Discussion Unit 4

1. Write an example of a function whose derivative can be found by using the following rules:

a) Product rule and special function differentiation rules.

$$f(x) = x^2 \ln(x)$$

Using product rule, let $h(x) = x^2$ and $g(x) = \ln(x)$

$$f'(x) = h(x)g'(x) + g(x)h'(x)$$

$$= x^2 \frac{d}{dx}\ln(x) + \ln(x)\frac{d}{dx}x^2$$

$$= \frac{x^2}{x} + \ln(x).2x$$

$$= x + \ln(x).2x$$

$$= x[1 + 2\ln(x)]$$

b) Power rule, quotient rule, and chain rule.

$$f(x) = \frac{x^2}{\sqrt{x+2}}$$

Using quotient rule, let $g(x) = x^2$ and $h(x) = (x+2)^{1/2}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$= \frac{\left[(x+2)^{1/2}\frac{d}{dx}x^2\right] - \left[x\frac{d}{dx}(x+2)^{1/2}\right]}{[(x+2)^{1/2}]^2}$$

Solving $h'(x)$ requires application of chain rule, while $g'(x)$ requires power rule.

For $h'(x) = \frac{d}{dx}(x+2)^{1/2}$, let $u = x + 2$. Therefore, $\frac{d}{dx}(x+2)^{1/2} = \frac{d}{dx}u^{1/2}$.

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{du}u^{1/2} = \frac{1}{2\sqrt{u}}$$

$$\frac{d}{dx}(x+2) = 1$$

$$h'(x) = \left(\frac{1}{2\sqrt{u}}\right).(1) = \frac{1}{2\sqrt{u}} = \frac{1}{2\sqrt{x+2}}$$

$$= \frac{\left[2x\sqrt{x+2}\right] - \left[\left(\frac{x^2}{2\sqrt{x+2}}\right)\right]}{x+2}$$

$$= \frac{\frac{\left(2x\sqrt{x+2}\right)\left(2\sqrt{x+2}\right) - x^2}{2\sqrt{x+2}}}{x+2}$$

$$= \frac{4x(x+2) - x^2}{2(x+2)\left(\sqrt{x+2}\right)} = \frac{4x^2 + 8x - x^2}{2(x+2)\left(\sqrt{x+2}\right)}$$

$$= \frac{3x^2 + 8x}{2(x+2)(\sqrt{x+2})}$$

c) Chain rule twice

$$f(x) = \ln(sin^2(x))$$

Chain Rule Using Leibniz's Notation

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Let $u = sin^2(x)$, therefore:

$$f(x) = \ln(u)$$

$$\frac{d}{du}\ln(u) = \frac{1}{u}$$

$\frac{d}{dx}sin^2(x)$ will require applying chain rule for the second time. Let $v = \sin(x)$

$$\frac{du}{dx} = \frac{du}{dv}.\frac{dv}{dx}$$

$$\frac{d}{dv}u^2 = 2u$$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{du}{dx} = 2u.cos(x) = 2\sin(x)\cos(x)$$

$$\frac{dy}{dx} = \frac{1}{u}.2\sin(x)\cos(x)$$

$$= \frac{1}{sin^2(x)}.2\sin(x)\cos(x)$$

$$= \frac{1}{\cancel{\sin(x)}.\sin(x)}.2\cancel{\sin(x)}\cos(x)$$

$$= \frac{2\cos(x)}{\sin(x)}$$

$$= 2\cot(x)$$

d) Implicit differentiation and special function differentiation rule

$$x + e^y = 2$$

$$x\frac{d}{dx} + \frac{d}{dx}e^y = \frac{d}{dx}2$$

$$1 + e^y\frac{dy}{dx} = 0$$

$$e^y\frac{dy}{dx} = -1$$

$$\frac{dy}{dx} = -\frac{1}{e^y} = -e^{-y}$$