HW Ch.3 (Part 3: Multipole Expansion)

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem.

1) **Problem 3.27 (15 pts)**
A sphere of radius R, centered at the origin, carries charge density

$$\rho(r,\theta) = k\frac{R}{r^2}\,(R-2r)\;\sin\theta$$

where k is a constant, and r, θ are the usual spherical coordinates. Find the approximate potential contribution from

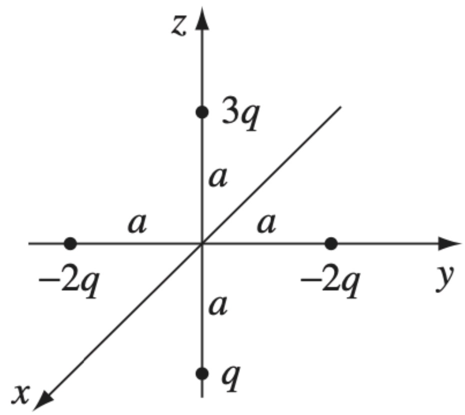
(a) **(5 pts)** the *monopole* term

(b) **(5 pts)** the *dipole* term

(c) **(5 pts)** the *quadrupole* term

for a point on the z-axis, *far* from the sphere, in other words take the limit as $r \to z$.

2) **Problem 3.29 (5 pts)**
Four particles are placed as shown in the figure below, each a distance a from the origin.



Find a simple approximate formula for the potential, valid at points *far* from the origin and show it can be expressed as

$$V \approxeq \frac{1}{4\pi\epsilon_0} \frac{2qa\cos\theta}{r^2}$$

3) **Problem 3.30** **(10 pts)**
In Example 3.9, we derived the exact potential for a spherical shell of radius R, which carries a surface charge $\sigma = k \cos\theta$

(a) **(5 pts)** Calculate the dipole moment of this charge distribution and show it can be expressed as

$$\mathbf{p} = \frac{4\pi R^3 k}{3}\hat{\mathbf{z}}$$

(b) **(5 pts)** Find the approximate potential from the dipole contribution, at points far from the sphere, and show it can be expressed as

$$V \approxeq \frac{kR^3}{3\epsilon_0}\frac{\cos\theta}{r^2}$$

Compare it to the exact answer (Eq. 3.87). What can you conclude about the higher multipoles ?

4) **Problem 3.33** **(15 pts)**
A "pure" dipole p is situated at the origin, pointing in the z direction.

(a) **(5 pts)** Show that the force on a point charge q at $(a, 0, 0)$ (Cartesian coordinates) can be expressed as

$$\mathbf{F} = q\mathbf{E} = -\frac{pq}{4\pi\epsilon_0 a^3}\hat{\mathbf{z}}$$

Hint: first find the electric field $\mathbf{E}(r, \theta)$, at the location of the point charge starting from Eq. 3.103

(b) **(5 pts)** Show that the force on q at $(0, 0, a)$ can be expressed as

$$\mathbf{F} = q\mathbf{E} = \frac{2pq}{4\pi\epsilon_0 a^3}\hat{\mathbf{z}}$$

(c) **(5 pts)** How much work does it take to move q from $(a, 0, 0)$ to $(0, 0, a)$?

5) **Problem 3.34** **(10 pts)**
Three point charges are located as shown in the figure, each a distance a from the origin.

(a) **(5 pts)** Find the approximate electric potential for the two lowest orders (*monopole* and *dipole*) of the multipole expansion *far* from the origin and show it can be expressed as

$$V \simeq \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{r}+\frac{a\cos\theta}{r^2}\right)$$

(b) **(5 pts)** Find the corresponding approximate electric field and show it can be expressed as

$$\mathbf{E}(r,\theta) \simeq \frac{q}{4\pi\epsilon_0}\left[-\frac{1}{r^2}\hat{\mathbf{r}}+\frac{a}{r^3}\left(2\cos\theta\,\hat{\mathbf{r}}+\sin\theta\,\hat{\boldsymbol{\theta}}\right)\right]$$