Mathematics Assignment Unit 2

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MATH 1302 - Discrete Mathematics

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1. We have a function $f: Z \to Z$ defined as $f(n) = \begin{Bmatrix} n+3 & if\ n\ is & odd \\ n-5 & if\ n\ is & even \end{Bmatrix}$. Find whether the function is injective and surjective. What would be the inverse of this function?

Writing the function into two-line notation:

$$f(n) = \begin{Bmatrix} 1 & 2 & 3 & 4 \\ 4 & -3 & 6 & -1 \end{Bmatrix}$$

The function can be considered as injective because every element of the codomain is the image of at most one element from the domain. On the same note, the function is injective because there are no repeated elements from the codomain in the range. In addition, the function can also be considered surjective because every element of the codomain is the image of at least one element from the domain, and that there are no missing elements from the codomain in the range. Since the function is both injective and surjective, we can therefore conclude that it is bijective.

2. Consider three sets A, B, and C each with three elements, and define a function 'f' from set A to B and a function 'g' from set B to C with the mapping of the elements you specify. After defining 'f' and 'g', discuss the composition of functions fog and gof on the sets A, B, and C that you have defined. Can both fog and gof be defined for the sets you have considered? Are they equal? Provide proof to support your answer.

Let's define the sets A, B, and C as follows: Set A = {1, 2, 3}, Set B = {3, 5, 7}, and Set C = {4, 6, 8}.

Defining the functions $f: A \ \to \ B$ and $g: B \ \to \ C$:

- Function $f: A \rightarrow B$ maps elements from set A to set B as follows: f(1) = 3, f(2) = 5, and f(3) = 7.
- Function $g: B \rightarrow C$ maps elements from set B to set C as follows: g(3) = 4, g(5) = 6, and g(7) = 8.

For the composition of fog: $A \rightarrow C$: fog(x) = f(g(x)), which implies that:

- fog(3) = f(g(3)) = f(4) = 4, undefined (since 4 is not in set A)
- fog(5) = f(g(5)) = f(6) = 6, undefined (since 6 is not in set A)
- fog(7) = f(g(7)) = f(8) = 8, undefined (since 8 is not in set A)

Therefore, fog is undefine since each codomain in set C is not in set A.

For the composition of gof: $C \rightarrow A$: gof(x) = g(f(x)), which implies that:

- gof(1) = g(f(1)) = g(3) = 4
- gof(2) = g(f(2)) = g(5) = 6
- gof(3) = g(f(3)) = g(7) = 8

Therefore, $gof = \{1 \rightarrow 4, 2 \rightarrow 6, 3 \rightarrow 8\}$

From the composition of functions above, it is evident that only gof can be defined for the given sets A, B, and C, while fog was undefined due to missing element in set A.

3. There are 5 students; Ani, Leon, Linh, Liam, and Abdul with scores in computer science as 75, 60, 85, 95, and 60 respectively. They are graded by the University as B, C, B+, A, and C respectively. Define the mappings from students to marks and marks to grades explicitly. What are the domains and ranges of the functions? Can you make a

composite function out of this? If yes, make the composite function and find if the composition of functions is commutative. Explain the reason.

Mapping:

Students to Marks mapping can be represented as a function $f: Students \rightarrow Marks$, where, Students = {Ani, Leon, Linh, Liam, Abdul} becomes the domain, while Marks = {75, 60, 85, 95, 60} becomes the range. Therefore, the mapping will be as follows:

- f(Ani) = 75
- f(Leon) = 60
- f(Linh) = 85
- f(Liam) = 95
- f(Abdul) = 60

Marks to Grades mapping can be represented as a function $g: Marks \rightarrow Grades$, where Marks = {75, 60, 85, 95, 60} becomes the domain, while Grades = {B, C, B+, A, C} becomes the range. Therefore, the mapping (based on the given example) will as follows:

- g(75) = B
- g(60) = C
- g(85) = B+
- g(95) = A

Composite Function:

A composite function $\boldsymbol{h}: \boldsymbol{Students} \rightarrow \boldsymbol{Grades}$ can be formed by applying gof as h(student) = g(f(student)). For instance, $\boldsymbol{h(Ani) = g(f(Ani)) = g(75) = B}$.

Checking Commutativity Property:

The composition of functions is not commutative in this case. This means that $\boldsymbol{h}(\boldsymbol{student}) \neq \boldsymbol{f}(\boldsymbol{g}(\boldsymbol{student}))$ for some students. For instance, by taking $\boldsymbol{h}(\boldsymbol{Linh}) = \boldsymbol{g}(\boldsymbol{f}(\boldsymbol{Linh})) = \boldsymbol{g}(\mathbf{85}) = \boldsymbol{B}+$, while $\boldsymbol{f}(\boldsymbol{g}(\boldsymbol{Linh})) = \boldsymbol{f}(\boldsymbol{B}+) = \boldsymbol{undefined}$ since there is no mapping defined for grades back to marks. Therefore, we can conclude that the order in which the functions are applied matters, and the composition is not commutative.

5. Explain the following concepts using relevant examples: sequence, recursive function, closed formula for the sequence, arithmetic sequence, and geometric sequence. Ensure that you do not take the same examples discussed in the textbooks and online resources.

i. The sequence 5, 6, 6, 7, 8, 8, 9, 10, 10, 11, 12, 12...... can be generated by a closed formula. Find the formula and the next four terms of the sequence.

The sequence appears has consecutive integers and some repeated elements, which are the even integers, each repeated two times. The closed formula for generating this sequence can be described as follows:

1. Start with the first even number, which is 6.
2. For each even number n starting from 6, include n and n+1 in the sequence.

So, the formula for generating the sequence is: $a_n = \begin{Bmatrix} 6 + \frac{(n-1}{2} & if\ n\ is & odd \\ 6 + \frac{n}{2} & if\ n\ is & even \end{Bmatrix}$, where

$n = 1,2,3,4,5, \dots \dots ....$

Now, the next four terms of the sequence will be determine as follows:

1. For $n = 13$, an odd number, $a_{13} = 6 + \frac{(13-1)}{2} = 6 + \frac{12}{2} = 6 + 6 = 12$

2. For $n = 14$, an even number, $a_{14} = 6 + \frac{14}{2} = 6 + 7 = 13.$

3. For $n = 15$, an odd number, $a_{15} = 6 + \frac{(15-1)}{2} = 6 + \frac{14}{2} = 6 + 7 = 13$

4. For $n = 16$, an even number, $a_{16} = 6 + \frac{16}{2} = 6 + 8 = 14.$

Therefore, the next four terms of the sequence are: 12, 13, 13, 14.

ii. Find the next three terms of the series, provide a closed formula, and calculate the sum to the first n terms for the following series: $\mathbf{6 + 36 + 216 + \cdots}$ … … … … ….

A closer look at the sequence reveals that each term is obtained by multiplying the previous term by 6, thus requiring the formula for the geometric series: $x_n = ar^{n-1}$, where x_n=the nth term, a = the first term of the series, r = the common ratio (in this case, 6), and n = is the number of terms (1,2,3,4,5…….). The next three terms will be determined as follows:

Now, the next three terms of the sequence will be determined as follows:

1. For $n = 4$, $x_4 = (6)(6^{4-1}) = (6)(6^3) = 6 \times 216 = 1296.$

2. For $n = 5$, $x_4 = (6)(6^{5-1}) = (6)(6^4) = 1296 = 7776.$

3. For $n = 6$, $x_4 = (6)(6^{6-1}) = (6)(6^5) = 7776 = 46656.$

Therefore, the next three terms of the sequence are: 1296, 7776, and 46656.

iii. Find the next three terms of the series, provide a closed formula, and calculate the sum to the first n terms for the following series: 21+24+27+……………….

A closer look at the sequence reveals that it is an arithmetic series with the closed formula: $a_n = a + (n-1)d$, where a = the first term, n = the nth term, and d = common difference (in this case, d = 3.

Now, the next three terms of the sequence will be determined as follows:

1. For $n = 4$, $x_4 = 21 + (4-1)3 = 21 + 9 = 30$.
2. For $n = 5$, $x_5 = 21 + (5-1)3 = 21 + 12 = 33$.
3. For $n = 6$, $x_6 = 21 + (6-1)3 = 21 + 15 = 36$.

Therefore, the next three terms of the sequence are: 30, 33, and 36.