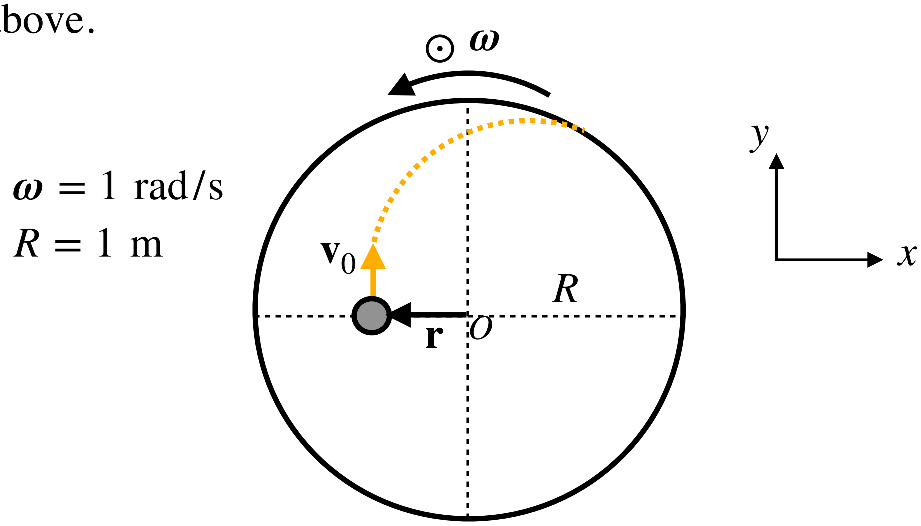
Advanced Classical Mechanics (PHYS 615) **Fall 2024**
Extra Credit Project 3 (due on date of Exam I)

The points obtained in this project will be added towards Exam I. To obtain full credit, your computer source code + calculations done on paper **MUST** be provided along with the plots.

A student is performing measurements with a hockey puck on a large merry-go-round with a smooth (frictionless) horizontal, flat surface. The merry-go-round has a constant angular velocity $\boldsymbol{\omega} = \omega\hat{z}$ and rotates counterclockwise as seen from above.



(a) **(5 pts)** The effective acceleration on the puck (as observed in the rotating frame) after it is given an initial push is:

$$\boldsymbol{a}_{\rm eff} = \frac{\mathbf{F}_{\rm eff}}{m} = -\underbrace{\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})}_{\text{centrifugal term}} - \underbrace{2\boldsymbol{\omega}\times\mathbf{v}}_{\text{coriolis term}}$$

where $\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ and $\mathbf{v} = v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}}$ an arbitrary position and velocity of the puck some time $t > 0$ after it is given an initial push.

Determine the x- and y- components of the effective acceleration ($a_x = dv_x/dt,\ \ a_y = dv_y/dt$).

(b) **(10 pts)** The equations in (a) are coupled differential equations for which there is no exact solutions and numerical methods must be used. Use Euler's method (introduced in Project #2) to solve the equations for $[v_x(t), v_y(t)]$ and $[x(t), y(t)]$

The velocity and position:

$$\mathbf{v}_{\text{eff}} = \int \mathbf{a}_{\text{eff}} dt, \quad \mathbf{r}_{\text{eff}} = \int \mathbf{v}_{\text{eff}} dt$$

are given by numerical integration:

$$v_{x,n+1} = v_{x,n} + a_{x,n}\Delta t$$
$$v_{y,n+1} = v_{y,n} + a_{y,n}\Delta t$$

$$x_{n+1} = x_n + v_{x,n}\Delta t$$
$$y_{n+1} = y_n + v_{y,n}\Delta t$$

where n represents number of step-sizes and $\Delta t = t_{n+1} - t_n$ is the time step-size required to advance the numerical integrations

Perform the numerical calculation to determine the motion $[x(t), y(t)]$ and make separate plots of $y(t)$ vs. $x(t)$ to show the different paths of the puck for the following initial conditions:

(i) <u>initial condition ($t = 0$):</u>
$(x_0, y_0) = (-0.5R,\ 0)$
$(v_{x,0}, v_{y,0}) = (0,\ 1.5\ \text{m/s})$

(ii) <u>initial condition ($t = 0$):</u>
$(x_0, y_0) = (-0.5R,\ 0)$
$(v_{x,0}, v_{y,0}) = (0,\ 0.8\ \text{m/s})$

(iii) initial condition ($t = 0$):
$(x_0, y_0) = (-0.5R,\ 0)$
$(v_{x,0}, v_{y,0}) = (0,\ 0.45\ \text{m/s})$

(iv) initial condition ($t = 0$):
$(x_0, y_0) = (-0.5R,\ 0)$
$(v_{x,0}, v_{y,0}) = (0,\ 0.328\ \text{m/s})$

(v) initial condition ($t = 0$):
$(x_0, y_0) = (-0.5R,\ 0)$
$(v_{x,0}, v_{y,0}) = (0.47\cos(45°),\ 0.47\sin(45°))$ m/s

(vi) initial condition ($t = 0$):
$(x_0, y_0) = (-0.5R,\ 0)$
$(v_{x,0}, v_{y,0}) = (0.283\cos(45°),\ 0.283\sin(45°))$ m/s

where (v) and (vi) have initial velocities set at an angle of 45 deg. relative to the x-axis, which will have more interesting trajectories.

Set the following constants to: $\omega = 1\ \text{rad/sec}, R = 1\ \text{m}$

When plotting the trajectories, also draw a circle of radius R to indicate the boundary of the merry-go-round, which will help visualize the trajectory of the hockey puck as well as when it goes out of bounds. If possible, you may also find the time T the puck takes to reach the boundary.