PROBLEM SET 2

1) The critical point is the unique point on the original van der Waals isotherms where both the first and second derivatives of P with respect to V (at fixed T) are zero. Use this fact to show that

$$V_c = 3Nb, \qquad P_c = \frac{1}{27}\frac{a}{b^2}, \qquad \text{and} \qquad kT_c = \frac{8}{27}\frac{a}{b}$$

Hint: Use Text Eq. 5.49 (p. 180) discussed in lecture (you'll need to take 1st, 2nd partial derivatives, set them to 0, solve and substitute)

2) In the van der Waals equation of state, define dimensionless volumes, pressures, and temperatures by the equations $v = V/V_c$, $p = P/P_c$, and $t = T/T_c$, where V_c, P_c and T_c are the critical volume, pressure, and temperature associated with the van der Waals equation of state (discussed in lecture). Show that in terms of these dimensionless quantities, the van der Waals equation of state is given by

$$\left(p + \frac{3}{v^2}\right)(v - 1/3) = \frac{8}{3}t$$

3) An ideal *monatomic* gas has initial pressure P_0 and occupies initial volume V_0. The gas undergoes an *adiabatic* expansion in which the volume is doubled. Calculate in terms of P_0 and V_0

a) the final pressure of the gas

b) work done by the gas during the expansion

c) the change in its enthalpy during the expansion

4) **Text, Problem 1.36**

5) The equation of state of a particular (non-ideal!) gas is given by $(P + b)V = nRT$, where b is a constant. The internal energy of the gas is given by $U = U_0 + naT + bV$, where a and U_0 are two more constants.

a) Derive an expression for the differential work done on the gas when its volume is increased from V to V + dV at temperature T. Your result should depend on T, V, dV, and the constants R and b.

b) Determine the corresponding differential increase in U (note that dT is not zero!) and then use the first law of thermodynamics to find the differential heat transfer into the gas.

c) Using the result of part b, show that the constant volume heat capacity for this gas is given by $C_V = na$.

d) Show that for a constant pressure process for this gas, $dV = [nR/(P + b)]dT$, and then following a similar procedure as in parts a), b), and c), determine the constant pressure heat capacity for the gas. Finally, show that $C_p - C_V = nR$, as for an ideal gas.

6) The pressure associated with the electromagnetic radiation within a closed container of volume V and temperature T depends only on the temperature and is given by $P = aT^4/3$, where a is a constant. The internal energy associated with the radiation is given by $U = aT^4V$.

a) With the help of the first law of thermodynamics, show that if the volume of the container is doubled in an isothermal process, then the heat transfer into the container is given by $Q = (4/3)aT^4V$.

b) In an adiabatic process, dQ=0 so that PdV = -dU. Use this relation to derive a differential relation connecting V and T in an adiabatic process;

then integrate this relation to show that in an adiabatic process, VT^3 is constant for this system.

7) **Text, Problem 1.49** – you should assume here that the result of the combustion, liquid water, has negligible volume. Also, note that the work done on the gas is positive (its volume is reduced), so that the work done on the atmosphere is negative.

8) **Text, Problem 1.53**