

### PROBLEM SET 3

- 1) **(5 pts)** It is desired to freeze 50 kg of water in a refrigerator operating at 0 °C. The heat leaving the refrigerator is expelled into a room that is at a temperature of 20 °C. Calculate the minimum power required to freeze all of the ice in one hour. Note that the heat of fusion of water is  $L=334$  kJ/kg.
  
- 2) **(5 pts)** It is desired to increase the efficiency of a Carnot engine by either increasing the temperature of the high temperature reservoir by an amount  $\Delta T$  or by decreasing the temperature of the low temperature reservoir by the same  $\Delta T$ . Which choice will result in the greatest increase in efficiency? Hint: examine the ratio of the two new efficiencies.
  
- 3) **(5 pts) Text Problem 4.1** – refer to problem 7 of homework set 1 (text problem 1.34). Note that  $Q_{in}$  is the sum of the two positive heat transfers calculated in that problem, In part b, to calculate the Carnot efficiency, use the highest and lowest temperatures that occur in the cycle.
  
- 4) **(5 pts) Text, problem 4.4** – note that the specific heat capacity of water is 4190 J/kg-K and that water has density  $\rho=1000$  kg/m<sup>3</sup>.
  
- 5) **(10 pts)** A heat pump is a refrigerator that is used to heat the interior of an enclosure, rather than to cool it; i.e., it is used to remove heat from a low temperature reservoir and transfer it to a high temperature reservoir. Let  $T_C$  and  $T_H$  be the temperatures of the low and high temperature reservoirs and let  $Q_{out}$  and  $Q_{in}$  be the heat transferred out of the low temperature reservoir and the heat transferred into the high temperature reservoir (so that both  $Q$ 's are positive).
  - a) Define the heat pump coefficient of performance by  $K = Q_{in}/W$ , where  $W$  is the work done by the refrigerator. Note that this definition of  $K$  is in accord with the general definition of coefficient of performance or efficiency as “what

you get out” divided by “what you put in”. Show that for a Carnot heat pump,  $K$  is given by  $K = T_H / (T_H - T_C)$

b) Suppose that on a cold winter day, the temperature outside is  $0^\circ\text{C}$ , and its desired to maintain the temperature inside a house equipped with a Carnot heat pump at  $20^\circ\text{C}$ . Calculate the coefficient of performance of this heat pump.

6) **(20 pts) Text Problem 4.18** - Otto cycle, discussed in class; determine efficiency; first determine heat transferred to the system from the hot reservoir ( $Q_h$ ), and heat expelled ( $Q_c$ ) into the cold reservoir and then use Eq. (4.3)

7) **(5 pts) Text Problem 3.10**

8) **(5 pts) Text Problem 3.11**

9) **(5 pts) Text Problem 3.28** – note that air is mostly made up of diatomic molecules, and that increasing the volume at constant pressure also changes the temperature. Choose the initial temperature (room temperature) to be  $20^\circ\text{C}$ .

10) **(10 pts)** Consider a thermally insulated resistor with resistance  $R=20\ \Omega$  and mass  $m=5.0\ \text{g}$ . The resistor is made of a material with specific heat capacity  $c=850\ \text{J}/(\text{g}\cdot\text{K})$  and carries a current of  $2.0\ \text{A}$  for a time period of  $1.0\ \text{s}$ .

a) Calculate the increase in the temperature of the resistor if it has initial temperature  $T_{\text{in}}=20^\circ\text{C}$ . Recall that the power dissipated by a resistor is given by  $P = I^2 R$ .

b) Determine the increase in the resistor's entropy over the period that the current passes through it.

11) **(15 pts)** An ideal gas with adiabatic index  $\gamma$  is taken around a complete thermodynamic cycle consisting of three steps. Starting at point A, the pressure is increased at constant volume  $V_1$  from  $P_1$  to  $P_2$  at point B. From

point B to point C, the gas is allowed to expand adiabatically from volume  $V_1$  and pressure  $P_2$  to volume  $V_2$  and the original pressure  $P_1$ . Finally, from point C to point A, the volume of the gas is decreased at constant pressure  $P_1$  back to the original volume  $V_1$ .

a) Make a PV diagram of the complete cycle.

b) For each step of the cycle, determine the change in the entropy of the gas. Sum your results to find the total entropy change for the whole cycle. Note that every step is reversible.

c) Use the adiabatic relations to eliminate the pressure from your result for part b. Show that the resulting expression gives a total entropy change for the complete cycle equal to zero.

12) **(10 pts)** Suppose that the molar specific heat capacity of an ideal gas at constant volume is a linear function of temperature given by  $C_V = A + BT$ , where A and B are constants. Show that the change in entropy per mole in a process that takes the gas from volume  $V_{in}$  and temperature  $T_{in}$  to volume  $V_f$  and temperature  $T_f$  is given by

$$\Delta S/n = A \ln(T_f/T_{in}) + B\Delta T + R \ln(V_f/V_{in})$$

**Extra Credit Problem** (to count towards Exam II)  
(15 points undergraduates, 10 points graduates)

7) **Text Problem 4.20** - Diesel cycle, discussed in class; determine diesel efficiency similarly to **Text Problem 4.18**;

first show that:  $\frac{Q_c}{Q_h} = \frac{1}{\gamma} \frac{V_1(P_4 - P_1)}{P_2(V_3 - V_2)}$ , where  $\gamma = \frac{f+2}{f}$ ;

then using the fact that 3-4, and 1-2 are adiabatic, relate the initial and final pressures to the volumes and show that the expression above can be re-written as:

$$\frac{Q_c}{Q_h} = \frac{1}{\gamma} \frac{V_1}{V_3 - V_2} \left[ \left( \frac{V_3}{V_1} \right)^\gamma - \left( \frac{V_2}{V_1} \right)^\gamma \right]$$

finally, to express the efficiency in terms of the compression ratio, divide the numerator and denominator of each volume ratio by  $V_2$ , then factor out the compression ratio,  $V_1/V_2$ , to show that:

$$\frac{Q_c}{Q_h} = \left( \frac{V_2}{V_1} \right)^{\gamma-1} \cdot \frac{1}{\gamma} \frac{(V_3/V_2)^\gamma - 1}{(V_3/V_2) - 1}, \text{ where the first term is the efficiency of}$$

the Otto cycle, determined in Text Problem 4.18. The efficiency is then

$$\text{determined from Eq. (4.3), } \epsilon = 1 - \frac{Q_c}{Q_h}$$

*Programming Part:*

Now, make a plot of the “correction factor”  $\frac{1}{\gamma} \frac{(V_3/V_2)^\gamma - 1}{(V_3/V_2) - 1}$  versus the cutoff

ratio,  $V_3/V_2$  for a diatomic ideal gas ( $\gamma = 7/5$ ). Set the y-axis range [0, 1.6], x-axis range [1, 5]. Is the efficiency of the diesel larger than or less than that of the Otto Cycle? *hint:* Will the correction factor make the efficiency of the diesel engine larger than or less than the efficiency of the Otto cycle?