Неделя 15.

Элементы гидродинамики

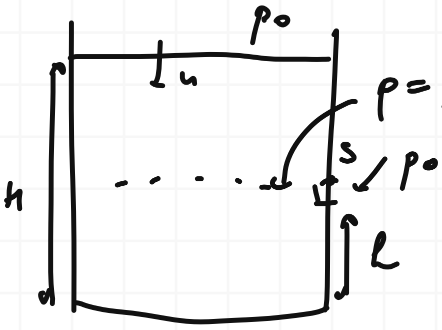
№25

$S = 1\ см^2 =$
$= 1 \cdot 10^{-4}\ м^2$

$h = 0{,}5\ м$

$H = 1\ м$

$F - ?$



$p = p_0 + \rho g (H - h)$

$(u \approx 0)$

$$\cancel{\frac{p_0}{\rho}} + \cancel{\frac{u^2}{2}}^{\,0} + gH = \cancel{\frac{p_0}{\rho}} + \frac{v^2}{2} + gh$$

$$v^2 = 2g(H-h)$$

$$v = \sqrt{2g(H-h)}$$

$$\underline{\underline{F}} = |\dot{p} v| = \frac{|dm|}{dt} v = \left[dm = \rho S v\, dt \right] = \rho S v^2 =$$

$$= \cancel{10^3} \cdot \cancel{10^{-4}} \cdot \cancel{2} \cdot \cancel{10} \cdot \cancel{\frac{1}{2}} \ \underline{\underline{= 1\ (Н)}}$$

№26

$h = 0{,}1\ мм = 1 \cdot 10^{-4}\ м$

$\eta = 10^{-1}\ Па \cdot с$

$v = 10\ \frac{см}{с} = 0{,}1\ \frac{м}{с}$

$\frac{F}{S} - ?$

$$F = \eta S \frac{v}{h}\ ;\ \underline{\underline{\frac{F}{S}}} = \underline{\underline{\eta \frac{v}{h}}} = 10^{-1} \cdot 10^{-1} \cdot 10^{4} = \underline{\underline{100\ Н/м^2}}$$

№27

$Re - ?$

$d = 2\ см = 0{,}02\ м$

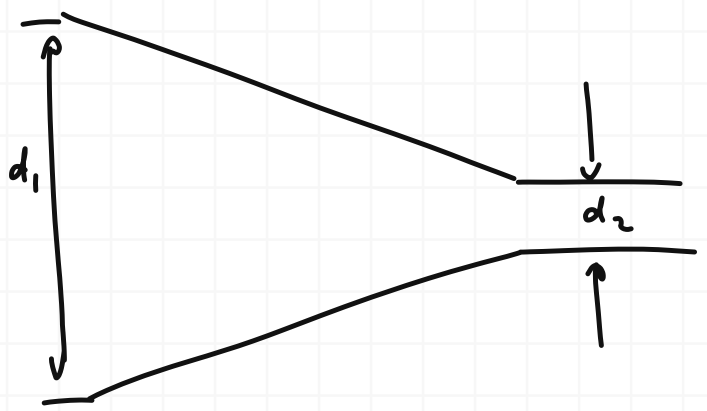
$\eta = 1{,}5 \cdot 10^{-3}\ Па \cdot с$

$Q = 30\ л/мин =$
$= 0{,}5 \cdot 10^{-3}\ м^3/с$

$$Re = \frac{\rho v d}{\eta} = \frac{\rho Q d}{\eta S} = \frac{4 \rho Q \cancel{d}}{\eta \pi d^{\cancel{2}}} = \frac{4 \rho Q}{\pi \eta d} \approx 1000 < 2300$$

<u>ламинарное</u>

14.42



$p_1 = \rho g h_1$

$p_2 = 0$

$v = \frac{Q}{S}$; $v_1 = \frac{4Q}{\pi d_1^2}$; $v_2 = \frac{4Q}{\pi d_2^2}$

$$\cancel{\rho} g h_1 + \frac{8 \cancel{\rho} Q^2}{\pi^2 d_1^4} = \frac{8 \cancel{\rho} Q^2}{\pi^2 d_2^4}$$

$$\frac{\pi^2 g h_1}{8Q^2} = \frac{1}{d_2^4} - \frac{1}{d_1^4}$$

$$d_2 = \sqrt[4]{\frac{\frac{8Q^2}{\pi^2 g h} d_1^4}{\frac{8Q^2}{\pi^2 g h} + d_1^4}} = \sqrt[4]{\frac{d_1^4}{1 + \frac{\pi^2 g h_1}{8Q^2} d_1^4}} = \frac{d_1}{\sqrt[4]{1 + \frac{\pi^2 g h_1}{8Q^2} d_1^4}} = \underline{\underline{0{,}5\ \text{см}}}$$

14.21

$r_1 = 1\ \text{мм} = 1 \cdot 10^{-3}\ \text{м}$

$v_0 = \frac{10\ \text{см}}{\text{с}} = 0{,}1\ \frac{\text{м}}{\text{с}}$

$r_2 = 1\ \text{см} = 0{,}01\ \text{м}$

$\eta = 0{,}01\ \text{П}$

$v(r)$ – ?

$F_{сопр}(r) = \eta \cdot 2\pi r \cdot \frac{dv}{dr}$

$dF_{сопр} = 0$ (потому что давл. на концах одинаковые)

$F_{сопр}(r) = \underbrace{\eta \cdot 2\pi}_{const} \cdot r \frac{dv}{dr} = const$

$r \frac{dv}{dt} = const = \tilde{c}$

$\frac{v}{\tilde{c}} = \ln\left(\frac{r_2}{r_1}\right)$; $\tilde{c} = \frac{v_0}{\ln\left(\frac{r_2}{r_1}\right)}$

$$\underline{\underline{v(r) = v_0 \frac{\ln\left(\frac{r_2}{r}\right)}{\ln\left(\frac{r_2}{r_1}\right)}}}$$

$$\underline{\underline{f}} = 2\pi\eta \cdot \frac{v_0}{\ln\left(\frac{r_2}{r_1}\right)} = \underline{\underline{7{,}7 \cdot 10^{-5}\ \frac{\text{Н}}{\text{м}}}}$$

14.46

$R = 1\text{ см} = 0{,}01\text{ м}$

$\Delta p / l - ?$ (турбулентн.)

$Re = 2000$

$\eta = 10^{-3}$ Па·с

$$F_{сопр} = 2\pi r \cdot \eta \cdot \frac{dv}{dr}$$

$$dF_{сопр} + 2\pi r \frac{\Delta p}{l} dr = 0 \quad \text{(усл-е равновесия)}$$

$$2\pi\eta r \frac{dv}{dr} + \frac{\Delta p}{l} \pi r^2 = const = \tilde{c}$$

$$dv = -\frac{\Delta p}{l} \frac{r\,dr}{2\eta} + \tilde{c}\frac{dr}{r}$$

$$v(r) = \frac{\Delta p}{l} \cdot \frac{R^2 - r^2}{4\eta} + \tilde{c}\ln\left(\frac{r}{R}\right) = \frac{\Delta p}{l}\frac{R^2 - r^2}{4\eta}$$

из нач. условий подбираем $\tilde{c} = 0$

найдем ср. ск. жидк.:

$$v = \int_0^R \frac{v(r)\cdot 2\pi r\,dr}{\pi R^2} = \frac{2}{R^2}\int_0^R v(r) r\,dr = \frac{2}{R}\frac{\Delta p}{l \cdot 4\eta}\int_0^R (R^2 r\,dr - r^3 dr) =$$

$$= \frac{\Delta p}{l \cdot 4\eta}\left(\frac{R^4}{2} - \frac{R^4}{4}\right) = \frac{\Delta p}{4 l \eta}\cdot\frac{R^4}{4} = \frac{R^4}{16\eta}\cdot\frac{\Delta p}{l}$$

$$Re = \frac{\rho v R}{\eta}\ ; \quad v = \frac{Re\,\eta}{\rho R}$$

$$v = \frac{Re\,\eta}{\rho R} = \frac{R^4}{16\eta}\frac{\Delta p}{l}\ ; \quad \frac{\Delta p}{l} = \frac{8 Re\,\eta^2}{\rho R^3} = 16\ \frac{Н}{м^2}$$

14.29

$R = 20\text{ см} = 0{,}2\text{ м}$

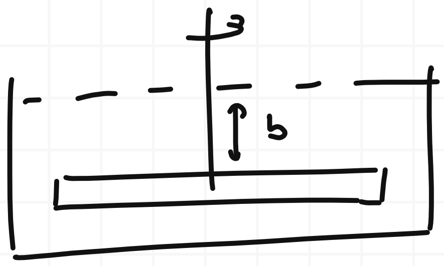
$n = 300$ об/мин $= 5\text{ с}^{-1}$

$a = 5\text{ мм} = 0{,}005\text{ м}$

$b = 10\text{ мм} = 0{,}010\text{ м}$

$\eta = 10^{-1}\ \frac{кг}{м\cdot с}$

$\omega = 2\pi n$



$$dF_{сопр} = \eta\frac{dS\,v}{a} + \eta\frac{dS\,v}{b} = \eta\,dS\,v\left(\frac{1}{a} + \frac{1}{b}\right) =$$

$$= [dS = 2\pi r\,dr] = 2\pi\eta r\,dr \cdot v\left(\frac{1}{a} + \frac{1}{b}\right)$$

$$dN = v(r)\,dF = \eta\frac{a+b}{ab}v^2 dS$$

$$\underline{\underline{N}} = \eta \frac{a+b}{ab} \cdot 2\pi \int_0^R \overbrace{v^2}^{=\omega^2 r^2} r\, dr = \eta \frac{a+b}{ab} \cdot 2\pi \omega^2 \int_0^R r^3 dr =$$

$$= \eta \frac{a+b}{ab} \cdot \cancel{2}\pi\omega^2 \frac{R^4}{\cancel{2}} = \frac{a+b}{ab} \pi \eta \cdot \overset{2}{\cancel{4}}\pi^2 n^2 \frac{R^4}{\cancel{2}} = \underline{\underline{2\frac{a+b}{ab} \pi^3 \eta n^2 R^4 = 74\ \text{Вт}}}$$