Discussion Assignment 5

It helps to convert a very complex differentiation problem into a simpler one (Herman & Strang, 2020). For instance, to differentiate $f(x) = (x+1)^{\sqrt[3]{x+1}}$, expressing it as $\ln f(x) = \sqrt[3]{x+1}.\, ln(x+1)$ would make it simpler to solve because the variable in the exponent has been removed. Although $f(x) = (x+1)^{\sqrt[3]{x+1}}$ is a composite function, applying chain rule when differentiating would be proven impossible because the exponent is a variable. It would have worked if the exponent is a constant.

Moreover, logarithmic differentiation is preferable when working with a function, which do not have a differentiation rule (UT Calculus, 2023). For instance, $f(x) = (x+1)^x$ is neither a power function (because there is an x in the exponent) nor an exponential function (because there is an x in the base). According to UT Calculus (2023), logarithmic differentiation works beautifully when differentiating a function of the form $(f(x))^{g(x)}$ because once we take logs, we can pull the power down and use the product rule.

Finally, one may prefer logarithmic differentiation in situations where it is easier to differentiate the logarithm of a function than to differentiate the function itself (Wells, 2012). For example, when having a function such as $f(x) = g(x)^{h(x)}$, where one function is an exponent of the other. Taking the logarithm of the function reduces the function to the product the logarithms of the individual expressions, $lnf(x) = h(x) \ln g(x)$). This allows us to apply product rule of differentiation, which exists and is easier to perform, rather than to differentiate the function itself.

In conclusion, one might prefer to use logarithmic differentiation for finding derivatives of functions that feature another function as an exponent for different reasons. First, it helps convert a very complex differentiation problem into a simpler one. Second, logarithmic

differentiation is preferable when working with a function that does not have a differentiation rule. Finally, one may opt for logarithmic differentiation in situations where it is easier to differentiate the logarithm of a function than to differentiate the function itself. It probably goes without saying that familiarity with product rule of the logarithmic laws and its application is a prerequisite for effectively executing logarithmic differentiation.

References

Wells, C. J. (2012, 6). *Logarithmic differentiation*. TechnologyUK. https://www.technologyuk.net/mathematics/differential-calculus/logarithmic-differentiation.shtml

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