HW Ch.4 Electric Fields in Matter : Part 2

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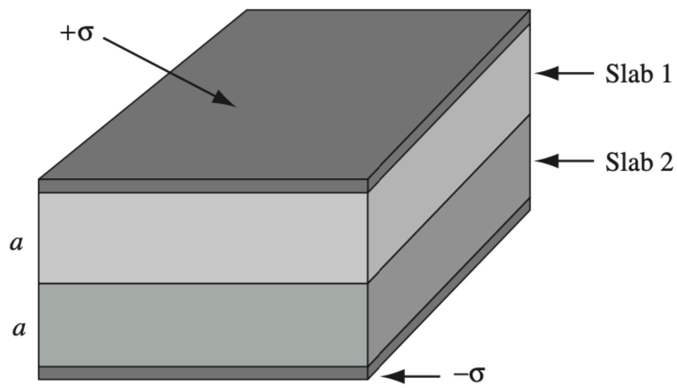
1) **Problem 4.18 (30 pts)**
The space between the plates of a parallel-plate capacitor is filled with two slabs of linear dielectric material. Each slab has thickness a, so the total distance between the plates is $2a$. Slab 1 has a dielectric constant of $\epsilon_r = 2$, and slab 2 has a dielectric constant of $\epsilon_r = 1.5$. The free charge density on the top plate is σ and on the bottom plate $-\sigma$.



(a) **(5 pts)** Find the electric displacement **D** in each slab.
(*hint*: apply $\int \mathbf{D} \cdot d\mathbf{a} = Q_{f_{\text{enc}}}$ to each slab/metal-plate interface)

(b) **(5 pts)** Find the electric field **E** in each slab and show it can be expressed as $E_1 = \sigma/2\epsilon_0, \;\; E_2 = 2\sigma/3\epsilon_0$

(c) **(5 pts)** Find the polarization **P** in each slab
(*Answer*: $P_1 = \sigma/2,\ P_2 = \sigma/3$)

(d) **(5 pts)** Find the potential difference between the plates
(*Answer*: $V = 7\sigma a/6\epsilon_0$)

(e) **(5 pts)** Find the location and amount of all volume and surface bound charge

(f) **(5 pts)** Now that you know all the charge (free and bound), recalculate the **E**field in each slab, and confirm your answer to (b).

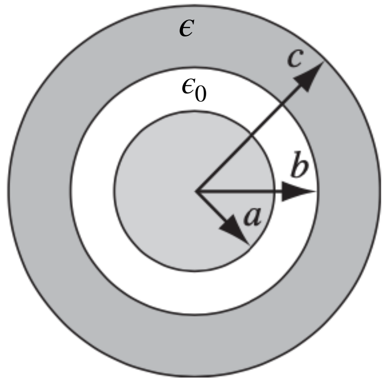
2) **Problem 4.20** **(10 pts)**

A sphere of linear dielectric material has embedded in it a uniform *free* charge density ρ. Find the potential at the center of the sphere (relative to infinity), if its radius is R and the dielectric constant is ϵ_r, and show it can be expressed as

$$V = \frac{\rho R^2}{3\epsilon_0}\left(1 + \frac{1}{2\epsilon_r}\right)$$

hint: start with finding the electric displacement $\mathbf{D}$ and then the field $\mathbf{E}$

3) **Problem 4.21** **(15 pts)**
A certain coaxial cable consists of a copper wire, radius a, surrounded by a concentric copper tube of inner radius c. The space between is partially filled (from b out to c) with material of dielectric constant (relative permittivity) $\epsilon_r = \epsilon/\epsilon_0$, as shown.



(a) **(5 pts)** Let Q be the charge on a length ℓ of the inner conductor and show that the electric field at different regions of the cylinder is given by

$$E(a < s < b) = \frac{Q}{2\pi\epsilon_0 s\ell}\,, \qquad E(b < s < c) = \frac{Q}{2\pi\epsilon s\ell}$$

(b) **(5 pts)** Show that the electric potential between the inner (s=a) and outer (s=c) cylindrical conductors is given by

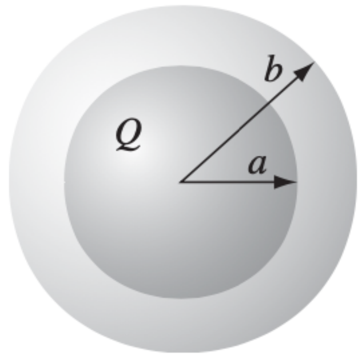
$$V = \frac{Q}{2\pi\epsilon_0\ell}\left[\ln\left(\frac{a}{b}\right) + \frac{\epsilon_0}{\epsilon}\ln\left(\frac{c}{b}\right)\right]$$

(hint: use $V = -\int \mathbf{E} \cdot d\mathbf{l}$ from the outer to inner conductor)

(c) **(5 pts)** Using the result from (b), show that the capacitance per unit length of this cable and show it can be expressed as

$$\frac{C}{\ell} = \frac{Q}{V\ell} = \frac{2\pi\epsilon_0}{\ln(b/a) + (1/\epsilon_r)\ln(c/b)}$$

4) **Problem 4.26** **(15 pts)**
A spherical conductor, of radius a, carries a charge Q. It is surrounded by linear dielectric material of susceptibility χ_e, out to radius b.



(a) **(5 pts)** Find the electric displacement in each region and show it can be expressed as: $\mathbf{D}(r < a) = 0$, $\mathbf{D}(r > a) = \dfrac{Q}{4\pi r^2}\hat{\mathbf{r}}$

(b) **(5 pts)** Find the electric field in each region and show it can be expressed as:

$$\mathbf{E}(r < a) = 0, \quad \mathbf{E}(a < r < b) = \frac{Q}{4\pi\epsilon r^2}\hat{\mathbf{r}}, \quad \mathbf{E}(r > b) = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$

(c) **(5 pts)** Find the energy of this configuration (use Eq. 4.58) and show it can be expressed as: $W = \dfrac{Q^2}{8\pi\epsilon_0(1+\chi_e)}\left(\dfrac{1}{a} + \dfrac{\chi_e}{b}\right)$