

Discussion Assignment 7

Based on what you have studied in this unit, reflect and answer the following questions:

- **What are the main characteristics of the normal distribution?**
 1. Its graph is bell-shaped
 2. It has two parameters (two numerical descriptive measures): the mean (μ) and the standard deviation (σ)
 3. It has the z-score tells, which tells how many standard deviations the value x is above (to the right of) or below (to the left of) the mean, μ
- **Why is it advantageous to use the Z scores?**

The z-score can be used to compare data that are scaled differently.

For a standard normal distribution, about 68% of values falls within one standard deviation of the mean; about 95% falls within two standard deviations; and about 99.7% falls within three standard deviations. Suppose the distribution of math scores in the SAT follow a normal distribution with a mean $\mu=700$ and a standard deviation $\sigma=150$.

1. **Calculate the Z-score for an SAT Math score of 600. Interpret it in your own word.**

$$z = \frac{x - \mu}{\sigma} = \frac{600 - 700}{150} = -0.6667$$

Interpretation:

A Z-score of -0.6667 indicates that a SAT Math score of 600 is approximately 0.6667 standard deviations below the mean (average) SAT Math score, which is 700 in this case. In simple terms, this means that the score of 600 is below the average performance in SAT Math. The negative sign tells us that it's below the mean. The magnitude (0.6667) gives us a sense of how far below the mean it is in standard deviation units. In this context, being 0.6667 standard deviations below the mean suggests that the score is somewhat below the typical performance of test-takers, but it's not extremely low.

2. **Find the interval $[a, b]$ that captures 68% of the SAT Math score in this example.**

About 68% of the x values lie within one standard deviation of the mean. Therefore, about 68% of the x values lie between $-1\sigma = (-1)(150) = -150$ and $1\sigma = (1)(150) = 150$ of the mean 700. The values $700 - 150 = 550$ and $700 + 150 = 850$ are within one standard deviation from the mean 700. The z-scores are -1 and +1 for 550 and 850, respectively.

3. **Find the interval $[a, b]$ that captures 95% of the SAT Math score in this example.**

About 95% of the x values lie within two standard deviations of the mean. Therefore, about 95% of the x values lie between $-2\sigma = (-2)(150) = -300$ and $2\sigma = (2)(150) = 300$ of the mean 700. The values $700 - 300 = 400$ and $700 + 300 = 1000$ are within two standard deviations from the mean 700. The z-scores are -2 and $+2$ for 400 and 1000, respectively.

4. **Find the interval $[a, b]$ that captures 99.7% of the SAT Math score in this example.**

About 99.7% of the x values lie within three standard deviations of the mean. Therefore, about 99.7% of the x values lie between $-3\sigma = (-3)(150) = -450$ and $3\sigma = (3)(150) = 450$ of the mean 700. The values $700 - 450 = 250$ and $700 + 450 = 1150$ are within three standard deviations from the mean 700. The z-scores are -2 and $+2$ for 250 and 1150, respectively.