Неделя 3. ЗСИ. Реактивное движение.

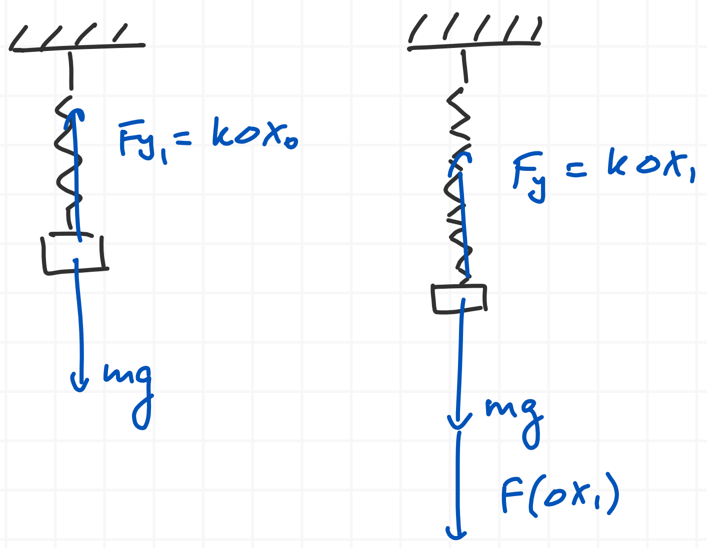
№6

$k = 400$ Н/м

$\Delta x_0 = 2$ см $= 0{,}02$ м

$\Delta x_1 = 6$ см $= 3\Delta x_0$

A — ?



$$mg = k\Delta x_0$$

$$F(\Delta x) = k\Delta x - mg = k(\Delta x - \Delta x_0)$$

$$dA = F(\Delta x)\cdot d(\Delta x) = k(\Delta x - \Delta x_0)\, d(\Delta x)$$

$$A = k\int_{\Delta x_0}^{\Delta x_1} (\Delta x - \Delta x_0)\, d(\Delta x - \Delta x_0) = \frac{1}{2}k(\Delta x_1 - \Delta x_0)^2 = 2k\Delta x_0^2 =$$

$$= 2\cdot 400\,\frac{Н}{м}\cdot(0{,}02)^2 = \underline{0{,}32\ Дж}$$

Ответ: 0,32 Дж.

№7

$a = 4$ нм

r — ?

$F_0 = F(r) = 0$

$$U(r) = U_0\left[\left(\frac{a}{r}\right)^{12} - \left(\frac{a}{r}\right)^{6}\right]; \quad U_0 > 0$$

$$F(r) = 0 \text{ когда } \frac{dU(r)}{dr} = 0$$

$$dU(r) = U_0\left(a^{12}(-12)r_0^{-13} - a^6(-6)r_0^{-7}\right) = 0$$

$$+\cancel{12}^{\,2}\, a^6 r_0^{-13} = +\cancel{6}\, r_0^{-7}$$

$$2a^6 = r_0^6$$

$$\underline{r_0} = \sqrt[6]{2}\cdot a = \sqrt[6]{2}\cdot 4\ нм = \underline{4{,}49\ нм}$$

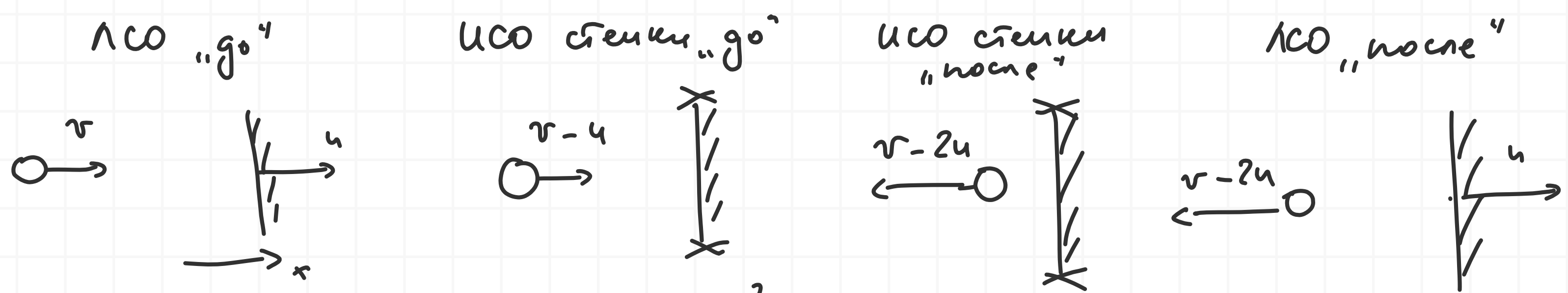
4.70

Δk — ?

Δp — ?

v, u

$\frac{v}{u}$ — ? (остановка)



$$p_{T_0} = mv \qquad k_{T_0} = \frac{mv^2}{2}$$

$$p_T = -mv + 2mu \qquad k_T = \frac{m(v-2u)^2}{2}$$

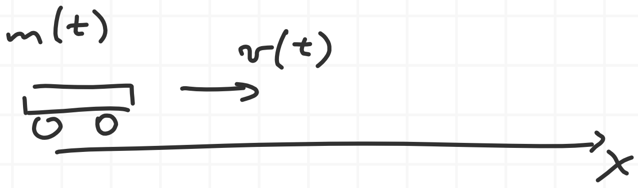
$$\underline{\Delta p} = p_T - p_{T_0} = -mv + 2mu - mv = \underline{2m(u - v)}$$

$$\underline{\Delta k_T} = \frac{m}{\cancel{2}}\left(\cancel{v^2} + \cancel{4}^{\,2}u^2 - \cancel{4}^{\,2}vu - \cancel{v^2}\right) = \underline{2mu(u - v)}$$

остановка: $v - 2u = 0$; $v = 2u$; $\underline{\frac{v}{u} = 2}$

3.11 1) дворник, кот. не сметает снег

μ, m_0, v_0



ЗСИ: $m_0 v_0 = m(t) v(t)$; $v(t) = \frac{m_0}{m(t)} v_0 = \frac{m_0}{m_0+\mu t} v_0$

через ур-е Мещерского: $m\frac{dv}{dt} = -\mu v$; $-\int_{v_0}^{v(t)} \frac{dv}{v} = \int_0^t \frac{\mu}{m_0+\mu t}\,dt$

$$-\ln\left|\frac{v(t)}{v_0}\right| = \int \frac{d\left(\frac{\mu}{m_0}t\right)}{1+\frac{\mu}{m_0}t} = \ln\left|\frac{1+\frac{\mu}{m_0}t}{1}\right| = \ln\left|\frac{m_0+\mu t}{m_0}\right|$$

$$v(t) = v_0 \frac{m_0}{m_0+\mu t}$$

2) дворник сметает снег

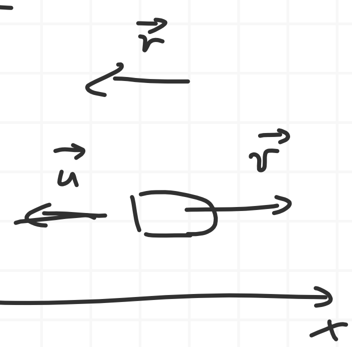
Тележка закреплена на рельсах, а значит, интересует лишь проекция на ось рельс. Дворник смет. перп., $\vec{u}\frac{dm}{dt} \perp$ оси, в пр. не учит.

$$m\frac{d\vec{v}}{\cancel{dt}} = -\vec{v}\frac{dm}{\cancel{dt}} - \vec{u}\frac{dm}{dt}$$

$m\,dv = -v\,dm$; $\frac{dv}{v} = -\frac{dm}{m} = -\frac{\mu\,dt}{m} = -\frac{\mu}{m}dt$; $\ln\frac{v(t)}{v_0} = -\frac{\mu}{m}t$

$$v(t) = v_0 e^{-\frac{\mu}{m}t}$$

3.41



$v_0 = 0$
$u = const$

$m_0 = const$ (масса ракеты)

$dm_п = -dm_т$ (расход)

$m_0\frac{dv}{dt} = -u\,dm_T - v\,dm_п = (v-u)\,dm_T$

$m_0\,dv = (v-u)\,dm_T$

ракеты без топл

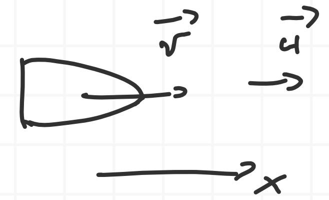
$\int\frac{d(v-u)}{v-u} = \int\frac{dm_T}{m_0}$; $\ln\left(\frac{v-u}{-u}\right) = \frac{m-m_0}{m_0}$

$v-u = -ue^{\frac{m}{m_0}-1}$; $\underset{max}{v} = u\left(1-e^{\underset{min=0}{\frac{m}{m_0}}-1}\right) = [m_{топл}=0] = u(1-e^{-1})$

$$v_{max} = u\left(1-\frac{1}{e}\right)$$

№ 3.43

1) по прямой



$m d\vec{v} = -\vec{u} dm$; $m dv = -u dm$; $\int_v^0 \frac{dv}{u} = \int_{m_0}^{m_{ост}} \frac{dm}{m}$;

$\frac{v}{u} = \ln\left(\frac{m_{ост}}{m_0}\right)$; $\frac{m_{ост}}{m_0} = \exp\left(\frac{v}{u}\right)$

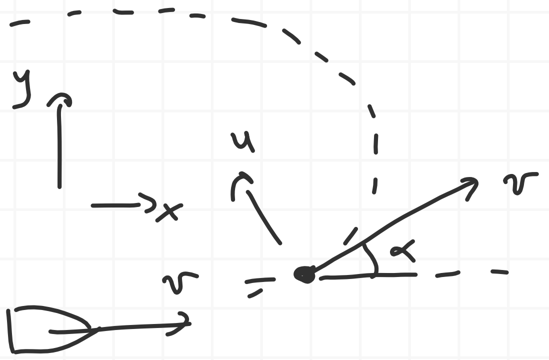
$m d\vec{v} = \vec{u} dm$; $m dv = u dm$; $\int_0^v \frac{dv}{u} = \int_{m_{ост}}^{m} \frac{dm}{m}$

$\frac{v}{u} = \ln\left(\frac{m}{m_{ост}}\right)$; $\frac{m}{m_{ост}} = \exp\left(\frac{v}{u}\right)$

$$\frac{m}{m_0} = \exp^2\left(\frac{v}{u}\right) = e^{\frac{2v}{u}}$$

$$\Delta m = m_0 - m = m_0\left(1 - e^{\frac{2v}{u}}\right)$$

2) по дуге



$m d\vec{v} = \vec{u} dm$ $\quad \int \frac{dm}{m} = \frac{dv_x}{u_x} = \frac{d(v\cos\alpha)}{u\cos\left(\frac{\pi}{2}+\alpha\right)} = -\frac{v}{u}\int_0^{\pi} \frac{d(\cos\alpha)}{\sqrt{1-\cos^2\alpha}}$

$tg\,\alpha = \frac{v_y}{v_x}$ $\quad \ln\frac{m}{m_0} = -\frac{v}{u}\int_1^{-1}\frac{dx}{\sqrt{1-x^2}} = -\frac{v}{u}\left(\arcsin(-1) - \arcsin(1)\right)$

$= -\frac{v}{u}(\pi - 0) = -\pi\frac{v}{u}$

$$\frac{m}{m_0} = e^{-\pi\frac{v}{u}}$$

$$\Delta m = m_0 - m = m_0\left(1 - e^{-\pi\frac{v}{u}}\right)$$

сравним:

$$-e^{\frac{2v}{u}} \;?\; -e^{-\pi\frac{v}{u}}$$

$$-\pi\frac{\cancel{v}}{\cancel{u}} \;\underset{<}{?}\; \frac{2\cancel{v}}{\cancel{u}}$$

менять скорость <u>по прямой</u> выгодней

3.60

$h_1 - ?$

$\tau = 50 с$

$\mu = const$

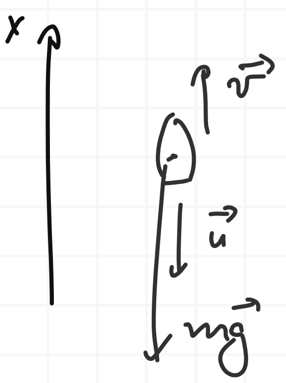
$u = 1000$ м/с

$\frac{m_0}{m} = \alpha = 10$

$g = 10$ м/с²

$h_2 - ?$

1) топливо сгорает постепенно



$$m\frac{d\vec{v}}{dt} = -\mu\vec{u} + m\vec{g} \qquad -dm = \mu dt$$

x: $m dv = \mu u dt - mg dt$

$(m_0 - \mu t) dv = \mu u dt - (m_0 - \mu t) g dt$

$\int dv = \int\frac{\mu u dt}{m_0 - \mu t} - g\int dt = u\int\frac{\frac{\mu}{m_0}dt}{1 - \frac{\mu}{m_0}t} - g\int dt$

$v(t) = -u\ln\frac{\frac{\mu}{m_0}t - 1}{-1} - gt = -u\ln\left(1 - \frac{\mu}{m_0}t\right) - gt$

$$H = \int v(t)dt = -u\int \ln\left(1-\frac{\mu}{m_0}t\right)dt - g\int_0^t t\,dt = +u\frac{m_0}{\mu}\int_0^t \ln\left(1-\frac{\mu}{m_0}t\right)d\left(1-\frac{\mu}{m_0}t\right) - \ldots =$$

$$= \frac{m_0 u}{\mu}\left(1-\frac{\mu}{m_0}\tau\right)\left(\ln\left(1-\frac{\mu}{m_0}\tau\right)-1\right) + \frac{m_0 u}{\mu}\cdot 1\cdot \underbrace{(\overset{0}{\ln 1}-1)}_{+1} - \frac{1}{2}g\tau^2 =$$

$$\parallel \quad m_0 = \alpha m \,; \quad \mu = \frac{m_0 - m}{\tau} = \frac{(\alpha-1)m}{\tau} = \frac{9m_0}{10\tau} \,; \quad \frac{\mu}{m_0} = \frac{9}{10\tau} \,; \quad \frac{\mu\tau}{m_0} = 0{,}9 \,; \quad \frac{m_0}{\mu} = \frac{10\tau}{9} \quad \parallel$$

$$= \frac{10u\tau}{9}\left(1-0{,}9\right)\left(\ln(1-0{,}9)-1\right) + \frac{10u\tau}{9} - \frac{1}{2}g\tau^2 =$$

$$= \frac{10u\tau}{9}\left(0{,}1\left(\ln(0{,}1)-1\right)+1\right) - \frac{1}{2}g\tau^2 = \frac{1000\cdot 50}{0{,}9}\left(0{,}1(\ln 0{,}1 - 1)+1\right) - 5\cdot 50^2 =$$

$$= \underline{24{,}7 \text{ км}}$$

затем двигатель выключается.

$$v(\tau) = -u\ln\left(1-\frac{\mu\tau}{m_0}\right) - g\tau = -u\ln(0{,}1) - g\tau = -1000\cdot\ln(0{,}1) - 10\cdot 50 = 1803 \text{ м/с}$$

$$h_1 = H + \frac{v^2(\tau)}{2g} = 24{,}7\text{ км} + \frac{1803^2}{2\cdot 10}\text{ м} = 24{,}7 + 162{,}5 \text{ км} = \underline{187{,}2 \text{ км}}$$

2) $\tau \to 0$

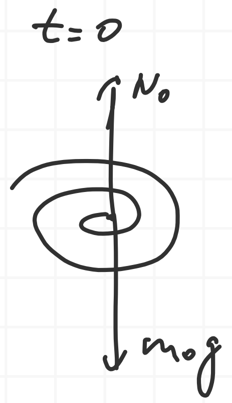
$$h_2 = \frac{v^2(\tau)}{2g} = \frac{(u\ln(10))^2}{2g} = \frac{(1000\ln(10))^2}{20} = \underline{265{,}1 \text{ км}}$$

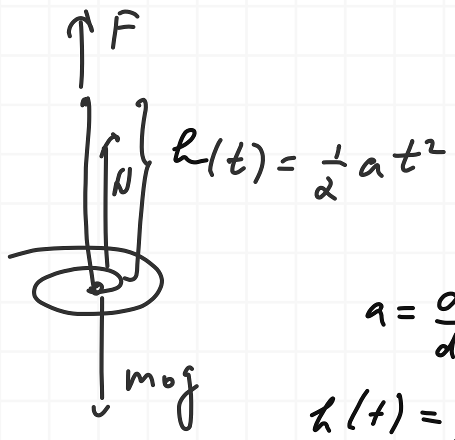
2.76

m_0, l, k

N_{max} – ?

$a = kt$





$$a = \frac{dv}{dt}\,; \quad v = \int a\,dt = k\int t\,dt = \frac{1}{2}kt^2$$

$$h(t) = \int v\,dt = \frac{1}{2}k\int t^2 dt = \frac{1}{6}kt^3$$

$$\frac{dp}{dt} = \frac{m\,dv}{dt} + \frac{v\,dm}{dt} = \frac{h}{l}m_0 a + \frac{v\frac{dh}{l}m_0}{dt} = \frac{h}{l}m_0 a + \frac{m_0 v^2}{l} =$$

$$= km_0\frac{h(t)}{l}t + \frac{m_0k^2t^4}{4l} = km_0\cdot\frac{kt^4}{6l} + \ldots = \frac{k^2m_0t^4}{6l} + \frac{k^2m_0t^4}{4l} =$$

$$= \frac{k^2m_0t^4}{2l}\cdot\frac{5}{6} = \frac{5}{12}\frac{k^2m_0t^4}{l}$$

$$\frac{dp}{dt} = N - m_0 g\,; \quad N = \frac{5}{12}\frac{k^2m_0t^4}{l} + m_0 g$$

N_{max} при t_{max}

t_{max}: $h(t_{max}) = l = \frac{1}{6}kt_{max}^3\,; \quad t_{max} = \sqrt[3]{\frac{6l}{k}}$

$$\underline{N_{max}} = m_0 g + \frac{5}{_2\cancel{12}}\frac{k^2 m_0}{\cancel{l}}\cdot\frac{\cancel{6l}}{\cancel{k}}\sqrt[3]{\frac{6l}{k}} = \underline{m_0 g + \frac{5}{2}m_0\sqrt[3]{6k^2l}}$$

T.1

$M = 150$ кг

$L = 40$ м

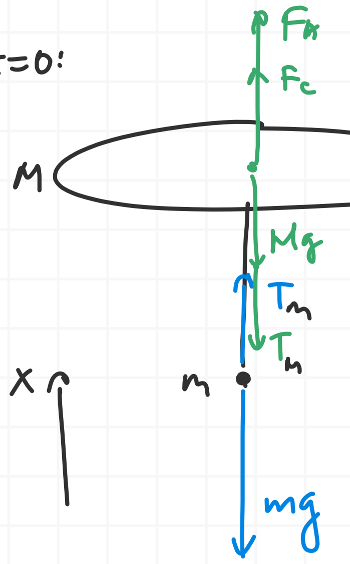
$m = 50$ кг

$u = 0{,}2$ м/с

$k = 1{,}0\ \frac{Н \cdot с}{м}$

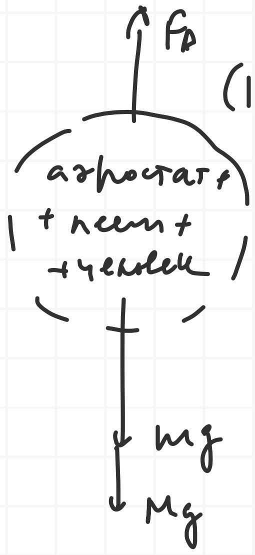
H – ?

$t=0$:



$\vec u = \vec v_1 - \vec v$

$\vec v_1 = \vec u + \vec v$



(1) $\vec F_A = -(M+m)\vec g$ (усл. равн.)

(2) $\vec T_m = -\vec T_M$

3) ЗСИ: $m\vec v_{10} + M\vec v_0 = 0$

$m(\vec u + \vec v_0) + M\vec v_0 = 0$

$mu_x + (m+M)v_{0x} = 0$ (3)

4) $M\frac{d\vec v}{dt} = \vec F_A - k\vec v + \vec T_M + M\vec g$; $m\frac{d(\vec u + \vec v)}{dt} = m\frac{d\vec v}{dt} = \vec T_m + m\vec g$

+

$$(M+m)\frac{d\vec v}{dt} \overset{(2)}{=} \vec F_A - k\vec v + (M+m)\vec g$$

$$(M+m)\frac{d\vec v}{dt} \overset{(1)}{=} -k\vec v \ ; \ (M+m)\frac{dv}{dt} = -kv ; \ \int\frac{dv}{v} = -\frac{k}{M+m}\int dt$$

$$\ln\frac{v}{v_0} = -\frac{k}{M+m}t \ ; \ v = v_0\exp\left(-\frac{k}{M+m}t\right) \quad (4)$$

5) $v_{0x} = -\frac{mu_x}{m+M} = [u_x = u] = -\frac{50}{50+150}\,0{,}2 = -0{,}05\ \frac{м}{с}$

$$\alpha = \frac{k}{M+m}$$

6) $H = ut + \int_0^t v_x\,dt = ut + \frac{(M+m)v_{0x}}{k}\left(1 - e^{-\frac{k}{m+M}t}\right) (=)$

$$// \int_0^t v_{0x}e^{-\alpha t}dt = -\frac{v_{0x}}{\alpha}\int_0^t e^{-\alpha t}d(-\alpha t) = -\frac{v_{0x}}{\alpha}\left(e^{-\alpha t} - 1\right) = \frac{v_{0x}}{\alpha}\left(1 - e^{-\alpha t}\right) //$$

$$(=) \left[t = \frac{L}{u}\right] = L + \frac{(M+m)v_{0x}}{k}\left(1 - \exp\left(-\frac{k}{m+M}\cdot\frac{L}{u}\right)\right) =$$

$$= 40 - \frac{200\cdot 0{,}05}{1{,}0}\left(1 - \exp\left(-\frac{\cancel{1{,}0}\cdot\cancel{40}}{\cancel{200}\cdot\cancel{0{,}2}}\right)\right) = 40 - 10\left(1 - \frac{1}{e}\right) = 33{,}7\ м$$