

## PROBLEM SET 8

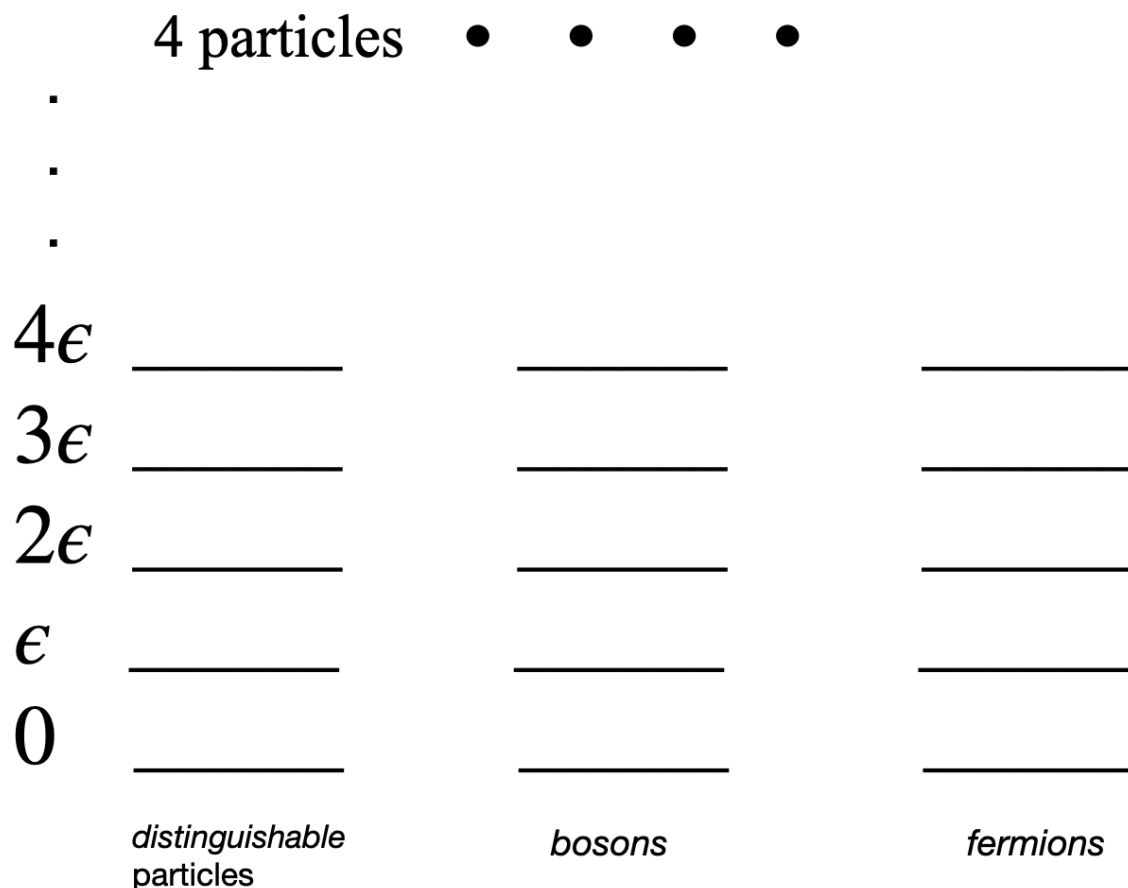
Show **ALL WORK** to get full/partial credit. Begin each problem on a new page, and clearly label each part of the problem.

1) **(10 pts) Text, problem 7.8**

2) **(50 pts)** A system has a series of evenly spaced, non-degenerate energy levels with energies  $\epsilon_0 = 0$ ,  $\epsilon_1 = \epsilon$ ,  $\epsilon_2 = 2\epsilon$ ,  $\epsilon_3 = 3\epsilon$ , etc. It is desired to fill these levels with a total of 4 particles. Consider three different cases: (1) the particles are *distinguishable*, (2) the particles are *bosons*, and (3) the particles are *fermions*.

(Hint: for each part make an energy level diagram, to explicitly describe the possible configurations. There is not limit in the energy diagram)

a) The ground state of a system is the multi-particle state which has the **lowest** possible energy. For each case above, populate the diagram that shows what is the ground-state distribution among the various energy levels. What is the total energy,  $\epsilon_{\text{g.s}}$  of the ground state for each case?



b) Now suppose that the system of 4 particles has a total energy  $\epsilon$  **above** the ground state energy,  $\epsilon_{\text{g.s.}}$ , determined in part a). For each case above, populate a diagram (similar to the one above) to show what are the possible distributions of the 4 particles in the various energy levels for this total energy and then determine how many micro-states there are for each such arrangement.

c) Repeat part b if the total energy **above** the ground state energy is  $2\epsilon$ , rather than  $\epsilon$ . (Note that there may be more than one arrangement (macro-state) for each case)

3) **(10 pts)** For a temperature  $T=300$  K, determine the equilibrium number of indistinguishable particles per state ( $\bar{n}_{\text{FD}}$  or  $\bar{n}_{\text{BE}}$ ) for an energy level with energy  $\epsilon$  if

a) the particles are *bosons* and (i)  $\epsilon - \mu = 0.001$  eV and (ii)  $\epsilon - \mu = 0.1$  eV.

b) the particles are *fermions* and (i)  $\epsilon - \mu = -0.1$  eV and (ii)  $\epsilon - \mu = +0.1$  eV.

4) **(10 pts)** Suppose that at temperature  $T=3T_{\text{F}}$ , the chemical potential of a system of fermions has the value  $\mu = -5.6\epsilon_{\text{F}}$ . For each of the energies below, calculate the Fermi-Dirac function  $\bar{n}_{\text{FD}}(\epsilon)$ .

a)  $\epsilon = 0$

b)  $\epsilon = \epsilon_{\text{F}}/2$

c)  $\epsilon = \epsilon_{\text{F}}$

d)  $\epsilon = 2\epsilon_F$

5) **(10 pts) Text Problem 7.9** – in the first part, compare the quantum volume with the volume per molecule of the gas at a temperature of 20°C and a pressure of 1 atm. In the second part, you should assume that as the temperature is lowered, the pressure is also lowered so that the volume per molecule stays constant. Then find the temperature at which the volume per molecule and the quantum volume become equal.

6) **(10 pts) Text Problem 7.12**