PROBLEM SET 0

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem.

1) **(20 pts)** If a projectile is fired from the origin of the coordinate system with an initial velocity v_0 and in a direction making an angle α with the horizontal

   a) **(5 pts)** show that the time t' required for the projectile to cross a line l passing through the origin and making an angle $\beta < \alpha$ with the horizontal can be expressed as $t' = \dfrac{l\cos(\beta)}{v_0\cos(\alpha)}$

   b) **(5 pts)** using the time t' it takes the projectile to cross the line determined in part (a), show that the line can be expressed as $l = \dfrac{2v_0^2\cos(\alpha)}{g\cos^2(\beta)}\sin(\alpha - \beta)$

   c) **(5 pts)** assuming that β is *constant*, show that the angle at which the range l is *maximized* is given by $\alpha_0 = \dfrac{\pi}{4} + \dfrac{\beta}{2}$

   d) **(5 pts)** using the results from the previous parts (b) and (c), show that the maximum range for the line l is given by $l_{max} = \dfrac{v_0^2}{g(1 + \sin\beta)}$

2) **(5 pts)** Consider a projectile fired vertically in a constant gravitational field, g. For the same initial velocities, v_0, determine the time required for the projectile to reach its *maximum* height for

a) **(2 pts)** zero resisting force

b) **(3 pts)** a resisting force proportional to the instantaneous velocity of the projectile

3) **(20 pts)** A projectile of mass m under the influence of a resistive force, $F_r = -mkv^2$ (k is a positive constant) is projected vertically upward in a constant gravitational field g with an initial speed v_0.

a) **(5 pts)** show that the speed of the projectile as its thrown upwards can be expressed as:

$$v(t) = v_t \tan(\ v_t\ (\alpha - kt)\ ),$$

where $v_t = \sqrt{g/k}$ is the terminal speed and $\alpha v_t = \tan^{-1}(v_0/v_t)$

b) **(5 pts)** show that the maximum height h reached by the projectile is $h = \frac{1}{k}\ln\left(1/\cos(\alpha v_t)\right)$

c) **(5 pts)** as the projectile falls show that its vertical position as a function of its speed can be expressed as $y(v) = \frac{1}{k}\ln\left[v_t \Big/ \sqrt{v_t^2 - v^2}\right]$.
Hint: At maximum height, reset initial conditions to $t = 0$ and $v(t = 0) = 0$, and note that this is a different problem than the upward moving projectile, and must be treated separately from the previous parts.

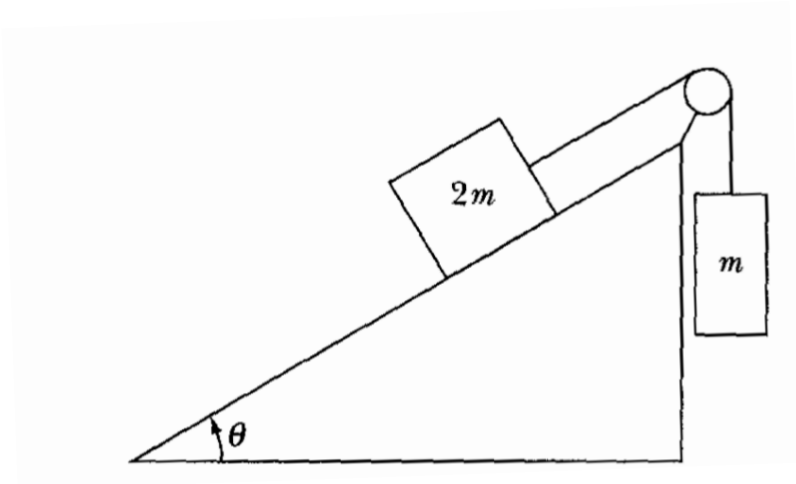
d) **(5 pts)** show that the speed of the projectile when it returns to the initial position is $v = v_t v_0 \Big/ \sqrt{v_0^2 + v_t^2}$. ***Hint:*** start from part (c) and also use the result from part (b) for the total distance the projectile travels back to its initial position

4) **(5 pts)** A particle of mass m slides down an inclined plane under the influence of gravity. If the motion is resisted by a force $f = kmv^2$, show that the time required to move a distance d after starting from rest is

$$t = \frac{\cosh^{-1}(e^{kd})}{\sqrt{kg \sin\theta}},$$

where θ is the angle of inclination of the plane.

5) **(5 pts)** Two blocks of unequal mass are connected by a string over a smooth pulley. If the coefficient of kinetic friction is μ_k, what angle θ of the incline allows the masses to move at a constant speed ?



6) **(15 pts)** A boat with initial speed v_0 is launched on a lake. The boat is slowed by the water by a force $F = -\alpha e^{\beta v}$.

a) **(5 pts)** find an expression for the speed $v(t)$

b) **(5 pts)** find the time for the boat to stop

c) **(5 pts)** find the distance for the boat to stop

7) **(10 pts)** A skier of mass m starts from rest down a hill inclined at θ. He skis a distance L down the hill and then coasts for a distance l along level snow until he stops. ***Hint:*** Use energy considerations and work done by friction

a) **(5 pts)** find a general expression for the coefficient of kinetic friction between the skis and the snow in terms of $L, l,$ and θ

b) **(5 pts)** find a general expression for the velocity the skier have at the bottom of the hill in terms of μ_k, l and θ

8) **(20 pts)** A particle of mass m moving in one dimension has a potential energy $U(x) = U_0[2(x/a)^2 - (x/a)^4]$, where U_0 and a are positive constants.

a) **(5 pts)** find the force $F(x)$, which acts on the particle

b) **(5 pts)** sketch $U(x)$ and find the x positions of stable and unstable equilibrium (use 1st and 2nd derivatives of $U(x)$ to find the points)

c) **(5 pts)** what is the minimum speed the particle must have at the origin to escape to infinity ?

d) **(5 pts)** at $t = 0$ the particle is at the origin and its velocity is positive and equal in magnitude to the escape speed of part (c); show that the general expression for the particle position can be expressed as $x(t) = a \tanh(\alpha t)$, where $\alpha^2 = \dfrac{2U_0}{ma^2}$