

Learning Journal Unit 8

Godfrey Ouma

University of the People

MATH 1201: College Algebra

Yousef Ajeeb

November 2, 2023

Learning Journal Unit 8

Complete the following questions utilizing the concepts introduced in this unit. The assignment will be graded by your instructor.

1. Evaluate the cube root of $z = 27cis(240^\circ)$. Then raise them to the cube. Show the steps of your reasoning.

To find the root of a complex number in polar form, use the formula given as

$z^{\frac{1}{n}} = r^{\frac{1}{n}} \left[\cos\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2k\pi}{n}\right) \right]$ where $k = 0, 1, 2, 3, \dots, n-1$. We add $\frac{2k\pi}{n}$ to $\frac{\theta}{n}$ in order to obtain the periodic roots.

$$z = 27cis(240^\circ) = 27 \left[\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right) \right]$$

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}} \left[\cos\left(\frac{\frac{4\pi}{3} + 2k\pi}{3}\right) + i\sin\left(\frac{\frac{4\pi}{3} + 2k\pi}{3}\right) \right]$$

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}} \left[\cos\left(\frac{4\pi}{9} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{4\pi}{9} + \frac{2k\pi}{3}\right) \right]$$

There will be three roots: $k = 0, 1, 2$.

When $k = 0$, we have:

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}} \left[\cos\left(\frac{4\pi}{9} + \frac{2(0)\pi}{3}\right) + i\sin\left(\frac{4\pi}{9} + \frac{2(0)\pi}{3}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \left[\cos\left(\frac{4\pi}{9}\right) + i\sin\left(\frac{4\pi}{9}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \left[\cos\left(\frac{4\pi}{9}\right) + i\sin\left(\frac{4\pi}{9}\right) \right]$$

$$z^{\frac{1}{3}} = 3cis\left(\frac{4\pi}{9}\right) = 3cis80^\circ$$

When $k = 1$, we have:

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}} \left[\cos\left(\frac{4\pi}{9} + \frac{2(1)\pi}{3}\right) + i \sin\left(\frac{4\pi}{9} + \frac{2(1)\pi}{3}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \left[\cos\left(\frac{4\pi}{9} + \frac{2\pi}{3}\right) + i \sin\left(\frac{4\pi}{9} + \frac{2\pi}{3}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \left[\cos\left(\frac{10\pi}{9}\right) + i \sin\left(\frac{10\pi}{9}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \operatorname{cis}\left(\frac{10\pi}{9}\right) = 3 \operatorname{cis}200^\circ$$

When $k = 2$, we have:

$$z^{\frac{1}{3}} = 27^{\frac{1}{3}} \left[\cos\left(\frac{4\pi}{9} + \frac{2(2)\pi}{3}\right) + i \sin\left(\frac{4\pi}{9} + \frac{2(2)\pi}{3}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \left[\cos\left(\frac{4\pi}{9} + \frac{4\pi}{3}\right) + i \sin\left(\frac{4\pi}{9} + \frac{4\pi}{3}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \left[\cos\left(\frac{16\pi}{9}\right) + i \sin\left(\frac{16\pi}{9}\right) \right]$$

$$z^{\frac{1}{3}} = 3 \operatorname{cis}\left(\frac{16\pi}{9}\right) = 3 \operatorname{cis}320^\circ$$

2. Evaluate $\left[\sqrt[5]{3} \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right) \right]^{10}$

Expanding the inner bracket:

$$\left[\sqrt[5]{3} \left(\frac{\sqrt{3}}{2} + \frac{i}{2} \right) \right]^{10} = \left[\frac{\sqrt[10]{3^7}}{2} + \frac{i\sqrt[5]{3}}{2} \right]^{10}$$

Since De Moivre's Theorem applies to complex numbers written in polar form, we must first write $(1 + i)$ in polar form r . Let us find r .

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{\left(\frac{\sqrt[10]{3^7}}{2}\right)^2 + \left(\frac{\sqrt[5]{3}}{2}\right)^2}$$

$$r = \sqrt{\left(\frac{\sqrt[5]{3^7}}{4}\right)^2 + \left(\frac{\sqrt[5]{3}}{4}\right)^2} \approx \sqrt[5]{3^2}$$

Then we find θ . Using the formula $\tan\theta = \frac{x}{y}$ gives $\tan\theta = \frac{\frac{\sqrt[10]{3^7}}{2}}{\frac{\sqrt[5]{3}}{2}} = \sqrt{3}$

$$\theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Use De Moivre's Theorem to evaluate the expression:

$$(a + ib)^n = r^n[\cos(n\theta) + i \sin(n\theta)]$$

$$\left(\frac{\sqrt[10]{3^7}}{2} + \frac{i\sqrt[5]{3}}{2}\right)^{10} = \left(\sqrt[5]{3^2}\right)^{10} \left[\cos\left(10 \cdot \frac{\pi}{3}\right) + i \sin\left(10 \cdot \frac{\pi}{3}\right)\right]$$

$$\left(\frac{\sqrt[10]{3^7}}{2} + \frac{i\sqrt[5]{3}}{2}\right)^{10} = 81 \left[\cos\left(\frac{10\pi}{3}\right) + i \sin\left(\frac{10\pi}{3}\right)\right]$$

$$\left(\frac{\sqrt[10]{3^7}}{2} + \frac{i\sqrt[5]{3}}{2}\right)^{10} = 81 \left[-\frac{1}{2} + \frac{-i\sqrt{3}}{2}\right]$$

$$\left(\frac{\sqrt[10]{3^7}}{2} + \frac{i\sqrt[5]{3}}{2}\right)^{10} = \left[-\frac{81}{2} + \frac{-i81\sqrt{3}}{2}\right]$$

3. Find $\frac{z_1}{z_2}$ in polar form

$$z_1 = 21cis(135^\circ)$$

$$z_2 = 3cis(75^\circ)$$

If $z_1 = r_1(\cos\theta_1 + \sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + \sin\theta_2)$, then the quotient of these numbers

is:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2), \text{ where } z_2 \neq 0$$

Therefore, the moduli are divided, while the angles are subtracted.

$$z_1 = 21cis(135^\circ) = 21(\cos 135^\circ + \sin 135^\circ)$$

$$z_2 = 3cis(75^\circ) = 3(\cos 75^\circ + \sin 75^\circ)$$

$$\frac{z_1}{z_2} = \frac{21}{3} [\cos(135^\circ - 75^\circ) + i\sin(135^\circ - 75^\circ)]$$

$$\frac{z_1}{z_2} = 7cis(135^\circ - 75^\circ)$$

$$\frac{z_1}{z_2} = 7cis(60^\circ)$$