Learning Journal Unit 3

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MATH 1211: Calculus

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Learning Journal Unit 3

1. Using the limit definition of derivative, find $f'(3)$ for the function $f(x) = x^3 + 17x^2 - 12x + 2$.

$$f'(x) = \log_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{[(x+h)^3 + 17(x+h)^2 - 12(x+h) + 2] - [x^3 + 17x^2 - 12x + 2]}{h}$$

$$= \lim_{h\to 0} \frac{[(x^3 + 3x^2h + 3xh^2 + h^3) + 17(x^2 + 2xh + h^2) - 12(x+h) + 2] - [x^3 + 17x^2 - 12x + 2]}{h}$$

$$= \lim_{h\to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 17x^2 + 34xh + 17h^2 - 12x - 12h + 2 - x^3 - 17x^2 + 12x - 2}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h + 3xh^2 + h^3 + 34xh + 17h^2 - 12h}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2 + 34x + 17h - 12}{\cancel{h}}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 + 34x + 17h - 12$$

$$= 3x^2 + 3x(0) + (0)^2 + 34x + 17(0) - 12$$

$$= 3x^2 + 34x - 12$$

$$f'(3) = 3(3)^2 + 34(3) - 12$$

$$= 117$$

2. Find the slope of the secant line between the values 3 and 4 for the function $f(x) = x^4$.

$$slope = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(4) - f(3)}{4 - 3}$$

$$= \frac{(4)^4 - (3)^4}{4 - 3}$$

$$= \frac{256 - 81}{1}$$

$$= 175$$

3. Applying limit definition of derivative, find the slope of the tangent line $f(x) = x^4$ at $x = 3$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(3+h)^4 - (3)^4}{h}$$

$$= \lim_{h \to 0} \frac{f(3+h)^4 - (3)^4}{h}$$

$$= \lim_{h \to 0} \frac{81 + 108h + 54h^2 + 12h^3 + h^4 - 81}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(108 + 54h + 12h^2 + h^3)}{\cancel{h}}$$

$$= \lim_{h \to 0} 108 + 54h + 12h^2 + h^3$$

$$= 108 + 54(0) + 12(0)^2 + (0)^3$$

$$= 108$$

4. What do you understand from question numbers 2 and 3 about secant lines and tangent lines? Explain any difference or any similarity.

In question 2, the slope of the secant line between points 3 and 4 on $f(x) = x^4$ is 175, representing the average rate of change of the function over that interval. In question 3, the slope of the tangent line to $f(x) = x^4$ at $x = 3$ is found 108. Secant lines give an average rate of change between two points on a curve, while tangent lines provide the instantaneous rate of change (slope) at a specific point on the curve. Tangent lines show how the function behaves at a precise location, while secant lines demonstrate the average behavior over an interval.

5. **The total cost of producing x winter jackets in dollars is given by $\boldsymbol{C(x) = 100 + 10x + 0.2x^2}$ using the limit definition of the derivative, find average cost per jacket in [10,12]. Then find the average cost of making 1500 jackets.**

$$A(x) = C'(x) = \lim_{h\to 0} \frac{C(x+h) - C(x)}{h}$$

$$A(x) = \lim_{h\to 0} \frac{[100 + 10(x+h) + 0.2(x+h)^2] - [100 + 10x + 0.2x^2]}{h}$$

$$= \lim_{h\to 0} \frac{[100 + 10x + 10h + 0.2x^2 + 0.4xh + h^2] - [100 + 10x + 0.2x^2]}{h}$$

$$= \lim_{h\to 0} \frac{100 + 10x + 10h + 0.2x^2 + 0.4xh + h^2 - 100 - 10x - 0.2x^2}{h}$$

$$= \lim_{h\to 0} \frac{10h + 0.4xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(10 + 0.4x + h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 10 + 0.4x + h$$

$$= 10 + 0.4x + (0)$$

$$= 0.4x + 10$$

$$A(12) = 0.4(12) + 10 = \$14.8$$

$$A(10) = 0.4(10) + 10 = \$14$$

The average cost of making 1500 jackets

$$A(1500) = 0.4(1500) + 10$$

$$= 600 + 10$$

$$= \$610$$

6. Apply the derivative rules, find $f'(x)$ for $f(x) = (x + 2)(x^2 + 16x + 4)$

Using product rule, let $h(x) = x + 2$ and $g(x) = x^2 + 16x + 4$

$$f'(x) = h(x)g'(x) + g(x)h'(x)$$

$$= (x + 2)\frac{d}{dx}(x^2 + 16x + 4) + (x^2 + 16x + 4)\frac{d}{dx}(x + 2)$$

$$= (x + 2)(2x + 16) + (x^2 + 16x + 4)(1)$$

$$= 2x^2 + 16x + 4x + 32 + x^2 + 16x + 4$$

$$= 3x^2 + 36x + 36$$

7. Apply the derivatives rules, find $f'(x)$ for $f(x) = \frac{x^4-5x^3+2x^2-5}{3x^2}$.

Using quotient rule, let $g(x) = x^4 - 5x^3 + 2x^2 - 5$ and $h(x) = 3x^2$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$= \frac{\left[3x^2 \frac{d}{dx}(x^4 - 5x^3 + 2x^2 - 5)\right] - \left[(x^4 - 5x^3 + 2x^2 - 5)\frac{d}{dx}3x^2\right]}{[3x^2]^2}$$

$$= \frac{[3x^2(4x^3 - 15x^2 + 4x)] - [6x(x^4 - 5x^3 + 2x^2 - 5)]}{9x^4}$$

$$= \frac{(12x^5 - 45x^4 + 12x^3) - (6x^5 - 30x^4 + 12x^3 - 30x)}{9x^4}$$

$$= \frac{12x^5 - 45x^4 + 12x^3 - 6x^5 + 30x^4 - 12x^3 + 30x}{9x^4}$$

$$= \frac{6x^5 - 15x^4 + 30x}{9x^4}$$

$$= \frac{\cancel{3x}(2x^4 - 5x^3 + 10)}{\cancel{3x}(3x^3)}$$

$$= \frac{2x^4 - 5x^3 + 10}{3x^3}$$

8. Find the equation of the tangent line to the graph of $f(x) = (x + 2)(x^3 - 5x - 1)$ at $x = 1$.

$$gradient = f'(x)$$

Using product rule, let $h(x) = x + 2$ and $g(x) = x^3 - 5x - 1$

$$f'(x) = h(x)g'(x) + g(x)h'(x)$$

$$= (x + 2)\frac{d}{dx}(x^3 - 5x - 1) + (x^3 - 5x - 1)\frac{d}{dx}(x + 2)$$

$$= (x + 2)(3x^2 - 5) + (x^3 - 5x - 1)(1)$$

$$= 3x^3 - 5x + 6x^2 - 10 + x^3 - 5x - 1$$

$$= 4x^3 + 6x^2 - 10x - 11$$

$$f'(1) = 4(1)^3 + 6(1)^2 - 10(1) - 11$$

$$= -11$$

At $x = 1, f(x) = (1 + 2)((1)^3 - 5(1) - 1)$

$$= -15$$

The tangent touches the curve at point $(1, -15)$.

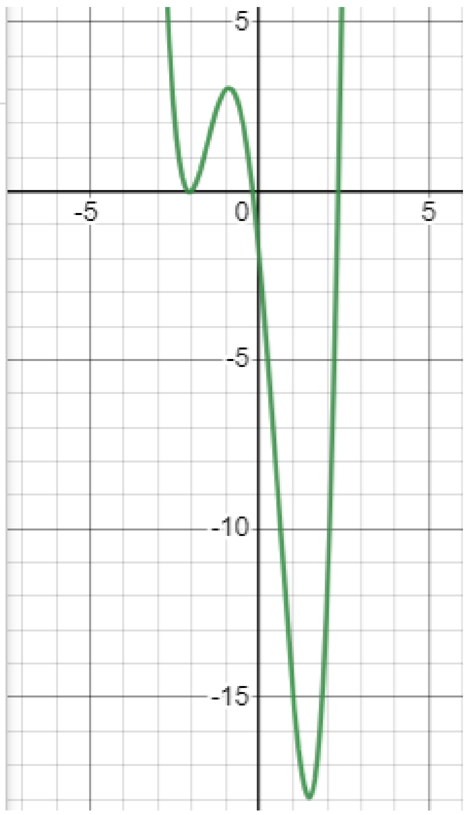
Using slope-point formula, equation of the tangent is:

$$y + 15 = -11(x - 1)$$

$$y = -11x - 26$$

9. Use Desmos graphing calculator to graph the function $f(x) = (x + 2)(x^3 - 5x - 1)$

$f(x) = (x + 2)(x^3 - 5x - 1)$



10. The concentration of antibiotic in the bloodstream t hours after being injected is given by the function C(t) is measured in milligrams per litre of blood. Find the rate of change of $\boldsymbol{C(t)} = \frac{t^2+4}{2t}$ .Determine the rate of change for $\boldsymbol{t} = 2$ hours.

Rate of change of $C(t) = C'(t)$

Using quotient rule, let $g(t) = t^2 + 4$ and $h(t) = 2t$

$$C'(t) = \frac{h(x)g'(t) - g(t)h'(t)}{[2t]^2}$$

$$= \frac{\left[(2t)\frac{d}{dt}(t^2 + 4) - (t^2 + 4)\frac{d}{dt}(2t)\right]}{[h(t)]^2}$$

$$= \frac{[(2t)(2t) - (t^2 + 4)(2)]}{4t^2}$$

$$= \frac{[4t^2 - 2t^2 - 8]}{4t^2}$$

$$= \frac{2t^2 - 8}{4t^2}$$

$$= \frac{t^2 - 4}{2t^2}$$

The rate of change for $t = 2$ hours.

$$C'(2) = \frac{(2)^2 - 4}{2(2)^2} = 0$$