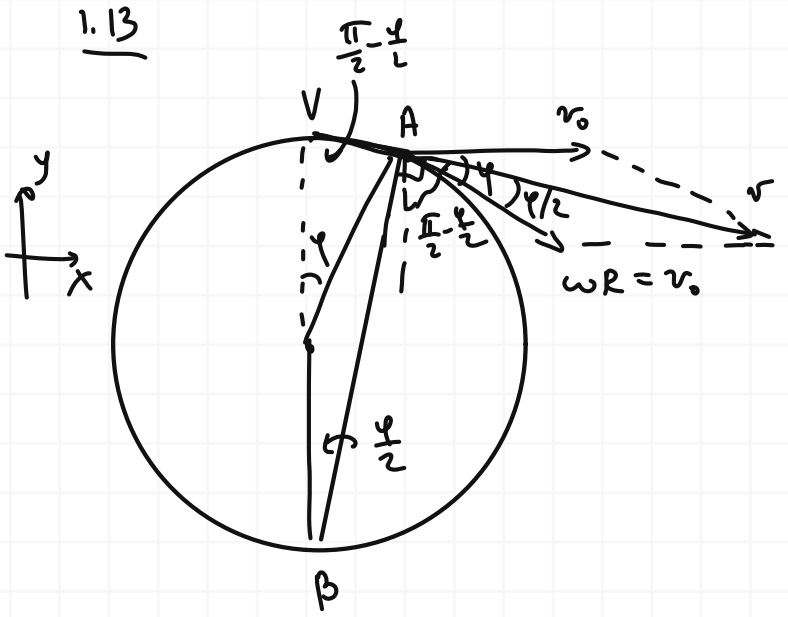
$$a_{c\tau} = \frac{v_a^2\cos\beta}{4l\sin^3\beta}$$

$$a_{cn} = \frac{v_c^2}{R_c} = \frac{v_a^2}{4\sin^2\beta\, l} = \frac{v_a^2}{4l\sin^2\beta}$$

$$a_c = \sqrt{a_{c\tau}^2 + a_{cn}^2} = \frac{v_a^2}{4l\sin^3\beta}$$

Ответ: $a_c = \frac{v_a^2}{4l\sin^3\beta}$, $\uparrow\uparrow \vec{v}_B$

1.13



1) проекц. нед: $v_0 = \omega R$

$v_x = v_0 + v_0 \cos\varphi = v_0(1+\cos\varphi)$

2) $v_y = -v_0 \sin\varphi$

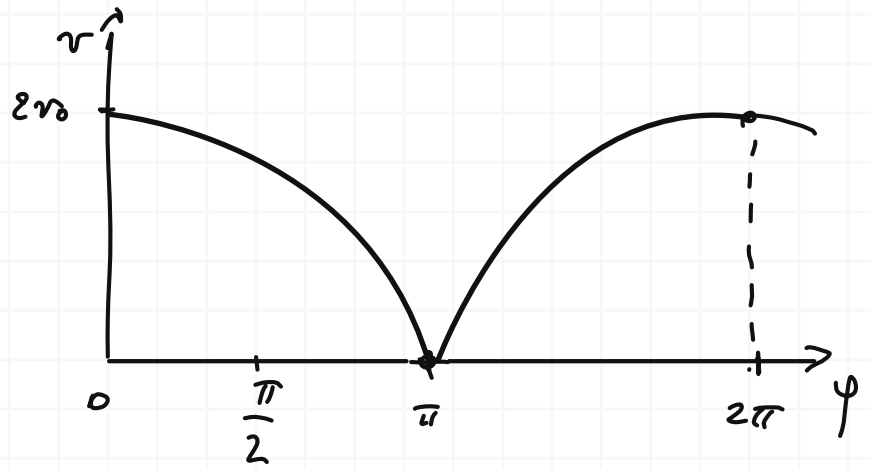
3) $\alpha + \frac{\pi}{2} - \varphi = \frac{\pi}{2} + \frac{\varphi}{2}$; $\alpha = \frac{3}{2}\varphi$

4) $\vec{v} \perp AB$ (см. след угол)

$\vec{v}$ действительно проходит через V

5) $\omega|BA| = 2R\cos\frac{\varphi}{2}\cdot\omega$; $v_{полн} = 2v_0\cos\frac{\varphi}{2} = 2\omega R\cos\frac{\varphi}{2}$; $v_{полн} = |BA|\omega$

6)



$v(\varphi)$ циклична с периодом 2π