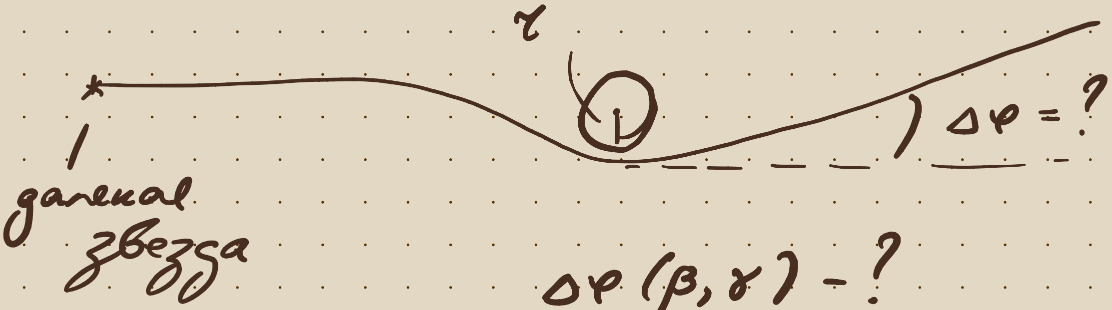
Д/З №12 Отклонение лучей света в поле Солнца



$\Delta\varphi(\beta, \gamma)$ - ?

$r \in (-\infty, r_{min})$

Можно считать для пол периода и $\times 2$

Заново написать ЗСЭ, ЗСИ, u^μ и взять инт-л

▷ $ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\omega^2$

$$A(r) = 1 + \gamma \frac{r_g}{r} \qquad B(r) = 1 - \frac{r_g}{r} + \frac{1}{2}(-\gamma + \beta)\frac{r_g^2}{r^2}$$

В-ры Киллинга: $\xi_1^\mu = \delta_0^\mu \quad \xi_2^\mu = \delta_\varphi^\mu$

$\Rightarrow E = \xi_1^\mu u_\mu = B u^0 = const$

$L = \xi_2^\mu u_\mu = r^2 u^\varphi = const$

$\Rightarrow$ движ. в пл-ти $\Rightarrow$ выберем такую с.о.:

$\theta = \frac{\pi}{2} \quad u^\theta = 0$

$$u_\mu u^\mu = 0 \Rightarrow -B(u^0)^2 + A(u^r)^2 + r^2(u^\varphi)^2 = 0$$

$$u^0 = \frac{E}{B} \quad u^\varphi = \frac{L}{r^2} \Rightarrow -B - \frac{E^2}{B^2} + A(u^r)^2 + r^2\frac{L^2}{r^2} = 0$$

$$A(u^r)^2 = \frac{E^2}{B} - \frac{L^2}{r^2} \Rightarrow \frac{dr}{ds} = \pm\left(\frac{1}{A}\left(\frac{E^2}{B} - \frac{L^2}{r^2}\right)\right)^{\frac{1}{2}}$$

$$\frac{d\varphi}{dr} = \frac{d\varphi}{ds}\frac{ds}{dr} = \pm\frac{L}{r^2}\left(\frac{1}{A}\left(\frac{E^2}{B} - \frac{L^2}{r^2}\right)\right)^{-\frac{1}{2}} =$$

$$= \pm \frac{1}{r^2}\left(\frac{1}{A}\left(\frac{1}{B}\left(\frac{E}{L}\right)^2 - \frac{1}{r^2}\right)\right)^{\frac{1}{2}}$$

r_0 - миним. расст. до Солнца на тракторі.

$$\Rightarrow \frac{E^2}{B(r_0)} - \frac{L^2}{r_0^2} = 0 \Rightarrow \left(\frac{E}{L}\right)^2 = \frac{B(r_0)}{r_0^2}$$

$$\frac{d\varphi}{dr} = \pm \frac{1}{r^2} A^{\frac{1}{2}}\left(\frac{B(r_0)}{B(r)}\frac{1}{r_0^2} - \frac{1}{r^2}\right)^{-\frac{1}{2}}$$

$$\delta\varphi = \int_{r_0}^{\infty} \frac{1}{r^2} \frac{A^{\frac{1}{2}}(r)}{\left(\frac{B(r_0)}{B(r)}\frac{1}{r_0^2} - \frac{1}{r^2}\right)^{\frac{1}{2}}} dr - \frac{\pi}{2}$$

$$\Delta\varphi = 2\delta\varphi$$

$$\int_{r_0}^{\infty} \frac{\left(1 + \gamma\frac{r_g}{r} + \ldots\right)^{\frac{1}{2}}}{r\left(\frac{r^2}{r_0^2}\frac{B(r_0)}{B(r)} - 1\right)^{\frac{1}{2}}} dr \; - ?$$

$$\frac{r^2}{r_0^2}\frac{B(r_0)}{B(r)} - 1 \simeq \frac{r^2}{r_0^2}\left(1 - \frac{r_g}{r_0} + \frac{1}{2}(\beta - \gamma)\frac{r_g^2}{r_0^2}\right) \cdot$$

$$\cdot \left(1 + \frac{r_g}{r} + \frac{1}{2}(2 - \beta + \gamma)\frac{r_g^2}{r^2}\right) - 1 =$$

$$= -1 + \frac{r_g^2(2 - \beta + \gamma)}{2r_0^2} B(r_0) + \frac{r_g B(r_0)}{r_0^2} r +$$

$$+ r^2\left(\frac{r_g^2(\beta - \gamma)}{2r_0^4} + \frac{1}{r_0^2} - \frac{r_g}{r_0^3}\right)$$

$$\frac{r^2}{r_0^2}\frac{B(r_0)}{B(r)} - 1 = (Cr + D)(r - r_0) = Cr^2 - Cr_0 r +$$

$$+ Dr - Dr_0$$

$$\frac{B(r_0)}{r_0^2 B(r)} - \frac{1}{r^2} = C - \frac{Cr_0}{r} + \frac{D}{r} - \frac{Dr_0}{r^2}$$

$$r \longrightarrow 0 : \quad -Dr_0 = -1 + \frac{r_g^2(\alpha - \beta + \gamma)}{2r_0^2} B(r_0)$$

$$D = -\frac{r_g^2(\alpha - \beta + \gamma)}{2r_0^3} B(r_0) + \frac{1}{r_0}$$

$$r \longrightarrow \infty : \quad C = \frac{B(r_0)}{r_0^2}$$

$$\left(1 + \gamma\frac{r_g}{r}\right)^{\frac{1}{2}} \simeq 1 + \frac{\gamma}{2}\frac{r_g}{r} + \ldots$$

$$\int_{r_0}^{\infty} \frac{1 + \frac{\gamma}{2}\frac{r_g}{r}}{r(Cr + D)^{\frac{1}{2}}(r - r_0)^{\frac{1}{2}}}\,dr = \int_{r_0}^{\infty} \frac{1 + \frac{\gamma}{2}\frac{r_g}{r}}{r^{\frac{3}{2}}\left(C + \frac{D}{r}\right)^{\frac{1}{2}}(r - r_0)^{\frac{1}{2}}}\,dr \simeq$$

$$\simeq \int_{r_0}^{\infty} \frac{dr}{r^{\frac{3}{2}}(r - r_0)^{\frac{1}{2}}}\left(\frac{1}{\sqrt{C}} + \left(\frac{r_g\gamma}{2\sqrt{C}} - \frac{D}{2C^{\frac{3}{2}}}\right)\frac{1}{r} +\right.$$

$$\left. + \left(\frac{3D^2}{8C^{\frac{5}{2}}} - \frac{r_g\gamma D}{4C^{\frac{3}{2}}}\right)\frac{1}{r^2}\right) = \frac{2}{\sqrt{C}\,r_0} + \frac{4}{3r_0^2} \times$$

$$\times \left(\frac{r_g\gamma}{2\sqrt{C}} - \frac{D}{2C^{\frac{3}{2}}}\right) + \frac{16}{15r_0^3}\left(\frac{3D^2}{8C^{\frac{5}{2}}} - \frac{r_g\gamma D}{4C^{\frac{3}{2}}}\right)$$

$$\Delta\varphi = -\pi + \frac{4}{\sqrt{C}\,r_0} + \frac{\gamma}{3r_0^2}\left(\frac{r_g\gamma}{2\sqrt{C}} - \frac{D}{2C^{\frac{3}{2}}}\right) +$$

$$+ \frac{32}{15r_0^3}\left(\frac{3D^2}{8C^{\frac{5}{2}}} + \frac{r_g\gamma D}{4C^{\frac{3}{2}}}\right)$$

Подставить сюда C и D и получим ответ