Discussion Assignment 5

The population of a culture of bacteria is modelled by the logistic equation

$$p(t) = \frac{14250}{1 + 29e^{-0.62t}}$$

1. **To the nearest tenth, how many days will it take the culture to reach 75% of its carrying capacity?**

To find how many days it will take for the culture to reach 75% of its carrying capacity using the logistic equation, we set p(t) equal to 75% of the carrying capacity, which is 0.75 times the carrying capacity:

$$\frac{75}{100} \times 14250 = \frac{14250}{1 + 29e^{-0.62t}}$$

To solve for 't':

$\frac{75}{100} \times 14250 = \frac{14250}{1+29e^{-0.62t}}$ (divide both sides of the equation by 14250).

$\frac{75}{100} = \frac{1}{1+29e^{-0.62t}}$

By cross multiplication we obtain

$(1 \times 100) = 75(1 + 29e^{-0.62t})$

$100 = 75 + 2175e^{-0.62t}$

$25 = 2175e^{-0.62t}$

$\frac{25}{2175} = e^{-0.62t}$

Taking the natural logarithm (ln) of both sides:

$In\left(\frac{25}{2175}\right) = In\ e^{-0.62t}$

$-0.62t = In\left(\frac{25}{2175}\right)$

$t = \frac{In\left(\frac{25}{2175}\right)}{(-0.62)} \approx 35.2$ days

Therefore, it will take almost 35.2 days for the culture to reach 75% of its carrying capacity.

2. **What is the carrying capacity?**

Carrying capacity (K) is the limiting population that the environment can sustain indefinitely. In this case, the carrying capacity is 14250.

3. **What is the initial population for the model?**

The initial population (P_o) for the model is the value of p(t) at t = 0:

$$p(0) = \frac{14250}{1+29e^{-0.62t}}$$

$$p(0) = \frac{14250}{1+29} = \frac{14250}{30} = 475$$

Therefore, the initial population for the model is 475.

4. **Why a model like $p(t) = P_o e^{kt}$, where P_o is the initial population, would not be plausible?**

The model $\boldsymbol{p(t) = P_o e^{kt}}$ is not plausible in this case because it represents exponential growth, which does not account for environmental limitations. Bacterial growth, like many biological populations, is constrained by factors such as available resources, competition, and the carrying capacity of the environment. The logistic model is more appropriate for such situations because it considers these limiting factors and approaches a stable equilibrium (the carrying capacity).

5. **What are the virtues of the logistic model?**

The virtues of the logistic model lie in its reflection of the real-world dynamics of population growth. In a world with finite resources and environmental limitations, this model provides a realistic representation. The concept of a carrying capacity, a fundamental idea in population ecology, is seamlessly integrated into the logistic model. The resulting S-shaped curve is a mirror to the growth patterns seen in numerous natural populations, further grounding this model in reality. Additionally, the logistic model predicts the stability of populations as they approach their carrying capacity, an observation consistently witnessed in actual ecosystems. Perhaps its most potent virtue is its predictive power, offering a valuable tool for forecasting population growth trends. It equips us to make informed decisions about resource management and conservation efforts, allowing us to better navigate the delicate balance between the needs of a population and the limitations of its environment.

6. **Graph**

$$y = \frac{14250}{1+29e^{-0.62x}} \{0 < x < 15\} \{0 < y < 15000\}$$

$$y = 14300 \{0 < x < 15\}$$