Unit 4 Written Assignment Solutions:

1. Chains Inc. is in the business of making and selling chains. Let $c(t)$ be the number of miles of chain produced after t hours of production. Let $p(c)$ be the profit as a function of the number of miles of chain produced and let $q(t)$ be the profit as a function of the number of hours of production. Suppose the company can produce 3 miles of chain per hour and suppose their profit on the chains is $4000 per mile of chain. Find and interpret (use complete sentences) each of the following (include units), $c'(t)$, $p'(c)$, and $q'(t)$. How does $q'(t)$ relates to $p'(c)$ and $c'(t)$?

   Answer: $q(t) = p(c(t))$, that is, $q(t)$ is a composition function of p and c functions, and $q'(t) = p'(c(t))c'(t)$= $12,000. This means, the company Chains Inc. is making a profit of $12,000 for every hour of production when they produce 3 miles of chains per hour and make $4,000 profit for every mile of chain produced.

2. Use Desmos to graph the function $y^3 + yx^2 + x^2 - 3y^2 = 0$ and estimate the slope of the tangent line at $(-1, 1)$. Then find dy/dx using implicit differentiation and plug in $x = -1$ and $y = 1$. Compare and discuss the estimated slope with the slope you found analytically.

   Answer: Estimate will vary.
   And $\frac{dy}{dx} = \frac{-2x-2xy}{3y^2+x^2-6y}$, at $(-1, 1)$ $\frac{dy}{dx} = -2$
   Discussion will vary.

3. Let $f(x) = (3x^2 + 1)^2$. Find $f'(x)$ in 3 different ways by following the instructions below in parts a, b and c, show all intermediate steps for full credit:

a) Develop the identity f(x), then take the derivative
b) View $f(x)$ as $(3x^2 + 1)(3x^2 + 1)$ and use product rule to find $f'(x)$.
c) Apply the chain rule directly to the expression $f(x) = (3x^2 + 1)^2$.
d) Are your answers in parts a, b, c the same? Why or why not?

Answer:

a) $f(x) = (3x^2 + 1)^2 = 9x^4 + 6x^2 + 1$, and $f'(x) = 36x^3 + 12x$
b) $f(x) = (3x^2 + 1)(3x^2 + 1)$, and $f'(x) = (6x)(3x^2 + 1) + (3x^2 + 1)(6x) = 18x^3 + 6x + 18x^3 + 6x = 36x^3 + 12x$
c) $f(x) = (3x^2 + 1)^2$, and $f'(x) = 2(3x^2 + 1)(6x) = 12x(3x^2 + 1) = 36x^3 + 12x$
d) All the answers in parts a, b and c are the same, each one gave the correct derivative of the function.

4. Find dy/dx for the equation $3y^2 - \cos y = x^3$

   Answer: $\frac{dy}{dx} = \frac{3x^2}{6y+\sin y}$

5. Find the equation of the tangent line that passes through point $(1, 2)$ to the graph $8y^3 + x^2y - x = 3$. Show detailed work for full credit.

Answer: First find y' by implicit derivation, $y' = \frac{1-2xy}{x^2+24y^2}$, then plug in $x = 1$ and $y = 2$ to find the slope. The slope is $\frac{-3}{97}$ .Then use the equation, $y = m(x - x_1) + y_1$, where m is the slope and (x_1, y_1) is the point it is passing through. Plugging in the values and simplifying we get, $y = \frac{-3}{97}x + \frac{197}{97}$ is the equation of the tangent line.

6. Find dy/dx for the equation $xy^2 - x^2\,y = 2$

   Answer: $\frac{dy}{dx} = \frac{2xy-y^2}{2xy-x^2}$

7. Find $f'(x)$ for the function $f(x) = \sqrt[4]{-3x^4 - 2}$

   Answer: $f'(x) = \frac{1}{4}(-3x - 2)^{\frac{-3}{4}}(-12x^3)$ which can be simplified to
   $f'(x) = -\frac{3x^3}{(-3x^4-2)^{3/4}}$

8. Find $f'(x)$ for the function $f(x) = (5x^2 + 3)^4$

   Answer: $f'(x) = 40x(5x^2 + 3)^3$

9. Find $f'(x)$ for the function $f(x) = \sin^2(\cos(4x))$

   Answer: $y' = -8\,sin4x\;sin(cos(4x))\;cos(cos4x)$

10. Find $f'(x)$ for the following functions:
    a) $f(x) = \cos(\ln(4x^3))$ b) $f(x) = e^{(4x^3+5)^2}$

    Answer: a) $f'(x) = \frac{-3\sin(ln4x^3)}{x}$ b) $f'(x) = 24x^2(4x^3 + 5)e^{(4x^3+5)^2}$