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Experimental Options for Measuring Spin-1 Tensor Observables from Electron Scattering

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Abstract. Recent developments in high-luminosity solid tensor polarized targets are allowing for new high-precision measurements of spin-one observables. This paper explores the options for extracting these observables from electron scattering data and discusses the benefits and constraints of each.

1 Introduction

Upcoming experiments to measure the deep-inelastic tensor structure function b_1 [1] and the quasi-elastic tensor asymmetry A_{zz} [2] utilizing inclusive electron scattering have motivated the development of a new solid tensor polarized target. The primary measured observable for both experiments is the tensor asymmetry A_{zz}, which can be measured in a variety of methods depending on the polarization values chosen for the experiment. This paper defines the various ways to measure the A_{zz} observable, along with additional vector and tensor spin asymmetries, so that these and future experiments can be optimized for maximum physics extraction potential. We will first define the various states in a simplified form that does not take into account effects from the acceptance, luminosity, or detector efficiency terms in order to show clearly sense of how A_{zz} can be extracted using the various methods, and then a more complete derivation will be shown taking these additional effects into account.

2 Definitions

Following the work of [3], beginning with their Eq. (23-28) the cross section for a polarized beam scattering off of a polarized deuteron target is given by

$$\begin{aligned}\frac{d^2\sigma}{dkd\Omega} &= \sigma(h_e, P, Q)\\ &= \sigma_u\left[1+h_e(A_e+PA^V_{ed}+QA^T_{ed})+PA^V_d+\frac{1}{2}QA_{zz}\right],\end{aligned} \tag{1}$$

where σ_u is the unpolarized cross section, h_e is the electron beam helicity, $P = \rho_{+1} - \rho_{-1}$ is the target vector polarization, $Q = \rho_{+1} + \rho_{-1} - 2\rho_0$ is the target tensor polarization, and we convert the notation of [3] to that of the upcoming Jefferson Lab experiments by using $\alpha^T_d = \frac{1}{2}A_{zz}$. We also include the beam single-spin asymmetry A^e for completeness, even though it is not included in [3].

In the case of an unpolarized electron beam where $h_e = 0$, this simplifies to

$$\sigma(0, P, Q) = \sigma_u\left[1 + PA^V_d + \frac{1}{2}QA_{zz}\right]. \tag{2}$$

As written in the proposals for the b_1 and A_{zz} experiments, extraction of A_{zz} can be obtained by solving the above equation for A_{zz}, which gives

$$A_{zz} = \frac{2}{Q}\left(\frac{\sigma(0, P, Q) - \sigma_u}{\sigma_u} - PA^V_d\right). \tag{3}$$

In the case where $PA^V_d \to 0$, this simplifies to the primary extraction used in [1, 2],

$$A_{zz} = \frac{2}{Q}\left(\frac{\sigma(0, P, Q) - \sigma_u}{\sigma_u}\right). \tag{4}$$

3 Multiple Polarization States

3.1 Two Target Polarization States with $h_e = 0$

As a more general alternative to the case of measuring A_{zz} using tensor enhanced and unpolarized cross-sections, we instead define two arbitrary P and Q states such that

$$\sigma_1 = \sigma(0, P_1, Q_1) = \sigma_u\left[1 + P_1A^V_d + \frac{1}{2}Q_1A_{zz}\right] \tag{5}$$

$$\sigma_2 = \sigma(0, P_2, Q_2) = \sigma_u\left[1 + P_2A^V_d + \frac{1}{2}Q_2A_{zz}\right]. \tag{6}$$

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We can then use this to relate the cross section asymmetry to the various observables by

$$\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}=\frac{(P_1-P_2)A_d^V+(Q_1-Q_2)\frac{1}{2}A_{zz}}{1+P_1A_d^V+\frac{1}{2}Q_1A_{zz}+1+P_2A_d^V+\frac{1}{2}Q_2A_{zz}}, \tag{7}$$

which when solved for A_{zz} gives

$$A_{zz}=2\left[\frac{\sigma_1-\sigma_2}{Q_1\sigma_2-Q_2\sigma_1}\right]\left[1+A_d^V\frac{(P_2\sigma_1-P_1\sigma_2)}{(\sigma_1-\sigma_2)}\right]. \tag{8}$$

We can also continue expanding the terms in Eq. (8) to get

$$A_{zz}=2\left[\frac{\sigma_1-\sigma_2}{Q_1\sigma_2-Q_2\sigma_1}\right]-2A_d^V\left[\frac{P_1\sigma_2-P_2\sigma_1}{Q_1\sigma_2-Q_2\sigma_1}\right], \tag{9}$$

which reduces to Eq. (3) in the conditions described in Section 2 where $P_1=P$, $Q_1=Q$, and $P_2=Q_2=0$.

In the limiting case where $P_nA_d^V\to 0$, this simplifies to

$$A_{zz}=\frac{2(\sigma_1-\sigma_2)}{Q_1\sigma_2-Q_2\sigma_1}. \tag{10}$$

If we additionally define the conditions described in Section 2 where $Q_1=Q$, $\sigma_1=\sigma(0,P,Q)$, $Q_2=0$, and $\sigma_2=\sigma_u$, this reduces to Eq. (4) as expected.

In the special case where $P_1=P_2=P$, then

$$A_{zz}=2\left[\frac{\sigma_1-\sigma_2}{Q_1\sigma_2-Q_2\sigma_1}\right]\left[1+A_d^VP\right] \tag{11}$$

and in the special case where $P_1=-P_2=P$, then

$$A_{zz}=2\left[\frac{\sigma_1-\sigma_2}{Q_1\sigma_2-Q_2\sigma_1}\right]\left[1-A_d^VP\left(\frac{\sigma_1+\sigma_2}{\sigma_1-\sigma_2}\right)\right]. \tag{12}$$

The $P\left(\frac{\sigma_1+\sigma_2}{\sigma_1-\sigma_2}\right)$ term approaches the definition of $1/A_d^V$ in the case where the tensor polarization is the same in both states ($Q_1=Q_2$), which causes A_{zz} to approach zero. As such, this case should be avoided if attempting to extract A_{zz}.

3.2 Three Target Polarization States with $h_e=0$

We can also look at a case where we have three different target polarization states, $\sigma_+=\sigma(0,+P,Q)$, $\sigma_u=\sigma(0,0,0)$, and $\sigma_-=\sigma(0,-P,Q)$ where A_{zz} is determined from the two enhanced states individually giving an A_{zz}^+ and an A_{zz}^-,

$$A_{zz}^+=2\left[\frac{\sigma_+-\sigma_u}{Q\sigma_u}\right]\left[1-A_d^V\frac{P\sigma_u}{(\sigma_+-\sigma_u)}\right] \tag{13}$$

$$A_{zz}^-=2\left[\frac{\sigma_--\sigma_u}{Q\sigma_u}\right]\left[1+A_d^V\frac{P\sigma_u}{(\sigma_--\sigma_u)}\right]. \tag{14}$$

Combining these as $2A_{zz}=A_{zz}^++A_{zz}^-$ reduces to

$$A_{zz}=\frac{\sigma_++\sigma_--2\sigma_u}{Q\sigma_u}. \tag{15}$$

This measurement is of particular benefit in that it completely eliminates the contaminating A_d^V asymmetry.

Alternatively, we can define the three polarization states such that each of the three spin densities are maximized. We can maximize ρ_+ with $\sigma_+=\sigma(0,+P,Q)$, ρ_- with $\sigma_-=\sigma(0,-P,Q)$, and ρ_0 with $\sigma_0=\sigma(0,0,-2Q)$. These related directly to A_{zz} by propagating Eq. 2 such that

$$A_{zz}=\frac{1}{Q}\left(\frac{\sigma_++\sigma_--2\sigma_0}{\sigma_++\sigma_-+\sigma_0}\right). \tag{16}$$

This approach was used by [4] to extract A_d^T in the case where $P\approx Q\approx 1$.

3.3 Four Target Polarization States with $h_e=0$

We can further define four polarization states such that the target is always polarized as

$$\begin{aligned}\sigma_1&=\sigma(0,P_1,Q_1)\to\sigma(0,+P,Q)\\ \sigma_2&=\sigma(0,P_2,Q_2)\to\sigma(0,-P,Q)\\ \sigma_3&=\sigma(0,P_3,Q_3)\to\sigma(0,+P,0)\\ \sigma_4&=\sigma(0,P_4,Q_4)\to\sigma(0,-P,0)\end{aligned} \tag{17}$$

with the assumption that the optimal operation would have $P_1=P_3=+P$, $P_2=P_4=-P$, $Q_1=Q_2=Q$, and $Q_3=Q_4=0$. Keeping the terms general, we can define a tensor asymmetry by grouping the tensor enhanced $Q>0$ states and the tensor suppressed $Q\approx 0$ states such that

$$\begin{aligned}&\frac{(\sigma_1+\sigma_2)-(\sigma_3+\sigma_4)}{\sigma_1+\sigma_2+\sigma_3+\sigma_4} \qquad (18)\\ &=\frac{A_d^V(P_1+P_2-P_3-P_4)+\frac{1}{2}A_{zz}(Q_1+Q_2-Q_3-Q_4)}{4+A_d^V(P_1+P_2+P_3+P_4)+\frac{1}{2}A_{zz}(Q_1+Q_2+Q_3+Q_4)}.\end{aligned}$$

Putting this in terms of A_{zz} gives

$$\begin{aligned}A_{zz}=&\left[(Q_1+Q_2)(\sigma_3+\sigma_4)-(Q_3+Q_4)(\sigma_1+\sigma_2)\right]^{-1}\\ &\times\big[4(\sigma_1+\sigma_2-\sigma_3-\sigma_4)\\ &\qquad+2A_d^V[(P_1+P_2)(\sigma_3+\sigma_4)\\ &\qquad-(P_3+P_4)(\sigma_1+\sigma_2)]\big].\end{aligned} \tag{19}$$

Optimizing the polarization states such that $P_1=P_3=+P$, $P_2=P_4=-P$, $Q_1=Q_2=Q$, and $Q_3=Q_4=0$, this reduces to

$$A_{zz}=\left[\frac{2}{Q}\right]\left[\frac{\sigma(P,Q)+\sigma(-P,Q)}{\sigma(P,0)+\sigma(-P,0)}-1\right] \tag{20}$$

and is an alternative for eliminating the diluting A_d^V asymmetry. This extraction technique is particularly useful in cases where the tensor polarization can be rapidly changed while maintaining a large vector polarization, which is the case for the upcoming Jefferson Lab experiments.

4 Experimental Measurements

This section builds upon previous work [5] that was used for the initial A_{zz} and b_1 proposals [2, 1]. As these experiments will be utilizing an ND_3 target immersed in liquid helium, there will be a dilution factor between the total measured rates and the physics rates that contain just the deuteron information of interest. The total rates for ND_3 are

$$\begin{aligned} R &= \mathcal{A}\left[\mathcal{L}_{\mathrm{He}}\sigma_{\mathrm{He}} + \mathcal{L}_{\mathrm{N}}\sigma_{\mathrm{N}} + \mathcal{L}_{\mathrm{D}}\sigma_{\mathrm{D}}\right] \\ &= \mathcal{A}\Bigg[\mathcal{L}_{\mathrm{He}}\sigma^u_{\mathrm{He}} + \mathcal{L}_{\mathrm{N}}\sigma^u_{\mathrm{N}} + \mathcal{L}_{\mathrm{D}}\sigma^u_{\mathrm{D}}\Bigg(1 \\ &\qquad + h_e\left(A_e + PA^V_{ed} + QA^T_{ed}\right) + PA^V_d + \frac{1}{2}QA_{zz}\Bigg)\Bigg] \end{aligned} \tag{21}$$

where $\mathcal{A}$ is the acceptance $(\Delta\Omega\Delta E')$, σ^u_A is the unpolarized cross section of a given nucleus A, and $\mathcal{L}_A$ is the luminosity. The general form of the luminosity is

$$\mathcal{L}_A = \frac{I_{beam}}{e}\mathcal{N}\frac{\rho_A}{M_A}z_A p_{f_A}, \tag{22}$$

where $\frac{I_{beam}}{e} = \frac{q}{te} = \frac{n_e}{t}$ is the rate of incident electrons and q is the integrated beam charge over time t, $\mathcal{N}$ is Avogadro's number, ρ_A is the density, M_A is the atomic or molecular mass, z is the target thickness, and p_{f_A} is the packing fraction of the material. When using solid deuterated ammonia beads surrounded by liquid helium, the luminosities come out to

$$\begin{aligned} \mathcal{L}_{\mathrm{He}} &= \frac{q}{te}z_{\mathrm{tgt}}\mathcal{N}\frac{\rho_{\mathrm{He}}}{M_{\mathrm{He}}}\left(1-p_f\right), \\ \mathcal{L}_{\mathrm{N}} &= \frac{q}{te}z_{\mathrm{tgt}}\mathcal{N}\frac{\rho_{\mathrm{ND_3}}}{M_{\mathrm{ND_3}}}p_f, \\ \mathcal{L}_{\mathrm{D}} &= 3\frac{q}{te}z_{\mathrm{tgt}}\mathcal{N}\frac{\rho_{\mathrm{ND_3}}}{M_{\mathrm{ND_3}}}p_f. \end{aligned} \tag{23}$$

The factor of 3 in $\mathcal{L}_{\mathrm{D}}$ takes into account that there are three deuterium atoms in each ammonia molecule. Given that some small amount of beam charge drift is likely to occur during the experiment, we separate the charge rates from the luminosity such that

$$\begin{aligned} \mathcal{L}'_{\mathrm{He}} &= \mathcal{L}_{\mathrm{He}}/(q/t) = \frac{1}{e}\mathcal{N}\frac{\rho_{\mathrm{He}}}{M_{\mathrm{He}}}z_{\mathrm{tgt}}\left(1-p_f\right), \\ \mathcal{L}'_{\mathrm{N}} &= \mathcal{L}_{\mathrm{N}}/(q/t) = \frac{1}{e}\mathcal{N}\frac{\rho_{\mathrm{ND_3}}}{M_{\mathrm{ND_3}}}z_{\mathrm{tgt}}p_f, \\ \mathcal{L}'_{\mathrm{D}} &= \mathcal{L}_{\mathrm{D}}/(q/t) = 3\frac{1}{e}\mathcal{N}\frac{\rho_{\mathrm{ND_3}}}{M_{\mathrm{ND_3}}}z_{\mathrm{tgt}}p_f. \end{aligned} \tag{24}$$

As noted in the previous section, to extract A_{zz} we will be measuring events from multiple target states with $h_e = 0$. Also including detector efficiencies ϵ_n, the total number of counts for each state are

$$\begin{aligned} N_n &= R_n t_n \\ &= \mathcal{A}\epsilon_n q_n\Bigg[\mathcal{L}'_{\mathrm{He}}\sigma^u_{\mathrm{He}} + \mathcal{L}'_{\mathrm{N}}\sigma^u_{\mathrm{N}} \\ &\qquad + \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}\left(1 + P_nA^V_d + \frac{1}{2}Q_nA_{zz}\right)\Bigg]. \end{aligned} \tag{25}$$

If we include measurements with arbitrary h_e, we find the more general relation

$$\begin{aligned} &\frac{N_n}{\mathcal{A}\left[\mathcal{L}'_{\mathrm{He}}\sigma^u_{\mathrm{He}} + \mathcal{L}'_{\mathrm{N}}\sigma^u_{\mathrm{N}} + \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}\right]} \\ &\quad = \epsilon_n q_n + \epsilon_n q_n f\Bigg(h_e(A_e + P_nA^V_{ed} + Q_nA^T_{ed}) + P_nA^V_d \\ &\qquad + \frac{1}{2}Q_nA_{zz}\Bigg) \end{aligned} \tag{26}$$

where n represents each individual polarization state and

$$f = \frac{\mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}}{\mathcal{L}'_{\mathrm{He}}\sigma^u_{\mathrm{He}} + \mathcal{L}'_{\mathrm{N}}\sigma^u_{\mathrm{N}} + \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}} \tag{27}$$

is the dilution factor.

4.1 Two Target Polarization States with $h_e = 0$

In the case of two arbitrary polarization states with total counts $N_n(P_n, Q_n)$ and $h_e = 0$, we again begin with a traditional asymmetry and develop an equation to extract the physical quantity of interest, A_{zz}.

$$\begin{aligned} \frac{N_1 - N_2}{N_1 + N_2} = &\Bigg[\left[\mathcal{L}'_{\mathrm{He}}\sigma^u_{\mathrm{He}} + \mathcal{L}'_{\mathrm{N}}\sigma^u_{\mathrm{N}} + \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}\right](\epsilon_1 q_1 - \epsilon_2 q_2) \\ &+ \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}\Bigg[\left(P_1A^V_d + \frac{1}{2}Q_1A_{zz}\right)\epsilon_1 q_1 \\ &- \left(P_2A^V_d + \frac{1}{2}Q_2A_{zz}\right)\epsilon_2 q_2\Bigg]\Bigg] \\ \times &\Bigg[\left[\mathcal{L}'_{\mathrm{He}}\sigma^u_{\mathrm{He}} + \mathcal{L}'_{\mathrm{N}}\sigma^u_{\mathrm{N}} + \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}\right](\epsilon_1 q_1 + \epsilon_2 q_2) \\ &+ \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}\Bigg[\left(P_1A^V_d + \frac{1}{2}Q_1A_{zz}\right)\epsilon_1 q_1 \\ &+ \left(P_2A^V_d + \frac{1}{2}Q_2A_{zz}\right)\epsilon_2 q_2\Bigg]\Bigg]^{-1} \end{aligned} \tag{28}$$

Solving this for the tensor asymmetry we find

$$\begin{aligned} A_{zz} = &\frac{2}{f}\left[\frac{N_1\epsilon_2 q_2 - N_2\epsilon_1 q_1}{Q_1N_2\epsilon_1 q_1 - Q_2N_1\epsilon_2 q_2}\right] \\ &\times\left[1 - fA^V_d\left(\frac{P_1N_2\epsilon_1 q_1 - P_2N_1\epsilon_2 q_2}{N_1\epsilon_2 q_2 - N_2\epsilon_1 q_1}\right)\right]. \end{aligned} \tag{29}$$

In the simplifying case where the efficiencies are identical ($\epsilon_1 = \epsilon_2 = \epsilon$) and the beam charge for each state is $q_1 = q_2 = q/2$, this reduces to

$$\frac{N_1 - N_2}{N_1 + N_2} = \frac{fA^V_d(P_1 - P_2) + f\frac{1}{2}A_{zz}(Q_1 - Q_2)}{2 + fA^V_d(P_1 + P_2) + f\frac{1}{2}A_{zz}(Q_1 + Q_2)}. \tag{30}$$

and can be rearranged to find the tensor asymmetry

$$A_{zz} = \frac{2}{f}\left[\frac{N_1 - N_2}{Q_1N_2 - Q_2N_1}\right]\left[1 - fA^V_d\left(\frac{P_1N_2 - P_2N_1}{N_1 - N_2}\right)\right]. \tag{31}$$

4.2 Three Target Polarization States with $h_e = 0$

Expanding into the case of three polarization states, we find

$$\begin{aligned}\frac{N_1+N_2-2N_0}{N_1+N_2+N_0} = \Bigg[&(\epsilon_1 q_1 + \epsilon_2 q_2 - 2\epsilon_0 q_0) \\ &+ f\left(P_1 A_d^V + \frac{1}{2}Q_1 A_{zz}\right)\epsilon_1 q_1 \\ &+ f\left(P_2 A_d^V + \frac{1}{2}Q_2 A_{zz}\right)\epsilon_2 q_2 \\ &- 2f\left(P_0 A_d^V + \frac{1}{2}Q_0 A_{zz}\right)\epsilon_0 q_0\Bigg] \\ \times \Bigg[&(\epsilon_1 q_1 + \epsilon_2 q_2 + \epsilon_0 q_0) \\ &+ f\left(P_1 A_d^V + \frac{1}{2}Q_1 A_{zz}\right)\epsilon_1 q_1 \\ &+ f\left(P_2 A_d^V + \frac{1}{2}Q_2 A_{zz}\right)\epsilon_2 q_2 \\ &+ f\left(P_0 A_d^V + \frac{1}{2}Q_0 A_{zz}\right)\epsilon_0 q_0\Bigg]^{-1}\end{aligned} \tag{32}$$

from which we can extract the tensor asymmetry

$$\begin{aligned}A_{zz} \\ = \left[\frac{2}{f}\right]&\left[\frac{q_0\epsilon_0(N_1+N_2)-(q_1\epsilon_1+q_2\epsilon_2)N_0}{(Q_1q_1\epsilon_1+Q_2q_2\epsilon_2)N_0 - Q_0q_0\epsilon_0(N_1+N_2)}\right]\Bigg[1 \\ &+ fA_d^V\left(\frac{P_0q_0\epsilon_0(N_1+N_2)-(P_1q_1\epsilon_1+P_2q_2\epsilon_2)N_0}{q_0\epsilon_0(N_1+N_2)-(q_1\epsilon_1+q_2\epsilon_2)N_0}\right)\Bigg]\end{aligned} \tag{33}$$

In the case where $Q_0 = P_0 = 0$ and $P_1 = -P_2 = P$, we find that

$$\begin{aligned}A_{zz} = \left[\frac{2}{f}\right]&\left[\frac{q_0\epsilon_0(N_1+N_2)-(q_1\epsilon_1+q_2\epsilon_2)N_0}{(Q_1q_1\epsilon_1+Q_2q_2\epsilon_2)N_0}\right]\Bigg[1 \\ &- fPA_d^V\left(\frac{(q_1\epsilon_1-q_2\epsilon_2)N_0}{q_0\epsilon_0(N_1+N_2)-(q_1\epsilon_1+q_2\epsilon_2)N_0}\right)\Bigg].\end{aligned} \tag{34}$$

In the above general equation, we can completely eliminate the diluting asymmetry A_d^V if $\epsilon_1 q_1 = \epsilon_2 q_2 = \epsilon q/2$. This reduced form is

$$A_{zz} = \left[\frac{4}{f}\right]\left[\frac{q_0\epsilon_0(N_1+N_2)-q\epsilon N_0}{q\epsilon(Q_1+Q_2)N_0}\right]. \tag{35}$$

If a 2-cell target is utilized where one cell is always unpolarized and the second cell spends half its time in the (P, Q_1) state and half its time in the $(-P, Q_2)$ state, then $q_0 = q$, and if $\epsilon_0 = \epsilon$, then

$$A_{zz} = \left[\frac{4}{f}\right]\left[\frac{N_1+N_2-N_0}{(Q_1+Q_2)N_0}\right]. \tag{36}$$

Furthermore, if the same tensor enhancement is achieved in both polarized states such that $Q_1 = Q_2 = Q$, then this reduces to

$$A_{zz} = \left[\frac{2}{fQ}\right]\left[\frac{N_1+N_2-N_0}{N_0}\right]. \tag{37}$$

This form is effectively the same as that in [5], except that for the polarized state it explicitly spends half the time with positive vector polarization and half the time with negative vector polarization.

Alternatively, if the polarizations are optimized for maximum spin populations such that $P_1 = +P$, $Q_1 = Q_2 = Q$, $P_2 = -P$, $P_0 = 0$, and $Q_0 = -2Q$, then

$$\begin{aligned}A_{zz} = \left[\frac{1}{fQ}\right]&\left[\frac{q_0\epsilon_0(N_1+N_2)-(q_1\epsilon_1+q_2\epsilon_2)N_0}{q_0\epsilon_0(N_1+N_2)+\frac{1}{2}(q_1\epsilon_1+q_2\epsilon_2)N_0}\right]\Bigg[1 \\ &- fPA_d^V\left(\frac{(q_1\epsilon_1-q_2\epsilon_2)N_0}{q_0\epsilon_0(N_1+N_2)-(q_1\epsilon_1+q_2\epsilon_2)N_0}\right)\Bigg].\end{aligned} \tag{38}$$

Of particular note here is that the A_d^V term is both suppressed by the dilution factor f and disappears entirely as $q_1\epsilon_1 = q_2\epsilon_2$, making this a cleaner measurement of A_{zz} than the two-state term from Sec. 4.1. Additionally, if the integrated charge and efficiencies are identical ($q_0\epsilon_0 = q_1\epsilon_1 = q_2\epsilon_2 = q\epsilon$) then this form becomes the same as Eq. (16) as expected.

4.3 Four Target Polarization States with $h_e = 0$

Expanding into the case of four polarization states, we find

$$\begin{aligned}\frac{N_1+N_2}{N_3+N_4} = \Bigg[&(\epsilon_1q_1+\epsilon_2q_2) + fA_d^V(P_1\epsilon_1q_1+P_2\epsilon_2q_2) \\ &+ f\frac{1}{2}A_{zz}(Q_1\epsilon_1q_1+Q_2\epsilon_2q_2)\Bigg]\Bigg[(\epsilon_3q_3+\epsilon_4q_4) \\ &+ fA_d^V(P_3\epsilon_3q_3+P_4\epsilon_4q_4) \\ &+ f\frac{1}{2}A_{zz}(Q_3\epsilon_3q_3+Q_4\epsilon_4q_4)\Bigg]^{-1}\end{aligned} \tag{39}$$

Again solving for A_{zz}, we find the general formula for A_{zz} from four polarization states to be

$$\begin{aligned}A_{zz} \\ = \left[\frac{2}{f}\right]&\Big[[(\epsilon_3q_3+\epsilon_4q_4)(N_1+N_2) \\ &-(\epsilon_1q_1+\epsilon_2q_2)(N_3+N_4)]\,[(Q_1\epsilon_1q_1+Q_2\epsilon_2q_2)(N_3+N_4) \\ &- (Q_3\epsilon_3q_3+Q_4\epsilon_4q_4)(N_1+N_2)]^{-1}\Big] \\ \times&\Big[1+fA_d^V\,[(P_3\epsilon_3q_3+P_4\epsilon_4q_4)(N_1+N_2) \\ &-(P_1\epsilon_1q_1+P_2\epsilon_2q_2)(N_3+N_4)]\,[(\epsilon_3q_3+\epsilon_4q_4)(N_1+N_2) \\ &- (\epsilon_1q_1+\epsilon_2q_2)(N_3+N_4)]^{-1}\Big]\end{aligned} \tag{40}$$

Note that the diluting A_d^V term here is doubly suppressed in the case where P_2 and P_4 are negative by f and subtraction of the small vector polarization differences. Additionally, systematic effects from detector efficiency or beam current drifts would mostly cancel out.

Utilizing the case where $P_1 = P_3 = P$, $P_2 = P_4 = -P$, $Q_1 = Q_2 = Q$, and $Q_3 = Q_4 = 0$, we find that this simplifies to

$$A_{zz} = \left[\frac{2}{Qf}\right]\left[\frac{(\epsilon_3 q_3 + \epsilon_4 q_4)(N_1 + N_2)}{(\epsilon_1 q_1 + \epsilon_2 q_2)(N_3 + N_4)} - 1\right] \times \Big[1 + fPA_d^V\left[(\epsilon_3 q_3 - \epsilon_4 q_4)(N_1 + N_2) - (\epsilon_1 q_1 - \epsilon_2 q_2)(N_3 + N_4)\right]\left[(\epsilon_3 q_3 + \epsilon_4 q_4)(N_1 + N_2) - (\epsilon_1 q_1 + \epsilon_2 q_2)(N_3 + N_4)\right]^{-1}\Big] \tag{41}$$

where the diluting A_d^V is now quadruply suppressed by f, P, and the differences of the differences of the efficiencies. In addition, it's clear from the above that the uncertainty due to vector polarization will be similarly suppressed by f and A_d^V such that the primary uncertainties will be from statistical, dilution factor, and tensor polarization. In the even more ideal case where the efficiencies are equal ($\epsilon_1 q_1 = \epsilon_2 q_2 = \epsilon_3 q_3 = \epsilon_4 q_4 = \epsilon q$), then the diluting A_d^V term is completely eliminated and the tensor asymmetry becomes

$$A_{zz} = \left[\frac{2}{f}\right]\left[\frac{(N_1 + N_2) - (N_3 + N_4)}{Q(N_3 + N_4)}\right]. \tag{42}$$

4.4 Four Polarization States with $|h_e| > 0$

In the case where $h_e \neq 0$, we can find the four-polarization state to see its effect on these additional diluting terms. In this case, the number of counts for each state would be

$$N_n = R_n t_n = \mathcal{A}\epsilon_n q_n \Big[\mathcal{L}'_{\mathrm{He}}\sigma^u_{\mathrm{He}} + \mathcal{L}'_{\mathrm{N}}\sigma^u_{\mathrm{N}} + \mathcal{L}'_{\mathrm{D}}\sigma^u_{\mathrm{D}}\Big(1 + h_e(A_\parallel P_n + A_T^{ed} Q_n) + A_d^V P_n + \frac{1}{2}A_{zz}Q_n\Big)\Big] \tag{43}$$

Here we can solve the ratio $\frac{N_1+N_2}{N_3+N_4}$ for A_{zz}, which gives the result

$$A_{zz} = \left[\frac{2}{f}\right]\left[\frac{(\epsilon q)_{34}(N_1 + N_2) - (\epsilon q)_{12}(N_3 + N_4)}{(Q\epsilon q)_{12}(N_3 + N_4) - (Q\epsilon q)_{34}(N_1 + N_2)}\right] \times \Big[1 + f(h_e A_\parallel + A_d^V)\frac{(P\epsilon q)_{34}(N_1 + N_2) - (P\epsilon q)_{12}(N_3 + N_4)}{(\epsilon q)_{34}(N_1 + N_2) - (\epsilon q)_{12}(N_3 + N_4)} - f h_e A_T^{ed}\frac{(Q\epsilon q)_{12}(N_3 + N_4) - (Q\epsilon q)_{34}(N_1 + N_2)}{(\epsilon q)_{34}(N_1 + N_2) - (\epsilon q)_{12}(N_3 + N_4)}\Big] \tag{44}$$

where $(\epsilon q)_{ij} = \epsilon_i q_i + \epsilon_j q_j$, $(Q\epsilon q)_{ij} = Q_i\epsilon_i q_i + Q_j\epsilon_j q_j$, and $(P\epsilon q)_{ij} = P_i\epsilon_i q_i + P_j\epsilon_j q_j$.

There are a few things that stand out here regarding the importance of the diluting asymmetries. Both the A_d^V and $A_\parallel$ asymmetries are triply-suppressed by the dilution factor f, the vector polarization P, and the difference between the detector efficiency and beam charge drifts (keeping in mind that the upcoming JLab experiments are striving towards $P_1 = P_3 = P$, $P_2 = P_4 = -P$, $Q_1 = Q_2 = Q$, and $Q_3 = Q_4 = 0$). For the b_1 measurement, the dilution factor is expected to be on the order of 0.3, though in the A_{zz} experiment it ranges from a maximum of 0.7 on the elastic and QE peaks and dips to as low as 0.1 in the SRC region. The vector polarization is expected to average between 0.3 to 0.4 during the experiment. The level of efficiency and charge drifts is expected to be approximately the same as during the Transversity experiment, which was 0.00355. Note that this is the estimate of $\Delta\epsilon q$ from Transversity, and the A_{zz} is further suppressed by $\Delta(\Delta\epsilon q)$, meaning the 0.00355 estimate is likely a vast overestimate. Regardless, using the maximum/overestimate of these terms, both asymmetries are suppressed by a factor of $fP\Delta(\epsilon q) \approx 0.7 \cdot 0.4 \cdot 0.00355 = 0.001 = 0.1\%$ relative to A_{zz}, which is fairly negligible even assuming maximum asymmetries equal to 1. For b_1, both of these asymmetries are expected to be on the order of a few$\times 10^{-4}$, which further emphasizes that these terms are negligible for this experiment. Additionally, the $A_\parallel$ term is further suppressed by the average integrated beam helicity, which is expected to be similar to that seen during the Transversity experiment on the order of 3.8×10^{-5}. Even using a gross over-estimate of integrated beam helicity of 2×10^{-4}, it becomes immediately obvious that the $A_\parallel$ is completely negligible during these measurements of A_{zz}.

Of larger concern is the A_T^{ed} asymmetry, as the ratio in that term times the dilution factor is approximately equal to $\frac{2}{A_{zz}}$, making the entire term approximately equal to $-2h_e\frac{A_T^{ed}}{A_{zz}}$. Using our same over-estimate of the intergrated beam helicity, this term would only be suppressed by $2h_e \approx 2 \cdot 0.0002 = 0.0004 = 0.04\%$. The term would start coming into play if $\frac{A_T^e d}{A_z z} > 10^2$ (for a $\approx 4\%$ contamination) or 10^3 (for a $\approx 40\%$ contamination).

From [3], this diluting asymmetry is related to the beam-target double polarization asymmetry

$$A_T^{ed} = \frac{1}{4h_e Q\sigma_0}\left[\sigma(h,P,Q) + \sigma(h,-P,Q) - (\sigma(-h,P,Q) + \sigma(-h,-P,Q))\right] \tag{45}$$

Given that we are already planning to measure the $(\pm P, Q)$ states and will have access to the beam helicity states, we will very likely be able to measure this asymmetry simultaneously alongside the A_{zz} measurements for no additional beam time cost.

Since we're already measuring the following states,

$$\begin{aligned}\sigma_1 &= \sigma(P_1, Q_1) \rightarrow \sigma(+P, Q)\\ \sigma_2 &= \sigma(P_2, Q_2) \rightarrow \sigma(-P, Q)\\ \sigma_3 &= \sigma(P_3, Q_3) \rightarrow \sigma(+P, 0)\\ \sigma_4 &= \sigma(P_4, Q_4) \rightarrow \sigma(-P, 0)\end{aligned} \tag{46}$$

we can tag the beam helicity to simultaneously measure the states

$$\begin{aligned}\sigma_{1+} &= \sigma(h_e, P_1, Q_1) \rightarrow \sigma(+h_e, +P, Q)\\ \sigma_{1-} &= \sigma(-h_e, P_1, Q_1) \rightarrow \sigma(-h_e, +P, Q)\\ \sigma_{2+} &= \sigma(h_e, P_2, Q_2) \rightarrow \sigma(+h_e, -P, Q)\\ \sigma_{2-} &= \sigma(-h_e, P_2, Q_2) \rightarrow \sigma(-h_e, -P, Q)\\ \sigma_{3+} &= \sigma(h_e, P_3, Q_3) \rightarrow \sigma(+h_e, +P, 0)\\ \sigma_{3-} &= \sigma(-h_e, P_3, Q_3) \rightarrow \sigma(-h_e, +P, 0)\\ \sigma_{4+} &= \sigma(h_e, P_4, Q_4) \rightarrow \sigma(+h_4, -P, 0)\\ \sigma_{4-} &= \sigma(-h_e, P_4, Q_4) \rightarrow \sigma(-h_e, -P, 0)\end{aligned} \tag{47}$$

We can measure A_T^{ed} twice using these terms, once with tensor enhancement Q using $\sigma_{1+/-}$ and $\sigma_{2+/-}$, and once with tensor suppressed $Q = 0$ using $\sigma_{3+/-}$ and $\sigma_{4+/-}$. Given that the tensor polarization is not required for this asymmetry, both of those measurements should come out the same, so in determining the extraction below I'll only consider the first terms.

Let's consider the beam asymmetry measurement of

$$\begin{aligned}&A_T^{ed}\\ &= \left[\frac{1}{[h_{1-}Q_{1-}\epsilon_{1-}q_{1-}N_{1+} - h_{1+}Q_{1+}\epsilon_{1+}q_{1+}N_{1-}]}\right]\\ &\quad\times\Bigg[\epsilon_{1+}q_{1+}N_{1-}\\ &\quad - \epsilon_{1-}q_{1-}N_{1+}\\ &\quad + fA_{\parallel}(h_{1+}P_{1+}\epsilon_{1+}q_{1+}N_{1-} - h_{1-}P_{1-}\epsilon_{1-}q_{1-}N_{1+})\\ &\quad + fA_d^V(P_{1+}\epsilon_{1+}q_{1+}N_{1-} - P_{1-}\epsilon_{1-}q_{1-}N_{1+})\\ &\quad + f\frac{1}{2}A_{zz}(Q_{1+}\epsilon_{1+}q_{1+}N_{1-} - Q_{1-}\epsilon_{1-}q_{1-}N_{1+})\Bigg]\end{aligned} \tag{48}$$

Given the rapidity of the beam helicity changes compared with the target polarization, it's likely safe to assume that $P_{1+} = P_{1-} = P_n$, $Q_{1+} = Q_{1-} = Q_n$, and $\epsilon_{1+}q_{1+} = \epsilon_{1-}q_{1-} = \epsilon_n q_n$ such that

$$\begin{aligned}A_T^{ed} &= \left[\frac{1}{Q_n\epsilon_n q_n[h_{n-}N_{n+} - h_{n+}N_{n-}]}\right]\times\\ &\Bigg[\epsilon_n q_n(N_{n-} - N_{n+})\\ &\quad + fA_{\parallel}P_n\epsilon_n q_n(h_{n+}N_{n-} - h_{n-}N_{n+})\\ &\quad + fA_d^V P_n\epsilon_n q_n(N_{n-} - N_{n+})\\ &\quad + f\frac{1}{2}A_{zz}Q_n\epsilon_n q_n(N_{n-} - N_{n+})\Bigg]\end{aligned} \tag{49}$$

Additionally, if $h_{n+} = -h_{n-} = h$, then

$$\begin{aligned}A_T^{ed} = \left[\frac{1}{hQ_n}\right]\Bigg[\left(\frac{N_{n+} - N_{n-}}{N_{n+} + N_{n-}}\right)\Bigg(1 + fA_d^V P_n\\ + f\frac{1}{2}A_{zz}Q_n\Bigg)\Bigg] - fA_{\parallel}\frac{P_n}{Q_n}\end{aligned} \tag{50}$$

Here, the asymmetries A_d^V and A_{zz} are suppressed by the dilution factor and polarization, which together will suppress these terms to $\sim 9\%$ relative A_T^{ed} before applying their values. It's also obvious here that we cannot measure A_T^{ed} in the case where $Q_n = 0$, as A_T^{ed} would be undefined in that case. This means that we could only measure A_T^{ed} using the σ_1 and σ_2 cases, which would reduce the total number of events measured by about a factor of 1/2 compared to the A_{zz} measurements. It's also clear that $A_{\parallel}$ might not be suppressed much at all in this case, making it another important observable to measure. $A_{\parallel}$ seems to be a beam-target asymmetry, so we'll start with

$$
\begin{aligned}
\frac{N_{1+}+N_{2-}-N_{2+}-N_{1-}}{N_{1+}+N_{1-}+N_{2+}+N_{2-}} = \Bigg[&\epsilon_{1+}q_{1+} \\
&+ \epsilon_{2-}q_{2-} \\
&- \epsilon_{2+}q_{2+} \\
&- \epsilon_{1-}q_{1-} \\
&+ fA_{\parallel}h_{1+}P_{1+}\epsilon_{1+}q_{1+} \\
&+ fA_{\parallel}h_{2-}P_{2-}\epsilon_{2-}q_{2-} \\
&- fA_{\parallel}h_{2+}P_{2+}\epsilon_{2+}q_{2+} \\
&- fA_{\parallel}h_{1-}P_{1-}\epsilon_{1-}q_{1-} \\
&+ fA_d^V P_{1+}\epsilon_{1+}q_{1+} \\
&+ fA_d^V P_{2-}\epsilon_{2-}q_{2-} \\
&- fA_d^V P_{2+}\epsilon_{2+}q_{2+} \\
&- fA_d^V P_{1-}\epsilon_{1-}q_{1-} \\
&+ fA_T^{ed}h_{1+}Q_{1+}\epsilon_{1+}q_{1+} \\
&+ fA_T^{ed}h_{2-}Q_{2-}\epsilon_{2-}q_{2-} \\
&- fA_T^{ed}h_{2+}Q_{2+}\epsilon_{2+}q_{2+} \\
&- fA_T^{ed}h_{1-}Q_{1-}\epsilon_{1-}q_{1-} \\
&+ f\frac{1}{2}A_{zz}Q_{1+}\epsilon_{1+}q_{1+} \\
&+ f\frac{1}{2}A_{zz}Q_{2-}\epsilon_{2-}q_{2-} \\
&- f\frac{1}{2}A_{zz}Q_{2+}\epsilon_{2+}q_{2+} \\
&- f\frac{1}{2}A_{zz}Q_{1-}\epsilon_{1-}q_{1-}\Bigg] \\
\times \Bigg[&\epsilon_{1+}q_{1+} \\
&+ \epsilon_{2-}q_{2-} \\
&+ \epsilon_{2+}q_{2+} \\
&+ \epsilon_{1-}q_{1-} \\
&+ fA_{\parallel}h_{1+}P_{1+}\epsilon_{1+}q_{1+} \\
&+ fA_{\parallel}h_{2-}P_{2-}\epsilon_{2-}q_{2-} \\
&+ fA_{\parallel}h_{2+}P_{2+}\epsilon_{2+}q_{2+} \\
&+ fA_{\parallel}h_{1-}P_{1-}\epsilon_{1-}q_{1-} \\
&+ fA_T^{ed}h_{1+}Q_{1+}\epsilon_{1+}q_{1+} \\
&+ fA_T^{ed}h_{2-}Q_{2-}\epsilon_{2-}q_{2-} \\
&+ fA_T^{ed}h_{2+}Q_{2+}\epsilon_{2+}q_{2+} \\
&+ fA_T^{ed}h_{1-}Q_{1-}\epsilon_{1-}q_{1-} \\
&+ fA_d^V P_{1+}\epsilon_{1+}q_{1+} \\
&+ fA_d^V P_{2-}\epsilon_{2-}q_{2-} \\
&+ fA_d^V P_{2+}\epsilon_{2+}q_{2+} \\
&+ fA_d^V P_{1-}\epsilon_{1-}q_{1-} \\
&+ f\frac{1}{2}A_{zz}Q_{1+}\epsilon_{1+}q_{1+} \\
&+ f\frac{1}{2}A_{zz}Q_{2-}\epsilon_{2-}q_{2-} \\
&+ f\frac{1}{2}A_{zz}Q_{2+}\epsilon_{2+}q_{2+} \\
&+ f\frac{1}{2}A_{zz}Q_{1-}\epsilon_{1-}q_{1-}\Bigg]^{-1}
\end{aligned}
\tag{51}
$$

Given the rapidity of beam helicity flips, we can safely assume that $P_{1+} = P_{1-} = P_1$, $P_{2+} = P_{2-} = P_2$, $Q_{1+} = Q_{1-} = Q_1$, $Q_{2+} = Q_{2-} = Q_2$, $\epsilon_{1+}q_{1+} = \epsilon_{1-}q_{1-} = \epsilon_1 q_1 =$, and $\epsilon_{2+}q_{2+} = \epsilon_{2-}q_{2-} = \epsilon_2 q_2 =$ such that

$$
\begin{aligned}
\frac{N_{1+}+N_{2-}-N_{2+}-N_{1-}}{N_{1+}+N_{1-}+N_{2+}+N_{2-}} = \Bigg[&fA_{\parallel}P_1\epsilon_1 q_1 h_{1+} \\
&+ fA_{\parallel}P_2\epsilon_2 q_2 h_{2-} \\
&- fA_{\parallel}P_2\epsilon_2 q_2 h_{2+} \\
&- fA_{\parallel}P_1\epsilon_1 q_1 h_{1-} \\
&+ fA_T^{ed}Q_1\epsilon_1 q_1 h_{1+} \\
&+ fA_T^{ed}Q_2\epsilon_2 q_2 h_{2-} \\
&- fA_T^{ed}Q_2\epsilon_2 q_2 h_{2+} \\
&- fA_T^{ed}Q_1\epsilon_1 q_1 h_{1-}\Bigg] \\
\times \Bigg[&2\epsilon_1 q_1 \\
&+ 2\epsilon_2 q_2 \\
&+ fA_{\parallel}P_1\epsilon_1 q_1 h_{1+} \\
&+ fA_{\parallel}P_1\epsilon_1 q_1 h_{1-} \\
&+ fA_{\parallel}P_2\epsilon_2 q_2 h_{2+} \\
&+ fA_{\parallel}P_2\epsilon_2 q_2 h_{2-} \\
&+ fA_T^{ed}Q_1\epsilon_1 q_1 h_{1+} \\
&+ fA_T^{ed}Q_1\epsilon_1 q_1 h_{1-} \\
&+ fA_T^{ed}Q_2\epsilon_2 q_2 h_{2+} \\
&+ fA_T^{ed}Q_2\epsilon_2 q_2 h_{2-} \\
&+ 2fA_d^V P_1\epsilon_1 q_1 \\
&+ 2fA_d^V P_2\epsilon_2 q_2 \\
&+ 2f\frac{1}{2}A_{zz}Q_1\epsilon_1 q_1 \\
&+ 2f\frac{1}{2}A_{zz}Q_2\epsilon_2 q_2\Bigg]^{-1}
\end{aligned}
\tag{52}
$$

Again cross-multiplying, this gets us

$$
\begin{aligned}
A_{\parallel} \\
= \Bigg[& \frac{1}{f[P_1\epsilon_1 q_1(h_{1+}(N_{2+}+N_{1-}) - h_{1-}(N_{1+}+N_{2-})) + P_2\epsilon_2 q_2(h_{2-}(N_{2+}+N_{1-}) - h_{2+}(N_{1+}+N_{2-}))]} \Bigg] \\
& \times \Bigg[[\epsilon_1 q_1 + \epsilon_2 q_2][(N_{1+}+N_{2-}) - (N_{2+}+N_{1-})] \\
& + f A_T^{ed} Q_1 \epsilon_1 q_1 [h_{1-}(N_{1+}+N_{2-}) \\
& - h_{1+}(N_{2+}+N_{1-})] \\
& + f A_T^{ed} Q_2 \epsilon_2 q_2 [h_{2+}(N_{1+}+N_{2-}) \\
& - h_{2-}(N_{2+}+N_{1-})] \\
& + f A_d^V [P_1\epsilon_1 q_1 + P_2 \epsilon_2 q_2][(N_{1+}+N_{2-}) \\
& - (N_{2+}+N_{1-})] \\
& + f\frac{1}{2} A_{zz} [(N_{1+}+N_{2-}) - (N_{2+}+N_{1-})][Q_1\epsilon_1 q_1 \\
& + Q_2\epsilon_2 q_2] \Bigg]
\end{aligned} \tag{53}
$$

Already we can see that A_T^{ed} and A_{zz} will be suppressed by the dilution factor and the tensor polarization, and A_d^V by the dilution factor and difference of the vector polarization. If we make a further estimate that $h_{1+} = -h_{1-} = h_1$ and $h_{2+} = -h_{2-} = h_2$, then

$$
\begin{aligned}
A_{\parallel} \\
= & \left[\frac{1}{f(P_1\epsilon_1 q_1 h_1 - P_2 \epsilon_2 q_2 h_2)} \right] \left[\frac{(N_{1+}+N_{2-}) - (N_{2+}+N_{1-})}{(N_{2+}+N_{1-}) + (N_{1+}+N_{2-})} \right] \\
& \times \Bigg[[\epsilon_1 q_1 + \epsilon_2 q_2] \\
& + f A_d^V [P_1 \epsilon_1 q_1 + P_2 \epsilon_2 q_2] \\
& + f \frac{1}{2} A_{zz} [Q_1 \epsilon_1 q_1 + Q_2 \epsilon_2 q_2] \Bigg] \\
& + A_T^{ed} \left[\frac{Q_2 \epsilon_2 q_2 h_2 - Q_1 \epsilon_1 q_1 h_1}{P_1 \epsilon_1 q_1 h_1 - P_2 \epsilon_2 q_2 h_2} \right]
\end{aligned} \tag{54}
$$

Using $\sigma_{3+/-}$ and $\sigma_{4+/-}$ where $Q_3 = Q_4 = 0$ would eliminate both the A_{zz} and A_T^{ed} terms and leave just A_d^V doubly suppressed by the dilution factor and difference of vector polarizations, or by a factor of $f\Delta P \approx 0.3 \cdot 0.1 \approx 0.03 \approx 3\%$ relative to $A_\parallel$ before applying the A_d^V value. Changing to that case would then greatly simplify the above to

$$
\begin{aligned}
A_{\parallel} \\
= & \left[\frac{1}{f(P_3\epsilon_3 q_3 h_3 - P_4 \epsilon_4 q_4 h_4)} \right] \left[\frac{(N_{3+}+N_{4-}) - (N_{4+}+N_{3-})}{(N_{4+}+N_{3-}) + (N_{3+}+N_{4-})} \right] \\
& \times \Bigg[\epsilon_3 q_3 \\
& + \epsilon_4 q_4 \\
& + f A_d^V (P_3 \epsilon_3 q_3 + P_4 \epsilon_4 q_4) \Bigg]
\end{aligned} \tag{55}
$$

Applying the last ideal case scenario where $P_3 = -P_4 = P$, $h_3 = h_4 = h$, and $\epsilon_3 q_3 = \epsilon_4 q_4 = \epsilon q$, we get

$$
A_{\parallel} = \left[\frac{1}{fPh} \right] \left[\frac{(N_{3+}+N_{4-}) - (N_{4+}+N_{3-})}{(N_{4+}+N_{3-}) + (N_{3+}+N_{4-})} \right] \tag{56}
$$

At this point, we've nearly determined how to measure all the diluting asymmetries with our data, so we may as well finish it off to find the final one, A_d^V. This one appears to be just a target asymmetry, so we will extract it using

$$
\begin{aligned}
\frac{N_1 - N_2}{N_1 + N_2} = & \Bigg[\epsilon_1 q_1 \\
& - \epsilon_2 q_2 \\
& + f A_d^V (P_1 \epsilon_1 q_1 - P_2 \epsilon_2 q_2) \\
& + f A_\parallel (h_1 P_1 \epsilon_1 q_1 - h_2 P_2 \epsilon_2 q_2) \\
& + f A_T^{ed} (h_1 Q_1 \epsilon_1 q_1 - h_2 Q_2 \epsilon_2 q_2) \\
& + f \frac{1}{2} A_{zz} (Q_1 \epsilon_1 q_1 - Q_2 \epsilon_2 q_2) \Bigg] \\
& \times \Bigg[\epsilon_1 q_1 \\
& + \epsilon_2 q_2 \\
& + f A_d^V (P_1 \epsilon_1 q_1 \\
& + P_2 \epsilon_2 q_2) \\
& + f A_\parallel (h_1 P_1 \epsilon_1 q_1 \\
& + h_2 P_2 \epsilon_2 q_2) \\
& + f A_T^{ed} h_1 (Q_1 \epsilon_1 q_1 \\
& + h_2 Q_2 \epsilon_2 q_2) \\
& + f \frac{1}{2} A_{zz} (Q_1 \epsilon_1 q_1 \\
& + Q_2 \epsilon_2 q_2) \Bigg]^{-1}
\end{aligned} \tag{57}
$$

Once again cross-multiplying gives us

$$
\begin{aligned}
A_d^V = & \frac{\epsilon_2 q_2 N_1 - \epsilon_1 q_1 N_2}{f(P_1 \epsilon_1 q_1 N_2 - P_2 \epsilon_2 q_2 N_1)} \\
& - A_\parallel \frac{(h_1 P_1 \epsilon_1 q_1 N_2 - h_2 P_2 \epsilon_2 q_2 N_1)}{(P_1 \epsilon_1 q_1 N_2 - P_2 \epsilon_2 q_2 N_1)} \\
& - A_T^{ed} \frac{(h_1 Q_1 \epsilon_1 q_1 N_2 - h_2 Q_2 \epsilon_2 q_2 N_1)}{(P_1 \epsilon_1 q_1 N_2 - P_2 \epsilon_2 q_2 N_1)} \\
& - \frac{1}{2} A_{zz} \frac{(Q_1 \epsilon_1 q_1 N_2 - Q_2 \epsilon_2 q_2 N_1)}{(P_1 \epsilon_1 q_1 N_2 - P_2 \epsilon_2 q_2 N_1)}
\end{aligned} \tag{58}
$$

Here again, if we use σ_3 and σ_4 where $Q_3 = Q_4 = 0$, both the A_T^{ed} and A_{zz} asymmetries become zero. Switching to these observations, we find

$$
\begin{aligned}
A_d^V = & \frac{\epsilon_4 q_4 N_3 - \epsilon_3 q_3 N_4}{f(P_3 \epsilon_3 q_3 N_4 - P_4 \epsilon_4 q_4 N_3)} \\
& - A_\parallel \frac{(h_3 P_3 \epsilon_3 q_3 N_4 - h_4 P_4 \epsilon_4 q_4 N_3)}{(P_3 \epsilon_3 q_3 N_4 - P_4 \epsilon_4 q_4 N_3)}
\end{aligned} \tag{59}
$$

Furthermore, if we assume that the integrated $h_3 = h4 = h$, then this further simplifies to

$$
A_d^V = \frac{\epsilon_4 q_4 N_3 - \epsilon_3 q_3 N_4}{f(P_3 \epsilon_3 q_3 N_4 - P_4 \epsilon_4 q_4 N_3)} - A_\parallel h \tag{60}
$$

where $A_{\|}$ is suppressed by the integrated beam helicity, which is expected to be well less than 2×10^{-4}. Making our final ideal case assumption where $P_3 = P$, $P_4 = -P$, and $\epsilon_3 q_3 = \epsilon_4 q_4 = \epsilon q$, we find

$$A_d^V = \left[\frac{1}{fP}\right]\left[\frac{N_3 - N_4}{N_4 + N_3}\right] - A_{\|} h. \tag{61}$$

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1.
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