# Неделя 13

Вынужд. колебания.
НсИСО.

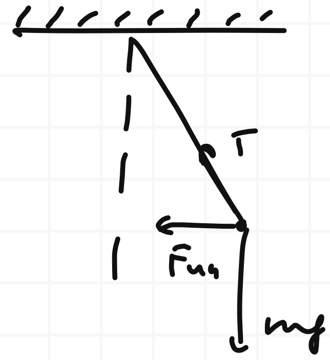
°22

$L = 9,8\,см = 0,098\,м$

$\Omega = 11\ рад/с$

$a = 1\,мм = 0,001\,м$

$A - ?$




пусть $x_0$ – коорд. подвеса в лев

$x_0 = a\cos\Omega t$

$\ddot{x}_0 = -a\Omega^2\cos\Omega t$

$$F_{ин} = -m\ddot{x}_0 = ma\Omega^2\cos\Omega t$$

(II з.Н)

$$ml\ddot{\varphi} = -(mg\overset{\approx\varphi}{\sin\varphi} + F_{ин}\overset{\approx 1}{\cos\varphi})$$

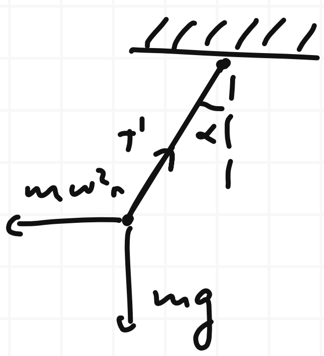
$$\ddot{\varphi} = -\frac{g}{l}\varphi + \frac{\Omega^2}{l}\cos\Omega t \quad \text{— вынужд. колеб.}$$

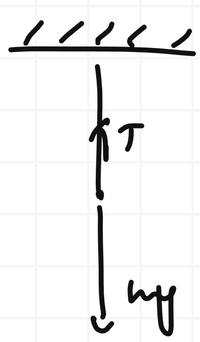
$$\varphi_{max} = \frac{\frac{a\Omega^2}{l}}{\frac{g}{l} - \Omega^2}$$

$$A = a + l\varphi_{max} = a + \frac{a\Omega^2}{\frac{g}{l} - \Omega^2} = \frac{a + \frac{g}{l}}{\frac{g}{l} - \Omega^2} = \frac{a}{1 - \Omega^2\frac{l}{g}}$$

$=$

°23




$T = mg$
(покоится)

$$\left.\begin{aligned} mg &= T'\cos\alpha \\ m\omega^2 r &= T'\sin\alpha \end{aligned}\right\} :$$

$$\underline{\underline{\overset{\approx\alpha}{tg\,\alpha} = \frac{\omega^2 r}{g} = \frac{v^2}{rg} = 3,2\cdot 10^{-3}}} \ll 1$$

α – малый, воспользуемся

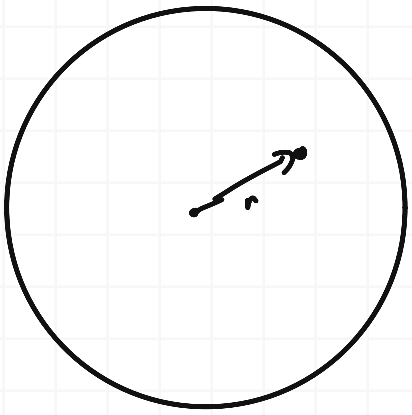
$$mg = T'\left(1 - \frac{\alpha^2}{2}\right)$$

$$m\omega^2 r = T'\alpha$$

$$\alpha = \frac{v^2}{rg}$$

$$\frac{|T'-T|}{T} = \frac{T'-mg}{mg} = \frac{T'}{mg} - 1 = \frac{1}{1-\frac{\alpha^2}{2}} = \frac{\alpha^2}{2} = 3,5\cdot 10^{-5}$$

12.38



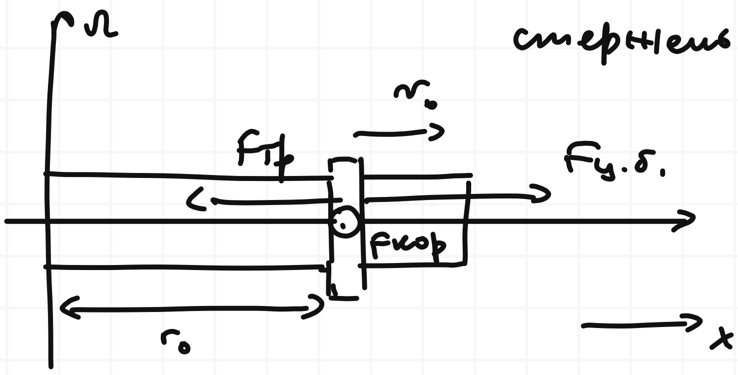

$$F_{ин} = F_{у.б} = m\omega^2 r$$

$$A = \int_R^0 -m\omega^2 r\,dr = \int_0^R m\omega^2 r\,dr = \frac{m\omega^2 R^2}{2} = 750 \text{ Дж}$$

12.48

$v = v_0 \frac{r}{r_0}$ — муфты отн. стержня

стержень неИСО




$$F_{кор} = -2m[\vec{\Omega}, \vec{v}] = 2m[\vec{v}, \vec{\Omega}]$$

$$F_{у.б.} = m\Omega^2 R$$

перейдем в СО вращ. стержня

$F_{ин} = F_{кор} + F_{уб}$

$$v = \frac{r}{r_0} v_0$$

$$\ddot{r} = \dot{v} = \frac{v_0}{r_0}\dot{r} = \left(\frac{v_0}{r_0}\right)^2 r$$

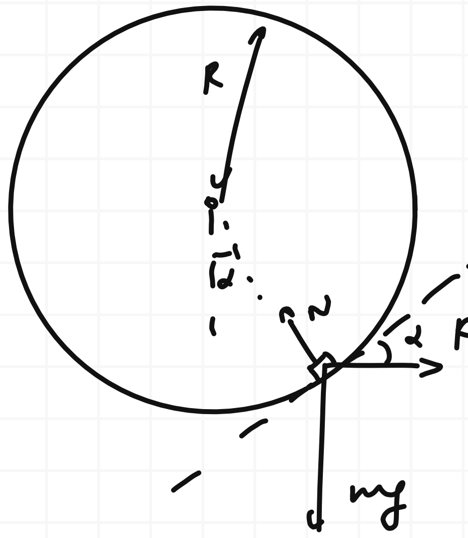
(II з.н)

$$\begin{cases} N - 2m\omega v = 0 \\ m\ddot{r} = -kN + m\omega^2 r \end{cases} \qquad \begin{cases} N = 2m\omega \frac{r}{r_0} v_0 \\ \left(\frac{v_0}{r_0}\right)^2 r + \left(2k\omega\frac{v_0}{r_0} - \omega^2\right) r = 0 \end{cases}$$

$$k = \frac{r_0}{2\omega v_0}\cdot\left(\omega^2 - \left(\frac{v_0}{r_0}\right)^2\right)$$

$$k = \frac{1}{2}\left(\frac{\omega r_0}{v_0} - \frac{v_0}{\omega r_0}\right) \qquad \omega r_0 > v_0$$

12.89




$F_{цб} = m\Omega^2 R\sin\alpha$

в СО вращ. кольца:

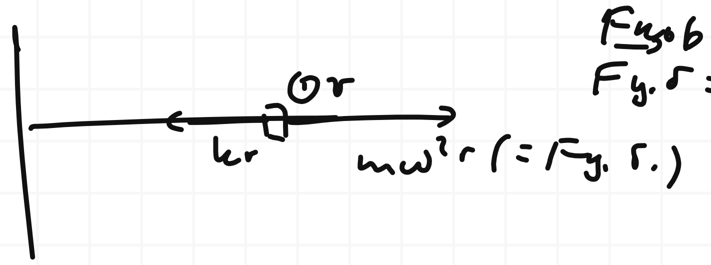
(II з.Н)

$$m\Omega^2 R\overset{\approx\alpha}{\sin\alpha}\,\overset{\approx1}{\cos\alpha} - mg\overset{\approx\alpha}{\sin\alpha} = Rm\ddot\alpha$$

$$\ddot\alpha + \underbrace{\left(\frac{g}{R} - \Omega^2\right)}_{\omega^2}\alpha = 0$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{R} - \Omega^2}} = 8{,}55\ с$$

12.86




$$\vec F_{кор} = -m\vec\omega\times\vec v$$

$$F_{ц.б} = m\omega^2\vec r$$

$$m\ddot{\vec r} = -k\vec r + m\omega^2\vec r - 2m\vec\omega\times\vec v = [k = m\omega^2 \text{ (численно)}] = -2m\vec\omega\times\vec r$$

$$\ddot{\vec r} = 2m\vec v\times\vec\omega$$

$$R = \frac{v^2}{2\omega v} = \frac{v}{2\omega}$$

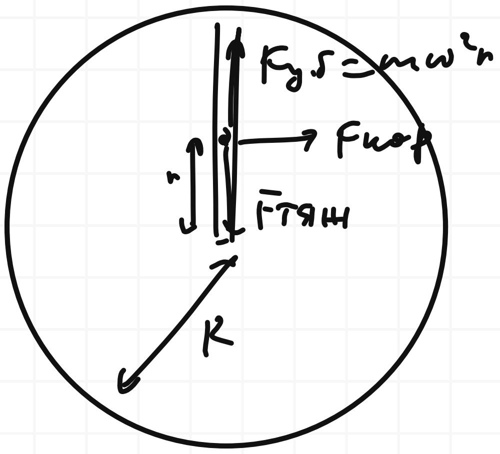
максим. удаление

$$l_{max} = r + 2R = r + \cancel{2}\cdot\frac{v}{\cancel{2}\omega} = r + \frac{v}{\omega} = 2\ (м)$$

$2 > 1{,}9$

значит, вылетит

12.70




$$F_{тяг} = \frac{\frac{4}{3}\pi r^3\rho m G}{r^2} = m\cdot\underbrace{\frac{4}{3}\pi\rho G r}_{g(r)} \ (=)$$

$$\overbrace{\frac{4}{3}\pi\rho GR}^{g}\,\frac{r}{R} = \frac{r}{R}g$$

$$(=)\ mg\frac{r}{R}$$

перейдем в СО Земли, запишем II з.Н.

$$\begin{cases} m\ddot{r} = m\omega^2 r - mg\dfrac{r}{R} \\ m\ddot{\alpha} = -2m\omega\dot{r} \end{cases} \qquad \begin{cases} \ddot{r} + \left(g\dfrac{r}{R} - \omega^2\right) r = 0 \\ \ddot{\alpha} + 2\omega\dot{r} = 0 \end{cases}$$

нач. усл: $r = R$, $r' = 0$ R – радиус Земли

$$r = R\cos(\tilde{\omega}t), \quad \tilde{\omega} = \sqrt{\frac{g}{R} - \omega^2}$$

$$\ddot{\alpha} = 2R\omega\tilde{\omega}\sin(\tilde{\omega}t)$$

$$\dot{\alpha} = -2R\omega\cos(\tilde{\omega}t) + c$$

$$\alpha = -2R\frac{\omega}{\tilde{\omega}}\sin(\tilde{\omega}t) + ct + c'$$

$c, c'$ – константы

нач. усл: $\alpha = 0$; $\dot{\alpha} = 0$ $\rightsquigarrow$ $c_1 = 2\omega R$, $c_2 = 0$

$$\alpha(\tau) = -2\frac{\omega}{\tilde{\omega}}R\sin(\tilde{\omega}\tau) + 2\omega R\tau = -\frac{2\omega R_3}{\tilde{\omega}} + \frac{\pi\omega R_3}{\tilde{\omega}}$$

$$\tilde{\omega} = \sqrt{\frac{g}{R}}$$

$$\alpha_{min} = \alpha(\tau) = \omega R\sqrt{\frac{R}{g}}(\pi - 2) = \omega\frac{R\sqrt{R}}{\sqrt{g}}(\pi - 2)$$