Неделя 6.

Реальные газы.
Течение газов.
Эффект Джоуля-Томсона

6.41

Ван-дер-Ваальс

$T = const$

$V_0 \to V_0/2$

$V_0/2 \to V_0$ (в вакуум)

$a,\ b,\ T_0$

$\nu = 1$ моль

ΔS - ?

$$\Delta S = c_V \ln\frac{T_2}{T_1} + R\ln\frac{V_2 - b}{V_1 - b}$$

1-2: изотерм. сж

$$\Delta S_{12} = R\ln\frac{V_0/2 - b}{V_0 - b}$$

2-3: расш. в вакуум → внутр. энергия сохр-ся

$$U_2 = U_3;\quad c_V \ln T_0 - \frac{2a}{V_0} = c_V \ln T - \frac{a}{2V_0}$$

$$c_V \ln\frac{T}{T_0} = \frac{a}{V_0}\left(\frac{1}{2} - 2\right) = -\frac{3a}{2V_0}$$

$$\Delta S_{23} = c_V \ln\frac{T}{T_0} + R\ln\frac{2V_0 - b}{V_0/2 - b} = -\frac{3}{a}V_0 - R\ln\frac{V_0/2 - b}{2V_0 - b}$$

$$\Delta S = \Delta S_{13} = \Delta S_{12} + \Delta S_{23} = R\ln\frac{2V_0 - b}{V_0 - b} - \frac{3}{2}\frac{a}{V_0}$$

6.73

$T_{кр}/T = 0{,}4$

$V_{кр}/V = 0{,}09$

$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$

$\frac{\Delta T_{Дж-Т}}{\Delta T_{адиаб.}}$ - ?

$$p = \frac{RT}{V-b} - \frac{a}{V^2}$$

адиабата:

$$dS = \frac{C_p}{T}dT + \left(\frac{\partial S}{\partial p}\right)_T dp = \frac{C_p}{T}dT - \left(\frac{\partial V}{\partial T}\right)_p dp = 0$$

$$dT_{ад} = \frac{T}{C_p}\left(\frac{\partial V}{\partial T}\right)_p dp,$$

$$V_{кр}T_{кр} = \frac{8a}{9R}, \quad a = \frac{9}{8}RV_{кр}T_{кр}$$

эфф. Джоуля-Томсона

$$\Delta T_{Дж-Т} = \frac{\frac{bRT}{(V-b)^2} - \frac{2a}{V^2}}{C_p(\partial p/\partial V)_T}\Delta p$$

$$\frac{\Delta T_{Дж-Т}}{\Delta T_{ад}} = \frac{\frac{bRT}{(V-b)^2} - \frac{2a}{V^2}}{T\left(\frac{\partial p}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p} = -\frac{\frac{bRT}{(V-b)^2} - \frac{2a}{V^2}}{T\left(\frac{\partial p}{\partial T}\right)_V} = \frac{\frac{2a}{V^2} - \frac{bRT}{(V-b)^2}}{RT/(V-b)} = \frac{2a\left(\frac{V-b}{V^2}\right)}{RT} - \frac{b}{V-b} =$$

$$= \frac{\frac{9}{4}RT_{кр}V_{кр}\frac{V - \frac{1}{3}V_{кр}}{V^2}}{RT} - \frac{\frac{1}{3}V_{кр}}{V - \frac{1}{3}V_{кр}} = \frac{3}{4}\frac{T_{кр}}{T}\frac{V_{кр}}{V}\left(3 - \frac{V_{кр}}{V}\right) - \frac{V_{кр}/V}{3 - V_{кр}/V} \approx 0{,}48$$

6.87

N_2, ВдВ → иг

$T_0 = 3T_{кр}$

$T_{кр} = 126\ K$

$V_{кр} = 114 \frac{см^3}{моль}$

$V_0 - ?$

$T - ?$

$$I = u + pV = const$$

$$\underbrace{c_V T_0 - \frac{a}{V_0} + p_0 V_0}_{ВдВ} = \underbrace{c_V T + pV}_{иг}$$

$$c_V T_0 - \frac{a}{V_0} + \left(\frac{R}{V_0 - b} - \frac{a}{V_0^2}\right) V_0 = c_V T + RT$$

$$dT = \frac{-\frac{2a}{V_0^2} + \frac{RT_0 b}{(V_0 - b)^2}}{c_V + R} \overset{иг}{=\!=} 0$$

$$(2a - RT_0 b) V_0^2 - 4ab V_0 + 2ab^2 = 0$$

$$D = 8ab^3 RT_0 = 4 \cdot 2ab^3 RT_0$$

$$V_0 = \frac{2ab \pm \sqrt{2ab^3 RT_0}}{2a - RT_0 b} = \frac{2a \pm \sqrt{2aRT_0}}{\frac{2a}{b} - RT_0} = \frac{2a \pm \sqrt{6a^2 \frac{8}{27}}}{\frac{27}{4}RT_{кр} - 3RT_{кр}} = \frac{2a \pm \frac{4}{3}a}{\frac{15}{4} \cdot \frac{8}{27} \frac{a}{b}} =$$

$$= \begin{cases} \frac{10}{3} \cdot \frac{9b}{10} = 3b = V_{кр} & ✓ \\ \frac{2}{3} \cdot \frac{9}{10} b = \frac{3}{5} b & \times \end{cases}$$

$V_{кр} = 3b$, $T_{кр} = \frac{8a}{27bR}$

$$a = \frac{27bRT_{кр}}{8} = \frac{27 V_{кр}}{3 \cdot 8} RT_{кр} = \frac{9}{8} V_{кр} T_{кр} R \Longrightarrow T = 351\ K$$

$V_0 = V_{кр} = 114 \frac{см^3}{моль}$

2.20

$M = 1$

число Маха

$p_B = 0{,}3$ атм

адиабат.

$M = v / v_{зв}$

$p_0 - ?$

Пусть T_0 — температура воздуха у носа ракеты

$$T_0 = \underbrace{T}_{атм}\left(1 + \frac{\gamma - 1}{2} M^2\right); \quad \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = \frac{\gamma + 1}{2}$$

адиабата: $\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{\gamma + 1}{2}\right)^{\frac{\gamma}{\gamma - 1}} \approx 1{,}89$

$p_0 \approx 0{,}568$ атм

$\underline{16^\circ}$

$$\left(\frac{P}{P_{кр}} + \frac{3}{(V/V_{кр})^2}\right)\left(\frac{V}{V_{кр}} - \frac{1}{3}\right) = \frac{8}{3}\frac{T}{T_{кр}}$$

$$\frac{P}{P_{кр}} = \frac{8(T/T_{кр})}{3(V/V_{кр} - \frac{1}{3})} - \frac{3}{(V/V_{кр})^2} = \pi = 3{,}14$$

$\underline{17^\circ}$

$\Delta S - ?$

иг

$P_1 = 4$атм

$P_2 = 1$атм

$$S = c_v \ln T + R \ln V \xlongequal{V = \frac{RT}{P}} c_v \ln T + R \ln T - R \ln P + c$$

$$S = c_p \ln T - R \ln P + c$$

$$I_1 = I_2: \quad c_p T_1 = c_p T_2; \quad T_1 = T_2$$

$$\Delta S = c_p \underbrace{\ln \frac{T_2}{T_1}}_{0} - R \ln \frac{P_2}{P_1} = R \ln \frac{P_2}{P_1} = 2R\ln 2 \approx 11{,}5\ \frac{Дж}{К}$$

$\underline{18^\circ}$

$T = 273$К

в вакуум

$v_{max} - ?$

$$\tilde{c}_p T = \tilde{c}_p \underbrace{T'}_{0} + \frac{v_{max}^2}{2} \approx \frac{v_{max}^2}{2}$$

$$\frac{C_p}{\mu} T = \frac{v_{max}^2}{2}$$

$$v_{max} = \sqrt{\frac{2C_pT}{\mu}} = \sqrt{\frac{7RT}{\mu}} \approx 740\ \frac{м}{с}$$