Learning Journal Unit 2

Godfrey Ouma

University of the People

MATH 1211: Calculus

Rakesh Das

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1. Use the limit laws to solve the problem below.

$$\lim_{x\to 3}\frac{x^4-x^2+3}{x^2+2}$$

Solving numerator:

$$\lim_{x\to 3}(x^4-x^2+3)=\lim_{x\to 3}x^4-\lim_{x\to 3}x^2+\lim_{x\to 3}3=81-9+3=75$$

Solving denominator:

$$\lim_{x\to 3}(x^2+2)=\lim_{x\to 3}x^2+\lim_{x\to 3}2=9+2=11$$

Recombining the numerator and denominator:

$$\lim_{x\to 3}\frac{x^4-x^2+3}{x^2+2}=\frac{75}{11}$$

2. Explain the continuity of a function at any point. Explain the procedure to check continuity using a simple example.

Using the steps below, I will be checking if the function $f(x)=\begin{cases}2x+1\ if\ x<1\\ 2\ if\ x=1\\ -x+4\ if\ x>1\end{cases}$ is continuous at $x=1$.

Step 1: Check to see if f(a) is defined. If f(a) is undefined, we need go no further. The function is not continuous at a. If f(a) is defined, continue to step 2.

$$a=1$$

$f(1)=2(1)+1=3$, therefore, $f(a)$ exists. Now I will move to step 2.

Step 2: Compute $\lim_{x\to a}f(x)$. In some cases, we may need to do this by first computing $\lim_{x\to a^-}f(x)$ and $\lim_{x\to a^+}f(x)$. If $\lim_{x\to a}f(x)$ does not exist (that is, it is not a real number), then the function is not continuous at a and the problem is solved. If $\lim_{x \to a} f(x)$ exists, then continue to step 3.

$\lim_{x \to a^-} f(x) = 2(1) + 1 = 3$ and $\lim_{x \to a^+} f(x) = -(1) + 4 = 3$

Since $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$, $\lim_{x \to a} f(x)$ exists. Therefore, I will move to step 3.

Step 3: Compare f(a) and $\lim_{x \to a} f(x)$. If $\lim_{x \to a} f(x) \neq f(a)$, then the function is not continuous at a. If $\lim_{x \to a} f(x) = f(a)$, then the function is continuous at a.

$\lim_{x \to a} f(x) = 3$ and $f(a) = 3$

Therefore, $\lim_{x \to a} f(x) = f(a)$

Since all three of the conditions in the definition of continuity are satisfied, we can conclude that f(x) is continuous at x = 1.

3. A rock is dropped from a height of 16 ft. It is determined that its height (in feet) above ground *t* seconds later (for $0 \leq t \leq 3$) is given by $s(t) = -2t^2 + 16$. Find the average velocity of the rock over [0.2,0.21] time interval.

The average velocity is given by the formula:

$$average\ speed = \frac{change\ in\ position}{change\ in\ time}$$

$$= \frac{s(0.21)-s(0.2)}{0.2-0.21}$$

$$= \frac{\{-2(0.21)^2 + 16\}-\{-2(0.2)^2 + 16\}}{0.21-0.20}$$

$$= \frac{15.9118-15.92}{0.01}$$

$$= \frac{-0.0082}{0.01}$$

$= 0.82$ feet/second.

4. Evaluate each of the following limits. Identify any vertical asymptotes of the function $f(x) = \frac{1}{(x-4)^2(x+3)}$

(i) $\lim_{x\to 4^+} f(x) = \lim_{x\to 4^+} \frac{1}{(x-4)^2(x+3)} = \frac{1}{(4-4)^2(x+3)} = \frac{1}{0(x+3)} = \frac{1}{0} = +\infty$

(ii) $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} \frac{1}{(x-4)^2(x+3)} = \frac{1}{(4-4)^2(x+3)} = \frac{1}{0(x+3)} = \frac{1}{0} = -\infty$

(iii) $\lim_{x\to 4} f(x)$ does not exist because $\lim_{x\to 4^+} f(x)$ and $\lim_{x\to 4^-} f(x)$ does not approach the same value.

The vertical asymptote will occur at a point where the denominator equals zero, which can be determined as follows:

$$(x-4)^2(x+3) = 0$$

$= (x-4)^2$ or $(x+3) = 0$

Therefore, the vertical asymptotes will be $x = 4$ and $x = -3$

5. Evaluate the trigonometric limit $\lim_{\theta\to 0} \frac{2sin^2\theta}{1-cos\theta}$

$$\lim_{\theta\to 0} \frac{2sin^2\theta}{1-cos\theta} = \lim_{\theta\to 0} \frac{2\cancel{(1-cos\theta)}(1+cos\theta)}{\cancel{1-cos\theta}}$$

$$= 2\lim_{\theta\to 0}(1+cos\theta)$$

$$= 2\left[\lim_{\theta\to 0} 1 + \lim_{\theta\to 0}(cos\theta)\right]$$

$$= 2[1+1] = 4$$

6. Find the value of *k* that makes the following function is continuous over the given interval.

$$f(x) = \begin{cases} 5x+4, & x \le k \\ 3x-4, & k < x < 7 \end{cases}$$

$$\lim_{x\to k^-} f(x) = 5k+4$$

$$\lim_{x \to k^+} f(x) = 3k - 4$$

For $\lim_{x \to k} f(x)$ to be continuous at $x = k$, $\lim_{x \to k^-} f(x) = \lim_{x \to k^+} f(x)$.

$$5k + 4 = 3k - 4$$

$$k = -4$$

Therefore, $k = -4$ is the value of k that makes the function continuous over the interval

&$k < x < 7$.

7. Determine at the point 5, if the following function is discontinuous. Classify any discontinuity as jump, removable, infinite, or other.

$$\boldsymbol{f(x) = \frac{|x - 5|}{x - 5}}$$

$$\lim_{x \to 5^-} \frac{-(x - 5)}{x - 5} = -1$$

$$\lim_{x \to 5^+} \frac{(x - 5)}{x - 5} = 1$$

Since $\lim_{x \to 5^-} f(x) \neq \lim_{x \to 5^+} f(x)$, we can conclude that there is a discontinuity at $x = 5$, which is a jump discontinuity since the left-hand limit is not equal to the right-hand limit.