

Discussion Unit 4

1. Write an example of a function whose derivative can be found by using the following rules:

a) Product rule and special function differentiation rules.

$$f(x) = x^2 \ln(x)$$

Using product rule, let $h(x) = x^2$ and $g(x) = \ln(x)$

$$\begin{aligned} f'(x) &= h(x)g'(x) + g(x)h'(x) \\ &= x^2 \frac{d}{dx} \ln(x) + \ln(x) \frac{d}{dx} x^2 \\ &= \frac{x^2}{x} + \ln(x) \cdot 2x \\ &= x + \ln(x) \cdot 2x \\ &= x[1 + 2\ln(x)] \end{aligned}$$

b) Power rule, quotient rule, and chain rule.

$$f(x) = \frac{x^2}{\sqrt{x+2}}$$

Using quotient rule, let $g(x) = x^2$ and $h(x) = (x+2)^{1/2}$

$$\begin{aligned} f'(x) &= \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2} \\ &= \frac{\left[(x+2)^{1/2} \frac{d}{dx} x^2\right] - \left[x \frac{d}{dx} (x+2)^{1/2}\right]}{(x+2)^{1/2}]^2} \end{aligned}$$

Solving $h'(x)$ requires application of chain rule, while $g'(x)$ requires power rule.

For $h'(x) = \frac{d}{dx} (x+2)^{1/2}$, let $u = x+2$. Therefore, $\frac{d}{dx} (x+2)^{1/2} = \frac{d}{dx} u^{1/2}$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{du} u^{1/2} = \frac{1}{2\sqrt{u}}$$

$$\frac{d}{dx}(x+2) = 1$$

$$h'(x) = \left(\frac{1}{2\sqrt{u}}\right) \cdot (1) = \frac{1}{2\sqrt{u}} = \frac{1}{2\sqrt{x+2}}$$

$$= \frac{\left[2x\sqrt{x+2}\right] - \left[\left(\frac{x^2}{2\sqrt{x+2}}\right)\right]}{x+2}$$

$$= \frac{(2x\sqrt{x+2})(2\sqrt{x+2}) - x^2}{2\sqrt{x+2}} \\ x+2$$

$$= \frac{4x(x+2) - x^2}{2(x+2)(\sqrt{x+2})} = \frac{4x^2 + 8x - x^2}{2(x+2)(\sqrt{x+2})}$$

$$= \frac{3x^2 + 8x}{2(x+2)(\sqrt{x+2})}$$

c) Chain rule twice

$$f(x) = \ln(\sin^2(x))$$

Chain Rule Using Leibniz's Notation

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Let $u = \sin^2(x)$, therefore:

$$f(x) = \ln(u)$$

$$\frac{d}{du} \ln(u) = \frac{1}{u}$$

$\frac{d}{dx} \sin^2(x)$ will require applying chain rule for the second time. Let $v = \sin(x)$

$$\frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dx}$$

$$\frac{d}{dv} u^2 = 2u$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{du}{dx} = 2u \cdot \cos(x) = 2 \sin(x) \cos(x)$$

$$\frac{dy}{dx} = \frac{1}{u} \cdot 2 \sin(x) \cos(x)$$

$$= \frac{1}{\sin^2(x)} \cdot 2 \sin(x) \cos(x)$$

$$= \frac{2 \cos(x)}{\sin(x)}$$

$$= 2 \cot(x)$$

d) Implicit differentiation and special function differentiation rule

$$x + e^y = 2$$

$$x \frac{d}{dx} + \frac{d}{dx} e^y = \frac{d}{dx} 2$$

$$1 + e^y \frac{dy}{dx} = 0$$

$$e^y \frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = -\frac{1}{e^y} = -e^{-y}$$