Discussion Assignment 8

1. **How can De Moivre's theorem be described?**

De Moivre's Theorem states that, for a positive integer n, z^n is found by raising the modulus to the nth power and multiplying the argument by n (Abramson, 2021). It is the standard method used in modern mathematics. According to De Moivre's Theorem:

If $z = r(\cos\theta + i\sin\theta)$ is a complex number, then $z^n = r^n[\cos(n\theta) + i\sin(n\theta)] = r^n cis(n\theta)$, where n is a positive integer.

2. **What is the scope of this theorem?**

The De Moivre's Theorem can be applied when obtaining the relationship between the powers of trigonometric functions (e.g.- cos4x, sin2 x) and trigonometric functions of multiple angles (e.g.- cos 7x, sin 3x). in this case, the theorem helps to raise complex numbers to the highest powers, proving the famous trigonometric identities (Unacademy, 2022). Similarly, the theorem can also be used to obtain the roots of the polynomial equations and solve any rational number—positive numbers, negative numbers, and fractions (Unacademy, 2022).

3. **Give two examples for roots and two examples for powers.**

Example on Roots:

Example 1 - Cube Root of a Complex Number:

To find the cube root of a complex number, $z = 8cis\left(\frac{\pi}{4}\right)$, using De Moivre's theorem:

$$z^{\frac{1}{3}} = 8^{\frac{1}{3}} cis\left(\frac{\frac{\pi}{4}}{3}\right)$$

$$z^{\frac{1}{3}} = 2cis\left(\frac{\pi}{12}\right)$$

Example 2 - Fifth Root of a Complex Number:

To find the fifth root of $w = 32cis\left(\frac{\pi}{3}\right)$ using De Moivre's theorem:

$$w^{\frac{1}{5}} = 32^{\frac{1}{5}} cis\left(\frac{\frac{\pi}{3}}{5}\right)$$

$$w^{\frac{1}{5}} = 2cis\left(\frac{\pi}{15}\right)$$

$$w^{\frac{1}{5}} \approx 2cis(0.2513)$$

Example on Powers:

Example 1 - Squaring a Complex Number:

To compute the square of $a = 5cis\left(\frac{\pi}{6}\right)$ using De Moivre's theorem:

$$a^2 = 5^2 cis\left(2 \cdot \frac{\pi}{6}\right)$$

$$a^2 = 25cis\left(\frac{\pi}{3}\right)$$

Example 2 - Raising a Complex Number to the Fourth Power:

To calculate $b = 3cis\left(\frac{\pi}{4}\right)$ raised to the fourth power using De Moivre's theorem:

$$b^4 = 3^4 cis\left(4 \cdot \frac{\pi}{4}\right)$$

$$b^4 = 81 cis\left(\frac{\pi}{4}\right)$$

References

Abramson, J. (2021). Algebra and trigonometry (2nd ed.). OpenStax, TX: Rice University. https://openstax.org/details/books/algebra-and-trigonometry-2e

Unacademy. (2022, April 3). *Uses of de Moivre's theorem.* https://unacademy.com/content/jee/study-material/mathematics/uses-of-de-moivres-theorem/