HW Ch.3 (Part 2: Separation of Variables)

Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem.

Cartesian Coordinates

1) **Problem 3.13** **(5 pts)**
Show that the potential in the infinite slot of Example 3.3 can be expressed as

$$V(x,y) = \frac{8V_0}{\pi} \sum_{n=2,6,10,...} \frac{e^{-n\pi x/a}\sin(n\pi y/a)}{n}$$

if the boundary condition at $x = 0$ consists of two metal strips: one, from $y = 0$ to $y = a/2$, is held at a constant potential V_0, and the other, from $y = a/2$ to $y = a$, is at potential $-V_0$

Hint: start from Eq. 3.30 and calculate the coefficients, C_n, (Eq. 3.34) for the first few values, $n = 1, 2, 3, 4$,.. and show that

$$C_n = \begin{cases} 8V_0/n, \ n = 2,6,10,14, \text{ etc. (in general, } 4j+2, \text{ for } j = 0,1,2,...) \\ 0, \qquad \text{otherwise} \end{cases}$$

2) **Problem 3.14** **(5 pts)**
For the infinite slot (Example 3.3), determine the charge density $\sigma(y)$ on the strip at $x = 0$, assuming it is a conductor at constant potential V_0, and show it can be expressed as

$$\sigma(y) = \frac{4\epsilon_0 V_0}{a} \sum_{n=1,3,5,...} \sin(n\pi y/a)$$

Spherical Coordinates

3) **Problem 3.19** **(20 pts)**
The potential at the surface of the sphere (radius R) is given by

$$V_0(R, \theta) = k \cos 3\theta,$$

where k is a constant. The second boundary condition is that the potential vanishes at larger distances: as $r \to \infty$, $V \to 0$. Assume that there is no charge inside or outside the sphere.

(a) **(5 pts)** To facilitate the computation of the potential inside and outside the sphere, it is helpful to express the potential $V_0(R, \theta)$ at the boundary in terms of the Legendre polynomials, as this will simplify the integration.

Use trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, and note that it as a linear combination of the 1st and 3rd order Legendre polynomials (see Table 3.1) , which has the general form

$$\cos 3\theta = \alpha P_3(\cos\theta) + \beta P_1(\cos\theta),$$

and show that the potential at the surface boundary can be expressed as:

$$V_0(R, \theta) = \frac{k}{5}[8P_3(\cos\theta) - 3P_1(\cos\theta)]$$

where $\alpha = 8/5$, $\beta = -3/5$ and $P_1 = \cos\theta$, $P_3 = (5\cos^3\theta - 3\cos\theta)/2$

(b) **(5 pts)** Staring from Eq. 3.65, find the potential **inside** the sphere and show it can be expressed as

$$V(r \leq R, \theta) = \frac{k}{5}\left[8(r/R)^3 P_3(\cos\theta) - 3(r/R)P_1(\cos\theta)\right]$$

(c) **(5 pts)** Staring from Eq. 3.65, find the potential **outside** the sphere and show it can be expressed as

$$V(r \geq R, \theta) = \frac{k}{5}\left[8(R/r)^4 P_3(\cos\theta) - 3(R/r)^2 P_1(\cos\theta)\right]$$

(d) **(5 pts)** Starting from Eq. 3.83, find the surface charge density, $\sigma_0(\theta)$ and show it can be expressed as

$$\sigma_0(\theta) = \frac{\epsilon_0 k}{5R}[-9P_1(\cos\theta) + 56P_3(\cos\theta)]$$