HW Ch.4 Electric Fields in Matter : Part 1

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1) **Problem 4.2 (15 pts)**
According to quantum mechanics, the electron cloud for a hydrogen atom in the ground state has a charge density

$$\rho(r) = \frac{q}{\pi a^3} e^{-2r/a} \ ,$$

where q is the charge of the electron and a is the Bohr radius.

(a) **(5 pts)** Calculate the electric field of the electron cloud, $E_e(r)$ and show it can be expressed as

$$E_e = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\left[1 - e^{-2r/a}\left(1 + 2\frac{r}{a} + 2\frac{r^2}{a^2}\right)\right]$$

Useful Integral: $\int e^{-2r'/a}(r')^2 dr' = -\frac{a}{2}e^{-2r'/a}\left((r')^2 + ar' + \frac{a^2}{2}\right) + C$

(b) **(5 pts)** In the presence of an external field, i.e., E_e (the electron cloud), the nucleus (proton in this case) will be shifted by some distance $r = d$ from the origin ($r = 0$), resulting in a polarized configuration, with the external field being $E = E_e(r = d)$. Using this fact, expand the exponential term of the external field in a power series of (d/a), assuming $r \ll a$ and truncate the 4th and higher order terms: $\geq (d/a)^4$, in the field to show that it can be expressed as

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{d^2}\left(\frac{4}{3}\frac{d^3}{a^3}\right)$$

(c) **(5 pts)** Starting from the result of part (b), calculate the atomic polarizability, α, and show it can be expressed as $\alpha = 3\pi\epsilon_0 a^3$

2) **Problem 4.9** **(10 pts)**
A dipole $\mathbf{p}$ is a distance r from a point charge q, and oriented so that $\mathbf{p}$ makes an angle θ with the vector $\mathbf{r}$ from q to $\mathbf{p}$. [Hint: start with Eq. 4.5]

(a) **(5 pts)** Calculate the force on $\mathbf{p}$ and show it can be expressed as

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}[\mathbf{p} - 3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]$$

Hint: start with calculating the electric field that q produces at the location of $\mathbf{p}$

(b) **(5 pts)** Calculate the force on q and show it can be expressed as

$$\mathbf{F} = q\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^3}[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}]$$

(this is consistent with the result of **Problem 3.36**)

3) **Problem 4.10** **(10 pts)**
A sphere of radius R carries a polarization

$$\mathbf{P}(\mathbf{r}) = k\mathbf{r},$$

where k is a constant and $\mathbf{r}$ is the vector from the center.

(a) **(5 pts)** Calculate the bound charges σ_b and ρ_b and show they have the values $\sigma_b = kR$ and $\rho_b = -3k$

(b) **(5 pts)** Find the field inside and outside the sphere, and show it takes on the values: $\mathbf{E}(r < R) = -(k/\epsilon_0)\mathbf{r}$ and $\mathbf{E}(r > R) = \mathbf{0}$

4) **Problem 4.15** **(10 pts)**
A thick spherical shell (inner radius a, and outer radius b) is made of dielectric material with a "frozen-in" polarization

$$\mathbf{P}(\mathbf{r}) = (k/r)\hat{\mathbf{r}},$$

where k is a constant and r is the distance from the center. (There is no *free* charge in the problem.) Find the electric field in all regions by two different methods:

(a) **(5 pts)** Locate all the bound charge, and use Gauss's law to calculate the field it produces in each region and show that it is:

$$\mathbf{E}(r < a) = \mathbf{0}, \quad \mathbf{E}(r > b) = \mathbf{0}, \quad \mathbf{E}(a < r < b) = -\,(k/\epsilon_0 r)\hat{\mathbf{r}}$$

(b) **(5 pts)** Use Eq. 4.23 to find $\mathbf{D}$, and then get $\mathbf{E}$ from Eq. 4.21. [Notice that the second method is much faster, and it avoids explicit reference to the bound charges.]