PROBLEM SET 1

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem. (**Max Score:** 80)

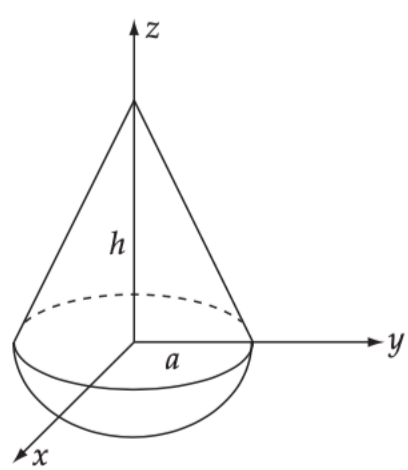
General Hint: Uniform mass distributions refer to a mass density or mass per unit length, area or volume, e.g., for a uniform volume: $\rho_m = M/V = dm/dV$

1) **(10 pts)** Find the center-of-mass (CM) of a uniformly solid cone of base diameter $2a$ and height h and a solid hemisphere of radius a where the two bases are touching.

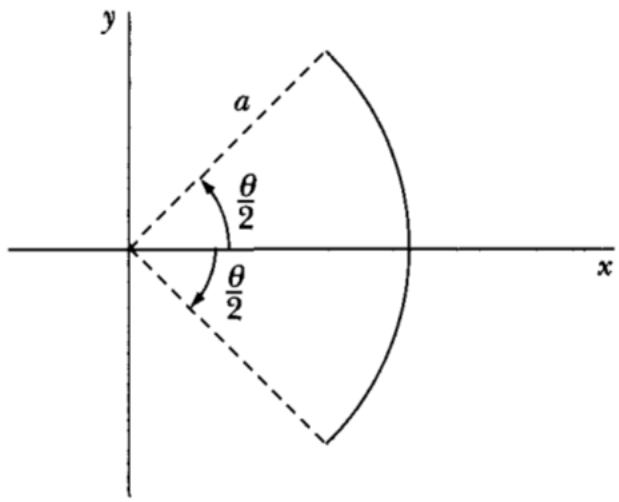
   Hint: calculate the CM of the shapes separately, using: $\vec{R} = \frac{1}{M}\int \vec{r}dm$

   then combine their results using the discrete formula, $\vec{R} = \frac{1}{M}\sum \vec{r}_i m_i$

   might be easier to use polar coordinates.

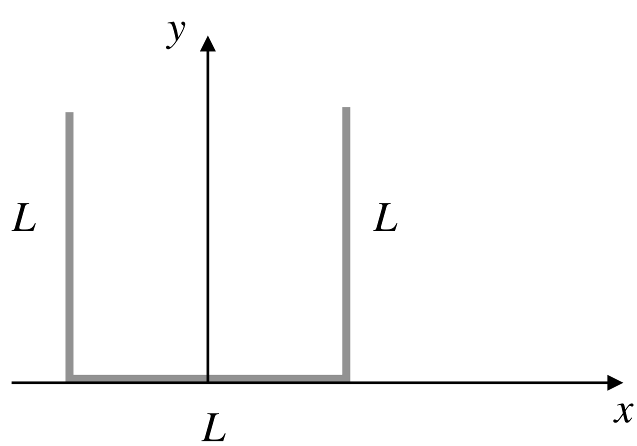


2) **(5 pts)** Find the center-of-mass (CM) of a uniform wire that subtends an arc θ if the radius of the circular arc is a.

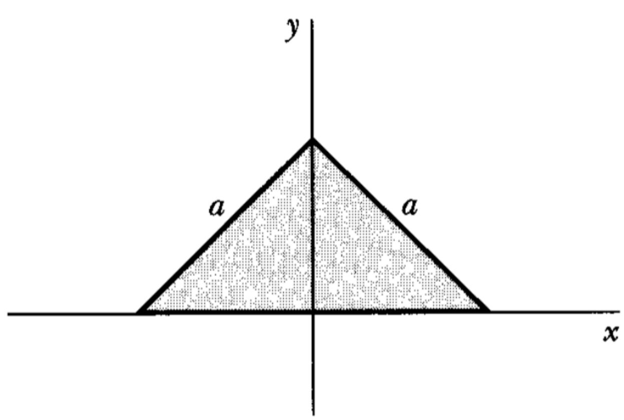


3) **(10 pts)** Find the center-of-mass (CM) of each of the two systems described below:

a) **(5 pts)** three uniform rods of equal mass M and equal length L arranged in a U shape

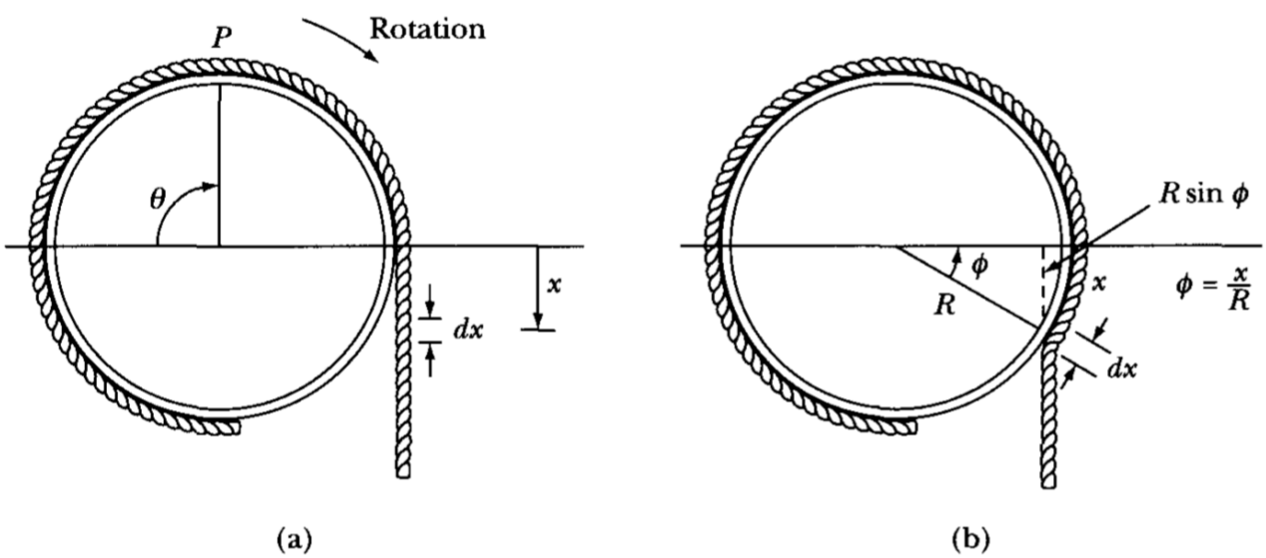


b) **(5 pts)** a uniform very thin plate of mass M is in the shape of an isosceles triangle with each side of length a

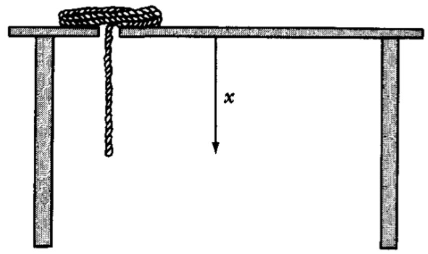


4) **(15 pts)** A rope of uniform linear density λ and mass m is wrapped one complete turn around a hollow cylinder of mass M and radius R. The cylinder rotates about its axis as the rope unwraps. The rope ends are at $x = 0$ (one fixed, one loose) when point P is at $\theta = 0$ [fig (a)], and the system is slightly displaced from equilibrium at rest [fig (b)]. Find the angular velocity as a function of angular displacement θ of the cylinder.

Hint: (i) first calculate the work done to displace the loose end of the rope *vertically*, (ii) then calculate the kinetic energy of both the rope and cylinder, as the rope unwinds and finally, (iii) use the work-energy theorem to equate the work done to change in kinetic energy and solve for angular velocity, $d\theta/dt$.



5) **(10 pts)** A smooth rope of uniform linear density $\lambda = dm/dx$ and length L is placed above a hole in a table. One end of the rope falls through the hole at $t = 0$, pulling steadily on the remainder of the rope.



a) The force of gravity acts on a section x of the rope, therefore change in momentum of the hanging section of rope involves changes in both its mass and speed with time. Apply $F_{net} = dp/dt$ to a section of mass m and length x of the rope and show that the differential equation can be expressed as

$$xg = xv\frac{dv}{dx} + v^2$$

hint: since we ultimately want to find $v(x)$, use the fact that there is an implicit time dependence of v, such that $dv/dt = (dv/dx)(dx/dt)$

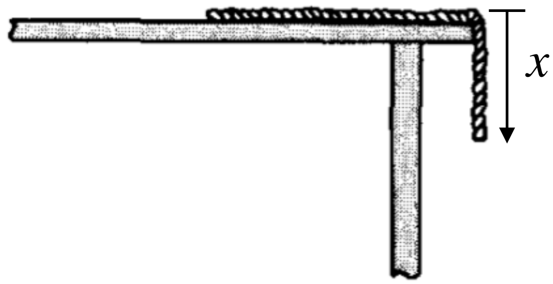
b) To solve the differential equation in part a), try out a power law solution of the form $v(x) = ax^n$, where n is an integer, a is a constant and substitute into the differential equation to show that

$$a^2(n+1)x^{2n} = xg$$

Since this expression must be true for all x, the exponent and coefficient of x must be the same on both sides of the equation. Therefore, use this fact to determine what the value of n should be and show that the general expression for the velocity and acceleration as a function of position are

$$v(x) = \sqrt{\frac{2gx}{3}}, \quad a = \frac{g}{3}$$

6) **(10 pts)** A flexible rope of mass m and length L slides from a *frictionless* table top. The rope is initially released from rest with a length x_0 hanging from the edge of the table.



a) Find the general differential equation for the acceleration of the rope and solve it.

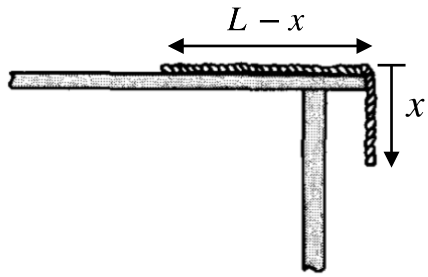
Hint: (i) If x is an arbitrary length of rope after its released from rest, then its mass is $(m/L)x$, and apply Newton's 2nd law to this section of the rope ; (ii) to solve the differential equation, think of what linear combination of a function $x(t)$ after taking two time derivatives, returns the same function back times a *positive* constant ?

b) Given the initial conditions, $x(t=0) = x_0$, $v_x\ (t=0) = 0$ m/s, show that the time at which the left end of the rope reaches the edge of the table can be expressed as

$$t = \frac{1}{\omega}\cosh^{-1}\left(\frac{L}{x_0}\right), \text{ where } \omega \equiv \sqrt{g/L}$$

Hint: (iii) use the following relation useful $\cosh u = (e^u + e^{-u})/2$ to simplify and solve for time

7) **(10 pts)** Consider the uniform rope of the previous problem *with friction*. The rope mass m and length L is stretched horizontally across a table top with a length x of the rope hanging vertically over the side of the table. The coefficient of kinetic friction between the rope and the table top is μ_k. The hanging end of the rope is given a quick pull to get it moving and then the rope is allowed to slide off the end of the table.



a) Give general expressions for the i) *gravitational* force on the hanging end of the rope, ii) the *kinetic frictional* force on the part of the rope on the table and iii) the *net* force acting on the rope as functions of the distance x

b) Determine the value of x_o, the minimum value of x that is required for the rope to keep moving once the motion is initiated. If $x < x_o$, the rope will stop moving immediately after it is pulled. If $x > x_o$, it will keep moving until the whole rope has slid off the table.

c) Assuming that the rope always moves as a whole and starting with initial velocity, $v_{x,i} \approx 0$ when $x = x_o$, show that the velocity of the rope at the moment when the whole rope is clear of the table (i.e., when $x = L$) is given by

$$v_x = \sqrt{\frac{gL}{\mu_k + 1}}$$

Hint: Use the general results from part a) to calculate the work done by the net force (gravity + friction) to move the part of the rope on the table. Then from the work-energy theorem, $\Delta K = W_{net}$, where $W_{net} = \int F_{net} dx$ to determine v_x.

8) **(10 pts)** A deuteron (nucleus of deuterium atom consisting of a proton and a neutron) with speed 14.9 km/s collides *elastically* with a neutron at rest. Use the approximation that the deuteron is twice the mass of the neutron.

Hint: use energy and momentum conservation

a) If the deuteron is scattered through a lab angle $\theta = 10°$, what are the final speeds of the deuteron and the neutron ?

b) What is the scattering angle of the neutron ?