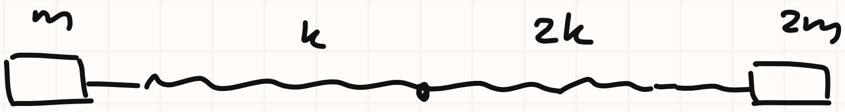
Неделя 12.

Гармонические колебания. Колебания твердых тел.

№20



$$k_{эфф} = \left(\frac{1}{k}+\frac{1}{2k}\right)^{-1} = \frac{2}{3}k = k_0 \;;\quad \frac{1}{\mu} = \frac{1}{m}+\frac{1}{2m} \;;\quad \mu = \frac{2}{3}m$$

$$\Delta\ddot{x} + \frac{k_0}{\mu}\Delta x = 0 \;;\quad \frac{k_0}{\mu} = \frac{\frac{\cancel{2}}{\cancel{3}}k}{\frac{\cancel{2}}{\cancel{3}}m} = \frac{k}{m} = \omega^2$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

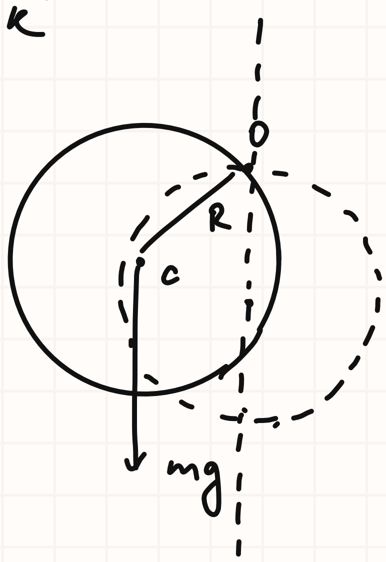
№21

$R = 10$ см $= 0{,}1$ м

$\omega_0 = 0{,}8\ с^{-1}$

φ_m мал.

$\varphi(t)$ - ?



$$\ddot{\varphi} + \underbrace{\frac{mgR}{I_c + mR^2}}_{\omega^2}\varphi = 0$$

$$\omega^2 = \frac{\cancel{m}gR}{\frac{1}{2}\cancel{m}R^2 + \cancel{m}R^2} = \frac{2g}{3R} = \frac{2\cdot 9{,}81}{3\cdot 0{,}1} = 65{,}4\ с^{-2}$$

$$\underline{\underline{\omega}} = \sqrt{65{,}4} = \underline{\underline{8{,}1\ с^{-1}}}$$

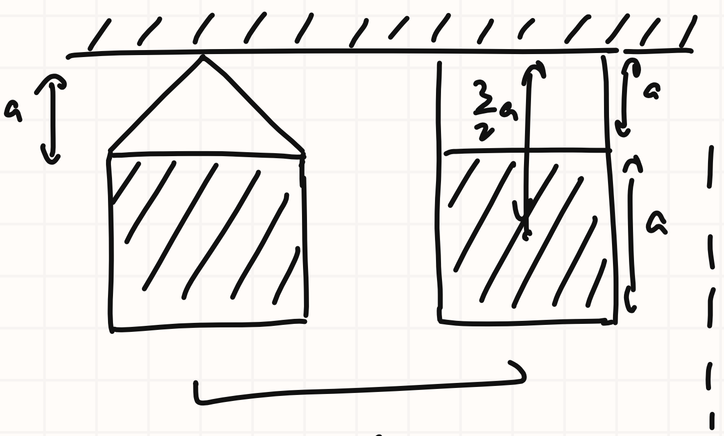
$$\varphi(t) = \varphi_m \sin\left(\sqrt{\frac{2g}{3R}}\,t\right) \qquad \varphi(t_0) = 0 \longrightarrow \text{триг. ф-я}$$

$$\dot{\varphi}(t) = \varphi_m\sqrt{\frac{2g}{3R}}\cos\left(\sqrt{\frac{2g}{3R}}\,t\right) \qquad \dot{\varphi}(t_0) = \omega_0$$

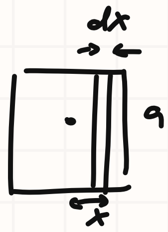
$$\omega_0 = \varphi_m \underbrace{\sqrt{\frac{2g}{3R}}}_{\omega}\cdot 1 \;;\quad \underline{\underline{\varphi_m}} = \frac{\omega_0}{\omega} = \frac{0{,}8}{8{,}1} = \underline{\underline{0{,}1\ рад}}$$

Ответ: $\varphi(t) = 0{,}1\sin(8{,}1t)$ (в СИ)

10.3



момент инерц. $\square a$ отн. центра:



$$I_\square = 2\int_0^{a/2}\left(dm\,x^2 + \frac{1}{12}dm\,a^2\right) =$$

$$= \left[dm = ds\,\frac{m}{a^2} = \frac{ma\,dx}{a^2} = \frac{m\,dx}{a}\right] =$$

$$= 2\int_0^{a/2}\left(\frac{m}{a}x^2dx + \frac{1}{12}ma\,dx\right) = \cancel{2}\cdot\left(\frac{m}{\cancel{a}}\cdot\frac{a^{\cancel{3}\,2}}{3\cdot\cancel{8}\,4} + \frac{1}{12}ma\cdot\frac{a}{\cancel{2}}\right) = \frac{1}{6}ma^2$$

для этих двух ситуаций T одинаков.

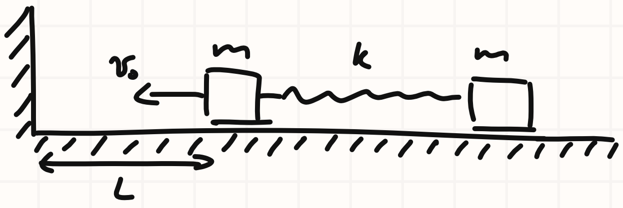
$$I = m\left(\frac{3}{2}a\right)^2 + I_\square = \frac{9}{4}ma^2 + \frac{1}{6}ma^2 = \frac{29}{12}ma^2 \;;\quad T = 2\pi\sqrt{\frac{I}{mg\cdot\frac{3}{2}a}} = 2\pi\sqrt{\frac{29}{18}\frac{a}{g}}$$

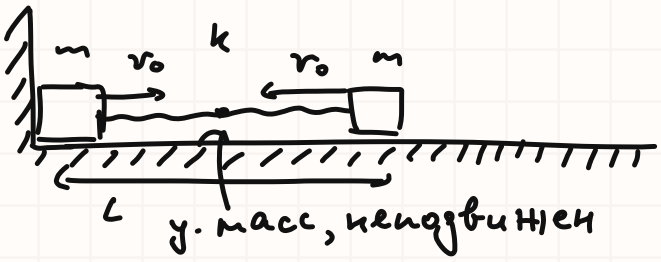
5.49

v_0, m, k

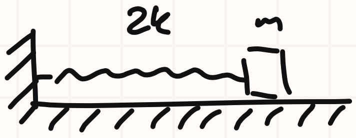
$t=0: L$

$t - ?$





сразу после столкновения



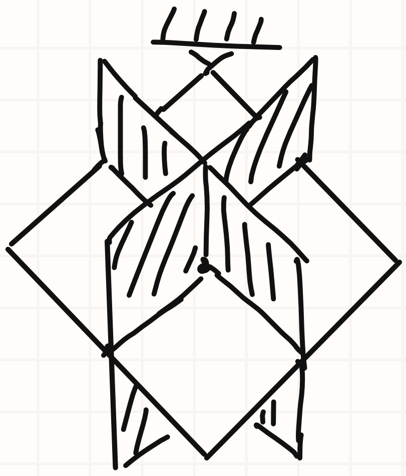
$$T = 2\pi\sqrt{\frac{m}{2k}}$$

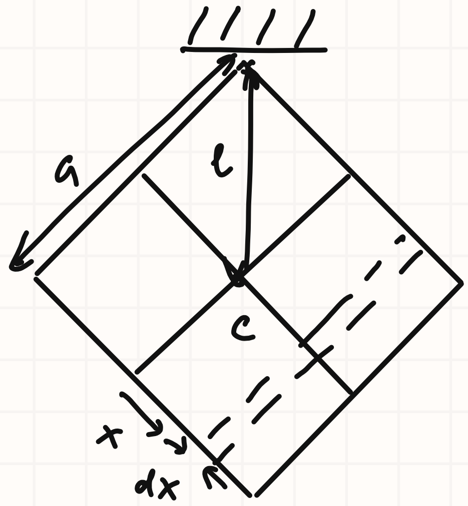
$$\tau = \frac{T}{2} + 2\frac{L}{v_0} = \pi\sqrt{\frac{m}{2k}} + \frac{2L}{v_0}$$

10.78

a

$T - ?$





$dS = dx \cdot a$

$\sigma = \frac{m}{a^2}$

$dm = \sigma\, dS = \frac{m\,dx}{a}$

$$I = ml^2 + I_{нн} + 2ml^2 + \frac{2ma^2}{12} = \frac{ma^2}{2} + I_{нн} + \frac{7}{6}ma^2 \;(=)$$

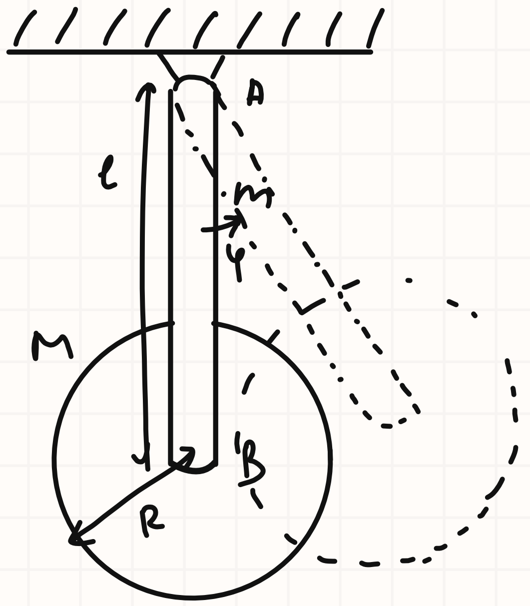
$$// \; I_{нн} = 2\int_0^{a/2} \frac{1}{12}(dm)a^2 + (dm)x^2 = 2\int \frac{1}{12}\frac{m\,dx}{\cancel{a}}\cdot a^{\cancel{3}} + \frac{m\,dx}{a}\cdot x^2 =$$

$$= \frac{\cancel{2}}{6}\cdot\frac{1}{\cancel{12}} ma\int_0^{a/2} dx + 2\frac{m}{a}\int_0^{a/2} x^2\,dx = \frac{1}{12}ma^2 + 2\frac{m}{\cancel{a}}\cdot\frac{a^{\cancel{3}\,2}}{\underset{12}{3\cdot 8\cancel{4}}} = \frac{1}{6}ma^2 \; //$$

$$(=)\left(\frac{1}{2} + \frac{1}{6} + \frac{7}{6}\right)ma^2 = \left(\frac{3}{6} + \frac{8}{6}\right)ma^2 = \frac{11}{6}ma^2$$

$$T = 2\pi\sqrt{\frac{I}{3mgl}} = 2\pi\sqrt{\frac{\sqrt{2}I}{3mga}} = 2\pi\sqrt{\frac{\sqrt{2}\cdot 11\,\cancel{m}a^{\cancel{2}}}{3\cancel{m}g\cancel{a}\cdot 6}} = 2\pi\sqrt{\frac{11\sqrt{2}}{18}\frac{a}{g}}$$

10.43



$$W_k = \frac{I\dot\varphi^2}{2} = \frac{1}{2}\left(\frac{1}{3}ml^2 + Ml^2\right)\dot\varphi^2$$

(диск не закреплен, а значит его вращения не происходит)

$$W_p = mg\frac{l}{2}(1-\cos\varphi) + Mgl(1-\cos\varphi) =$$
$$= \left(\frac{m}{2}+M\right)gl(1-\cos\varphi) \approx \left(\frac{m}{2}+M\right)gl\frac{\varphi^2}{2}$$

$W_k + W_p = const$ (ЗСЭ)

$$\cancel{\tfrac{1}{2}}\left(\frac{1}{3}m+M\right)l^{\cancel{2}}\dot\varphi^2 + \left(\frac{1}{2}m+M\right)g\cancel{l}\frac{\varphi^2}{\cancel{2}} = const$$

$$\left(\frac{1}{3}m+M\right)l\dot\varphi^2 + \left(\frac{1}{2}m+M\right)g\varphi^2 = const$$

$$\left(\frac{1}{3}m+M\right)l\cdot\cancel{2}\cancel{\dot\varphi}\ddot\varphi + \left(\frac{1}{2}m+M\right)g\cdot\cancel{2}\varphi\cdot\cancel{\dot\varphi} = 0$$

$$\left(\frac{m+3M}{3}\right)l\ddot\varphi + \left(\frac{m+2M}{2}\right)g\varphi = 0 \quad \Big|\cdot\frac{3}{(m+3M)l}$$

$$\ddot\varphi + \underbrace{\frac{3(m+2M)}{2(m+3M)}\cdot\frac{g}{l}}_{\omega^2}\varphi = 0$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2(m+3M)}{3(m+2M)}\cdot\frac{l}{g}}$$

10.53

T_1, T_2

$T - ?$

ур-е крутильных колебаний: $J\ddot\varphi + k\varphi = 0$

$\underbrace{\varphi}_{} = \underbrace{\varphi_1 + \varphi_2}_{}$

угол закручивания диска и стержней, соответственно

I стержней можно пренебречь (r мал)

I – момент инерции диска.

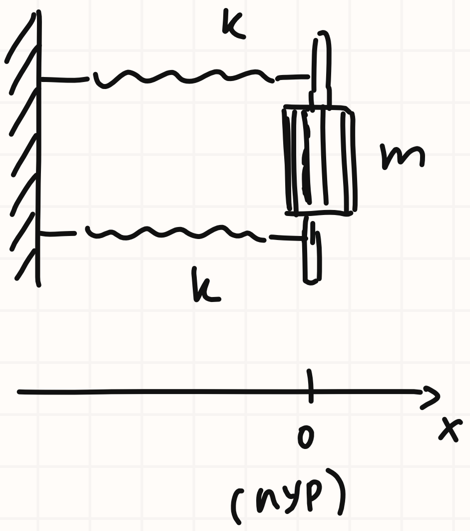
$$k_1\varphi_1 = k_2\varphi_2 = k(\varphi_1+\varphi_2) \longrightarrow \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\omega^2 = \frac{k}{I}; \quad T = 2\pi\sqrt{\frac{I}{k}}; \quad T^2 = 4\pi^2 I\left(\frac{1}{k_1}+\frac{1}{k_2}\right) = T_1^2 + T_2^2$$

Ответ: $T = \sqrt{T_1^2 + T_2^2}$

10.34

I, m, r, k



без проскальзывания: $\dot{\varphi} R = \dot{x}$

$$W_k = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} J\dot{\varphi}^2 = \frac{1}{2} m\dot{x}^2 + \frac{1}{2}\cdot\frac{1}{2} m\cancel{R^2}\cdot\left(\frac{\dot{x}}{\cancel{R}}\right)^2 = \frac{3}{4} m\dot{x}^2$$

$$W_p = 2\cdot\frac{1}{2} kx^2 = kx^2$$

$$W_k + W_p = const \quad (\text{ЗСЭ})$$

$$\frac{3}{4} m\dot{x}^2 + kx^2 = const$$

$$\frac{3}{4} m\cdot\cancel{2}\cancel{\dot{x}}\ddot{x} + k\cdot\cancel{2}x\cancel{\dot{x}} = 0 \quad \Big|\cdot\frac{4}{3m}$$

$$\ddot{x} + \underbrace{\frac{4k}{3m}}_{\omega^2} x = 0$$

$$\underline{\underline{T}} = \frac{2\pi}{\omega} = \cancel{2}\pi\sqrt{\frac{3m}{\cancel{4}k}} = \underline{\underline{\pi\sqrt{\frac{3m}{k}}}}$$