

Instituto Tecnológico y de Estudios Superiores de Monterrey

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## Induced Rotation of Ratchets in Passive Environments

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Master

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# Dedication

For those who come after.

# Acknowledgements

To my parents for their unwavering support, to my friends for making this journey easier, and especially to Dr. Antonio, who always believed in me and was an incredible mentor.

# Induced Rotation of Ratchets in Passive Environments

by  
Yeray Cruz Ruiz

## Abstract

Swimming at the mesoscale has been a topic of interest for the past two decades because of the complexity of motion at a low Reynolds number, a regime where the viscous forces dominate over the inertial ones. The scallop theorem is a principle that follows these complex ideas and states that a scallop, which has only one degree of freedom, must be unable to have net displacement in this regime due to the lack of time-reversal symmetry of the Navier-Stokes equations. Indifferent to our mathematical understanding, nature was capable of creating biological beings that are able to displace under these circumstances by developing what we call nonreciprocal motion. The *escherichia coli*, for example, has a flagellum that rotates in one direction, pushing the fluid backwards and therefore moving the bacteria forwards. Inspired by these ideas, researchers have taken interest in generating motion at those scales. One example are bacterial micromotors, where a dented ratchet is immersed into a bacterial bath where they convert energy from their surroundings into movement. This movement normally follows a ballistic trajectory and in time some of them, will collide into the ratchet transferring their kinetic energy, and therefore making the motor spin thanks to the geometry of the ratchet. Unfortunately, the nutrients they absorb will end and in time the system's medium will need to be replaced, stopping the whole process, therefore being an inefficient process. This is a type of active matter that gets energy from its medium. But, Can we do the same with passive matter?

In this work, we analyze paramagnetic colloids, manipulated by an external precessing magnetic field. The system is confined between the  $z$  axis and presents periodic boundary conditions in  $x$ , and  $y$  axis. When multiple particles are present, dipole-dipole interactions arise, leading to either attraction or repulsion depending on their head-to-tail alignment. The particles' internal fields also rotate, dynamically altering their interactions over time. We observe that from 0Hz to 3Hz, the colloids form pairs and start rotating with a shared center of mass, whereas from 3.25 to 7Hz particles have a moment of repulsion, creating a neighbor exchange between different pairs, obtaining a ballistic trajectory. We observe that from 0Hz to 3Hz, the colloids form pairs and start rotating with a shared center of mass, whereas from 3.25 to 7Hz particles have a moment of repulsion, creating a neighbor exchange between different pairs, obtaining a ballistic trajectory. To investigate whether this motion can perform work, we place a ratchet-like object with different parameters amidst the particles.

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# **Part I**

## **Introduction**



# **Chapter 1**

## **Introduction**





# **Part II**

## **Background**



# Chapter 2

## Physics at the mesoscale

### 2.1 Low Reynolds number regime

At the mesoscale, where objects such as bacteria and colloidal particles operate, the physical world is governed by a regime in which viscous forces dominate over inertial ones. This regime is characterized by a small Reynolds number (Re), a dimensionless quantity that compares inertial to viscous effects. Therefore, the force applied at that moment will describe the movement or displacement performed, not depending on any past force, this is a characteristic of an overdamped system. In his seminal lecture, *Life at Low Reynolds Number*, Purcell highlighted the surprising and often counterintuitive behaviors that emerge in such environments [1]. For instance, time-reversible motion — common at macroscopic scales — is ineffective for propulsion at low Re, necessitating non-reciprocal strategies like flagellar rotation or body undulation. This leads to the scallop theorem, that states that an animal with such degrees of freedom — in a viscous regime — will not have a net displacement.

This whole process can be described by the Navier-Stokes equation without the inertia terms, leaving us without any time depending terms as shown in 2.1.

$$-\nabla p + \eta \nabla^2 \vec{v} = 0 \tag{2.1}$$

This has been a topic of interest for researchers that are constantly looking for ways of transportation in those environments for specific tasks. Unfortunately this is not the only challenge we face when moving at the microscale.

## 2.2 Brownian Motion and Thermal Fluctuations

Even though this is a viscous regime, particles are not static. At small length scales, such as those of colloidal particles or bacteria, random thermal fluctuations become a dominant source of motion. This phenomenon, known as *Brownian motion*, was first explained quantitatively by Albert Einstein in 1905. He demonstrated that the irregular paths observed in microscopic particles suspended in fluid result from collisions with the molecules of the surrounding medium [2].

Einstein's work provided one of the first convincing arguments for the molecular nature of matter and led to a mathematical description of how these random movements accumulate over time. Specifically, he derived that the mean squared displacement (MSD) of a particle grows linearly with time:

$$\langle x^2(t) \rangle = 2Dt, \quad (2.2)$$

where  $D$  is the diffusion coefficient, a measure of how quickly particles spread out. Einstein further related this coefficient to measurable physical parameters through the expression:

$$D = \frac{k_B T}{6\pi\eta R}, \quad (2.3)$$

where  $k_B$  is Boltzmann's constant,  $T$  the absolute temperature,  $\eta$  the dynamic viscosity of the fluid, and  $R$  the radius of the spherical particle. This relation — often referred to as the *Einstein-Stokes equation* — is foundational in soft matter and colloidal physics.

In the systems considered in this thesis, Brownian motion plays a crucial role in the dynamics of passive colloids and must be accounted for even in the presence of external fields or active agents, such as bacteria.

While Einstein's formulation captures the long-term diffusive behavior of Brownian particles, it does not account for their instantaneous dynamics. To describe how particles move under both viscous damping and random thermal forces, we turn to a stochastic differential equation known as the *Langevin equation*.

### 2.2.1 Stochastic Representation

The Langevin equation [3] in one dimension, including inertial effects, damping, and thermal noise, can be written as:

$$m\ddot{x} = F - \lambda\dot{x} + \eta(t), \quad (2.4)$$

where  $m$  is the particle mass,  $\lambda$  is the damping coefficient, and  $\eta(t)$  is a stochastic force representing thermal noise. For simplicity, we write the velocity as  $v = \dot{x}$ , leading to:

$$m \frac{dv}{dt} = F - \lambda v + \eta(t). \quad (2.5)$$

Discretizing time with a small step  $dt$ , we apply the definition of a derivative:

$$m \frac{v(t + dt) - v(t)}{dt} = F - \lambda v(t) + \eta(t). \quad (2.6)$$

The thermal noise term  $\eta(t)$  is modeled as a Gaussian white noise process:

$$\eta(t) dt = g dW, \quad (2.7)$$

where  $dW$  is a Wiener process (increment of Brownian motion) such that:

$$\langle dW \rangle = 0, \quad \langle dW^2 \rangle = dt. \quad (2.8)$$

Solving for  $v(t + dt)$  gives:

$$v(t + dt) = v(t) + \frac{dt}{m}(F - \lambda v(t)) + g dW. \quad (2.9)$$

Now, assuming no deterministic force ( $F = 0$ ) — as is the case for free passive particles:

$$v(t + dt) = v(t) - \frac{\lambda dt}{m}v(t) + g dW. \quad (2.10)$$

Taking the expectation value (mean) of both sides:

$$\langle v(t + dt) \rangle = \langle v(t) \rangle - \frac{\lambda dt}{m} \langle v(t) \rangle + g \langle dW \rangle. \quad (2.11)$$

Since  $\langle dW \rangle = 0$ , the last term vanishes:

$$\langle v(t + dt) \rangle = \langle v(t) \rangle \left( 1 - \frac{\lambda dt}{m} \right). \quad (2.12)$$

In the limit of small  $dt$ , this leads to the differential equation:

$$\frac{d}{dt} \langle v(t) \rangle = -\frac{\lambda}{m} \langle v(t) \rangle, \quad (2.13)$$

whose solution is:

$$\langle v(t) \rangle = \langle v(0) \rangle e^{-\lambda t/m}. \quad (2.14)$$

This result shows that the average velocity of a particle in a viscous fluid decays exponentially due to damping. The characteristic timescale  $\tau = m/\lambda$  describes how quickly the particle forgets its initial velocity, after which the motion becomes diffusive.

### 2.2.2 Velocity Statistics and Energy Equipartition

To compute the variance of the velocity, we start again from the Langevin equation in discretized form, without external forces:

$$v(t + dt) = v(t) - \frac{\lambda}{m}v(t)dt + \frac{g}{m}dW. \quad (2.15)$$

Squaring both sides:

$$v(t + dt)^2 = \left[ v(t) - \frac{\lambda}{m}v(t)dt + \frac{g}{m}dW \right]^2, \quad (2.16)$$

$$\begin{aligned} &= v(t)^2 + \left( \frac{\lambda}{m} \right)^2 v(t)^2 dt^2 + \left( \frac{g}{m} \right)^2 dW^2 \\ &\quad - 2\frac{\lambda}{m}v(t)^2 dt + 2\frac{g}{m}v(t)dW - 2\frac{\lambda g}{m^2}v(t)dW. \end{aligned} \quad (2.17)$$

Now we take the expectation value:

$$\begin{aligned} \langle v(t + dt)^2 \rangle &= \langle v(t)^2 \rangle + \left( \frac{\lambda}{m} \right)^2 \langle v(t)^2 \rangle dt^2 + \left( \frac{g}{m} \right)^2 \langle dW^2 \rangle \\ &\quad - 2\frac{\lambda}{m} \langle v(t)^2 \rangle dt + 2\frac{g}{m} \langle v(t) \rangle \langle dW \rangle - 2\frac{\lambda g}{m^2} \langle v(t) \rangle \langle dW \rangle. \end{aligned} \quad (2.18)$$

Using the properties of the Wiener process:

$$\langle dW \rangle = 0, \quad \langle dW^2 \rangle = dt.$$

And neglecting second-order small terms ( $dt^2$ ), we obtain:

$$\langle v(t + dt)^2 \rangle = \langle v(t)^2 \rangle + \frac{g^2}{m^2}dt - 2\frac{\lambda}{m} \langle v(t)^2 \rangle dt. \quad (2.19)$$

Taking the continuous limit:

$$\frac{d}{dt} \langle v(t)^2 \rangle = -2\frac{\lambda}{m} \langle v(t)^2 \rangle + \frac{g^2}{m^2}. \quad (2.20)$$

This is a linear first-order ODE. Solving it with variation of constants yields:

$$\langle v(t)^2 \rangle = \frac{g^2}{2\lambda m} + De^{-2\lambda t/m}. \quad (2.21)$$

As  $t \rightarrow \infty$ , the exponential term vanishes and we get the stationary value:

$$\langle v(\infty)^2 \rangle = \frac{g^2}{2\lambda m}. \quad (2.22)$$

From the equipartition theorem, we know that the average kinetic energy is:

$$\frac{1}{2}m\langle v^2 \rangle = \frac{1}{2}k_B T.$$

Therefore:

$$\langle v^2 \rangle = \frac{k_B T}{m}. \quad (2.23)$$

Matching this to our stochastic result:

$$\frac{k_B T}{m} = \frac{g^2}{2\lambda m} \Rightarrow g^2 = 2\lambda k_B T, \quad g = \sqrt{2\lambda k_B T}. \quad (2.24)$$

This defines the noise amplitude in terms of temperature, viscosity, and Boltzmann's constant.

### 2.2.3 Overdamped Dynamics and Diffusion

In the overdamped limit, inertia is negligible, so the Langevin equation becomes:

$$0 = F - \lambda \frac{dx}{dt} + \eta(t). \quad (2.25)$$

Solving for the velocity:

$$\frac{dx}{dt} = \frac{1}{\lambda}(F + \eta(t)). \quad (2.26)$$

For free diffusion ( $F = 0$ ):

$$\frac{dx}{dt} = \frac{1}{\lambda}\eta(t). \quad (2.27)$$

Using stochastic calculus with  $\eta(t)dt = g dW$ , we write:

$$x(t + dt) = x(t) + \frac{g}{\lambda} dW. \quad (2.28)$$

**Mean Position** Taking the expectation:

$$\langle x(t + dt) \rangle = \langle x(t) \rangle + \frac{g}{\lambda} \langle dW \rangle = \langle x(t) \rangle. \quad (2.29)$$

So the mean position remains constant in free diffusion.

**Mean Square Displacement (MSD)** Squaring the position update:

$$x(t + dt)^2 = x(t)^2 + 2\frac{g}{\lambda}x(t)dW + \left(\frac{g}{\lambda}\right)^2 dW^2. \quad (2.30)$$

Taking the expectation:

$$\langle x(t + dt)^2 \rangle = \langle x(t)^2 \rangle + 2\frac{g}{\lambda} \langle x(t) \rangle \langle dW \rangle + \frac{g^2}{\lambda^2} \langle dW^2 \rangle, \quad (2.31)$$

$$= \langle x(t)^2 \rangle + \frac{g^2}{\lambda^2} dt. \quad (2.32)$$

In differential form:

$$\frac{d}{dt} \langle x(t)^2 \rangle = \frac{g^2}{\lambda^2}. \quad (2.33)$$

Using  $g^2 = 2\lambda k_B T$ , we substitute:

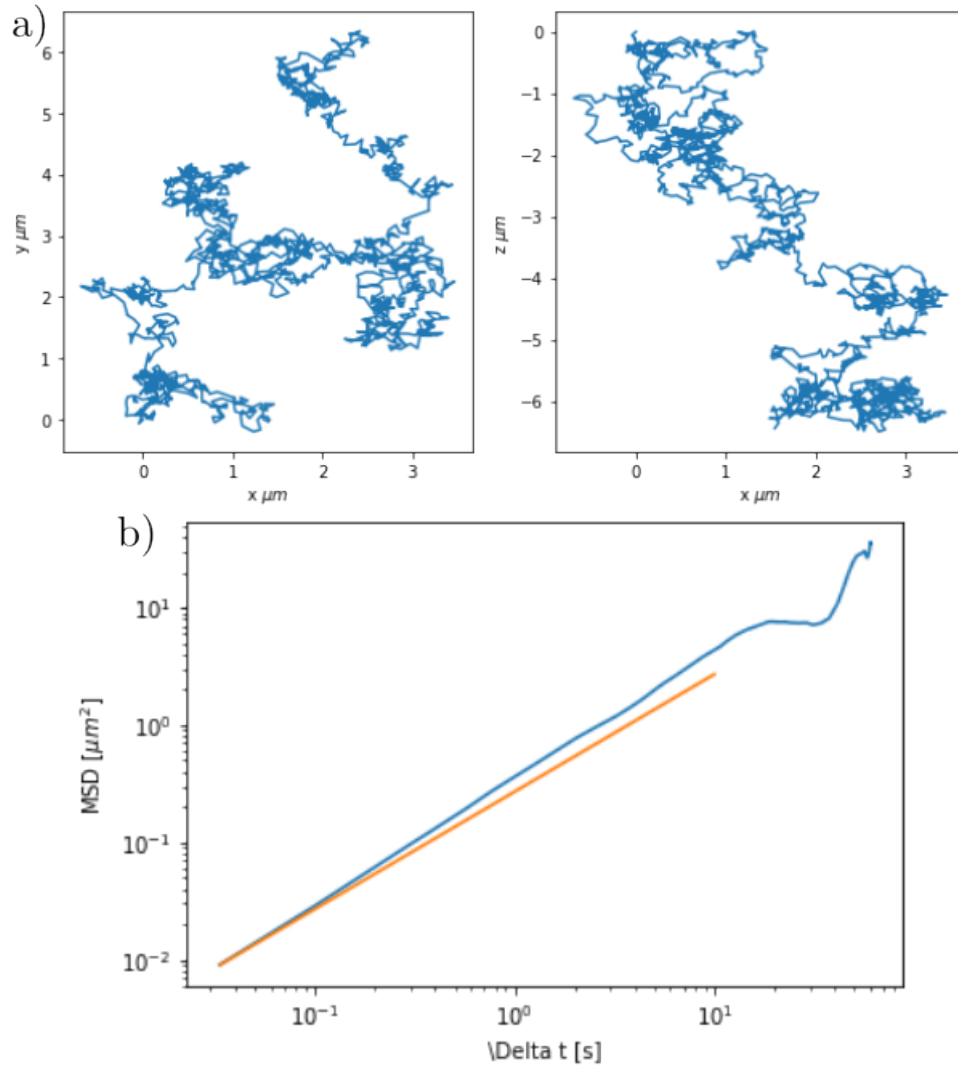
$$\frac{d}{dt} \langle x(t)^2 \rangle = \frac{2k_B T}{\lambda}. \quad (2.34)$$

Integrating gives the mean squared displacement (MSD):

$$\langle x(t)^2 \rangle = \frac{2k_B T}{\lambda} t. \quad (2.35)$$

This is the classical diffusion result, where the diffusion coefficient is  $D = \frac{k_B T}{\lambda}$ , consistent with Einstein's expression.





**Figure 2.1:** A colloidal particle undergoes random, thermally induced displacements in a fluid medium. **Panel a)** shows the trajectory of the particle. **Panel b)** shows the corresponding mean squared displacement (MSD) as a function of time, illustrating the linear relationship predicted by Einstein for diffusive behavior (orange) and the one obtained through a numerical simulation (blue).



# Chapter 3

## Ratchets and Rectification Mechanisms

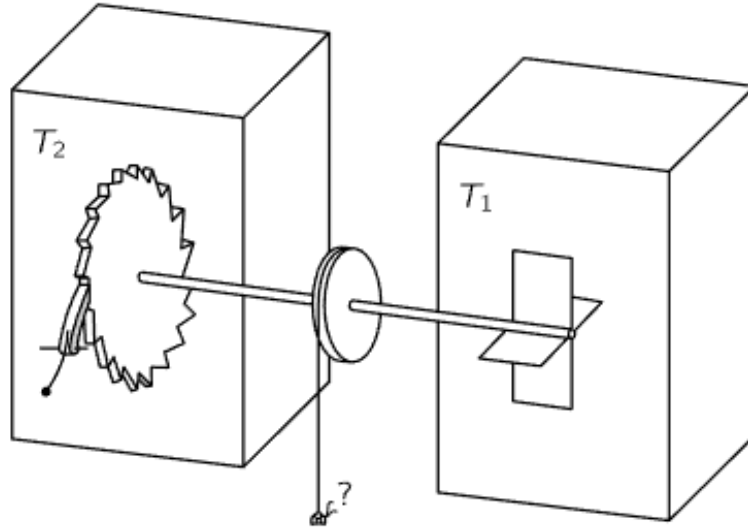
### 3.1 The Feynman-Smoluchowski ratchet

The inherent randomness of Brownian motion naturally leads to the question: can this disorder be transformed into order? In other words, can the random thermal motion of particles be used to produce directed movement or extract work? This question sits at the core of statistical mechanics and was famously explored by Richard Feynman in his lectures on physics, through a thought experiment known as the Feynman ratchet and pawl [4].

The Feynman ratchet consists of a set of vanes connected to a ratchet wheel, immersed in a fluid (Fig. 3.1). The idea is: random collisions from the surrounding molecules could push the vanes, but the pawl only allows rotation in one direction. At first glance, this asymmetric mechanism seems capable of converting random thermal motion into unidirectional rotation, apparently violating the second law of thermodynamics.

However, Feynman's analysis showed that when both the ratchet and the pawl are in thermal equilibrium with the same heat bath, the system cannot produce net work. The pawl itself undergoes thermal fluctuations and can occasionally lift off, allowing the ratchet to move backward. Over time, the forward and backward movements average out, and no net rotation occurs. This result reinforces the principle that thermal fluctuations alone cannot be rectified to perform work without a temperature gradient or an external energy input.

Despite this limitation, the Feynman ratchet introduced a powerful concept: asymmetry combined with non-equilibrium conditions can, in principle, produce directed motion. This idea is foundational in the study of Brownian motors, biomolecular machines, and active matter systems, including the systems explored in this thesis. In such systems, energy is continuously supplied, whether through bacterial metabolism or magnetic field modulation, creating the necessary non-equilibrium environment that allows motion rectification to occur.



**Figure 3.1:** Visual representation of the Feynman ratchet. Obtained from [4]

The failure of the Feynman ratchet in thermal equilibrium is intimately connected to another famous thought experiment: Maxwell’s demon. Both systems attempt to extract work from thermal fluctuations through selective processes—the ratchet through mechanical asymmetry, and the demon through information gathering.

Maxwell’s demon, proposed in 1867, imagines a microscopic being that can sort fast and slow molecules between two chambers, creating a temperature difference without apparent work. Similarly, the pawl in Feynman’s ratchet acts as a mechanical “demon,” attempting to select only forward fluctuations while blocking backward motion. However, both fail for the same fundamental reason: the cost of selection itself [5, 6].

Brillouin (1951) and later Landauer (1961) showed that Maxwell’s demon must expend energy to measure molecular velocities and erase information, with the minimum energy cost being  $kT \ln 2$  per bit erased. In the Feynman ratchet, the pawl must “decide” whether to allow motion, and this decision-making process—manifested as thermal fluctuations of the pawl itself—has an entropic cost that exactly cancels any work extracted [7, 8].

This connection reveals a deep principle: rectification requires either an information gradient (knowledge about the system) or an energy gradient (non-equilibrium conditions). As Parrondo and Español (1996) demonstrated, the Feynman ratchet can be viewed as an information engine where the pawl performs measurements on the ratchet’s position. When the pawl and ratchet are at the same temperature, the information gained equals the entropy produced, yielding no net work [9].

**Table 3.1:** Summary of operation of ratchet and pawl. Obtained from [4]

<b>Forward:</b>	Needs energy	$\epsilon + L\theta$ from vane.	$\therefore \text{Rate} = \frac{1}{\tau} e^{-(L\theta + \epsilon)/kT_1}$
	Takes from vane	$L\theta + \epsilon$	
	Does work	$L\theta$	
	Gives to ratchet	$\epsilon$	
<b>Backward:</b>	Needs energy	$\epsilon$ for pawl.	$\therefore \text{Rate} = \frac{1}{\tau} e^{-\epsilon/kT_2}$
	Takes from ratchet	$\epsilon$	
	Releases work	$L\theta$	
	Gives to vane	$L\theta + \epsilon$	

If system is reversible, rates are equal, hence

$$\frac{\epsilon + L\theta}{T_1} = \frac{\epsilon}{T_2}.$$

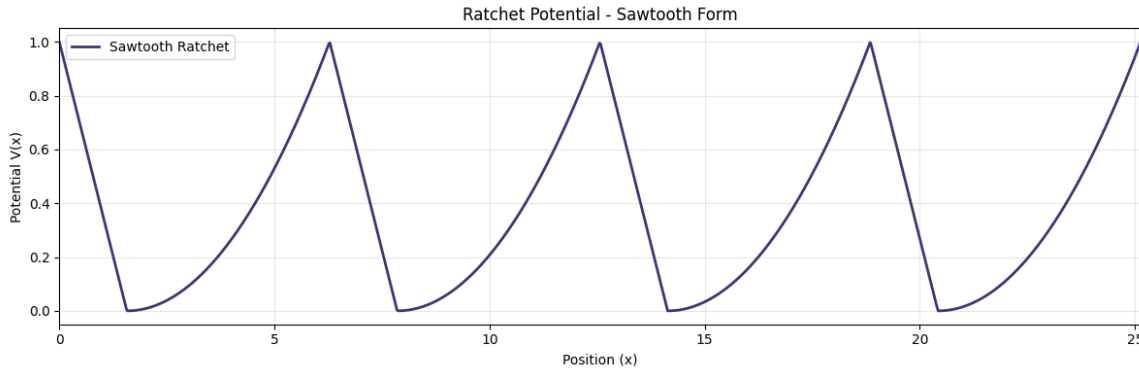
$$\frac{\text{Heat to ratchet}}{\text{Heat from vane}} = \frac{\epsilon}{L\theta + \epsilon}. \quad \text{Hence} \quad \frac{Q_2}{Q_1} = \frac{T_2}{T_1}.$$

## 3.2 Brownian ratchets

While Feynman's ratchet fails due to thermal equilibrium, rectified motion becomes possible when detailed balance is broken through external driving. Magnasco (1993) demonstrated this principle through the "rocking ratchet" mechanism [10]. In his model, a Brownian particle moves in an asymmetric periodic potential—essentially a sawtooth-shaped energy landscape with gentle slopes in one direction and steep walls in the other. The crucial innovation was applying an oscillating force  $F(t) = A \cos(\omega t)$  that rocks the potential back and forth. Although this force has zero time average—pushing equally left and right—the combination with the spatial asymmetry produces net directional motion. When the force tilts the potential forward, particles easily climb the gentle slopes and can overcome barriers. When tilted backward, particles encounter steep walls and remain trapped. This asymmetric response to symmetric driving breaks the forward-backward symmetry of thermal diffusion.

This mechanism differs fundamentally from Feynman's original ratchet: rather than attempting to rectify equilibrium fluctuations (which violates the second law), the rocking ratchet con-

tinuously injects energy through the oscillating force. The time-dependent driving maintains the system far from equilibrium, enabling the spatial asymmetry to generate directed transport. This principle—that asymmetry plus non-equilibrium driving yields rectification—underlies numerous biological processes and provides the theoretical foundation for the magnetically-driven ratchets explored in this thesis.



**Figure 3.2:** Example of a commonly used ratchet potential.

Following Magnasco’s rocking ratchet where an oscillating force drives particles in a static potential, Ajdari et al. (1994) analyzed rectification in periodic structures with either spatial or temporal asymmetry [11]. Using a sawtooth potential with asymmetric slopes (steep of height  $\epsilon/a$  and gentle of  $\epsilon/b$ ), they identified distinct transport regimes as a function of the AC force amplitude  $\gamma$ .

For small forces  $\gamma < \epsilon/a$ , particles remain trapped. In the intermediate regime  $\epsilon/a < \gamma < \epsilon/b$ , rectification occurs as particles can climb the gentle slope but not the steep one, yielding either integer velocities  $V = n$  when particles fully relax between cycles, or rational velocities  $V = n - 1/(m + 1)$  when incomplete relaxation creates a periodic pattern over multiple cycles.

This concept was later demonstrated experimentally by Faucheux et al. (1995), who created an “optical thermal ratchet” using focused laser beams [12]. To generate a circular trap without introducing external forces, they passed the laser through a plate to create circular polarization, maintaining constant laser intensity along the circular path. By rotating the laser frequency fast enough, particles could not experience any directed force from the beam itself, allowing them to move freely, then modulated the beam intensity to create an asymmetric potential mimicking a ratchet shape, as shown in Figure 3.3. They observed that particles localized in potential minima when the modulation was turned on, while for sufficiently long periods, particles moved either backward or forward with equal probability.

A different approach was developed by Lee et al. (2004), who employed holographic optical

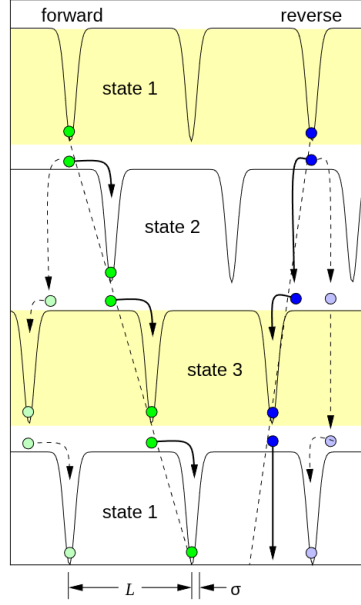


**Figure 3.3:** When the modulation is on, the beam intensity follows the shape of a ratchet potential with 4 periods per cycle. When the modulation is off, the intensity remains constant, allowing particles to diffuse freely. Obtained from [12].

trapping with a symmetric potential [13]. Their system consisted of discrete optical tweezers functioning as potential wells. Between each state, particles are released and free to diffuse, nevertheless they can rapidly jump between adjacent traps. Using this methodology, they achieved net particle displacement in one direction, as illustrated in Figure 3.4. Remarkably, when they reversed the direction of well displacement, particles followed the trap movement in the opposite direction. Due to this predictable response to trap motion, they named this system a "deterministic thermal ratchet."

Lebedev et al. (2009) modified this approach by using three linearly polarized beams acting only in the  $xy$ -plane, as shown in Figure 3.5 [14]. They applied forces to particles by phase-modulating two of the three lasers, which shifts the entire optical lattice. The modulation was generated using RF generators to produce potentials  $V_x$  and  $V_y$ . The sum  $V_x + V_y$  served as the input signal for beam 2, while the difference  $V_x - V_y$  was used for beam 3, creating the rocking forces. They studied this system using ultracold rubidium atoms as test particles. To track the atomic positions, they employed a CCD camera to determine the center of mass of the atomic cloud.

In their first experiment, they applied a biharmonic drive along only one axis. As expected, the system reduced to a one-dimensional harmonic ratchet, and they observed directed particle motion along the axis of the applied biharmonic force. In subsequent experiments, they applied



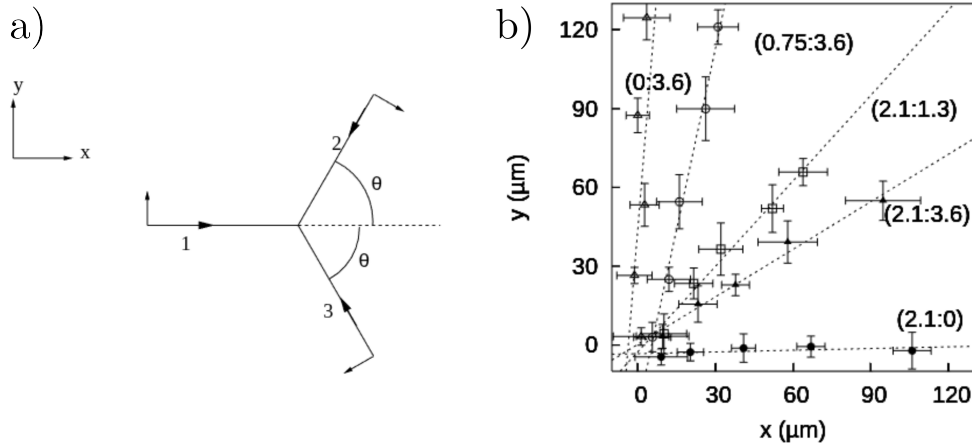
**Figure 3.4:** Discrete pattern of optical traps with displacement in one direction, minimum width of  $\sigma$  and a maximum width of  $L$ . Obtained from [13]

harmonic drives along both axes, with one frequency being twice the other. Their results showed that particles experienced directed motion along the axis with the faster harmonic, while motion along the other axis remained diffusive. In their final experiment, they applied biharmonic drives simultaneously along both axes to investigate whether directed transport could be achieved in two dimensions. By varying the relative weights of the forces along each axis, they demonstrated that it was possible to control the atomic displacement and direct atoms to specific target positions as shown in Figure 3.5 a).

Following the same lattice-based approach, Arzola et al. (2017) successfully recreated all five two-dimensional Bravais lattices using computer-generated holographic optical micromanipulation [15]. Their potential consisted of Gaussian wells distributed along a primary lattice, combined with a shifted replica of the same lattice with a reduced depth factor,  $Q$ , compared to the original. The asymmetry of the resulting potential depended directly on both the depth factor  $Q$  and the displacement between the original and replica lattices. They implemented the rocking mechanism by moving the particle cell along one axis.

The researchers performed three main experiments to validate their results, each with three different diffusion parameters (including no diffusion as one condition), calculating the current in the  $x$  and  $y$  axes with respect to the amplitude of movement. In the first experiment, they set the asymmetry parallel to the rocking force. They observed that since the wells were aligned with





**Figure 3.5:** **Panel a)** Lattice beam configuration in the  $xy$  plane.  $\theta = 60^\circ$ . **Panel b)** Position of the atomic cloud center of mass at different instants and forces when applying biharmonic at the same time in  $xy$ -plane. Obtained from [14].

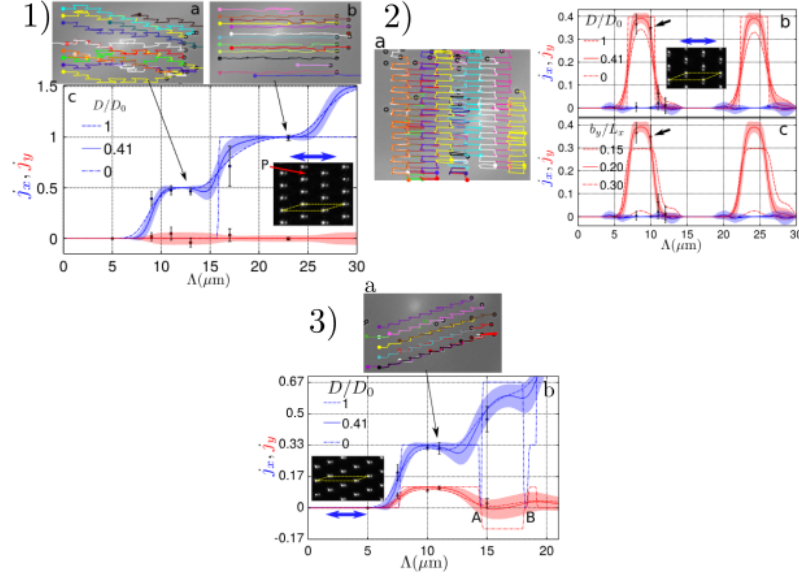
the particle movement, particles tended to move in the direction of the force, traveling from well to well, while experiencing no movement in the perpendicular direction. Current was present along the entire amplitude range.

The second experiment positioned the asymmetry perpendicular to the rocking force direction, yielding interesting results. Similar to the first experiment, current appeared along only one axis (the asymmetry axis), but the current behaved like a Gaussian bell curve. When the rocking movement amplitude approached the asymmetry lattice spacing, particles only moved back and forth between two wells in the  $x$  direction, experiencing no current in the  $y$  axis.

For the final experiment, they examined how the system behaved under a combination of both previous configurations. They created an oblique lattice with the asymmetry aligned with the rocking force direction, producing an oblique particle current. The results were essentially a combination of the two previous experiments: at certain amplitudes, current appeared in both axes, but as amplitude increased, the  $y$ -axis current decreased while the  $x$ -axis current increased. The results for each experiment can be seen in Figure 3.6.

Feynman demonstrated that a purely thermal Brownian ratchet cannot generate directed motion due to the second law of thermodynamics. However, introducing specific conditions to break symmetry has proven ideal for bypassing these limitations.

One successful approach to achieving this symmetry breaking involves rocking ratchets, where an external periodic drive is applied to an asymmetric potential. Previous experiments have demonstrated the potential of optics to create such asymmetries with high precision and



**Figure 3.6: Panel 1)** Experimental trajectories for asymmetry parallel to rocking movement with a) rocking amplitude of 13  $\mu\text{m}$  and b) 23  $\mu\text{m}$ . c) Graphs of simulations showing the current for each axis. **Panel 2)** Experimental trajectories for asymmetry perpendicular to rocking movement with a) rocking amplitude of 10  $\mu\text{m}$ . b) and c) Shows the current for each axis with different lattice parameters. **Panel 3)** Experimental trajectories for oblique lattice with asymmetry perpendicular to the rocking movement with a) rocking amplitude of 11  $\mu\text{m}$ . b) Current for each axis. Obtained from [15].

control. While obtaining directed motion at the microscale remains challenging, by applying these symmetry-breaking parameters it becomes possible not only to overcome thermal noise but to harness it for directional transport. Unfortunately, as shown in previous works, implementing these systems often requires specific and sophisticated experimental setups. Interestingly, nature has already developed an alternative solution by creating matter capable of self-propulsion, known as active matter, which achieves directed motion through entirely different mechanisms.

# Chapter 4

## Active Matter Systems

### 4.1 Fundamentals of Active Matter

The definition of active matter may vary a little within each author, but we reach a consensus. Active matter refers to systems composed of self-driven units that continuously consume energy from their environment to propel themselves [16, 17]. It is important to note it doesn't matter if the particles are artificial or organic. For this first part, we will introduce some macroscopic examples, and then we will focus on the closest related for this thesis.

### 4.2 Macroscopic Agents

It is common to confuse what can be considered active matter and what not. However, examples can be observed at plain sight in the macroscopic world. A flock of birds, for instance, is considered active matter because each bird is capable of self-propulsion. Unlike microscopic systems, they are not significantly affected by the thermal noise of their medium—in this case, air. A computational study by Reynolds (1987) modeled flocking behavior in birds by treating each individual as an autonomous agent whose trajectory was influenced by its local environment and neighboring individuals [18].

Reynolds introduced three fundamental rules governing flocking behavior: separation (collision avoidance), alignment (velocity matching with neighbors), and cohesion (attraction toward the average position of neighbors). This work laid the foundation for understanding how simple local interactions can give rise to complex patterns of collective motion.

Over time, further developments in the study of group dynamics have incorporated methods such as dynamical maximum entropy to predict the alignment of ordered groups [19]. Following a maximum entropy approach, network reshuffling was later studied [20]. These behaviors,

however, can change due to external factors, such as predation risk, which influences the formation and density of flocks [21]. In addition, individual characteristics can also affect collective behavior [22].

There are also self-organized groups of smaller flying animals whose collective behavior is even more complex than that of larger species. For example, bees are capable of dividing labor as a result of social interactions among individuals [23]. Remaining within the field of insects but shifting to a solid medium, ants exhibit similar patterns of labor division. Their organization can transition between ordered and disordered states depending on colony size. The disordered state typically occurs when the number of foraging ants is small, due to the infrequent discovery of food [24]. However, even when the colony is large enough to sustain a greater number of foragers, problems can still arise. In particular, some ants may remain stationary, occupying tunnels and thereby reducing the overall flow rate [25].

Another characteristic that arises from self-organization in ants—one not possessed by all animals—is their ability to self-assemble to perform specific tasks. Examples include the formation of rafts, bridges, or columns, where each individual uses its legs or jaws to grasp another and remains in place until the collective goal is achieved. Naturally, certain factors influence these self-assemblages, one of the most significant being colony size [26]. Interestingly, these structures possess some viscoelastic properties, when applied a certain stress, they tend to return to their original place, resulting in an elastic behavior, at the same time they adapt a viscous behavior due to objects being able to sink in the aggregation, the same way they would do in a viscous fluid [27]. This opens the possibility for ants to be used as smart materials.

There are also animals that form groups while inhabiting more complex environments, such as water. Regardless of the medium, different species often display similar behaviors and dependencies. For example, the dynamics of schooling fish can be predicted, typically resulting in three main configurations, with variations depending on group size [28–30].

These are just few examples of large animal groups that, interestingly, share common characteristics of self-organization. This process begins at the level of the individual but emerges as a collective, temporary order through direct or indirect interactions among organisms, ultimately allowing them to achieve a common goal [31].

The study of macroscopic agents reveals that self-organization is not limited to large animal groups. At smaller scales, microscopic agents such as bacteria, colloidal particles, and artificial swimmers also exhibit analogous self-organized behaviors, although these are driven by different physical principles, including thermal noise and low-Reynolds-number hydrodynamics.

### 4.3 Microscopic agents

As discussed in the previous section, the term active matter is not limited to microscopic systems but also applies to animal groups. However, the focus of this thesis will be on the former.

To begin, let us review some fundamentals. The key difference of active and passive ones, is that passive undergo mere diffusion motion, whereas active follows stochastic differential equations according to [32]:

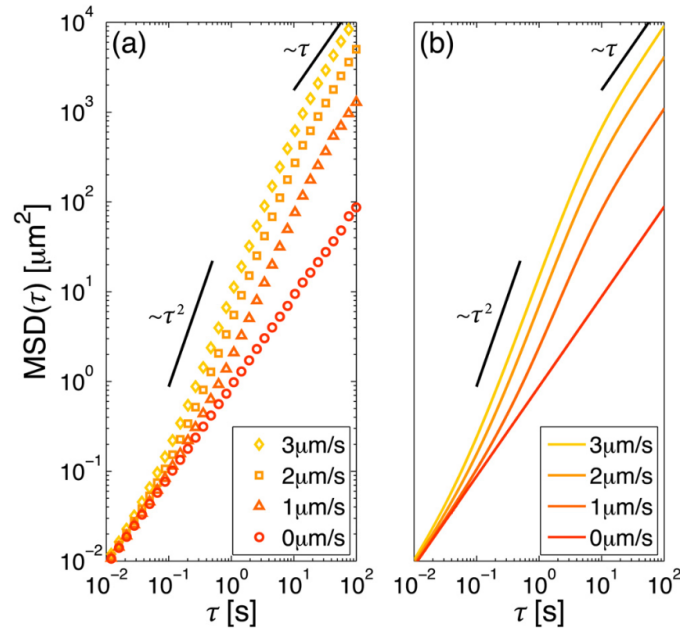
$$\frac{d}{dt}x(t) = v \cos \varphi(t) + \sqrt{2D_T}W_x, \quad (4.1)$$

$$\frac{d}{dt}y(t) = v \sin \varphi(t) + \sqrt{2D_T}W_y, \quad (4.2)$$

where  $W_x$ , and  $W_y$  represent their corresponding Wiener process. As stated before, if

$$\lim_{v \rightarrow 0} v \cos \varphi(t) + \sqrt{2D_T}W_x, \quad (4.3)$$

then the motion is merely diffusive and the particle is characterized as brownian particle. Figure 4.1 helps visualize how the motion varies according to their velocity. When the velocity is zero, the linear behavior of the MSD represents a diffusive behavior, whereas when it is different it is considered to be ballistic.



**Figure 4.1:** MSD for brownian particles with different velocities. **Panel a)** shows the result from simulations. **Panel b)** shows the theoretical calculation. Obtained from [32]

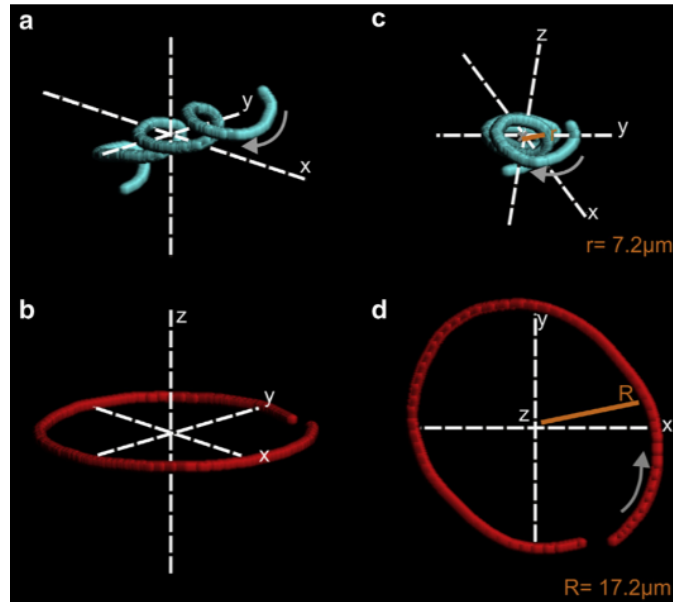
This was proven with the work done by Howse et al. (2007) that established the experimental basis for active Brownian particle dynamics by studying self-motile colloidal particles that use chemical reactions catalyzed on their surface to achieve autonomous propulsion. They demonstrated that at short times, these particles exhibit substantial directed motion with velocity dependent on fuel concentration, while at longer times, the motion transitions to a random walk with enhanced diffusion coefficients. This work provided a comprehensive experimental validation of the theoretical active Brownian particle model and established the standard mathematical framework for describing such systems through stochastic differential equations [33]. This, however is an artificial active particle, done by coating a polystyrene spheres with a layer of platinum, which will be the responsible of the propulsion method. This open a new term that appears active particles have, it is called chirality, which is the tendency to move in a circular path just as described by the random walk, however, this movement was only studied in a 2-dimensions confined space. In a research by Corkidi et al. (2008) spermatozoa behavior was studied in a 3-dimensions space mimicking their real environment in search for eggs, what they found was a helical trajectory, shown in Figure 4.2, being a pretty similar response, and obtaining a non-space dependant behavior [34]. Therefore we add a third term for the rotational diffusion given by:

$$\dot{\varphi} = \sqrt{2D_R}W_{\varphi}. \quad (4.4)$$

As discussed in section 2.1, organisms or objects must break symmetry in specific ways to achieve net movement when swimming in environments dominated by viscous forces. Nature has evolved elegant solutions to this challenge through various biological mechanisms. A prime example is *E. coli*, which employs helical flagella that can rotate both clockwise and counterclockwise, generating two distinct types of motion that researchers have termed "run"—swimming in a straight line—and "tumble"—rotation in a random direction. This behavior also depends on the medium's viscosity and random fluctuations [35]. The way *E. coli* achieves a "run" state is due to their flagella which has two different motions for its power and recovery strokes, simulating those of a human breast strokes.

## 4.4 Interaction with non-homogeneous environments

All this properties considered an ideal environment, where no obstacles are present. Active particles in real environments encounter surfaces, confinement, and flow fields that dramatically alter their behavior. A striking example is the work by Hill et al. (2007), who demonstrated that *E. coli* swimming near surfaces in shear flow exhibit sophisticated rheotaxis - the ability to sense



**Figure 4.2:** Trajectory of the spermatozoa for **a**, and **c** free space, and **b**, **d** confined 2-dimension space. Obtained from [34].

flow gradients and actively swim upstream through hydrodynamic surface interactions [36].

## 4.5 Active motors





# Chapter 5

## Magnetically Driven Colloidal Systems

### 5.1 Magnetic Colloids under static fields

### 5.2 Dynamic Magnetic Actuation

Active matter requires a continuous source of energy to avoid falling under the category of Brownian particles. However, for complex challenges—such as transportation—these systems can tend to be inefficient due to their energy depletion.

Fortunately, Ostinato et al. (2024) Dimer formation, exchange dynamics

Emergent currents without self-propulsion

In the study of motion at small scales, materials are often categorized as either active or passive. This distinction is based on whether the constituents are capable of autonomously converting energy into motion.

This thesis focuses on such externally driven passive systems, particularly on paramagnetic colloids subjected to dynamic magnetic fields. These systems provide a controlled platform to study non-equilibrium transport and rectified motion, drawing inspiration from active matter while remaining fundamentally passive in nature.



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# Curriculum Vitae

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