

## PROBLEM SET 6

### 1) (5 pts) TextProblem 6.5

2) (10 pts) A system has three non-degenerate energy levels with energies 0,  $\epsilon$ , and  $2\epsilon$ .

(*hint*: Set up similar to problem 6 of HW set 5. How many ways (microstates) can the particles populate the 3 energy levels such that the total energy is  $2\epsilon$ , then the entropy can be easily calculated)

a) Calculate the entropy of the system if the three levels are populated by two distinguishable particles such that the total energy is  $U=2\epsilon$ .

b) Calculate the entropy of the system if the three levels are populated by three distinguishable particles such that the total energy is  $U=2\epsilon$ .

### 3) (5 pts) TextProblem 6.12

(*hint*: Think of ratio of probability between 1st excited state and ground state, and note that 1st excited state has 3 degenerate states with the same energy. Also, note that the energy given is really a difference between 1st and ground)

### 4) (5 pts) TextProblem 6.13

(*hint*: Think of ratio of probabilities as the ratio of  $P_{\text{neutron}}/P_{\text{proton}}$ , you are also told how to calculate the total energy difference. Also keep in mind that  $P_{\text{neutron}} + P_{\text{neutron}} = 1$ )

5) (25 pts) Consider a system with two non-degenerate energy levels with energies  $\epsilon_1 = 0$  and  $\epsilon_2 = \epsilon$ . Suppose that the system contains  $N$  distinguishable particles at temperature  $T$ .

a) Determine the partition function of the system and the occupancies  $N_1$  and  $N_2$  of the two levels.

- b) Find the average energy per particle given by  $\langle u \rangle = U/N$ , where  $U$  is the total internal energy of the system and  $N$  the total number of particles.
- c) Show that at very small temperatures,  $\langle u \rangle \approx \epsilon e^{-\alpha}$ , where  $\alpha = \frac{\epsilon}{kT}$ , and that as the temperature becomes very large,  $\langle u \rangle \rightarrow \frac{1}{2}\epsilon$
- d) Show that the volume heat capacity per particle is given by  $C/N = k\alpha^2 e^{-\alpha} / (1 + e^{-\alpha})^2$
- 6) **(20 pts) Text Problem 6.31** – to perform the required integrals, make a change of variable and then carry out an integration by parts.
- 7) **(5 pts) Text Problem 6.33** – evaluate the requested quantities at  $T = 20^\circ \text{C}$ . (*Hint*: Careful with the units, convert to K first)
- 8) **(5 pts)** Calculate the average energy (in eV) and rms velocity of an electron at the temperatures  $T_1 = 10^3 \text{ K}$  and  $T_2 = 10^5 \text{ K}$
- 9) **(20 pts)** Suppose that instead of the Maxwell-Boltzmann distribution, the distribution of molecular speeds in a gas was given by the expression  $N(v) = A v e^{-v/v_0}$ , where  $A$  and  $v_0$  are constants.
- a) Determine the constant  $A$  so that the total number of molecules in the gas is  $N$ .
- b) In terms of  $v_0$ , find the average speed, the rms speed, and the most likely speed of the molecules in the gas.