Неделя 14.

Элементы теории упругости.

024

$S_1 = S$

$S_2 = 2S_1 = 2S$

$E_1 = E$

$E_2 = 2E_1 = 2E$

$\frac{w_1}{w_2} - ?$

$$w = \frac{\sigma^2}{2E} = \left[\sigma = \frac{F}{S}\right] = \frac{F^2}{S^2}\cdot\frac{1}{2E}$$

$$\frac{w_1}{w_2} = \frac{\frac{\cancel{F^2}}{\cancel{S^2}}\cdot\frac{1}{2\cancel{E}}}{\frac{\cancel{F^2}}{4\cancel{S^2}}\cdot\frac{1}{\underset{2}{\cancel{4E}}}} = 8$$

13.18

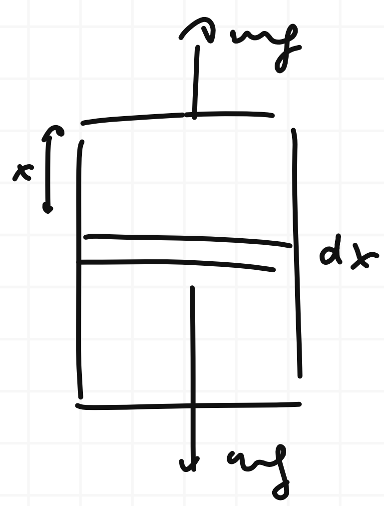
$$\frac{\Delta V}{V_n} = -\frac{p}{k}; \quad k = \frac{E_n}{3(1-2\mu_n)}$$

$$\frac{V_B}{V_n} = \frac{\rho_n}{\rho_B}$$

$$\rightarrow \frac{\rho}{\rho_B} - 1 = -p\cdot\frac{3(1-2\mu)}{E}$$

$$p = \frac{(\rho_B - \rho)E}{3\rho_B(1-2\mu)} = 1{,}93\cdot 10^5 \text{ Па} \approx 2\cdot 10^5 \text{ Па}$$

T.10



$$ma = 2F - mg = mg$$

$$a = g$$

$$\frac{m}{L}xa = mg - mg\frac{x}{L} - \pi r^2 T(x)$$

$$\pi r^2 T(x) = mg - \frac{m}{L}xa - mg\frac{x}{L}$$

$$\pi r^2 T(x) = mg\left(1 - \frac{2x}{L}\right)$$

$$T(x) = \frac{mg}{\pi r^2}\left(1 - \frac{2x}{L}\right)$$

$$\Delta L = \int_0^L \frac{dx\,(T(x))}{E} = 0$$

$T(x) = 0$ при $x = x_0 = \frac{L}{2}$

$$u = \int_0^T \frac{T^2(x)}{2E}\pi r^2 dx = \frac{m^2 g^2}{2E\pi r^2}\int_0^L \left(1 - \frac{2x}{L}\right)^2 dx = \frac{m^2 g^2}{6E\pi r^2}$$

13.36

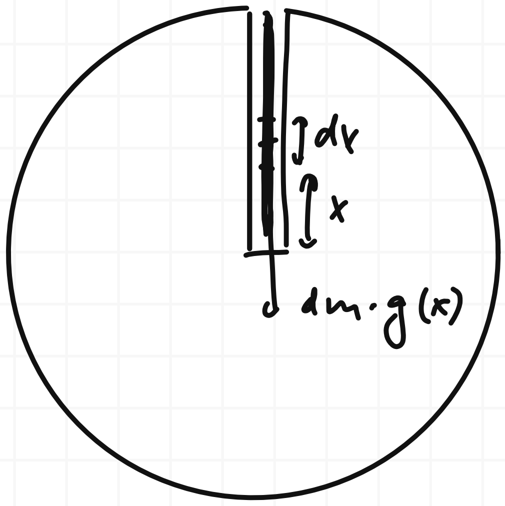
$R = 290$ км $= 29 \cdot 10^4$ м

$g_0 = 0{,}17\ \frac{м}{с^2}$

$\rho_w = 19300\ \frac{кг}{м^3}$

$E = 4 \cdot 10^{11}$ Па

$\Delta l - ?$



Р-м кусочек высотой dx на расст. x от центра астероида.

$$dm = \frac{m}{l}dx = \frac{\rho_w S \cancel{l}}{\cancel{l}}dx = \rho_w S dx$$

II з.Н: $dF = dm \cdot g(x) =$
$$= \underbrace{\frac{4}{3}\pi\rho G x}\,\rho_w S dx = \frac{4}{3}\pi\rho\rho_w G S x dx$$

$$g(r) = \frac{4}{3}\pi\rho G \cdot r$$

$$g_0 = \frac{4}{3}\pi\rho G R$$

з-н Гука: $dF = ES\frac{d(\Delta l)}{dx}$

$$\frac{4}{3}\pi\rho\rho_w G\cancel{S}x dx = E\cancel{S}\frac{d\Delta l}{dx}$$

$$\int d\Delta l = \frac{4}{3}\frac{\pi\rho\rho_w G\int_0^R x (dx)^2}{E}$$

$$\Delta l = \frac{4}{3}\pi\rho\rho_w G\int_0^R \frac{x^2}{2}dx = \frac{4}{3}\pi\rho\rho_w G\frac{R^3}{6} = \underbrace{\frac{4}{3}\pi\rho G R}_{g_0}\cdot\rho_w\frac{R^2}{6E} = \frac{g_0\rho_w R^2}{6E} = 115\ м$$

13.42

$m = 1000$ кг

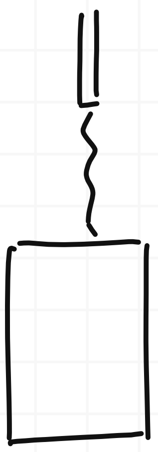
$v = 1\ \frac{м}{с}$

$E = 10^{11}\ \frac{Н}{м^2}$

$k = 4 \cdot 10^5\ \frac{Н}{м}$

$l = 10$ м

$F_{max} - ?$



не будем учитывать $F_{тяж}$ (силу, деформ, энергию), потому что в тросе и грузе возникн. „противод." силы, которые постоянны при всем движении

а нас интересует резкая остановка и силы, возникш. при ней

$k = \frac{ES}{l_0}$ мех. напр, кот. создаст динамич. сила (без учета mg)

$\sigma = E\varepsilon = E\frac{\Delta l}{l}$; $\Delta l = \frac{\sigma l_0}{E}$ (з-н Гука)

(ЗСЭ) $\frac{1}{2}mv_0^2 = \frac{1}{2}kx^2 + \frac{1}{2}k_T\Delta l^2$

$$mv_0^2 = \frac{ES}{l_0}\cdot\frac{\sigma^2 l_0^2}{E^2} + \frac{k\sigma^2 S^2}{k^2} = \frac{\sigma^2 l_0 S}{E} + \frac{\sigma^2 S^2}{k} = \sigma^2 S\left(\frac{l_0}{E} + \frac{S}{k}\right)$$

$$\sigma = \sqrt{\frac{mv_0^2}{S\left(\frac{l_0}{E}+\frac{S}{k}\right)}} = \sqrt{\frac{mv_0^2 Ek}{S(l_0k+ES)}} = v_0\sqrt{\frac{mEk}{S(kl_0+ES)}}$$

$$F = \sigma S = v_0\sqrt{\frac{mESk}{kl_0+ES}}$$

вспомним про силу тяжести

$$F_{max} = mg + v_0\sqrt{\frac{mESk}{kl_0+ES}} = 2{,}96\cdot10^4\ Н$$

13.49

$R = 100мм = 0{,}1м$

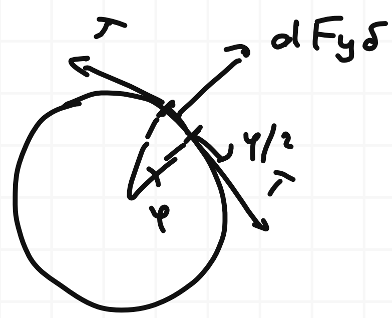
$\nu = 400\ об/с$

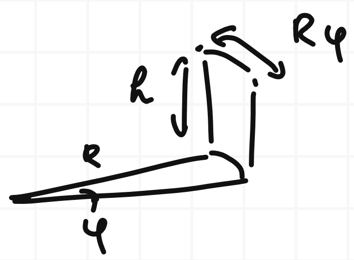
$E = 2\cdot10^{11}\ Па$

$\rho = 7800\ \frac{кг}{м^3}$

$T_{пр}$ - ?

ΔR - ?





$\omega = 2\pi\nu$

$dF_{цб} = dm\cdot\omega^2 R = (2\pi\nu)^2 R\rho R\varphi$

$dF_{цб} = T\varphi$ (II з.Н.)

$(2\pi\nu)^2\rho R^2\cancel{\varphi} = T\cancel{\varphi}$

$T = (2\pi\nu)^2\rho R^2 = 4\pi^2\nu^2\rho R^2 = 4{,}9\cdot10^8\ Па$

предельн. значение
(подойдет это и больше)

$\frac{\Delta L}{\omega} = \frac{\Delta L}{2\pi\nu}$; $\Delta L = \frac{2\pi\nu\, T}{E}$

$\cancel{L} + \Delta L = \pi(\cancel{R} + \Delta R)$; $\Delta L = \pi\Delta R$

$$\Delta R = \frac{2\pi RT}{ET} = \frac{2\rho(2\pi\nu)^2R^3}{E} = 0{,}49\ м$$