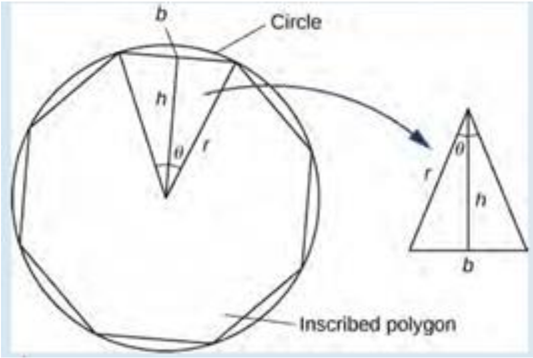
# Discussion Assignment 2

If an n-sided regular polygon is inscribed in a circle of radius *r*, as shown in the figure below, then n-isosceles triangles fill the circle.




Based on the statement and figure above answer the following:

1. **Express *h* and the base *b* of the isosceles triangle shown in terms of θ and *r*.**

$$h = rcos\frac{\theta}{2} \text{ and } b = 2rsin\frac{\theta}{2}$$

2. **Express the area of the isosceles triangle in terms of θ and *r*. Use trig identities as needed.**

$$area = \frac{1}{2}bh$$

$$= \frac{1}{2}.2rsin\frac{\theta}{2}.rcos\frac{\theta}{2}$$

$$= r^2sin\frac{\theta}{2}.cos\frac{\theta}{2}$$

Using double angle identity for sine and cosine,

$$sin2\theta = 2sin\theta cos\theta$$

$$sin\frac{\theta}{2}.cos\frac{\theta}{2} = \frac{1}{2}\sin\left(2.\frac{\theta}{2}\right) = \frac{1}{2}\sin(\theta)$$

Therefore, area of one triangle $= r^2.\frac{1}{2}\sin(\theta)$

Since n triangles are needed to form the circle, $\theta = \frac{2\pi}{n}$.

$$Area\ of\ a\ circle\ =\ total\ area\ of\ n\ triangles$$

$$= n\left[r^2.\frac{1}{2}\sin\left(\frac{2\pi}{n}\right)\right]$$

$$= \frac{nr^2}{2} . \sin\left(\frac{2\pi}{n}\right)$$

Multiply by $\frac{\frac{2\pi}{n}}{\frac{2\pi}{n}}$ as follows:

$$\frac{nr^2}{2} . \sin\left(\frac{2\pi}{n}\right) = \frac{\frac{2\pi}{n}}{\frac{2\pi}{n}} \times \frac{nr^2}{2} . \sin\left(\frac{2\pi}{n}\right)$$

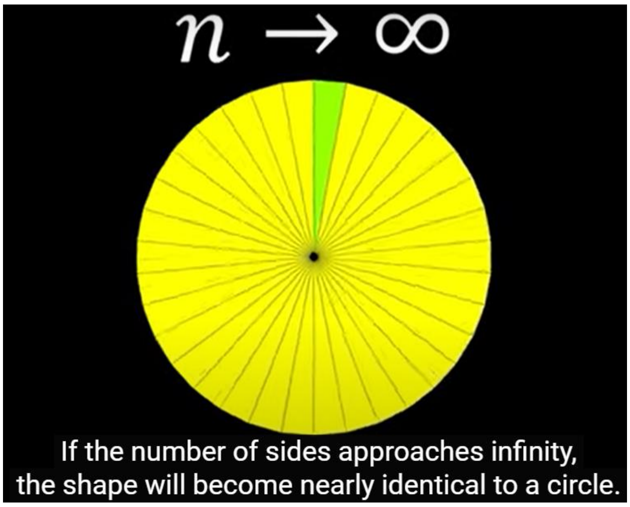
Cancelling out $n$ and 2 as follows:

$$= \left(\frac{\cancel{2}\pi}{\cancel{n}} \times \frac{\cancel{n}r^2}{\cancel{2}}\right) . \left[\frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}}\right]$$

$$= \pi r^2 \left[\frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}}\right]$$

3. **Describe what happens as *n* goes to infinity, (notice the polygon fills the circle, the angle θ goes to zero)**

As n approaches infinity, the number of triangles (n) form a perfect circle, as shown below.




Total area of the circle when applying limits becomes:

$$Area = \lim_{n\to\infty} \pi r^2 \left[\frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}}\right]$$

$$= \pi r^2 . \lim_{n\to\infty} \left[ \frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}} \right]$$

Since $\theta = \frac{2\pi}{n}$ as earlier mentioned, as $n \to \infty$, $\theta \to 0$. Therefore,

$$area\ of\ the\ circle = \pi r^2 . \lim_{\theta\to 0} \left(\frac{\sin\theta}{\theta}\right)$$

Given that $\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right) = 1$, $\pi r^2 . \lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\right) = \pi r^2 . 1$, hence the formula for calculating area of a circle would become $\pi r^2$.

References

Herman, E., & Strang, G. (2020). *Calculus volume 1*. Rice University. https://openstax.org/details/books/calculus-volume-1

Math with Alex. (Aug 9, 2023). *Deriving the area formula of a circle using limits.* https://www.youtube.com/watch?v=fnbSGxuHFJc