

**Advanced Classical Mechanics (PHYS 615)**  
**Extra Credit Project 1 (due on date of Exam I)**

**Fall 2024**

The points obtained in this project will be added towards Exam I. To obtain full credit, your computer source code + calculations done on paper **MUST** be provided along with the plots.

During our second lecture, we discussed projectile motion with drag force of the form  $F = -kmv$  from which the projectile flight time and range were derived

$$T = \frac{kV + g}{gk}(1 - e^{kT}), \quad R = \frac{U}{k}(1 - e^{kT})$$

where  $k$  is a drag coefficient and  $V \equiv v_0 \sin \theta$  and  $U \equiv v_0 \cos \theta$  are the initial y- and x-components of the projectile speed. The time of flight is a transcendental equation and can't be solved analytically. Two methods for solving this problem are (i) *Numerical* and (ii) *Perturbative*

*Numerical Approach:* **(20 pts)** To solve a transcendental equation, we use *Newtons's Method* numerical approach which is a root-finding algorithm, which has the recurring form:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

where  $x_{n+1}$  is the root after  $n$ -recursive steps. To use this method, the function must be of the form  $f(x) = 0$  and it requires an initial guess of the root  $x$  that satisfies the equation. In our case, for a given value of  $k$ , we find the root  $T$ , and used the  $(k, T)$  pair as input into the range equation.

Solve the transcendental equation using Newton's Method.

**- step 1:** need to put the equation in the form:  $f(k, T) = 0$

- **step 2:** for an arbitrary value of  $k \leq 0.08$ , find the root  $T$ , as follows

$$T_{n+1} = T_n - \frac{f(k, T_n)}{df(k, T_n)/dT}$$

Use the initial conditions to get the iteration process going, and try using  $n=100$  iterations

initial conditions ( $n=0$ ):

$$v_0 = 600 \text{ m/s}$$

$$\theta = 60^\circ$$

$$T_0 = 150 \text{ sec (initial guess will not necessarily satisfy } f(k, T_0) = 0)$$

after  $n$ -iterations, the algorithm should have found the root that satisfies the equation  $f(k, T) = 0$ ,

-**step 3:** now that you have a  $(k_i, T_i)$  pair for an  $i$ -th arbitrary value of  $k$ , use it as input into the range equation,  $R_i(k_i, T_i)$

-**step 4:** at this point, you should have a procedure established for determining the values  $(k_i, T_i, R_i)$ ; now repeat the procedure for a range of  $k$ -values from 0.0000001 to 0.08 with step-size of  $dk = 0.001$ ; \*\* note: since  $k$  is in the denominator,  $k \neq 0$

-**step 5:** make a plot of  $R$  vs.  $k$  to examine how the projectile range varies with the drag coefficient \*\* note: negative ranges are unphysical, therefore, when making the plots, set the lower limit on the y-axis to zero.  
Add proper labels and a legend to each of the plots.

*Perturbative Approach:* **(10 pts)** Recall, during lecture we applied a Taylor expansion to the time-of-flight ( $T$ ) and range ( $R$ ) using  $k$  as the *expansion parameter* since it is usually small and can be considered a perturbation. The series was truncated for powers of  $k^n$ ,  $n \geq 2$  only keeping the linear term:

$$R' \approx R \left( 1 - \frac{4kV}{3g} \right)$$

where  $R \equiv 2UV/g$  (projectile range without drag)

Let's now attempt to go one step further and include a *quadratic term* ( $k^2$ ) to check how well the perturbative approach using both a *linear* and *quadratic* term agrees with the numerical approach.

To keep the quadratic term  $k^2$  in the range expansion, let's begin with:

$$R' \approx \frac{U}{k} \left( kT - \frac{1}{2}k^2T^2 + \frac{1}{6}k^3T^3 - \dots \right)$$

\*\* notice that the  $k$  in the denominator cancels with a common factor  $k$

Now substitute the approximate expression for the flight time (which only kept  $k$  to a linear term) to the expression above

$$T \approx \frac{2V}{g} \left( 1 - \frac{kV}{3g} \right)$$

keeping only up to  $k^2$  terms, and re-arrange the equation to show that

$$R' \approx R \left[ 1 - 4 \left( \frac{kV}{3g} \right) + 9 \left( \frac{kV}{3g} \right)^2 \right]$$

Now, to check the perturbative method versus the numerical method, make a plot overlay of: i)  $R'$  vs.  $k$  (*perturbative **linear** approximation*),

ii)  $R'$  vs.  $k$  (*perturbative **quadratic** approximation*),

iii)  $R$  vs.  $k$  (***numerical** approximation*)

Add proper labels and a legend to each of the plots.

Write a brief paragraph commenting on your results. Is it what you expected to happen? Elaborate.