Неделя 8.

Произвольное движение твёрдого тела.
Гироскоп.

T6

$$\vec{M} = [\vec{\ell}\,; m\vec{g}]$$
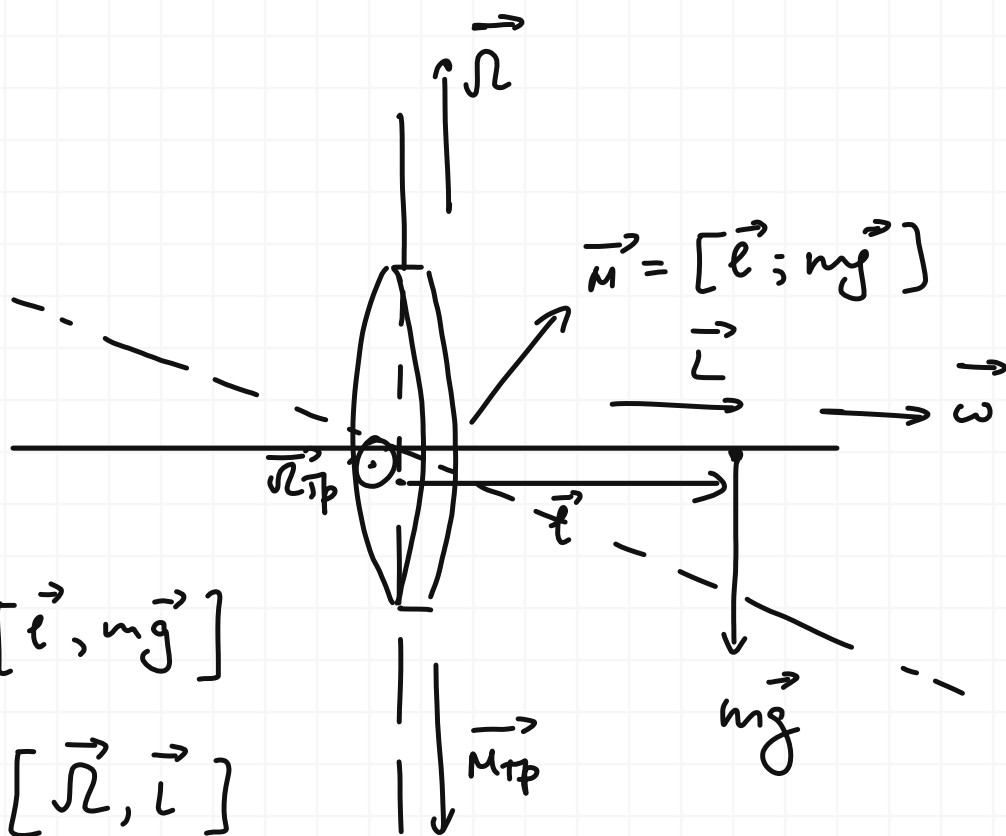
$\ell = 120\,\text{мм} = 0,120\,\text{м}$

$m = 306\,\text{г} = 0,306\,\text{кг}$

$\Delta\alpha = 10°$
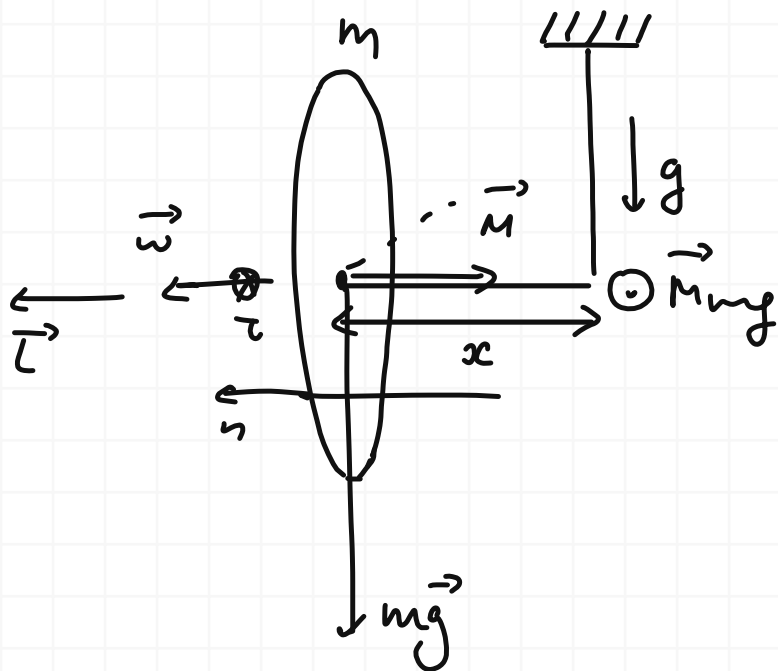
$$\vec{M} = [\vec{\ell}, m\vec{g}]$$

$$\vec{M} = [\vec{\Omega}, \vec{L}]$$

$$\vec{M}_{\text{тр}} = [\vec{\Omega}_{\text{тр}}, \vec{L}] \;;\; M_{\text{тр}} = \Omega_{\text{тр}} L \;;\; \Omega_{\text{тр}} = \frac{M_{\text{тр}}}{L} = \frac{\Delta\alpha}{\tau} = \frac{\Delta\alpha}{2\pi} \cdot \Omega$$

$$M = \Omega L = mg\ell \;;\; \Omega = \frac{mg\ell}{L}$$

$$M_{\text{тр}} = \frac{\Delta\alpha}{2\pi} \cdot \frac{mg\ell}{L} \cdot L = \frac{\Delta\alpha}{2\pi} \cdot mg\ell = \frac{10}{360} \cdot 0,306 \cdot 10 \cdot 0,120 = 0,0102$$
$$\text{Н·м}$$

T7.

m, M, x

$$\omega \longrightarrow \frac{1}{2}\omega$$

$$\frac{d\vec{L}}{dt} = \vec{M} + \vec{M}_{\text{ну}}$$

$$[\vec{\Omega}; \vec{L}] = \vec{M}_{\text{ну}}$$

$$\frac{d\vec{L}}{dt} = \vec{M} + \vec{M}_{\text{ну}}$$

на τ: $\quad \dfrac{dL_\tau}{dt} = -M_{\text{ну}} \;;\quad 0 - L_\tau(t) = -M_{\text{ну}}\,t \;;\quad L_\tau(t) = \Omega L t$

на n: $\quad \dfrac{dL_n}{dt} = -M \;;\quad L_n(t) = L_0 - Mt \;;\quad \dfrac{L_0}{2} = L_0 - M\tau \;;\quad \tau = \dfrac{L_0}{2M}$

$$\Omega L = M_{mg} = mgx$$

$$\Omega(L_0 - Mt) = mgx \quad ; \quad \frac{d\varphi}{dt} = \frac{mgx}{L_0 - Mt} ;$$

$$d\varphi = \frac{mgx}{L_0} \cdot \frac{\frac{L_0}{M} d(\frac{M}{L_0} t)}{1 - \frac{m}{L_0} t} = -\frac{mgx}{M} \frac{d(1 - \frac{M}{L_0} t)}{1 - \frac{m}{L_0} t}$$

$$\underline{\underline{\varphi = -\frac{mgx}{M} \ln \left| 1 - \frac{M}{L_0} \cdot \frac{L_0}{2M} \right| = \frac{m}{M} gx \ln 2}}$$

T 8

$m = 200 \text{ кг}$

$\ell = 1,5 \text{ м}$
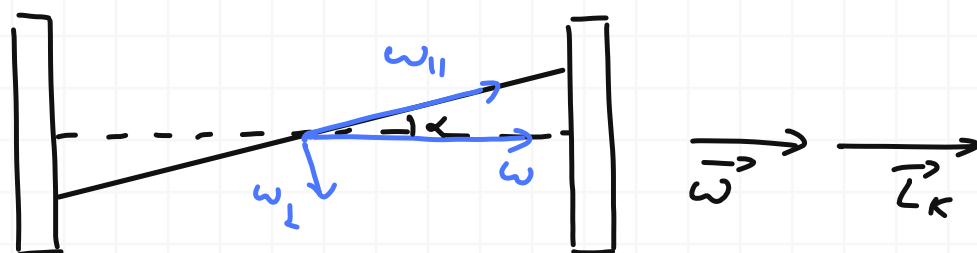
$\alpha = 1°$

$\omega = 50 \text{ c}^{-1}$

$m_0 = 1000 \text{ кг}$

$$\vec{\omega} = \vec{\omega}_{\shortparallel} + \vec{\omega}_\perp$$

$$\vec{L} = \vec{L}_\kappa + \vec{L}_c$$

$$\vec{M} = \left[\vec{\omega} ; \vec{L} \right] = \left[\vec{\omega}, \vec{L}_\kappa \right] + \left[\vec{\omega}, \vec{L}_c \right]$$

$$\vec{L}_c = J_\perp \omega_\perp \vec{n} + J_{\shortparallel} \omega_{\shortparallel} \vec{\tau} = J_\perp \omega_\perp \vec{n}$$

$$\vec{M} = J_\perp \omega_\perp \left[\vec{n} ; \vec{\omega} \right] = \left(\begin{array}{l} J_\perp = \frac{m\ell^2}{12} \omega \sin\alpha \\ \omega_\perp = \omega \sin\alpha \end{array} \right) =$$

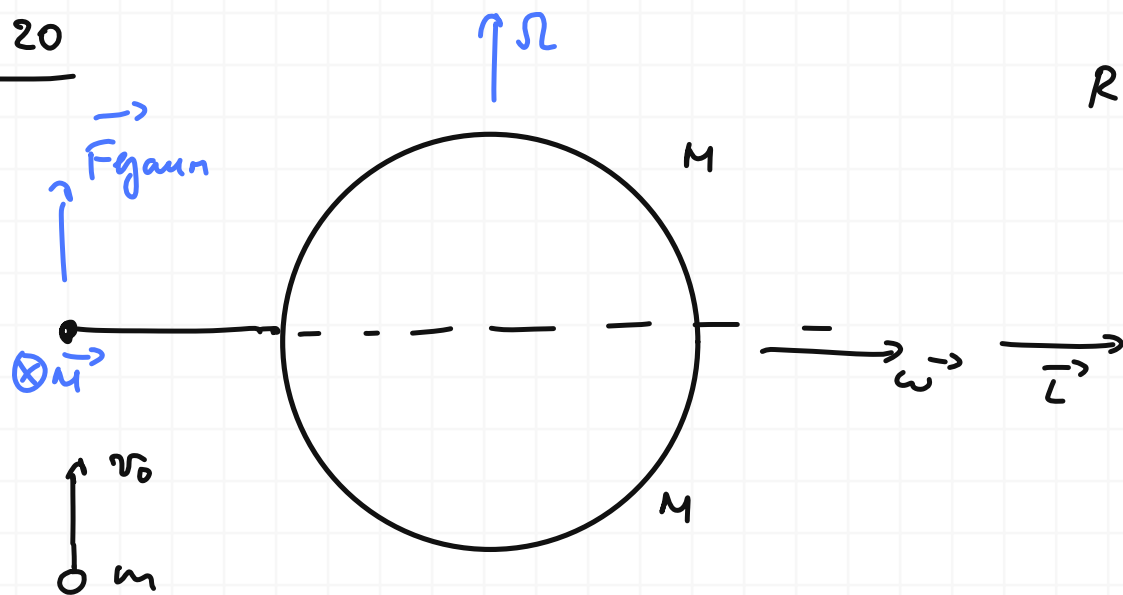$$= \frac{m\ell^2}{12} \omega \sin\alpha \left[\vec{n} ; \vec{\omega} \right] = \frac{1}{12} m\omega^2 \ell^2 \sin\alpha \cos\alpha$$

$N \to max$, когда $\vec{M} \parallel$ Земле

$$|M| = F\ell \implies F = \frac{M}{\ell}$$

$$N_{max} = F + \frac{1}{2} m_0 g = \frac{1}{12} m\omega^2 \ell^2 \alpha + \frac{1}{2} m_0 g$$

$$\underline{\underline{N_{max} = 6 \cdot 10^3 \text{ H}}}$$

11.20

R, ω, r_0 $\dfrac{R\omega}{v_0} = 50$

ЗСИ: $m v_0 = M u$

ЗСЭ: $\dfrac{1}{2} m v_0^2 + \dfrac{1}{2} \cdot \dfrac{2}{5} M R^2 \omega_0^2 = \dfrac{1}{2} M u^2 + \dfrac{1}{2} \cdot \dfrac{2}{5} M R^2 (\omega^2 + \Omega^2)$

ЗСМИ: $\dfrac{2}{5} M R^2 \omega_0 + \dfrac{1}{2} m v_0 R = \dfrac{2}{5} M R^2 \omega + \dfrac{2}{5} M R^2 \Omega$

$\omega = \omega_0$ (нет моментов сил, кот. меняли бы ω)

$m v_0 R = \dfrac{1}{5} M R^2 \Omega$

$u = \dfrac{m}{M} v_0$

$\Omega = \dfrac{5 m v_0}{M R}$

$m v_0^2 = M \dfrac{m^2}{M^2} r_0^2 + \dfrac{2}{5} M R^2 \Omega^2$

$m v_0^2 \dfrac{M - m}{M} = \dfrac{2}{5} M R^2 \Omega^2 = \dfrac{2}{5} M R^2 \cdot \dfrac{25 m^2 v_0^2}{M^2 R^2}$

$M - m = 10 m$

$M = 11 m$

$\varphi - ?$

$\vec{\omega} + \vec{\Omega}$ ← относит. которого все будут вращаться
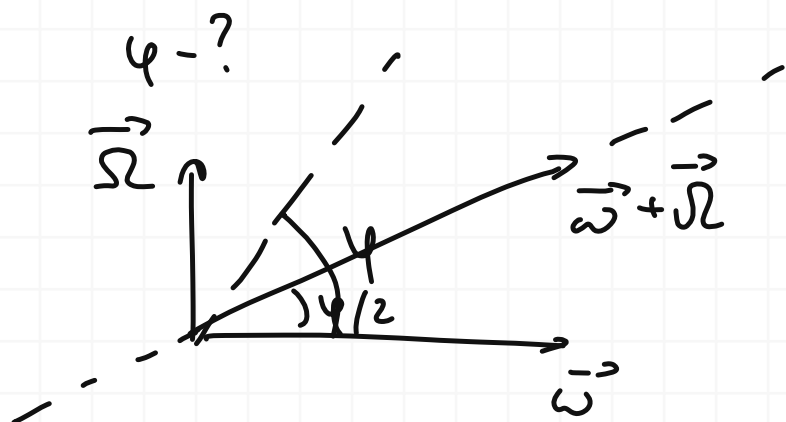
$\operatorname{tg} \dfrac{\varphi}{2} = \dfrac{\Omega}{\omega} = \dfrac{5 m v_0}{M R \omega} = \dfrac{5 m v_0}{11 m R \omega} = \dfrac{5}{11} \cdot \dfrac{1}{50} = \dfrac{1}{110}$

$\varphi = 2 \operatorname{arctg} \left(\dfrac{1}{110} \right) = \underline{\underline{1{,}042° \approx 1°}}$