Discussion Assignment 7

According to Herman and Strang (2020), Newton's method can also be used to approximate square roots. For example, to solve $\sqrt{3}$, which is hard to find since it is a rational number, we can let $f(x) = x^2 - 3$ and $x_0 = 3$. The choice of $x_0 = 3$ to approximate $\sqrt{3}$ is reasonable because $f(x) = x^2 - 3$ has a zero at 3. For $f(x) = x^2 - 3$, $f'(x) = 2x$. by applying Newton's Method,

$$x_n = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 3 - \frac{(3)^2 - 3}{2(3)} \approx 2$$

$$x_2 = 2.5 - \frac{(2.5)^2 - 3}{2(2.5)} \approx 1.85$$

$$x_3 = 2.0 - \frac{(2.0)^2 - 3}{2(2.0)} \approx 1.75$$

$$x_4 = 1.9 - \frac{(1.9)^2 - 3}{2(1.9)} \approx 1.739473$$

$$x_5 = 1.8 - \frac{(1.8)^2 - 3}{2(1.8)} \approx 1.733333333$$

$$x_6 = 1.75 - \frac{(1.75)^2 - 3}{2(1.75)} \approx 1.73214285714$$

$$x_7 = 1.74 - \frac{(1.74)^2 - 3}{2(1.74)} \approx 1.73206896552$$

$$x_8 = 1.73 - \frac{(1.73)^2 - 3}{2(1.73)} \approx 1.73205203212$$

$$x_9 = 1.731 - \frac{(1.731)^2 - 3}{2(1.731)} \approx 1.73205112652$$

$$x_{10} = 1.732 - \frac{(1.732)^2 - 3}{2(1.732)} \approx 1.73205080831$$

$$x_{11} = 1.73205 - \frac{(1.73205)^2 - 3}{2(1.73205)} \approx 1.73205080757$$

$$x_{12} = 1.732051 - \frac{(1.732051)^2 - 3}{2(1.732051)} \approx 1.73205080757$$

Since we obtained the same value for x_{11} and x_{12}, it is unlikely that the value x_n will change on any subsequent application of Newton's method. We conclude that $\sqrt{3} \approx$ 1.73205080757.

From the calculations above, it is evident that Newton's Method requires an initial guess (x_0), which can be refined through iterations to approach the actual root of the equation.

The derivative of f(x), denoted as f'(x), is computed, and the iterative formula $x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$ is employed to approximate the root. In this example, the initial guess $x_0 = 3$ is chosen because it's close to the actual square root of 3, and the function f(x) has a zero at x = 3.

The iterations start with x_0 and continue until convergence, producing subsequent values x_1, x_2, x_3, and so on, through the iterative formula. The calculated values progressively get closer to the actual square root.

In the example provided, the iterations gradually approach the value of $\sqrt{3} \approx$ 1.73205080757. The process is stopped once the successive iterations yield a negligible change in the value of x, indicating that further iterations won't significantly alter the result.

Newton's Method is a powerful numerical tool to approximate roots, including square roots, providing a means to calculate roots that may otherwise be challenging or impossible to obtain through direct arithmetic methods, especially for irrational or non-perfect square roots.

Reference

Herman, E., & Strang, G. (2020). *Calculus volume 1.* Rice University.

https://openstax.org/details/books/calculus-volume-1