Неделя 2. Динамика МТ. З-ны Ньютона.

04

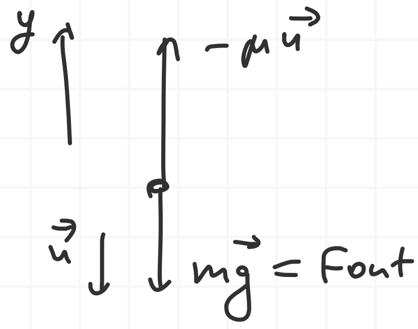
$M = 6т$

$u = 3\ км/с$

$\mu - ?$

$a = 2g$

$\vec{F} = \vec{F}_{out} - \mu\vec{u}$; $\vec{F}_{out} = m\vec{g}$



$Ma = F_y = -Mg + \mu u$

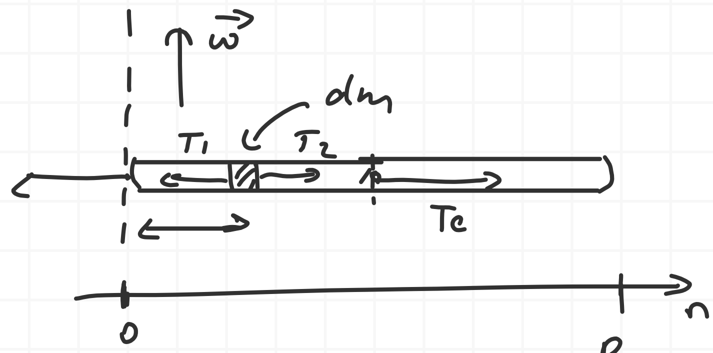
$2Mg + Mg = \mu u$

$\boxed{\mu} = \dfrac{3Mg}{u} = \dfrac{\cancel{3}\cdot 6\cdot 10^{\cancel{3}}\ кг \cdot 9{,}8\ \frac{м}{с^2}}{\cancel{3}\cdot\cancel{10^3}\ м/с} = \boxed{58{,}8\ \frac{кг}{с}}$

05

$T_c = 12Н$

$T_0 - ?$



Р-м мал. кусочек стержня массой dm:

$T_1 - T_2 = dm \cdot \omega^2 r$

$-dT = dm \cdot \omega^2 r$

$-dT = m\dfrac{dr}{R}\cdot\omega^2 r$

$$-\int_{T(r)}^{0} dT = \frac{m\omega^2}{R}\int_r^R r\,dr$$

$$T(r) = \frac{m\omega^2}{R}\cdot\frac{1}{2}(R^2 - r^2) = \frac{1}{2}m\omega^2 R\left(1 - \frac{r^2}{R^2}\right)$$

$$T(0) = \frac{1}{2}m\omega^2 R = T_0$$

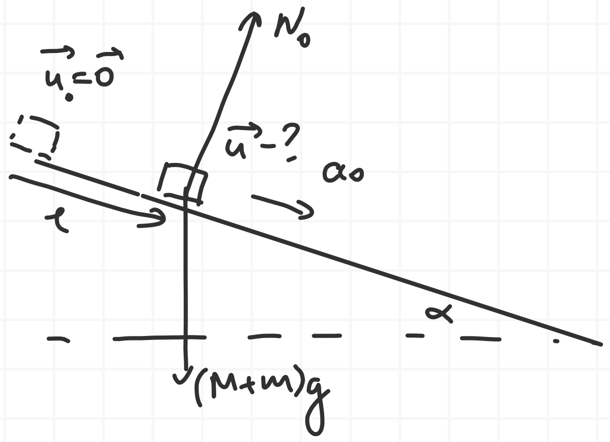
$$T\left(\frac{R}{2}\right) = \frac{1}{2}m\omega^2 R\left(1 - \frac{\cancel{R^2}}{4\cancel{R^2}}\right) = \frac{1}{2}m\omega^2 R\cdot\frac{3}{4} = \frac{3}{4}T_0 = T_c$$

$$\boxed{T_0} = \frac{4}{3}T_c = \frac{4}{\cancel{3}}\cdot\cancel{12}^{\,4}\ Н = \boxed{16Н}$$

4.10

$(\ell), m, M, \alpha$

$v - ?$



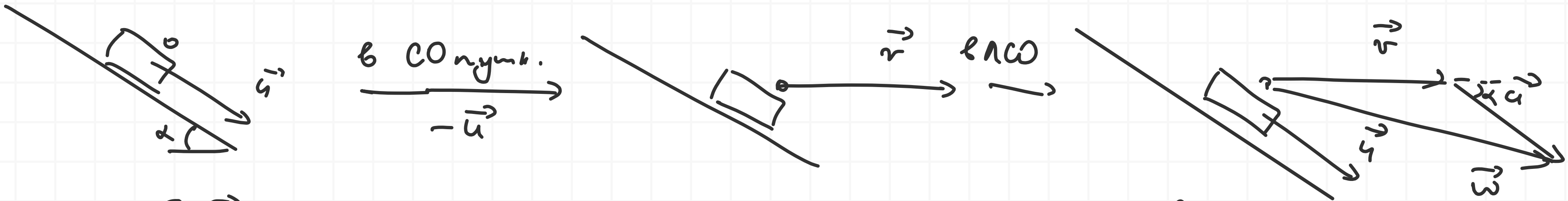
1) Найдем u — скорость пушки в лсо за мгновение до выстрела.

$\cancel{(M+m)}g\sin\alpha = \cancel{(M+m)}a_0$; $a_0 = g\sin\alpha$

$u = a_0 t$

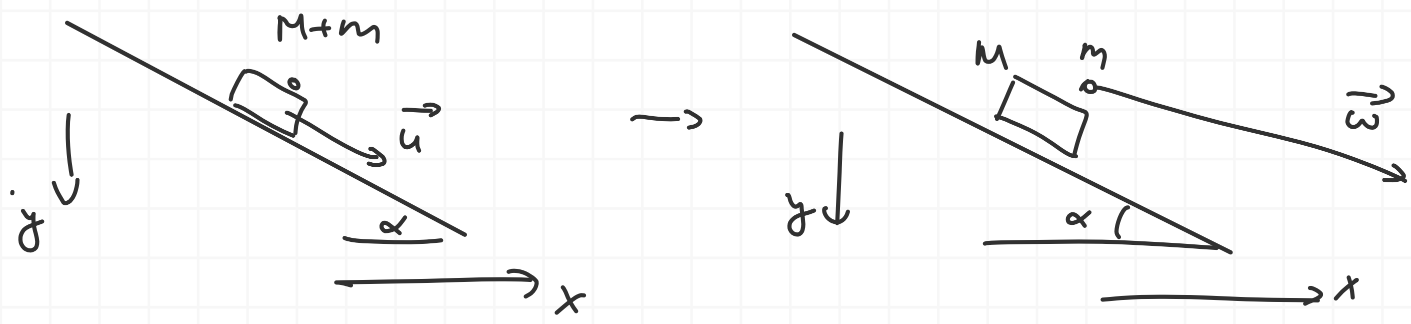
$\ell = \dfrac{a_0 t^2}{2}$; $t = \sqrt{\dfrac{2\ell}{a_0}}$ $\Big|$ $\rightarrow$ $u = a_0\sqrt{\dfrac{2\ell}{a_0}} = \sqrt{2a_0\ell} = \sqrt{2\ell g\sin\alpha}$

Заметим, что скорость снаряда в условии отсчитывается в СО пушки. Переведем v в ЛСО:



$\exists\ \vec{\omega}$ — скорость снаряда сразу после выстрела в ЛСО.

Запишем проекц. З. сохр. имп. для выстрела на ось x.



$\omega_y = u_y = u\sin\alpha$

$\omega_x = v + u_x = v + u\cos\alpha$

$$(M+\cancel{m})\,u\cos\alpha = m(v\,\cancel{+u\cos\alpha})$$

$$Mu\cos\alpha = mv$$

$$\boxed{v} = \frac{M}{m}u\cos\alpha = \boxed{\frac{M}{m}\sqrt{2gl\sin\alpha}\,\cos\alpha}$$

(если v — это скорость ядра в ЛСО)

ЗСИ: $Mu = mv\cos\alpha$

$$\boxed{v = \frac{M}{m}\frac{\sqrt{2gl\sin\alpha}}{\cos\alpha}}$$

2.18

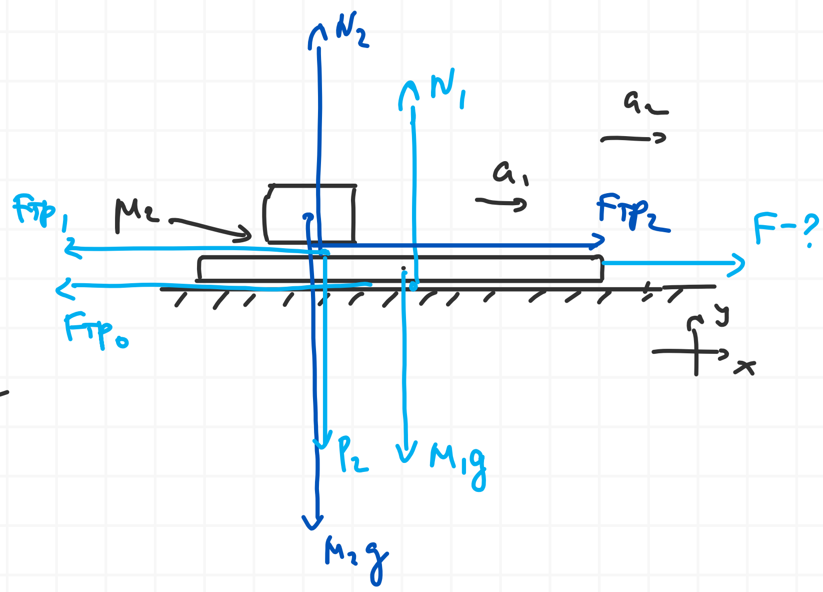
$M_1 = M = 1$ кг

$M_2 = m = 2$ кг

$\mu_1 = 0{,}5$

$\mu_2 = 0{,}25$

$F - ?$



1) II з. н.

доска:

$M_1a_1 = F - F_{тр_0} - F_{тр_1} = F - \mu_1N_1 - \mu_2P_2$

$N_1 = P_2 + M_1g$

$a_1 = \dfrac{F - \mu_1N_1 - \mu_2P_2}{M_1}$

брусок:

$N_2 = M_2g\ ;\quad M_2a_2 = F_{тр_2} = F_{тр_1} = \mu_2N_2$

$a_2 = \dfrac{\mu_2N_2}{M_2}$

3) III з. н.

$P_2 = N_2\ ;\quad F_{тр_1} = F_{тр_2}$

чтобы доска выскальзывала из-под бруска, $a_1 > a_2$:

$$\frac{F-\mu_1 N_1 - \mu_2 N_2}{M_1} > \frac{\mu_2 N_2}{M_2}\ ; \quad F > \frac{M_1}{M_2}\mu_2 N_2 + \mu_1 N_1 + \mu_2 N_2 =$$

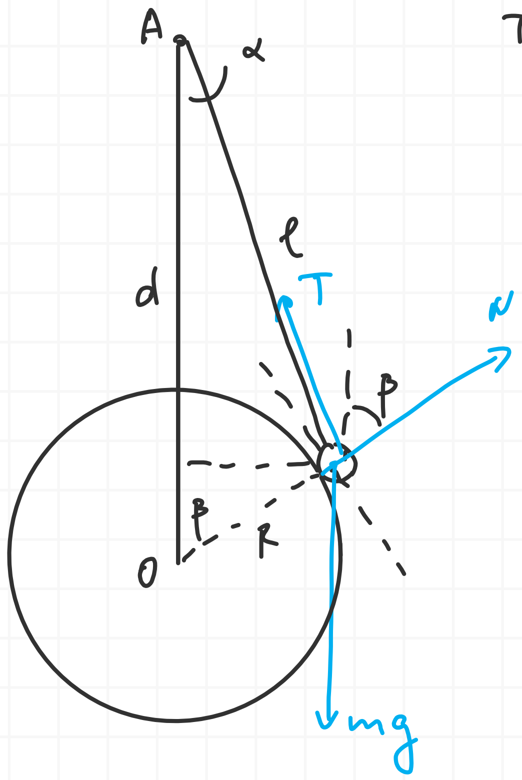
$$= \mu_2 M_1 g + \mu_2 M_2 g + \mu_1 (M_1 g + M_2 g) = (\mu_1 + \mu_2)(M_1 + M_2) g$$

$$\boxed{F > (\mu_1+\mu_2)(m+M)g = (0{,}5+0{,}25)(1+2)\cdot 10 = 22{,}5\ Н}$$

2.51

ℓ, R, d

$v - ?$



в состоянии покоя:

$$T\cos\alpha + N\cos\beta = mg \qquad (1)$$

$$R^2 = d^2 + \ell^2 - 2d\ell\cos\alpha\ ;$$

$$\cos\alpha = \frac{d^2+\ell^2-R^2}{2d\ell}$$

$$\ell^2 = d^2 + R^2 - 2dR\cos\beta$$

$$\cos\beta = \frac{d^2+R^2-\ell^2}{2dR}$$

$$\frac{\sin\beta}{\ell} = \frac{\sin\alpha}{R}\ ;$$

$$\sin\beta = \frac{\ell}{R}\sin\alpha \qquad (2)$$

$$T\sin\alpha = N\sin\beta$$

$$T\cancel{\sin\alpha} = N\cdot\frac{\ell}{R}\cancel{\sin\alpha}$$

$$N = \frac{R}{\ell}T \qquad (3)$$

З-н придаем $\vec{v}$ ⊗ и $N=0$: на шарик д. $a_n = \dfrac{v^2}{R\sin\phi} = \dfrac{v^2}{R\sin\beta}$

$$ma_n = \frac{mv^2}{R\sin\beta} = T\sin\alpha\ ; \quad v^2 = T\sin\alpha\,\frac{R\sin\beta}{m} \overset{(2)}{=} \frac{T\cancel{R}}{m}\cdot\frac{\ell}{\cancel{R}}\sin^2\alpha = \frac{T\ell}{m}\sin^2\alpha$$

(3) в (1):

$$mg = T\cos\alpha + N\cos\beta = T\cos\alpha + \frac{R}{\ell}T\cos\beta$$

$$\frac{m}{T}g = \cos\alpha + \frac{R}{\ell}\cos\beta = \frac{d^2+\ell^2-R^2}{2d\ell} + \frac{\cancel{R}}{\ell}\cdot\frac{d^2+R^2-\ell^2}{2d\cancel{R}} = \frac{1}{2d\ell}(d^2\cancel{+\ell^2}\cancel{-R^2}+d^2\cancel{+R^2}\cancel{-\ell^2}) =$$

$$= \frac{\cancel{2}d^{\cancel{2}}}{\cancel{2}d\ell} = \frac{d}{\ell}$$

$$\frac{m}{T} = \frac{d}{g\ell}\ ; \quad \frac{T}{m} = \frac{g\ell}{d} \qquad T = \frac{mg\ell}{d}\ ; \quad N = \frac{R}{\cancel{\ell}}\cdot\frac{mg\cancel{\ell}}{d} = \frac{mgR}{d}$$

$$v^2 = \frac{T\ell}{m}\sin^2\alpha = \frac{g\ell^2}{d}\cdot(1-\cos^2\alpha) = \frac{g\ell^2}{d}\left(1-\frac{d^2+\ell^2-R^2}{2d\ell}\right) = \frac{g\ell^{\cancel{2}}}{d}\cdot\frac{(2d\ell-d^2-\ell^2+R^2)}{2d\cancel{\ell}}$$

$$v = \sqrt{\frac{g\ell}{2d^2}(R^2-d^2-\ell^2+2d\ell)}$$

2.71

$F = F_0\left(1 - \frac{v}{u}\right)$

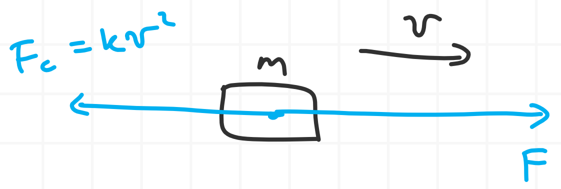
$F_0 = \frac{ku^2}{2}$

$v(\tau) = \frac{u}{4}$

$v(0) = 0$

m, k

$\tau - ?$



$$m\frac{dv}{dt} = F - F_c = \frac{ku^2}{2}\left(1 - \frac{v}{u}\right) - kv^2 = \frac{ku^2}{2}\cdot\frac{u-v}{u} - kv^2 =$$
$$= -k\left(\tfrac{1}{2}u(v-u) + v^2\right)$$

$$-\frac{2m}{k}\cdot\frac{dv}{dt} = \left(u(v-u) + 2v^2\right)$$

$$// \quad u^2\left(\frac{u(v-u)}{u^2} + \frac{2v^2}{u^2}\right) = u^2\left(\frac{v}{u} - 1 + 2\left(\frac{v}{u}\right)^2\right) //$$

$$-\frac{2m}{ku^2}\cdot\frac{dv}{dt} = 2\left(\frac{v}{u}\right)^2 + \left(\frac{v}{u}\right) - 1$$

$$\frac{dv}{2\left(\frac{v}{u}\right)^2 + \left(\frac{v}{u}\right) - 1} = \frac{u\,d\left(\frac{v}{u}\right)}{\dots} = -\frac{ku^2}{2m}dt$$

$$\left[\frac{v}{u} = x\right]$$

$$\frac{dx}{2x^2 + x - 1} = -\frac{ku}{2m}dt$$

// $2x^2 + x - 1 = 0$

$D = 1 + 4\cdot 2\cdot 1 = 9$

$\frac{-1 \pm 3}{4} = \begin{matrix}\frac{1}{2}\\ -1\end{matrix}$

$(2x-1)(x+1)$

$$\frac{1}{(2x-1)(x+1)} = \frac{a}{2x-1} + \frac{b}{x+1} = \frac{ax + a + 2bx - b}{(\quad)(\quad)}$$

$$\begin{cases} a + 2b = 0 \\ a - b = 1 \end{cases};\quad a = -2b;\quad -2b - b = -3b = 1;\ b = -\frac{1}{3};\quad a = +\frac{2}{3}$$

$$\frac{1}{3}\int\frac{d(2x-1)}{2x-1} - \frac{1}{3}\int\frac{d(x+1)}{x+1} = \frac{1}{3}\ln|2x-1| - \frac{1}{3}\ln|x+1| + c = \frac{1}{3}\ln\left|\frac{2x-1}{x+1}\right| + c \quad //$$

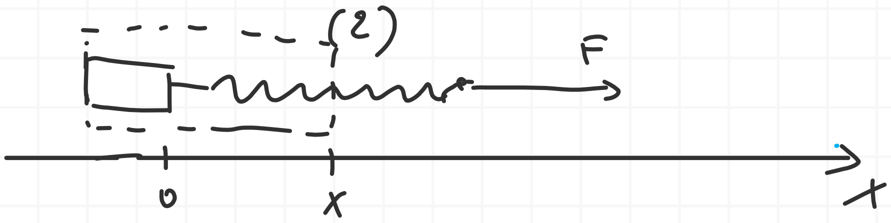
$$\frac{1}{3}\ln\left|\frac{2\frac{v}{u} - 1}{\frac{v}{u} + 1}\right|\Bigg|_0^{\frac{u}{4}} = \frac{1}{3}\left(\ln\left|\frac{2\cdot\frac{1}{4} - 1}{\frac{1}{4} + 1}\right| - \cancel{\ln\left|\frac{-1}{1}\right|}\right) = \frac{1}{3}\ln\frac{\cancel{4}\,2}{\cancel{2}\,5} = -\frac{ku}{2m}\tau$$

$$\tau = -\frac{1}{3}\ln\frac{2}{5}\cdot\frac{2m}{ku} = \frac{2}{3}\cdot\frac{m}{ku}\ln\frac{5}{2} \approx 0{,}61\,\frac{m}{ku}$$

2.59

k, ℓ_0, M, m, F

$\varepsilon_\ell = \dfrac{\Delta\ell}{\ell_0}$ - ?



1) $(m+M)a = F;\quad a = \dfrac{F}{m+M}$

2) $\left(m + M\dfrac{x}{\ell_0}\right)a = F_{упр}(x)$

3) $k(x) = k\dfrac{\ell_0}{x}$

4) $F_{упр}(x) = k(x)\,d(\Delta x);\quad d(\Delta x) = \dfrac{F_{упр}(x)}{k\ell_0}dx$

$$\Delta\ell = \int_0^{\Delta\ell} d(\Delta x) = \int_0^{\ell} \frac{\left(m + M\frac{x}{\ell_0}\right)a}{k\ell_0}dx = \frac{a}{k\ell_0}\left(m\int_0^{\ell}dx + \frac{M}{\ell_0}\int_0^{\ell+\Delta\ell}x\,dx\right) =$$

$$= \frac{F}{k\cancel{\ell_0}(m+M)}\left(m\cancel{\ell_0} + \frac{M}{\cancel{\ell_0}}\cdot\frac{1}{2}\cancel{\ell_0^2}\right) = \frac{F\left(\frac{1}{2}M+m\right)}{k(m+M)} = \frac{F(2m+M)}{2k(m+M)}$$

$$\varepsilon_\ell = \frac{\Delta\ell}{\ell_0} = \frac{F}{2k\ell_0}\cdot\frac{2m+M}{m+M}$$