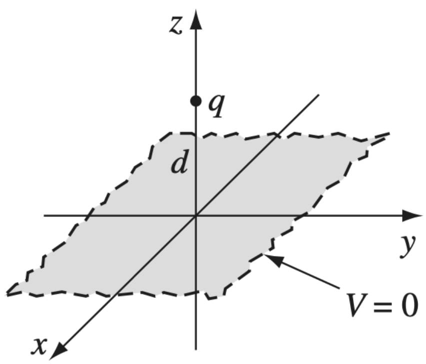
HW Ch.3 (Part 1: Method of Images)

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem.

The **1st uniqueness theorem** states that the potential in a volume $\mathscr{V}$ is uniquely determined if (i) the charge density thoughtout the region, and (ii) the value of V on all boundaries, are specified. Use the *method of images* in the problems below to solve for the potential in the region of interest.

1) **(25 pts)** A point charge q is held a distance d above an infinite grounded conducting plane inducing a charge, q_{ind} on the plane.



(a) **(5 pts)** Find a general expression for the potential, $V(\mathbf{r})$, in the region $z > 0$above the plane.

(b) **(5 pts)** Find the electric field, $\mathbf{E} = -\nabla V$, in the region $z > 0$ above the plane.

(c) **(5 pts)** Find the induced surface charge density, σ_{ch}, on the plane

(d) **(5 pts)** Find the electric force $\mathbf{F} = q\mathbf{E}$ between the induced charge on the plane and the point charge. *Hint*: be careful to exclude the field contribution from the point charge $+q$.

(e) **(5 pts)** Find the energy stored in the electric field between the point charge and the plane. *Hint*: Find the work required to bring the point charge from $z = \infty \rightarrow z = d$

2) **(20 pts)** A point charge q is situated a distance a from the center of a grounded conducting sphere of radius R.

(a) **(5 pts)** Show that the general expression for the potential, $V(\mathbf{r})$, in the region $r > R$outside the sphere is given by

$$V(r, \theta) = \frac{1}{4\pi\epsilon_0}\left[\frac{q_{\text{ind}}}{\sqrt{r^2 + b^2 - 2rb\cos\theta}} + \frac{q}{\sqrt{r^2 + a^2 - 2ra\cos\theta}}\right]$$

where $q_{\text{ind}} = -qR/a$ and $b = R^2/a$, determined from the boundary conditions. *Hint*: strategically select two points ($\theta = 0,\ \pi$) at the surface of the sphere, when solving for ($q_{\text{ind}},\ b$).

(b) **(5 pts)** Find the electric field components, E_r, E_θ, E_ϕ , in the region $r > R$

Hint: use the ∇operator in spherical coordinates to determine the individual components, (E_r, E_θ, E_ϕ)

(c) **(5 pts)** Show that the induced surface charge density on the sphere is given by

$$\sigma_{\text{ch}} = \frac{-q}{4\pi R}\frac{(a^2 - R^2)}{(R^2 + a^2 - 2Ra\cos\theta)^{3/2}}$$

(d) **(5 pts)** Using $\mathbf{F} = q\mathbf{E}$, show that the force between the induced charge on the sphere and the point charge is given by

$$F_r = -\frac{1}{4\pi\epsilon_0}\frac{q^2Ra}{(a^2 - R^2)^2}\ .\ \ (\textit{Hint}\text{: exclude the field from charge } +q)$$

3) **(15 pts)** A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say the wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy plane.)

[*Hint*: using the method of images, treat the conducting plane as an i*nfinite straight wire "image",* where now the problems reduced to calculating the electric field / potential due to two straight wires, one positive and one negative.]

(a) **(5 pts)** Calculate the total electric field $\mathbf{E}$ in the region above the plane (*Hint*: see problem 2.13 as reminder)

(b) **(5 pts)** Find the potential in the region above the plane and show it can be expressed as

$$V = \frac{\lambda}{4\pi\epsilon_0} \ln\left[\frac{y^2 + (z+d)^2}{y^2 + (z-d)^2}\right]$$

(*Hint*: see problem 2.22, 2.52 as reminder)

(c) **(5 pts)** Find the charge density σ_{ch} induced on the conduction plane and show it can be expressed as

$$\sigma_{\text{ch}}(y) = -\frac{\lambda d}{\pi(y^2 + d^2)}$$