Learning Journal Unit 5

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In this week's task, I found logarithmic differentiation creative in different situations where it is easier to differentiate the logarithm of a function than to differentiate the function itself (Wells, 2012). In each case, knowledge of the three laws of logarithm is necessary.

Applying the law of multiplication

The law of multiplication tells us that the logarithm of a number (m) raised to a power (the exponent) (n) can be found by multiplying the logarithm of the number by the exponent. This is expressed formally as: $ln(m)^n = nln(m)$ (Wells, 2012). It should be noted that functions sometimes contain expressions that have complex exponents, or exponents that are variables, or even exponents that are functions in their own right (Wells, 2012). In cases like $\mathrm{f(x)} = \mathrm{g(x)}^{\mathrm{h(x)}}$, where normal rules of differentiation may no longer apply, taking the logarithm of the function reduces the function to the product the logarithms of the individual functions, $\mathrm{lnf(x)} = \mathrm{h(x)}. \ln[\mathrm{g(x)}]$. This allows us to apply product rule of differentiation, which exists and is easier to perform, rather than to differentiate the function itself.

Applying the law of subtraction

The law of subtraction states that the logarithm of the quotient of two numbers (i.e. the result of dividing one number by another) can be found by subtracting the logarithm of the divisor (the number we are dividing by) from the logarithm of the dividend (the number we want to divide). This is expressed formally as: $ln\left(\frac{m}{n}\right) = \ln(m) = \ln(n)$ (Wells, 2012). In cases like $\mathrm{f(x)} = \frac{\mathrm{g(x)}}{\mathrm{h(x)}}$, which is a quotient of two functions. Taking the logarithm of the function reduces the function to difference between the logarithms of the individual functions, $\ln[\mathrm{f(x)}] = \ln[h(x)] - \ln[g(x)]$. This allows us to differentiate each logarithmic function separately, which exists and is easier to perform, rather than to differentiate the function itself using quotient rule.

Applying the law of addition

The law of addition states that the logarithm of the product of two numbers (i.e. the result of multiplying two numbers together) can be found by adding together their individual logarithms. This is expressed formally as: $logb\ (m \times n) = logb\ (m) + logb\ (n)$ (Wells, 2012). For instance, when having a function such as f(x) = g(x)h(x), which is a product of two functions. Taking the logarithm of the function reduces the function to adding the logarithms of the individual functions, ln [f(x)] = $\ln[h(x)] + \ln[g(x)]$. This allows us to differentiate each logarithmic function separately, which exists and is easier to perform, rather than to differentiate the function itself using product rule.

It is evident from the three scenarios above that knowledge of logarithm rules is essential when performing logarithmic differentiation. Therefore, logarithmic differentiation simplifies the complex function by eliminating the exponent and get it into a form that is easier to differentiate. On the other hand, differentiating the complex function may often involve tedious and error-prone calculations. In conclusion, logarithmic differentiation is a creative way of simplifying complex function with variables as exponent to make it easier to differentiate.

References

Wells, C. J. (2012, 6). *Logarithmic differentiation*. TechnologyUK. https://www.technologyuk.net/mathematics/differential-calculus/logarithmic-differentiation.shtml