Learning Journal Unit 4

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MATH 1211: Calculus

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Answer the following questions:

1. Explain each of the following questions:

(i) When is the chain rule of the derivative used?

We use chain rule when differentiating a composite function, where the derivative of the outer function is evaluated at the inner function times the derivative of the inner function (Herman & Strang, 2020). For instance, the expression $\ln(cos(x))$ can be considered a composite function, which is composed of two nested functions: the inner function $cos(x)$ and the outer function $\ln(x)$. The derivative of $\ln(cos(x))$ can be determined by applying chain rule using Leibniz's Notation $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$, which is a formula used to evaluate derivative of the outer function at the inner function times the derivative of the inner function.

(ii) What is the underlying algebraic principle behind the chain rule used to find the derivative. Give an example. (Ensure that you provide self-made examples and explanations, do not copy from the available resources).

Chain rule is based on the algebraic principle of composite functions, which is formed by applying one function to the output of another function. In the example given in part (a), $\ln(cos(x))$ is a composite function because it involves applying the function ln(x) to the result obtained by evaluating $cos(x)$. By applying chain rule using Leibniz's Notation $\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$, as stated in part (a), we can let $u = cos(x)$ to obtain $\ln(u)$. The chain rule will therefore be applied as follows:

Step 1: Differentiating the outer function: $\frac{d}{du}\ln(u) = \frac{1}{u}$

Step 2: Differentiating the inner function: $\frac{d}{dx}\cos(x) = -sinx$

Step 3: Finding product of differentiation of the outer and inner functions obtained in steps 1 and 2 respectively:

$$\frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$$

$$= \frac{1}{u} . -sinx$$

$$= -\frac{\sin(x)}{\cos(x)} = -tan\ (x)$$

2. Find an approximated equation of a tangent line to the folium curve $\boldsymbol{7x^3 + 5y^3 - 2x^2y + 5xy = 0}$ at the point $\boldsymbol{(-0.5, -0.5)}$ using implicit differentiation. Please show the step-by-step working of the problem.

Applying implicit differentiation:

$$\frac{d}{dx}7x^3 + \frac{d}{dx}5y^3 - \frac{d}{dx}2x^2y + \frac{d}{dx}5xy = \frac{d}{dx}0$$

$$21x^2 + 15y^2\frac{dy}{dx} - (4xy + 2x^2\frac{dy}{dx}) + (5y + 5x\frac{dy}{dx}) = 0$$

$$(15y^2 - 2x^2 + 5x)\frac{dy}{dx} = 4xy - 21x^2 - 5y$$

$$\frac{dy}{dx} = \frac{4xy - 21x^2 - 5y}{15y^2 - 2x^2 + 5x}$$

$$Slope\ of\ the\ tangent = f'(-0.5, -0.5) = \frac{4(-0.5).(-0.5) - 21(-0.5)^2 - 5(-0.5)}{15(-0.5)^2 - 2(-0.5)^2 + 5(-0.5)}$$

$$= -5$$

Equation of the tangent line:

$$y + 0.5 = -5(x + 0.5)$$

$$\boldsymbol{y = -0.5x - 3}$$

References

Herman, E., & Strang, G. (2020). *Calculus volume 1*. Rice University. https://openstax.org/details/books/calculus-volume-1