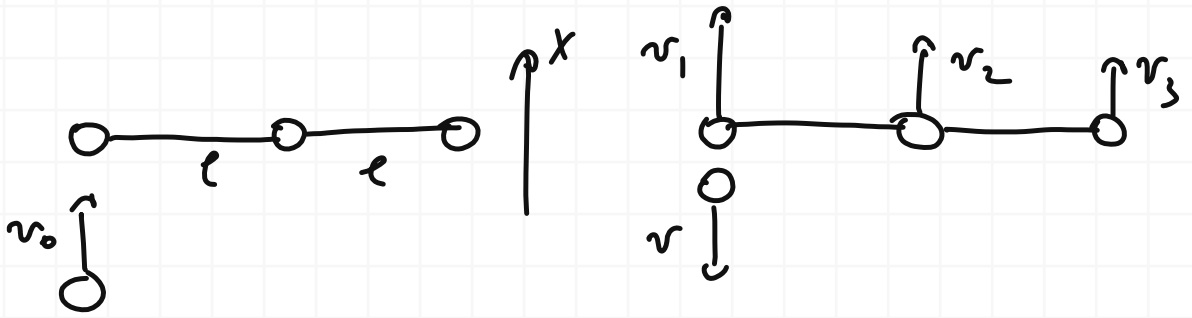
Неделя 7.

Плоское движение твердого тела. Качение.

6.15

v_0, ℓ



ЗСИ: x $\quad v_0 = v_x + v_{1x} + v_{2x} + v_{3x}$

$v_0 = v_x + 3v_c$; $v_x = v_0 - 3v_c$

v_c — скорость масс, т.е. II шарика

ЗСЭ: $\quad v_0^2 = v^2 + v_1^2 + v_2^2 + v_3^2$

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{3}{2} m v_c^2 + \frac{1}{2} J\omega^2$; $\quad J = 2 \cdot m\ell^2$

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + \frac{3}{2} m v_c^2 + m\ell^2\omega^2$

$v^2 = v_0^2 - 3v_c^2 - 2\omega^2\ell^2$

ЗСИМ: $\quad \Delta L = \Delta p \cdot \ell = J\omega$

$m(v_0 + v)\ell = 2m\ell^2\omega$; $\quad v = 2\omega\ell - v_0$

$v = -v_x = 3v_c - v_0$ ⟶ $3v_c - v_0 = 2\omega\ell - v_0$

$v^2 = v_0^2 - 3v_c^2 - 2\omega^2\ell^2$ $\qquad v_c = \frac{2}{3}\omega\ell$

$v = 2\omega\ell - v_0$

$(2\omega\ell - v_0)^2 = 4\omega^2\ell^2 + v_0^2 - 4v_0\omega\ell =$

$= v_0^2 - \frac{4}{3}\omega^2\ell^2 - 2\omega^2\ell^2 = v_0^2 - (3 + \frac{1}{3})\omega^2\ell^2$

$4\omega^2\ell^2 - 4v_0\omega\ell = (-3 - \frac{1}{3})\omega^2\ell^2$

$(7 + \frac{1}{3})\omega\ell = 4v_0$; $\quad \frac{22}{3}\omega\ell = 2v_0$; $\quad \omega = \frac{6}{11}\frac{v_0}{\ell}$

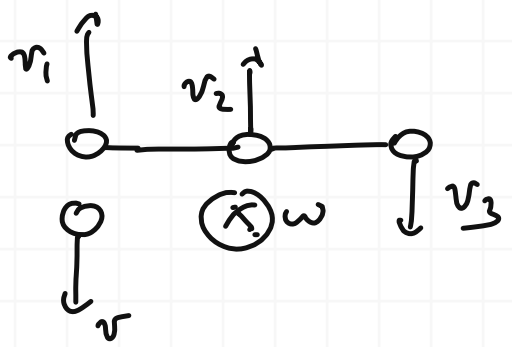
$v_c = \frac{2}{3}\omega\ell = \frac{2}{3} \cdot \frac{6}{11} v_0 = \frac{4}{11} v_0$

$$v_{1x} = v_c + \omega l = \frac{4}{11} v_0 + \frac{6}{11} v_0 = \frac{10}{11} v_0$$

$$v_{3x} = v_c - \omega l = \frac{4}{11} v_0 - \frac{6}{11} v_0 = -\frac{2}{11} v_0$$

$$v_{2x} = v_c = \frac{4}{11} v_0$$

$$v_{4x} = v_x = v_0 - 2\omega l = v_0 - \frac{12}{11} v_0 = -\frac{1}{11} v_0$$



$$v_1 = \frac{10}{11} v_0$$

$$v_2 = \frac{4}{11} v_0$$

$$v_3 = \frac{2}{11} v_0$$

$$v = \frac{1}{11} v_0$$

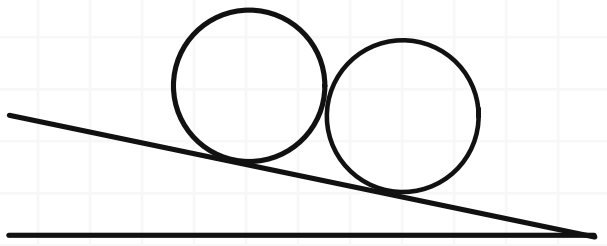
$$\omega = \frac{6}{11} \frac{v_0}{l}$$

ответ

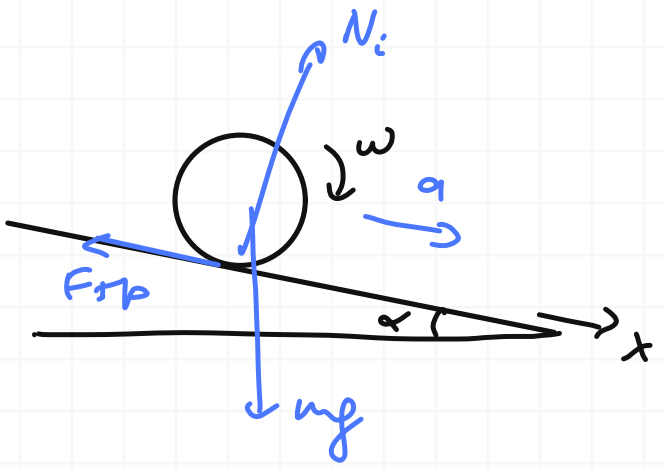
9.71

$$J_n = mr^2 = 2J_0$$

$$J_c = \frac{1}{2} mr^2 = J_0$$



нужно расположить цилиндр, который сам по себе скатывается вторым будет



$$N = mg\cos\alpha \; ; \; F_{тр} = kN = kmg\cos\alpha$$

$$ma = mg\sin\alpha - F_{тр} = mg(\sin\alpha - k\cos\alpha)$$

$$a = g(\sin\alpha - k\cos\alpha)$$

$$M = F_{тр} \cdot R = kmg\cos\alpha \cdot R = J\beta = \frac{d\omega}{dt} J$$

$$\begin{cases} \frac{dv}{dt} = g(\sin\alpha - k\cos\alpha) \\ kmg\cos\alpha R^2 = J\frac{R\,d\omega}{dt} = J\frac{dv}{dt} \end{cases}$$

без ускор.

$$\begin{cases} \frac{dv}{dt} = g(\sin\alpha - k\cos\alpha) \\ \frac{dv}{dt} = \frac{1}{J} kmg\cos\alpha R^2 \end{cases}$$

$$Jg(\sin\alpha - k\cos\alpha) = kmg\cos\alpha R^2 \qquad |: \cos\alpha$$

$$Jg(\operatorname{tg}\alpha - k) = kmgR^2$$

$$\operatorname{tg}\alpha - k = \frac{kmgR^2}{gJ}; \qquad \operatorname{tg}\alpha = k\left(1 + \frac{mR^2}{J}\right)$$

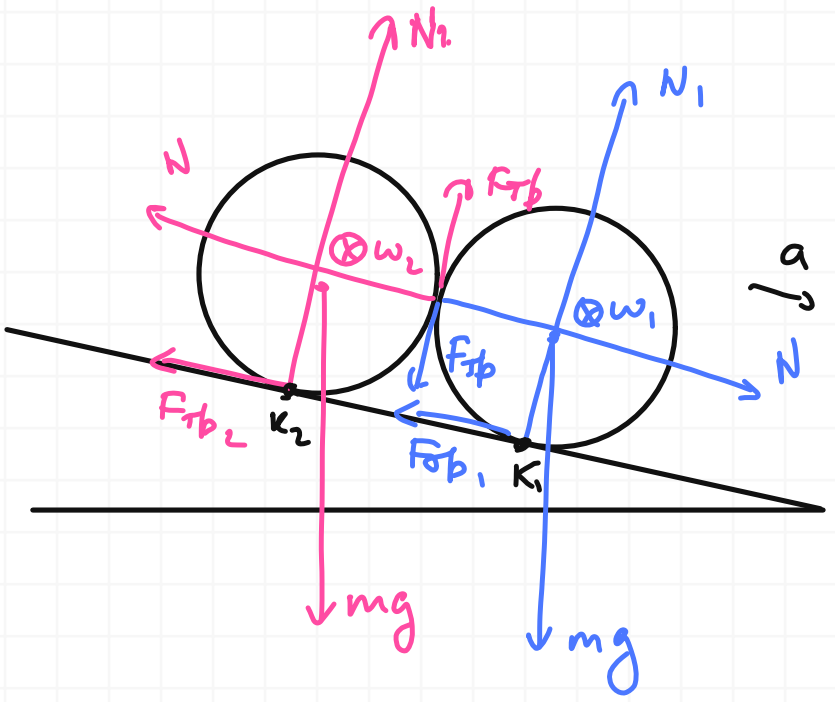
$$k_n = \frac{\operatorname{tg}\alpha}{1 + \frac{mR^2}{2J_0}} = \frac{\operatorname{tg}\alpha}{1 + \frac{mR^2}{mR^2}} = \frac{1}{2}\operatorname{tg}\alpha$$

$$k_n > k_c$$

$$k_c = \frac{\operatorname{tg}\alpha}{1 + \frac{mR^2}{J_0}} = \frac{\operatorname{tg}\alpha}{1 + \frac{mR^2}{\frac{1}{2}mR^2}} = \frac{1}{3}\operatorname{tg}\alpha$$

$$a_n < a_c$$

сплошной должен располагаться за полым



$$N_1 = N_2 = mg\cos\alpha$$

$$F_{тр_i} = k_i N_i = k_i mg\cos\alpha$$

$$k_1 = k_n = \frac{1}{2}\operatorname{tg}\alpha$$

$$k_2 = k_c = \frac{1}{3}\operatorname{tg}\alpha$$

$$F_{тр} = kN$$

$$M = J\beta = J\frac{d\omega}{dt}; \quad M = F_2 \cdot R; \quad J\frac{d\omega}{dt} = F_2 R$$

(k_1) $$J_1^{k_1}\frac{d\omega_1}{dt} = (mg\sin\alpha + N - kN)R$$

$$(mR^2 + mR^2)\frac{d\omega_1}{dt} = (N(1-k) + mg\sin\alpha)R$$

$$2m\frac{R\,d\omega_1}{dt} = 2m\frac{dv_1}{dt} = mg\sin\alpha + N(1-k)$$

(k_2) $$J_2^{k_2}\frac{d\omega_2}{dt} = (mg\sin\alpha - N - kN)R$$

$$\frac{3}{2}m\frac{dv_2}{dt} = mg\sin\alpha - N(1+k)$$

$$\frac{dv_1}{dt} = \frac{dv_2}{dt}$$

$$\frac{1}{2m}(mg\sin\alpha + N - kN) = \frac{2}{3m}(mg\sin\alpha - N - kN)$$

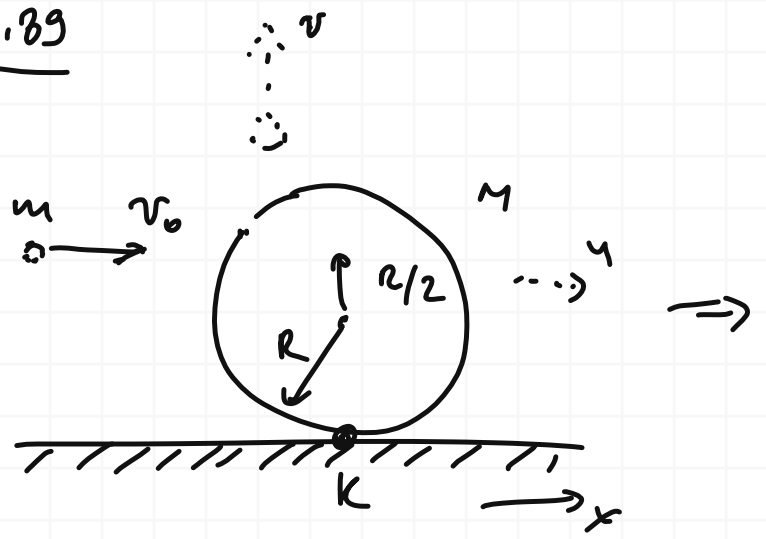
$$3mg\sin\alpha + 3N - 3Nk = 4mg\sin\alpha - 4N - 4Nk$$

$$mg\sin\alpha = 7N + Nk = N(k+7)$$

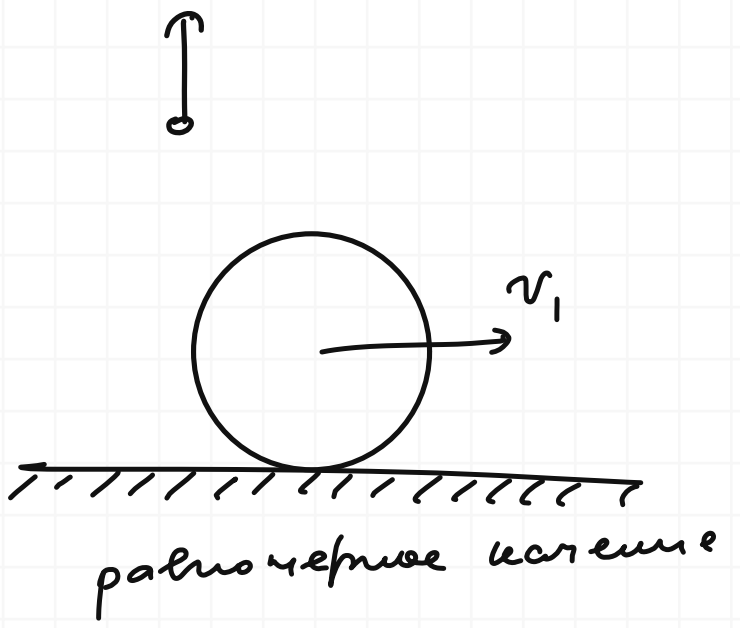
$$N = \frac{mg\sin\alpha}{k+7}$$

$$a = \frac{dv}{dt} = \frac{mg\sin\alpha + N(1-k)}{2m} = \frac{1}{2}g\sin\alpha\left(1 + \frac{1-k}{k+7}\right) = \frac{4g\sin\alpha}{k+7}$$

9.89



$\Rightarrow$



равномерное качение

ЗСМИ отн (·)K:

$$mv_0 \cdot \frac{3}{2}R = MuR + mv\frac{\sqrt{3}}{2}R + I_K\omega_K$$

$$MuR + I_K\omega_K = Mv_1R + I_K\omega_1$$

$$\frac{3}{2}mv_0R = \frac{\sqrt{3}}{2}mvR + Mv_1R + I\frac{v_1}{R} = \frac{\sqrt{3}}{2}mvR + Mv_1R + \frac{2}{5}Mv_1R$$

$$\frac{3}{2}mv_0R = \frac{\sqrt{3}}{2}mvR + \frac{7}{5}Mv_1R \quad ; \quad \frac{\sqrt{3}}{2}\eta mv = \frac{3}{2}\eta mv_0 - \frac{7}{5}Mv_1 \; ;$$

$$v = \sqrt{3}v_0 - \frac{14}{5\sqrt{3}}\frac{M}{m}v_1$$

9.187

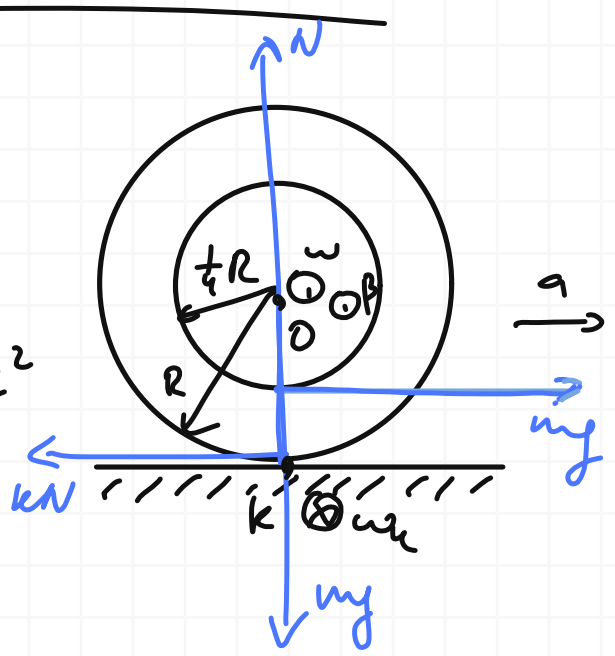
$S = 1M$

$R,\ \frac{1}{4}R$

$J_0 = \frac{1}{8}mR^2$

$k = \frac{3}{10}$

$S_x - ?$



(·)K: $\left(\frac{1}{8}\eta mR^2 + \eta mR^2\right)\frac{d\omega_K}{dt} =$

$= \eta mg \cdot \frac{3}{4}R$

$$\frac{9}{8}\frac{R\,d\omega_K}{dt} = \frac{9}{8}\frac{dv}{dt} = \frac{3}{4}g$$

Предположим, что катушка катится без проскальз.

$$a = \frac{dv}{dt} = \frac{\cancel{3}}{\cancel{4}}\cdot\frac{\cancel{8}^{2}}{\cancel{9}_{3}}g = \frac{2}{3}g$$

С другой стороны (II з.н.): $ma = mg - iN = mg - \frac{3}{10}mg = \frac{7}{10}g$

$a \neq a$, противоречие. Значит, скольжения нет.

по II з.н: $a = \frac{dv}{dt} = \frac{7}{10}g = (1-k)g$

$S = \frac{1}{2}at^2 = \frac{1}{2}\cdot\frac{7}{10}gt^2 = \frac{7}{20}gt^2$; $t^2 = \frac{20}{7}\frac{S}{g}$

(·)O: $M = J\beta$; $\frac{1}{4}mgR - kmgR = \frac{1}{8}mR^2\frac{d\omega}{dt}$

$(\frac{1}{4}-k)g = \frac{1}{8}R\beta$; $\beta = 8\frac{g}{R}(\frac{1}{4}-k) = \frac{g}{R}(2-8k) =$

$= \frac{g}{R}(2-\frac{8\cdot 3}{10\cdot 5}) = \frac{g}{R}(\frac{10-12}{5}) = -\frac{2}{5}\frac{g}{R}$

$S_x = \Delta\varphi \cdot R = \frac{1}{2}|\beta|t^2R = \frac{1}{2}\cdot\frac{2}{7}\frac{g}{R}\cdot\frac{20}{7}\frac{S}{g}\cdot R = \frac{4}{7}S = \frac{4}{7}M$

11.7

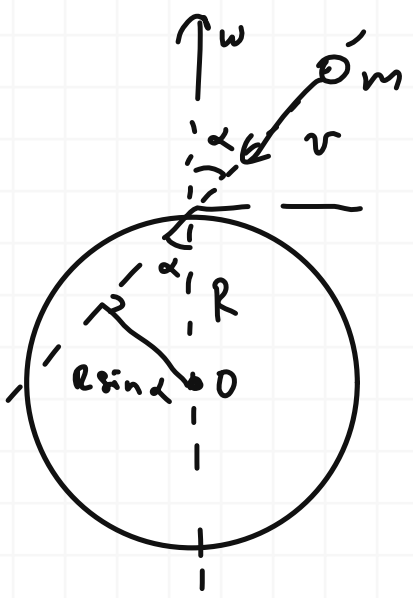
$\alpha = 45°$

$m = 1000т = 1\cdot10^6$ кг

$v = 20$ км/с

$M = 6\cdot10^{24}$ кг

$R = 6400$ км



$m\frac{dv}{dt} = F$

$\frac{dL}{dt} = M = FR\sin\alpha$

$dL = mR\sin\alpha\, dv$

$dL = L\,d\varphi$

$\int_0^{\varphi} d\varphi = \frac{mR\sin\alpha}{L}\int_0^{v} dv$

$\varphi = \frac{mvR\sin\alpha}{L} = \frac{mvR\sin\alpha}{\frac{2}{5}MR^2\omega} = \frac{5mv\sin\alpha\cdot T}{2MR\cdot 2\pi} = \frac{5\cdot10^{4}\cdot20\cdot10^3\cdot24\cdot3600}{\sqrt{2}\cdot2\cdot6\cdot10^{27}\cdot6{,}4\cdot10^{6}\cdot2\cdot\pi} =$

$= 1{,}27\cdot10^{-17}$ рад

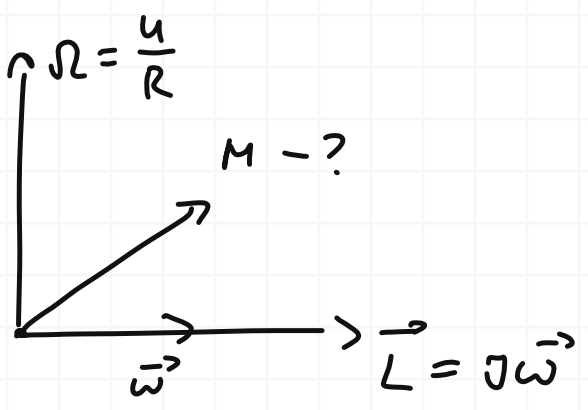
11.8

$u = 300$ км/ч $= \frac{300\cdot10^3}{3600} = \frac{5}{12}$ м/с

$R = 100$ м

$J = 7$ кг·м^2

$N = 1000$ об/(мин) $= \frac{50}{3}$ с$^{-1}$



$\omega = 2\pi N$ $L = 2\pi NJ$

$\vec{M} = [\vec{\Omega};\vec{L}]$; $M = \Omega L = \frac{u}{R}\cdot 2\pi NJ = \frac{5}{3}\cdot\frac{250\cdot2\cdot\pi\cdot50\cdot7}{100\cdot3} =$

$= 610{,}87 \approx 611$ Н·м^2