Documentation for American Option Pricing Code

Overview

This MATLAB code implements a predictor-corrector finite difference scheme to calculate the early exercise boundary (S_f) for an American put option, based on the method described in Zhu and Zhang's 2011 paper *"A new predictor-corrector scheme for valuing American puts"*. The code provides both the option pricing solution and convergence analysis for time and asset price discretizations.

Main Function: `american_option(N, M)`

Description

Computes the early exercise boundary for an American put option using a predictor-corrector finite difference scheme.

Input Parameters

- `N`: Number of time steps (time discretization)
- `M`: Number of asset price steps (price discretization)

Output

- `S_f`: Vector representing the early exercise boundary at each time step

Mathematical Model

The function solves the American put option pricing problem with the following parameters:

- Time to maturity (T): 1 year
- Volatility (σ): 0.3 (30%)
- Risk-free rate (r): 0.1 (10%)
- Dividend yield (D_0): 0

- Strike price (X): 100

The problem is transformed using:

$$\tau = \frac{T\sigma^2}{2}, \quad x = \log\left(\frac{S}{S_f}\right), \quad \gamma = \frac{2r}{\sigma^2}, \quad D = \frac{2D_0}{\sigma^2}$$

Numerical Scheme

1. **Initialization**: Sets up the grid and initial conditions
2. **Time Stepping**:
   - *Predictor Step*: Estimates the early exercise boundary using an explicit Euler scheme
   - *Corrector Step*: Refines the estimate using a Crank-Nicolson scheme
3. **Matrix Solution**: Solves the resulting tridiagonal system using sparse matrix operations

Boundary Conditions

$$P(x = 0, \tau) = 1 - S_f(\tau), \qquad P(x = x_{\max}, \tau) = 0$$

Convergence Analysis Script

Time Convergence Test

- Tests convergence with respect to time discretization (fixed $M = 100$, varying $N = [200, 400, 800, 1600, 3200]$)
- Displays:
  - Final early exercise boundary value ($S_f \cdot X$)
  - Differences between consecutive refinements
  - Ratios of consecutive differences (showing convergence rate)

Asset Price Convergence Test

- Tests convergence with respect to price discretization (fixed $N = 100$, varying $M = [200, 400, 800, 1600, 3200]$)
- Displays similar convergence metrics for the spatial dimension

Key Features

1. **Efficient Implementation**:
   - Uses sparse matrices for solving the tridiagonal system
   - Combines predictor-corrector approach for stability and accuracy
   - No iterations required at each time step
2. **Convergence Analysis**:
   - Provides quantitative measures of convergence for both time and space discretizations
   - Shows the order of convergence through difference ratios
3. **Flexibility**:
   - Easy to modify parameters (volatility, interest rate, dividend yield)
   - Can be extended to other option types with American-style exercise

Usage Example

```
% Run convergence tests
X = 100;
N_list = [200, 400, 800, 1600, 3200];
S_f_values = zeros(size(N_list));

for i = 1:length(N_list)
    N = N_list(i);
    S_f = american_option(N, 100);
    S_f_values(i) = S_f(end) * X;
end
```

References

Zhu, S.-P., & Zhang, J. (2011). *A new predictor-corrector scheme for valuing American puts.* Applied Mathematics and Computation, 217(9), 4439–4452.

Notes

- The code uses logarithmic transformation of the asset price for numerical stability
- The predictor-corrector scheme helps maintain stability while achieving good accuracy
- Convergence tables help verify the numerical implementation and demonstrate the method's reliability