

Written Assignment Unit 1

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MATH 1211: Calculus

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Written Assignment Unit 1

- 1. What linear function, $y = f(x)$ has $f(0) = 8$ and $f(7) = 14$?**

For $f(0) = 8$ and $f(7) = 14$, it implies that the linear function is passing through the points $(0,8)$ and $(7,14)$ respectively. The slope of the linear function can be determined

$$\text{as follows: } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 8}{7 - 0} = \frac{6}{7}.$$

Using the point-slope form, $y - y_1 = m(x - x_1)$.

$$y - 14 = \frac{6}{7}(x - 7)$$

Therefore, the linear function becomes $y = \frac{6}{7}x + 8$.

- 2. If $f(t) = 2t - t^2 + 3$, what is $\frac{f(t+h)-f(t)}{h}$?**

$$\frac{f(t+h) - f(t)}{h} = \frac{2(t+h) - (t+h)^2 + 3 - (2t - t^2 + 3)}{h}$$

$$= \frac{(-t^2 + 2t - 2th - h^2 + 2h + 3) - (2t - t^2 + 3)}{h}$$

$$= \frac{-2th - h^2 + 2h}{h}$$

$$= \frac{h(-2t - h + 2)}{h}$$

$$= -2t - h + 2$$

- 3. Find all solutions to the equation $-4\cos x = -\sin^2 x + 1$. Write your answer in radians in terms of π .**

$$-4\cos x = -\sin^2 x + 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$-4\cos x = -1 + \cos^2 x + 1$$

$$0 = \cos^2 x + 4\cos x$$

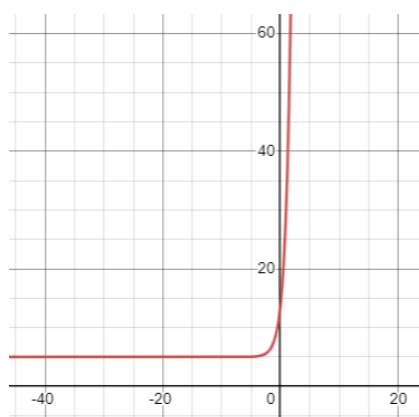
$$0 = \cos x(\cos x + 4)$$

$$\cos x = 0 \text{ or } \cos x + 4 = 0$$

For $\cos x = 0$, the solutions are $x = \frac{\pi}{2} + n\pi$ and $x = \frac{3\pi}{2} + n\pi$, where n is an integer.

For $\cos x + 4 = 0$, solving for $\cos x$ gives $\cos x = -4$, which has no real solutions because the cosine function has a range of $(-1, 1)$.

4. Sketch the graph of $y = 3^{x+2} + 5$. Find the domain, range, and horizontal asymptote. Include the horizontal asymptote in your graph.

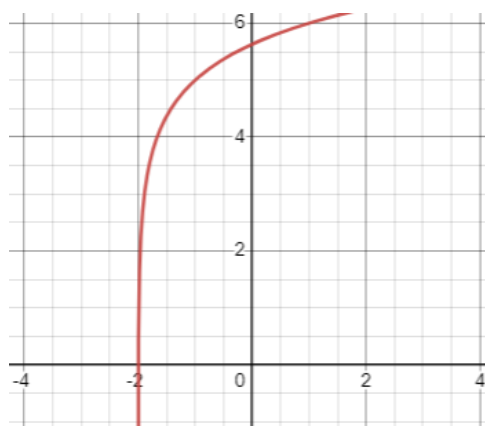


Domain: $(-\infty < x < \infty)$.

Range: $(5, \infty)$.

Horizontal asymptote: $y = 5$.

5. Sketch $\log_3(x + 2) + 5$. Find the domain, range, and vertical asymptote. Include the vertical asymptote in your graph.



Vertical asymptote: $x = -2$

Domain: $(-2, \infty)$

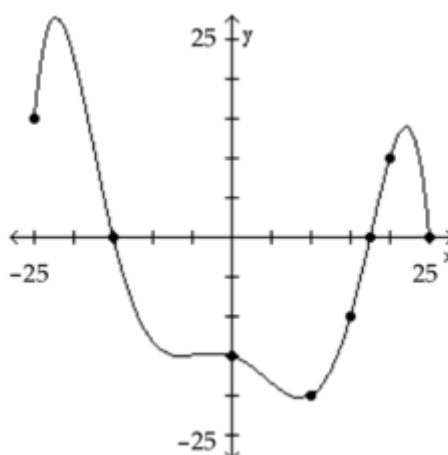
Range: $(-\infty, \infty)$

Vertical asymptote: $x = -2$

6. Find the domain of the function $g(x) = \frac{2x}{x^2 - 81}$.

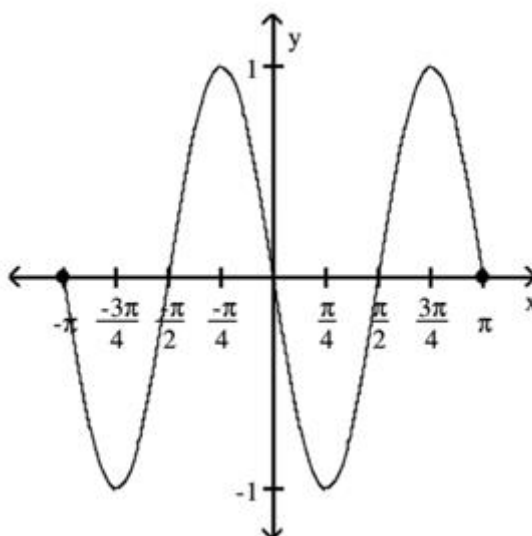
Solving the denominator: $x^2 - 81$, $x = \pm 9$. This means that $g(x)$ will become undefined at $x = 9$ and $x = -9$. Therefore, domain becomes $(-\infty, -9) \cup (-9, 9) \cup (9, +\infty)$.

7. From the graph below, find what is $f(-15)$ and for what numbers x is $f(x) = 0$.



From the graph, $f(-15) = 0$, and the values of x when $f(x) = 0$ is -15 and 17.5.

8. Determine whether the graph is that of a function. If it is, use the graph to find its domain, and range, the intercepts, if any, and any asymptote with respect to the x -axis, the y -axis, or the origin.



The graph is a function since the vertical line cuts it once.

Domain: $(-\pi, \pi)$

Range: $(1, -1)$

Horizontal asymptote: $y = 1$ and $y = -1$

9. Determine whether the function is even, odd, or neither.

a) $f(x) = 4x^3$

This is an odd function

b) $f(x) = \frac{-x^3}{4x^2+3}$

This is an even function

c) $f(x) = 3x^3 - 5$

This is neither an even function nor an odd function.

10. A cellular phone plan had the following schedule of charges: Basic service, including 100 minutes of calls is \$20.00/per month; 2nd 100 minutes of calls is \$0.075/minute; additional minutes of calls is \$0.10/minute.

a) What is the charge for 200 minutes of calls in one month?

100 minutes are covered by the basic service (\$20.00).

The next 100 minutes are charged at \$0.075 per minute.

Total charge for 200 minutes: $20.00 + (0.075 * 100) = \27.50

b) What is the charge for 250 minutes of calls in one month?

100 minutes are covered by the basic service (\$20.00).

The next 100 minutes are charged at \$0.075 per minute.

The remaining 50 minutes are charged at \$0.10 per minute.

Total charge for 250 minutes: $20.00 + (0.075 * 100) + (0.10 * 50) = \32.50

c) Construct a function that relates the monthly charge C for x minutes of calls?

For $x \leq 100$ minutes: $C = 20.00$

For $100 < x \leq 200$ minutes: $C = (20.00 + 0,075) * (x - 100)$

For $x > 200$ minutes: $C = 20.00 + (0.075 * 100) + 0.10(x - 200)$