Learning Journal Unit 5

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MATH 1280: Introduction to Statistics

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Solve the following defective rate problem using the concepts learned about the geometric distribution.

A machine that produces a special type of transistor (a component of computers) has a 2% defective rate. The production is considered a random process where each transistor is independent of the others.

1. **What is the probability that the 10th transistor produced is the first with a defect?**

To calculate the probability that the 10th transistor produced is the first with a defect, we can use the geometric distribution. The geometric distribution is used to model the number of trials needed to get the first success in a repeated Bernoulli trials experiment, where each trial has a constant probability of success.

In this case, the success is getting a defective transistor. The probability of success on each trial is 2%. So, the probability that the 10th transistor produced is the first with a defect is:

$$P(X = 10) = (1 - 0.02)^9 \times 0.02 = 0.01667$$

2. **What is the probability that the machine produces no defective transistors in a batch of 100?**

To calculate the probability that the machine produces no defective transistors in a batch of 100, we can use the binomial distribution. The binomial distribution is used to model the number of successes in a fixed number of Bernoulli trials.

In this case, the success is getting a defective transistor. The probability of success on each trial is 2%. So, the probability that the machine produces no defective transistors in a batch of 100 is:

$$P(X = 0) = (1 - 0.02)^{100} = 0.1326$$

3. **On average, how many transistors would you expect to be produced until the first with a defect? What is the standard deviation?**

The mean and standard deviation of the geometric distribution are given by:

$$\mu = \frac{1}{p}$$

$$\sigma = \sqrt{\frac{1}{p}\left(\frac{1}{p} - 1\right)}$$

where p is the probability of success on each trial.

In this case, the probability of success on each trial is 2%. So, the mean and standard deviation of the geometric distribution are:

$$\mu = \frac{1}{0.02} = 50$$

$$\sigma = \sqrt{\frac{1}{0.02}\left(\frac{1}{0.02} - 1\right)} = 49.5$$

Therefore, on average, we would expect to produce 50 transistors before the first with a defect. The standard deviation is 49.5, which means that about 68% of the time, the first defective transistor will be produced between 43 and 57 transistors.

4. **Another machine that also produces transistors has a 5% defective rate where each transistor is produced independent of the others. On average how many transistors would you expect to be produced with this machine before the first with a defect? What is the standard deviation?**

Using the same formulas as in Question 3, we can calculate the mean and standard deviation of the geometric distribution for the second machine, which has a defective rate of 5%:

$$\mu = \frac{1}{0.05} = 20$$

$$\sigma = \sqrt{\frac{1}{0.05}\left(\frac{1}{0.05} - 1\right)} = 19.5$$

Therefore, on average, we would expect to produce 20 transistors with the second machine before the first with a defect. The standard deviation is 19.5, which means that about 68% of the time, the first defective transistor will be produced between 16 and 24 transistors.

5. **Based on your answers to parts (c) and (d), how does increasing the probability of an event affect the mean and standard deviation of the wait time until success?**

Increasing the probability of an event decreases the mean and standard deviation of the wait time until success. This can be seen by comparing the mean and standard deviation for the two machines in Questions 3 and 4.

The machine with the higher defective rate (5%) has a lower mean and standard deviation than the machine with the lower defective rate (2%). This is because the more likely it is for an event to happen on each trial, the less time we expect to wait for the first success.