Неделя 10.

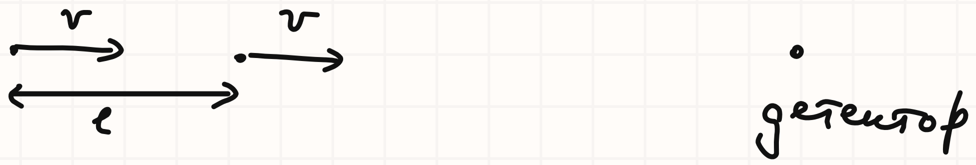
Кинематические эффекты теории относительности
Преобразования Лоренца

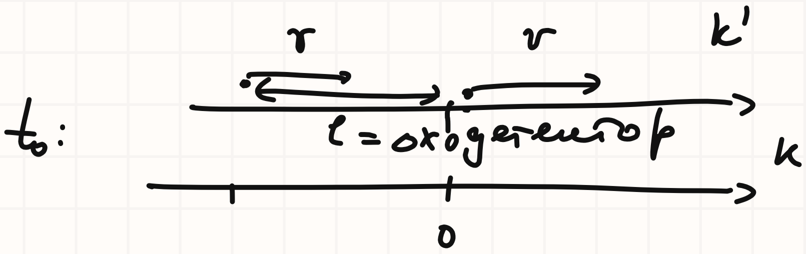
№16

$v = 0{,}99c$

$\Delta t = 10^{-4}$ с

$\ell - ?$



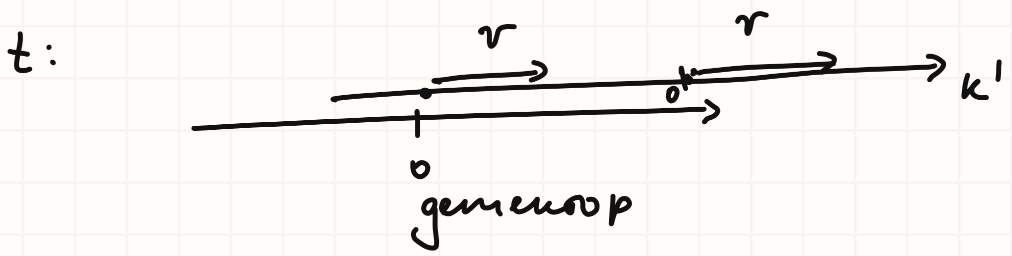


в этот момент t_0

$x_1 = x_1' = 0$

$x_1' = \dfrac{x^0 - vt_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$; $t_0 = 0$

$t_0' = \dfrac{0-0}{\sqrt{\ }} = 0$; $t_0' = 0$



$x_2 = 0$ $\quad t = \Delta t$

$x_2' = -\ell$

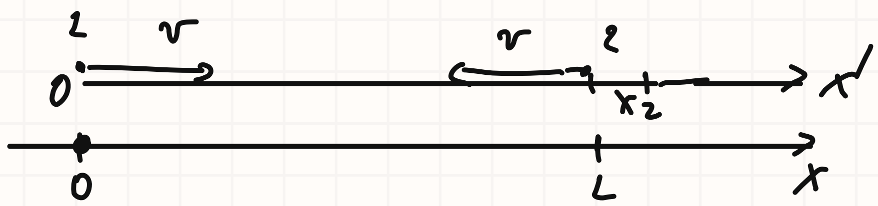
$$\boxed{+\ell = \frac{+vt}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{v\Delta t}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{0{,}99\cdot 3\cdot 10^8\cdot 10^{-4}}{\sqrt{1-(0{,}99)^2}} = 2{,}11\cdot 10^5 \text{ м}}$$

17°

L, $t = \frac{L}{c}$

$v - ?$

$t=0 \quad t'=0$



до столкновения

В ЛСО: $t = \dfrac{L}{2v} = \dfrac{L}{c}$; $v = \frac{1}{2}c$

В СО 1-й частицы $(V = v)$; $v_2' = v_{отн}$

$x_2 = L$; $x_2' = \dfrac{L - vt}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{L - \frac{1}{2}ct}{\sqrt{3}/2} = \dfrac{2}{\sqrt{3}}\left(L - \frac{1}{2}ct\right)$

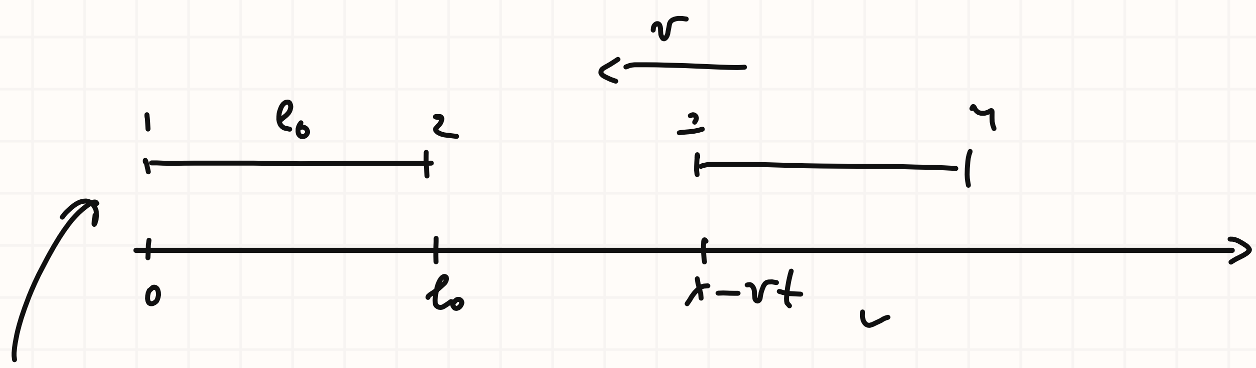
$t = \dfrac{t' + \frac{v}{c^2}x'}{\sqrt{1-\beta^2}} = \left[\begin{matrix}\text{в момент столкн.}\\ x'=0\end{matrix}\right] = \gamma t' = \dfrac{2}{\sqrt{3}}t'$

$$v_{отн} = \left|\frac{(-v) - v}{1 - \frac{v(-v)}{c^2}}\right| = \left|\frac{-2v}{1+\frac{v^2}{c^2}}\right| = \left|-\frac{c}{1+\frac{1}{4}}\right| = \frac{4}{5}c$$

8.4

l_0, τ

$v - ?$



пусть 12 — линейка, с которой связан наблюдатель.

t_1: 2 и 3 совп. $\qquad t_1 = \dfrac{x - v_0}{v}$

t_2: 1 и 4 совп. $\qquad t_2 = \dfrac{x + l_0\sqrt{1-\left(\frac{v}{c}\right)^2}}{v}$

$$\tau = t_2 - t_1 = \frac{l_0\sqrt{1-\left(\frac{v}{c}\right)^2}}{v} + \frac{l_0}{v}$$

$$\frac{v\tau}{l_0} = 1 + \sqrt{1-\left(\frac{v}{c}\right)^2}$$

$$\left(\frac{v\tau}{l_0} - 1\right)^2 = 1 - \left(\frac{v}{c}\right)^2$$

$$\left(\frac{v\tau}{l_0}\right)^2 - 2\frac{v\tau}{l_0} + \left(\frac{v}{c}\right)^2 = 0 \quad \Big| : \frac{l_0^2}{v}$$

$$v\tau^2 - 2l_0\tau + \frac{vl_0^2}{c^2} = 0$$

$$v\left(\tau^2 + \frac{l_0^2}{c^2}\right) = 2l_0\tau$$

$$\boxed{v = \frac{2l_0\tau}{\tau^2 + \left(\frac{l_0}{c}\right)^2} = \frac{2\frac{l_0}{\tau}}{1 + \left(\frac{l_0}{\tau c}\right)^2}}$$

8.79.

$k: \Delta t = 3$ нс

$k': \Delta t' = 5$ нс

$l' - ?$

$$l' = v\Delta t'$$

$$l = l\sqrt{1-\left(\frac{v}{c}\right)^2} = v\Delta t'$$

$$\left(\frac{v\Delta t}{l_0}\right)^2 = 1 - \left(\frac{v}{c}\right)^2$$

$$v^2\left(\left(\frac{\Delta t}{l'}\right)^2 + \left(\frac{1}{c}\right)^2\right) = 1$$

$$\frac{l'}{\Delta t'} = v = \frac{1}{\sqrt{\left(\frac{\Delta t}{l'}\right)^2 + \left(\frac{1}{c}\right)^2}}$$

$$(l')^2 = \frac{(\Delta t')^2}{\left(\frac{\Delta t}{l'}\right)^2 + \frac{1}{c^2}}$$

$$\Delta t^2 + \left(\frac{l'}{c}\right)^2 = (\Delta t')^2$$

$$(l')^2 = c^2(\Delta t'^2 - \Delta t^2)$$

$$l' = c\sqrt{\Delta t'^2 - \Delta t^2} = 3\cdot 10^8\sqrt{16\cdot 10^{-9}} = \underline{1{,}2\ \text{м}}$$

8.30

$v = 0{,}8c$

$\tau = 1c$

$$\Delta t = \tau\sqrt{1-\left(\frac{v}{c}\right)^2} = \underline{0{,}6\,c}$$

8.77

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{25}{\sqrt{625-576}} = \frac{25}{7}$$

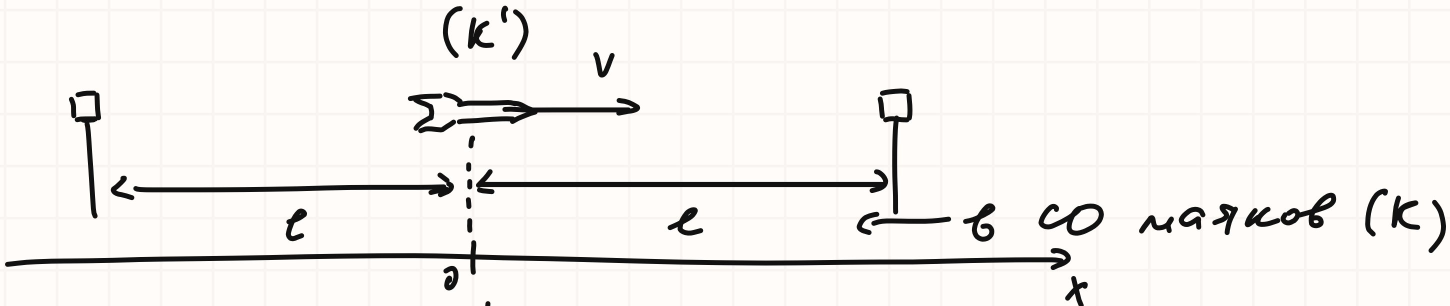
$$age_{\text{петр}} = 21 + 2\cdot 7 = \underline{35 \text{ лет}}$$

$$age_{\text{павел}} = 21 + 2\gamma 7 = 21 + 2\cdot\frac{25}{\cancel{7}}\cdot\cancel{7} = \underline{71 \text{ год}}$$

8.7

$V = 0{,}6c$

$\tau = 2$мес



1) $\tau = \frac{2\ell}{c}$; $\ell = \frac{1}{2}c\tau$

2) перейдем в K' (СО косм. корабля):

t'_1, t'_2 – t' рег.

$x'_1(t)$ – x' сигнала с 1 маяка

$$x_1(t) = -\ell + ct \qquad\qquad x_2(t) = \ell - ct$$

$$x'_1(t) = \frac{-\ell + ct - Vt}{\sqrt{1-\frac{V^2}{c^2}}} \qquad\qquad x'_2(t) = \frac{\ell - ct - Vt}{\sqrt{1-\frac{V^2}{c^2}}}$$

рег: $x'_1 = 0$; $x'_2 = 0$

$$-\ell + ct - Vt = 0 \qquad\qquad \ell - ct - Vt = 0$$

$$t_{1\,\text{рег}} = \frac{\ell}{c-V} = \frac{\cancel{c}\tau}{2\cdot\frac{2}{5}\cancel{c}} = \frac{5}{4}\tau \qquad\qquad t_{2\,\text{рег}} = \frac{\ell}{c+V} = \frac{\tau}{2\cdot\frac{8}{5}} = \frac{5}{16}\tau$$

$$X_{1\,\text{рег}} = \overset{-\frac{2}{4}c\tau}{-\ell} + \frac{5}{4}c\tau = \frac{3}{4}c\tau \qquad\qquad x_{2\,\text{рег}} = \ell - \frac{5}{16}c\tau = \frac{3}{16}c\tau$$

// $\beta = \frac{v}{c} = 0{,}6 = \frac{3}{5}$; $\sqrt{1-\beta^2} = \frac{4}{5}$; $\gamma = \frac{5}{4}$ //

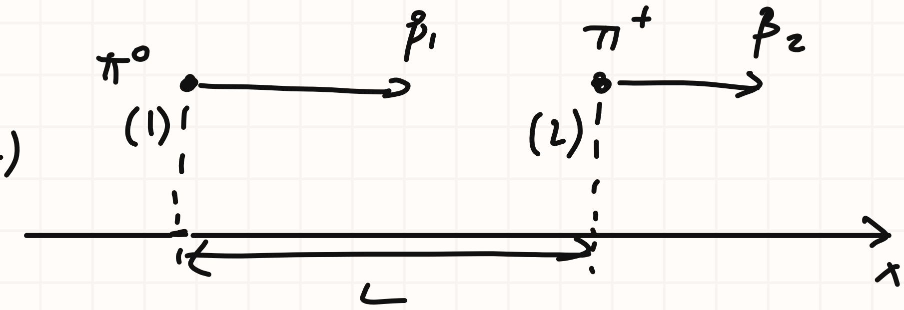
$$t_1' = \gamma\left(t_{1\,per} - \frac{v}{c^2}x_{1\,per}\right) = \frac{5}{4}\left(\frac{5}{4}\tau - \frac{3}{5\cancel{c}}\cdot\frac{3\cancel{c}}{4}\tau\right) = \frac{\cancel{5}}{\cancel{4}}\left(\frac{25-9}{\cancel{5}\cdot\cancel{4}}\right)\tau = \tau$$

$$t_2' = \gamma\left(t_{2\,per} - \frac{v}{c^2}x_{2\,per}\right) = \frac{5}{4}\left(\frac{5}{16}^{15} - \frac{3}{5}\cdot\frac{3}{16}\right)\tau = \frac{\cancel{8}}{4}\,\frac{\cancel{16}}{\cancel{5}\cdot\cancel{16}}\tau = \frac{1}{4}\tau$$

$$\Delta t = \tau - \frac{1}{4}\tau = \underline{\underline{\frac{3}{4}\tau}} = \underline{\underline{1{,}5\ \text{мкс}}}$$

8.98

$\tau_1 = 8{,}7\cdot 10^{-17}$ (c)
$v_1 = 0{,}8c$
$\tau_2 = 2\cdot 10^{-8}$ (c)
$v_2 = 0{,}6c$
$L = 6$ (нм)



$$\beta_{отн} = \frac{\beta_1 - \beta_2}{1-\beta_1\beta_2} = \frac{\frac{4}{5}-\frac{3}{5}}{1-\frac{3\cdot 4}{5\cdot 5}} = \frac{1\cdot 5\cdot\cancel{5}}{\cancel{5}\cdot 13} = \frac{5}{13}$$

$$\gamma_{отн} = \frac{1}{\sqrt{1-\left(\frac{5}{13}\right)^2}} = \frac{13}{12}$$

перейдем в СО π^+:

$$\tau'_{\pi^0} = \gamma_{отн}\cdot\tau_1 = \frac{13}{12}\cdot 8{,}7\cdot 10^{-17} = 9{,}425\cdot 10^{-17}\ c$$ — время жизни

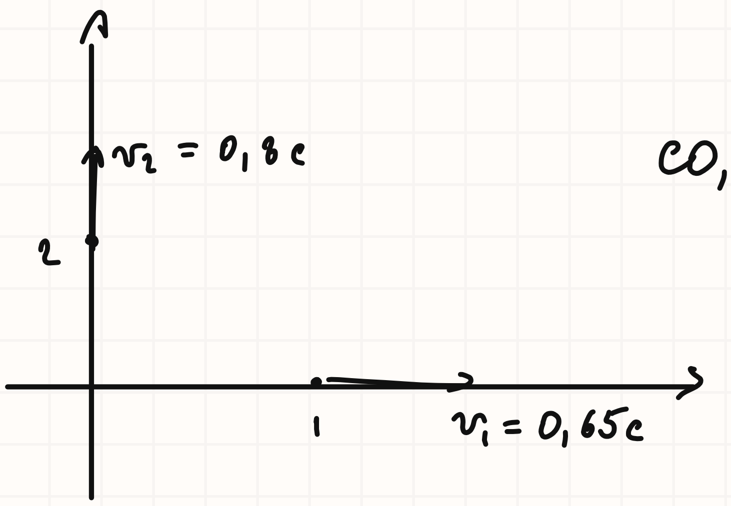
$$l' = \frac{L}{\gamma_{отн}} = \frac{6\cdot 12}{13} = \frac{72}{13}\ \text{нм}$$

$$t = \frac{l'}{\underbrace{\beta_{отн}\cdot c}_{v_{сближ}}} = 4{,}8\cdot 10^{-17}\ c$$ — время „догонения"

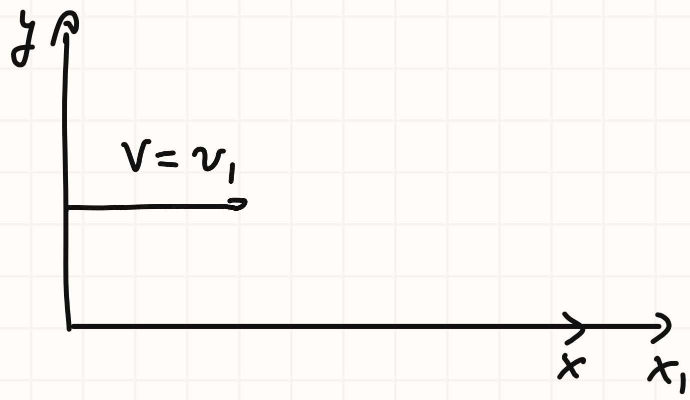
$t < \tau'_{\pi^0} \implies$ догонит!

8.89 $v_{отн}$ − ?

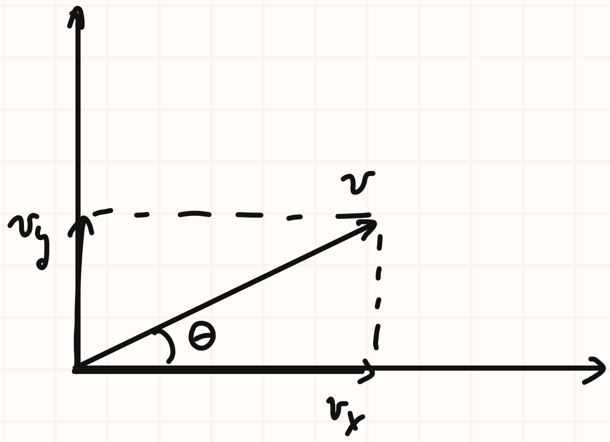
ЛСО: $x_1 = v_1 t$, $y_1 = 0$; $x_2 = 0$, $y_2 = v_2 t$



СО₁ :



воспользуемся преобразованием Лоренца



$$\begin{cases} v_x = v\cos\theta \\ v_y = v\sin\theta \end{cases}$$

$$v_x = \frac{v_x' + V}{1 + \frac{V v_x'}{c^2}} \qquad v_y = \frac{v_y'\sqrt{1 - \frac{V^2}{c^2}}}{1 + \frac{V v_x'}{c^2}}$$

где $V = v_1$, $v_x' = 0$, $v_y' = v_2$

$$\begin{cases} v\cos\theta = v_1 \\ v\sin\theta = v_2\sqrt{1 - \frac{v_1^2}{c^2}} \end{cases}$$

$$v = \sqrt{v_1^2 + v_2^2 - \frac{v_1^2 v_2^2}{c^2}}$$