Written Assignment 5

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MATH 1211: Calculus

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December 20, 2023

Written Assignment Unit 5

1. Find the derivative for the function $\boldsymbol{f(x) = 2e^x - 8^x}$.

Solution:

$$f'^{(x)} = \frac{d}{dx}2e^x - \frac{d}{dx}8^x$$

$$= 2e^x(1) - 8^x In(8)$$

$$= 2e^x - 8^x In(8)$$

$$\text{But } In(8) = In(2)^3 = 3In(2)$$

$$\boldsymbol{= 2e^x - 3In(2).(8^x)}$$

2. Find the derivative for the function $\mathbf{f(z) = z^5 - e^z Inz}$

Solution:

$$f'(z) = \frac{d}{dz}z^5 - \frac{d}{dz}e^z Inz$$

$$= 5z^4 - \left(e^z Inz + \frac{e^z}{z}\right)$$

$$\boldsymbol{= 5z^4 - e^z In(z) - \frac{e^z}{z}}$$

3. Find the tangent line to $\boldsymbol{f(x) = 7^x + 4e^x}$ at $\boldsymbol{x = 0}$.

Solution:

$$f'(x) = \frac{d}{dx}7^x + 4\frac{d}{dx}e^x$$

$$= 7^x . \ln(7) + 4e^x$$

Gradient of the tangent:

$$f'(0) = 7^{(0)} . \ln(7) + 4e^{(0)}$$

$$= 1.\ln(7) + 4(1)$$

$$\boldsymbol{\approx 5.95}$$

The point when tangent line touches the curve:

$$f(0) = 7^0 + 4e^0 = 5$$

The point is (0,5)

Using point-slope form to determine equation of the tangent line:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 5.95(x - 0)$$

$$\boldsymbol{y = 5.95x + 5}$$

4. Determine if $\boldsymbol{G(z) = (z - 6)ln(z)}$ is increasing or decreasing at the following points.

Solution:

$$G'(z) = \frac{d}{dz}[(z - 6).ln(z)]$$

$$= \frac{(z - 6)}{z} + In(z).(1)$$

$$= \frac{(z - 6)}{z} + ln(z)$$

(a) $z = 1$

$$G'(1) = \frac{(1 - 6)}{1} + ln(1)$$

$$= -5$$

Since $\boldsymbol{G'(1) = -5}$ (negative), $\boldsymbol{G(z)}$ is decreasing at $\boldsymbol{z = 1}$.

(b) $z = 5$

$$G'(5) = \frac{(5 - 6)}{5} + ln(5)$$

$$\approx 1.41$$

Since $\boldsymbol{G'(5) =\approx 1.41}$ (positive), $\boldsymbol{G(z)}$ is increasing at $\boldsymbol{z = 5}$.

(c) $z = 20$

$$G'(20) = \frac{(20 - 6)}{20} + ln(20)$$

$$\approx 3.70$$

Since $G'(20) \approx 3.70$ (positive), $G(z)$ is increasing at $z = 20$.

5. Find the derivative for the function $f(x) = (x+1)^x$

Solution:

Taking natural logarithm on both sides:

$$\ln f(x) = xln(x+1)$$

Differentiating both sides:

$$\frac{d}{dx}\ln f(x) = \frac{d}{dx}xln(x+1)$$

$$\frac{1}{f(x)}\frac{dy}{dx} = \frac{1}{x+1}.x + ln(x+1).(1)$$

Multiplying both sides by $f(x)$:

$$\cancel{f(x)} \times \frac{1}{\cancel{f(x)}}\frac{dy}{dx} = f(x).\left[\frac{x}{x+1} + \ln(x+1)\right]$$

$$\boldsymbol{\frac{dy}{dx} = (x+1)^x.\left[\frac{x}{x+1} + \ln(x+1)\right]}$$

6. Find the derivative for the function $f(x) = (x)^{x+1}$

Solution:

Taking natural logarithm on both sides:

$$\ln f(x) = (x+1).\ln(x)$$

Differentiating both sides:

$$\frac{d}{dx}\ln f(x) = \frac{d}{dx}(x+1).\ln(x)$$

$$\frac{1}{f(x)}\frac{dy}{dx} = \frac{1}{x}.(x+1) + ln(x).(1)$$

Multiplying both sides by $f(x)$:

$$\cancel{f(x)} \times \frac{1}{\cancel{f(x)}}\frac{dy}{dx} = f(x).\left[\frac{x+1}{x} + \ln(x)\right]$$

$$\boldsymbol{\frac{dy}{dx} = (x)^{x+1}.\left[\frac{x+1}{x} + \ln(x)\right]}$$

7. Find the derivative for the function $\boldsymbol{f(x) = \left(\sqrt{x}\right)^{x}}$

Solution:

Taking natural logarithm on both sides:

$$\ln f(x) = \ln\,(x)^{\frac{1}{2}x}$$

$$\ln f(x) = \frac{1}{2}x.\ln\,(\mathrm{x})$$

Differentiating both sides:

$$\frac{d}{dx}\ln f(x) = \frac{d}{dx}\left(\frac{1}{2}x.\ln\,(\mathrm{x})\right)$$

$$\frac{1}{f(x)}\frac{dy}{dx} = \frac{1}{\cancel{x}}.\left(\frac{1}{2}\cancel{x}\right) + ln(x).\left(\frac{1}{2}\right)$$

Multiplying both sides by $f(x)$:

$$\cancel{f(x)} \times \frac{1}{\cancel{f(x)}}\frac{dy}{dx} = f(x).\left[\frac{1}{2} + \frac{\ln\,(x)}{2}\right]$$

$$\boldsymbol{\frac{dy}{dx} = \left(\sqrt{x}\right)^{x}.\left[\frac{1}{2} + \frac{\ln\,(x)}{2}\right]}$$

8. Find $\boldsymbol{\frac{dy}{dx}}$ for $\boldsymbol{\left(\sqrt{3x^2+1}\right)(3x^4+1)^3}$

Solution:

$$\frac{dy}{dx} = \left(\sqrt{3x^2+1}\right)\frac{d}{dx}(3x^4+1)^3 + (3x^4+1)^3\frac{d}{dx}\left(\sqrt{3x^2+1}\right)$$

$$= \left(\sqrt{3x^2+1}\right).3(3x^4+1)^2.(12x^3) + (3x^4+1)^3.\frac{1}{2\sqrt{3x^2+1}}.6x$$

$$= \left(\sqrt{3x^2+1}\right).36x^3(3x^4+1)^2 + .\frac{\cancel{6}x.(3x^4+1)^3}{\cancel{2}\sqrt{3x^2+1}}$$

$$\boldsymbol{= \left(\sqrt{3x^2+1}\right).36x^3(3x^4+1)^2 + \frac{3x.(3x^4+1)^3}{\sqrt{3x^2+1}}}$$

9. Find $\frac{dy}{dx}$ for $y = 3x^{3x}$

Solution:

Taking natural logarithm on both sides:

$$\ln f(x) = 9x\ln(x)$$

Differentiating both sides:

$$\frac{d}{dx}\ln f(x) = \frac{d}{dx}(9x\ln(x))$$

$$\frac{1}{f(x)}\frac{dy}{dx} = \frac{1}{\cancel{x}}.(9\cancel{x}) + ln(x).9$$

$$\frac{1}{f(x)}\frac{dy}{dx} = 9 + ln(x).9$$

Multiplying both sides by $f(x)$:

$$\cancel{f(x)} \times \frac{1}{\cancel{f(x)}}\frac{dy}{dx} = f(x).[9 + ln(x).9]$$

$$\boldsymbol{\frac{dy}{dx} = 9x^{3x}.[ln(x) + 1]}$$