PROBLEM SET 1

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem. (**Max Score:** 80)

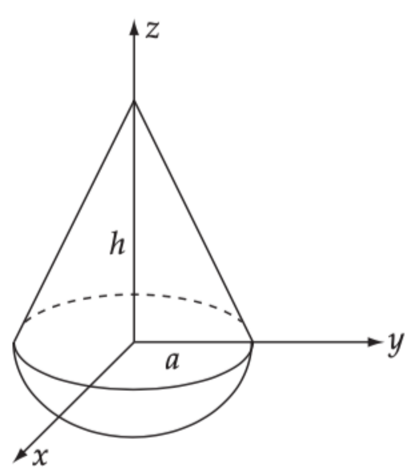
General Hint: Uniform mass distributions refer to a mass density or mass per unit length, area or volume, e.g., for a uniform volume: $\rho_m = M/V = dm/dV$

1) **(10 pts)** Find the center-of-mass (CM) of a uniformly solid cone of base diameter $2a$ and height h and a solid hemisphere of radius a where the two bases are touching.

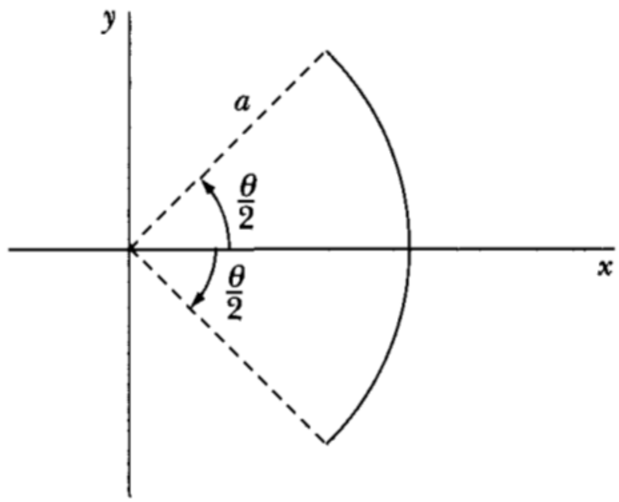
   Hint: calculate the CM of the shapes separately, using: $\vec{R} = \frac{1}{M}\int \vec{r}dm$

   then combine their results using the discrete formula, $\vec{R} = \frac{1}{M}\sum \vec{r}_i m_i$

   might be easier to use polar coordinates.

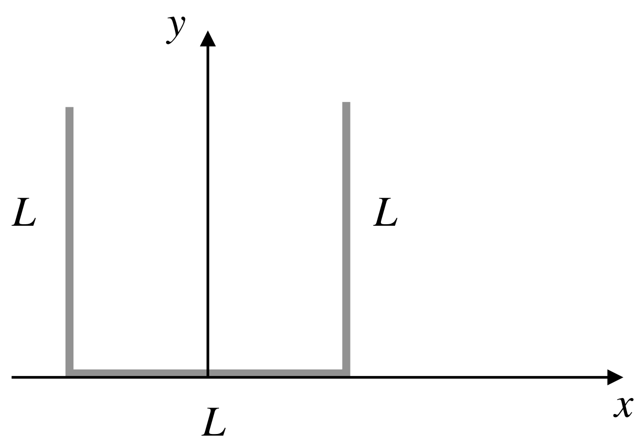


2) **(5 pts)** Find the center-of-mass (CM) of a uniform wire that subtends an arc θ if the radius of the circular arc is a.

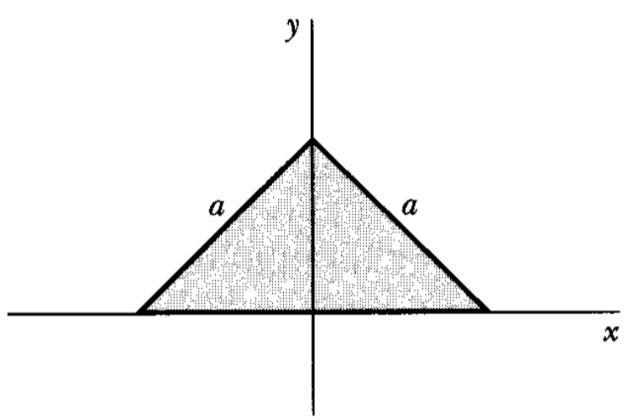


3) **(10 pts)** Find the center-of-mass (CM) of each of the two systems described below:

a) **(5 pts)** three uniform rods of equal mass M and equal length L arranged in a U shape

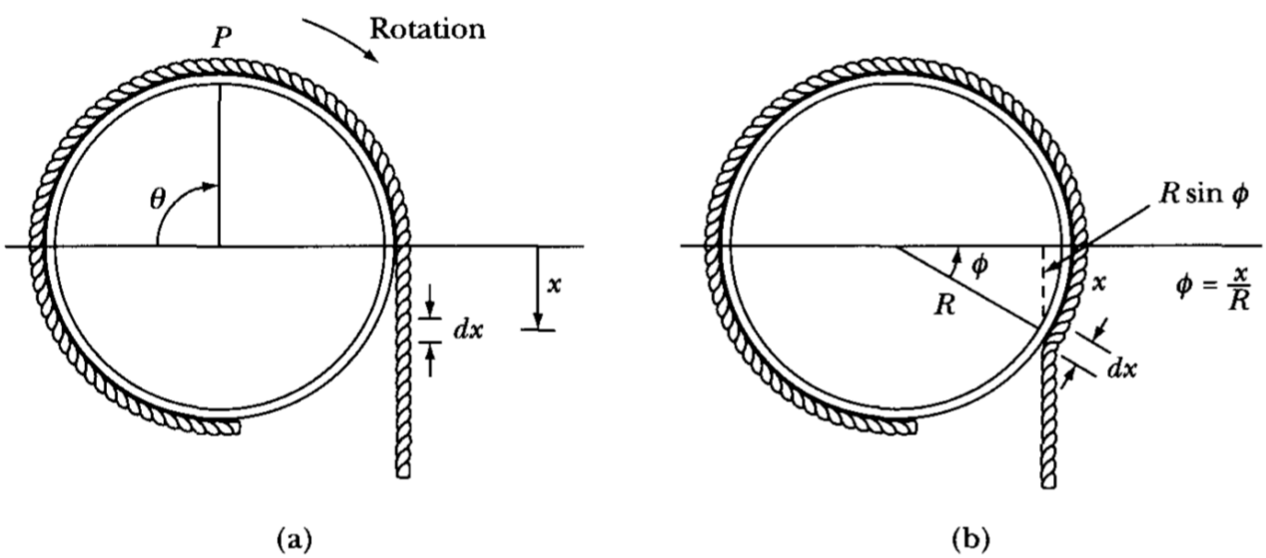


b) **(5 pts)** a uniform very thin plate of mass M is in the shape of an isosceles triangle with each side of length a



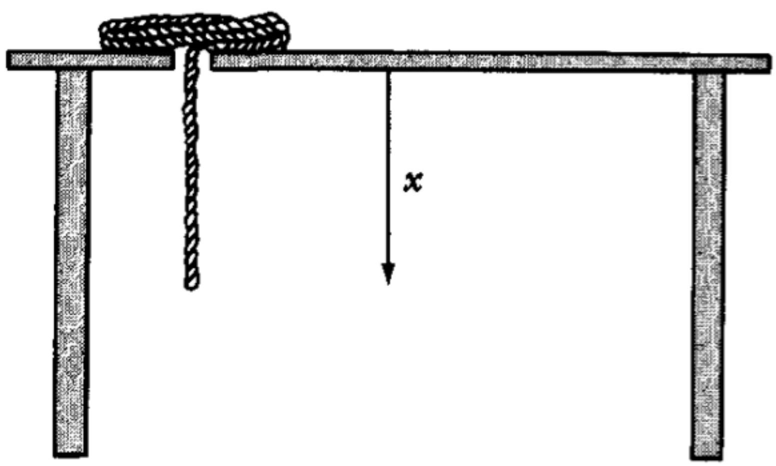
4) **(15 pts)** A rope of uniform linear density λ and mass m is wrapped one complete turn around a hollow cylinder of mass M and radius R. The cylinder rotates about its axis as the rope unwraps. The rope ends are at $x = 0$ (one fixed, one loose) when point P is at $\theta = 0$ [fig (a)], and the system is slightly displaced from equilibrium at rest [fig (b)]. Find the angular velocity as a function of angular displacement θ of the cylinder.

Hint: (i) first calculate the work done to displace the loose end of the rope *vertically*, (ii) then calculate the kinetic energy of both the rope and cylinder, as the rope unwinds and finally, (iii) use the work-energy theorem to equate the work done to change in kinetic energy and solve for angular velocity, $d\theta/dt$.

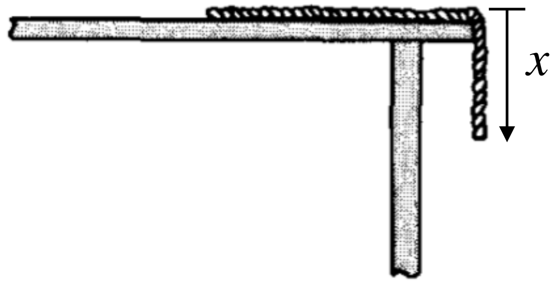


5) **(10 pts)** A smooth rope of uniform linear density λ and length L is placed above a hole in a table. One end of the rope falls through the hole at $t = 0$, pulling steadily on the remainder of the rope. Find the velocity and acceleration of the rope as a function of the distance to the end of the rope x. Ignore friction.

Hint: (i) the change in momentum of the hanging section of rope involves changes in both its mass and speed with time; (ii) the mass of the x section can be expressed as $m = \lambda x$, where λ is the uniform mass density of the rope



6) **(10 pts)** A flexible rope of mass m and length L slides from a *frictionless* table top. The rope is initially released from rest with a length x_0 hanging from the edge of the table.



a) Find the general differential equation for the acceleration of the rope and solve it.

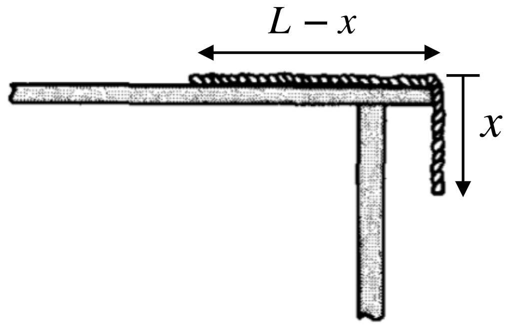
Hint: (i) If x is an arbitrary length of rope after its released from rest, then its mass is $(m/L)x$, and apply Newton's 2nd law to this section of the rope ; (ii) to solve the differential equation, think of what linear combination of a function $x(t)$ after taking two time derivatives, returns the same function back times a *positive* constant ?

b) Given the initial conditions, $x(t = 0) = x_0$, $v_x\ (t = 0) = 0$ m/s, show that the time at which the left end of the rope reaches the edge of the table can be expressed as

$$t = \frac{1}{\omega}\cosh^{-1}\left(\frac{L}{x_0}\right), \text{ where } \omega \equiv \sqrt{g/L}$$

Hint: (iii) use the following relation useful $\cosh u = (e^u + e^{-u})/2$ to simplify and solve for time

7) **(10 pts)** Consider the uniform rope of the previous problem *with friction*. The rope mass m and length L is stretched horizontally across a table top with a length x of the rope hanging vertically over the side of the table. The coefficient of kinetic friction between the rope and the table top is μ_k. The hanging end of the rope is given a quick pull to get it moving and then the rope is allowed to slide off the end of the table.



a) Give general expressions for the i) *gravitational* force on the hanging end of the rope, ii) the *kinetic frictional* force on the part of the rope on the table and iii) the *net* force acting on the chain as functions of the distance x

b) Determine the value of $x_{\min}$, the minimum value of x that is required for the chain to keep moving once the motion is initiated. If $x < x_{\min}$, the chain will stop moving immediately after it is pulled. If $x > x_{\min}$, it will keep moving until the whole chain has slid off the table.

c) Assuming that the chain always moves as a whole and starting with initial velocity, $v_x \approx 0$ when $x = x_{\min}$, show that the velocity of the chain at the moment when the whole chain is clear of the table (i.e., when $x = L$) is given by

$$v_x = \sqrt{\frac{gL}{\mu_k + 1}}$$

Hint: Use the general results from part a) to calculate the work done by gravity to move the hanging part of the rope, and the work by friction to move the part of the rope on the table. Then from the work-energy theorem, $\Delta K = W_g + W_f$, determine v_x

8) **(10 pts)** A deuteron (nucleus of deuterium atom consisting of a proton and a neutron) with speed 14.9 km/s collides *elastically* with a neutron at rest. Use the approximation that the deuteron is twice the mass of the neutron.

Hint: use energy and momentum conservation

a) If the deuteron is scattered through a lab angle $\theta = 10°$, what are the final speeds of the deuteron and the neutron ?

b) What is the scattering angle of the neutron ?