Advanced Classical Mechanics (PHYS 615) **Fall 2024**
Extra Credit Project 1 (due on date of Exam I)

The points obtained in this project will be added towards Exam I. To obtain full credit, your computer source code + calculations done on paper **MUST** be provided along with the plots.

During our second lecture, we discussed projectile motion with drag force of the form $F = -kmv$ from which the projectile flight time and range were derived

$$T = \frac{kV + g}{gk}(1 - e^{-kT}), \qquad R = \frac{U}{k}(1 - e^{-kT})$$

where k is a drag coefficient and $V \equiv v_0 \sin\theta$ and $U \equiv v_0 \cos\theta$ are the initial y- and x-components of the projectile speed. The time of flight is a transcendental equation and can't be solved analytically. Two methods for solving this problem are (i) *Numerical* and (ii) *Perturbative*

<u>*Numerical Approach:*</u> **(20 pts)** To solve a transcendental equation, we use *Newtons's Method* numerical approach which is a root-finding algorithm, which has the recurring form:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where x_{n+1} is the root after n-recursive steps. To use this method, the function must be of the form $f(x) = 0$ and it requires an initial guess of the root x that satisfies the equation. In our case, for a given value of k, we find the root T, and used the (k, T) pair as input into the range equation.

Solve the transcendental equation using Newton's Method.

- step 1: need to put the equation in the form: $f(k, T) = 0$

- step 2: for an arbitrary value of $k \leq 0.08$, find the root T, as follows

$$T_{n+1} = T_n - \frac{f(k, T_n)}{\mathrm{d}f(k, T_n)/dT}$$

Use the initial conditions to get the iteration process going, and try using *n=100* iterations

<u>initial conditions (*n=0*):</u>
$v_0 = 600 \text{ m/s}$
$\theta = 60°$
T_0 = 150 sec (initial guess will not necessarily satisfy $f(k, T_0) = 0$)

after n-iterations, the algorithm should have found the root that satisfies the equation $f(k, T) = 0$,

-step 3: now that you have a (k_i, T_i) pair for an i-th arbitrary value of k, use it as input into the range equation, $R_i(k_i, T_i)$

-step 4: at this point, you should have a procedure established for determining the values (k_i, T_i, R_i); now repeat the procedure for a range of k-values from 0.0000001 to 0.08 with step-size of $dk = 0.001$; ** note: since k is in the denominator, $k \neq 0$

-step 5: make a plot of R vs. k to examine how the projectile range varies with the drag coefficient ** note: negative ranges are unphysical, therefore, when making the plots, set the lower limit on the y-axis to zero.
Add proper labels and a legend to each of the plots.

<u>*Perturbative Approach:*</u> **(10 pts)** Recall, during lecture we applied a Taylor expansion to the time-of-flight (T) and range (R) using k as the *expansion parameter* since it is usually small and can be considered a perturbation. The series was truncated for powers of $k^n, n \geq 2$ only keeping the linear term:

$$R' \approx R\left(1 - \frac{4kV}{3g}\right)$$

where $R \equiv 2UV/g$ (projectile range without drag)

Let's now attempt to go one step further and include a *quadratic term* (k^2) to check how well the perturbative approach using both a *linear* and *quadratic* term agrees with the numerical approach.

To keep the quadratic term k^2 in the range expansion, lets begin with:

$$R' \approx \frac{U}{k}\left(kT - \frac{1}{2}k^2T^2 + \frac{1}{6}k^3T^3 - \ldots\right)$$

** notice that the k in the denominator cancels with a common factor k

Now substitute the approximate expression for the flight time (which only kept k to a linear term) to the expression above

$$T \approx \frac{2V}{g}\left(1 - \frac{kV}{3g}\right)$$

keeping only up to k^2 terms, and re-arrange the equation to show that

$$R' \approx R\left[1 - 4\left(\frac{kV}{3g}\right) + 9\left(\frac{kV}{3g}\right)^2\right]$$

Now, to check the perturbative method versus the numerical method, make a plot overlay of: i) R' vs. k (*perturbative* **linear** approximation),

ii) R' vs. k (*perturbative* **quadratic** approximation),

iii) R vs. k (**numerical** approximation)

Add proper labels and a legend to each of the plots.

Write a brief paragraph commenting on your results. Is it what you expected to happen ? Elaborate.