

glide plane in preference to the latter, on which glide has never been detected.

Double and triple glide can take place, according to the ordinary laws governing the glide plane and glide direction. Hardening on one set of glide planes hardens the whole crystal.

The critical shear stress at  $-43^{\circ}\text{C}$  is  $9.3\text{ gm wt. per sq. mm.}$

The very purest mercury gives, in the early stages of stretch, an average spacing of the glide planes which is  $0.0054\text{ cm.}$ , agreeing closely with that for ordinary pure mercury, and for mercury deliberately contaminated. The appearance of preferred glide planes separated by about 15,000 lattice spacings is thus not due to impurities or dissolved gas.

Under constant stress the single crystal flows at a rate which diminishes to a constant value.

---

## Quantum Theory of the Diplon

By H. BETHE and R. PEIERLS, University of Manchester

(Communicated by D. R. Hartree, F.R.S.—Received July 26, 1934)

### 1—INTRODUCTION

The work of Heisenberg,<sup>†</sup> Majorana,<sup>‡</sup> and Wigner<sup>§</sup> seems to show that the behaviour of protons and neutrons and their interaction in the nucleus may be described by the ordinary methods of quantum mechanics. It is of particular interest to study the simplest nuclear system, *i.e.*, the diplon, which almost certainly consists of a proton and a neutron. In dealing with such a two-body problem, the wave equation can be rigorously solved if the forces<sup>¶</sup> are known, and this problem therefore has the same importance for nuclear mechanics as the hydrogen atom has for atomic theory.

The force acting between a proton and a neutron has been investigated by Wigner (*loc. cit.*) who showed that in order to understand the high mass defect of  $\text{He}^4$  compared with  $\text{H}^2$  one must assume interaction forces with a range much smaller than the radius of  $\text{H}^2$ . Without knowing about these forces more than the binding energy of  $\text{H}^2$ , one can, then, investigate quantitatively the behaviour of  $\text{H}^2$  against various perturbations.

<sup>†</sup> 'Z. Physik,' vol. 77, p. 1 (1932) ; vol. 78, p. 156 (1932) ; vol. 80, p. 587 (1933).

<sup>‡</sup> 'Z. Physik,' vol. 82, p. 137 (1932).

<sup>§</sup> 'Phys. Rev.,' vol. 43, p. 252 (1933) ; 'Z. Physik,' vol. 83, p. 253 (1933).

## 2—WAVE EQUATION FOR $H^2$

The variables of our problem are the co-ordinates of neutron and proton and the  $z$ -components of their spins. We denote by  $\psi(xs, \xi\sigma)$  the wave function for the configuration in which the proton is found at the place  $x$  with the spin  $s$  and the neutron at  $\xi$  with spin  $\sigma$ . For this function three types of wave equation have been proposed :

$$\left(\frac{\hbar^2}{2M}(\nabla_x^2 + \nabla_\xi^2) + E\right) \psi(xs, \xi\sigma) = \begin{cases} V(x - \xi) \psi(xs, \xi\sigma) & (1A) \\ V(x - \xi) \psi(\xi\sigma, xs) & (1B) \\ V(x - \xi) \psi(\xi s, x\sigma) & (1C) \end{cases}$$

where  $M$  is the mass of one proton, which is, in sufficient approximation, equal to that of the neutron. Equation (1A) (ordinary interaction force) was discussed by Wigner (*loc. cit.*) who showed that it can describe correctly the behaviour of the nuclei up to  $He^4$ . It does not explain, however, the fact that the addition of more particles does not increase the mass defect per particle (*cf.* Majorana, *loc. cit.*). In order to explain this fact one has to assume a force of an exchange type leading to saturation, the simplest forms of which are (1B) and (1C). (1B) was proposed by Heisenberg, but it leads to saturation for the  $H^2$  nucleus already and therefore must lead to too low a mass defect for  $He^4$ . The equation (1C) proposed by Majorana, leads to  $He^4$  as a saturated configuration, and therefore seems to fulfil all requirements. We shall in the following adopt (1C), but we shall see that our results for the behaviour of the diplon at not too high energies are practically independent of this assumption.

The solution of (1) will be of the form

$$\psi(xs, \xi\sigma) = \chi\left(\frac{x + \xi}{2}\right) \alpha(s\sigma) \phi(x - \xi), \quad (2)$$

$$\phi(x - \xi) = u(r) P_{lm}(\vartheta\phi) \quad (2A)$$

where  $r = |\mathbf{r}|$  is the distance between the points  $x$  and  $\xi$ ;  $\vartheta\phi$  the direction of the vector  $\mathbf{r}$  and  $P_{lm}$  a spherical harmonic. Then  $u$  must satisfy the equation

$$\frac{\hbar^2}{M} \left( \frac{1}{r} \frac{d^2}{dr^2} (ru) - \frac{l(l+1)}{r^2} u \right) + Eu = (-1)^l V(r) u. \quad (3)$$

For a fixed value of  $l$ , this equation is equivalent to an ordinary



Schrödinger equation, the only difference against (1A) being that the "potential" is attractive for even and repulsive for odd values of  $l$ . If we had adopted (1B) the sign of the potential would also differ for the triplet and singlet state.

The function  $V$  has been investigated by Wigner who showed that  $V$  has appreciable values only if  $r < a \approx 1.0 \cdot 10^{-13}$  cm and at smaller distances becomes of the order  $10^8$  volts.† Wick‡ tried to determine  $V$  from the mass defects of heavy nuclei with the aid of (1c). The form of  $V$  obtained in this way is in reasonable agreement with that of Wigner.

In solving the equation for  $l = 0$ , one can therefore at small distances neglect the energy compared with the potential, so that the phase  $= \frac{1}{ru} \frac{d(ru)}{dr}$  of the wave function will have a definite value  $\alpha$  at distances small compared with the wave-length, but larger than  $a$ . Thus one can, instead of solving equation (3), work with the equation for free motion, but with the boundary condition that

$$-\frac{1}{ru} \frac{d(ru)}{dr} \rightarrow \alpha \quad \text{for } r = 0.$$

The value of  $\alpha$  can be deduced from the mass defect of  $H^2$ .

For  $l \neq 0$ , the potential  $V$  will have no appreciable effect at all, because the centrifugal force makes the wave function very small for distances small compared with the wave-length and the potential at still smaller distances will therefore not matter.§

If, therefore, (i) the wave-length of the particles is large compared with  $a$ , and (ii) their relative energy small compared with the values of

† The arguments of Wigner are based on (1A), but just because the range of the forces turns out so very small, the same results may be deduced, without appreciable modification, from (1c) as well. The reason is, that over the distances where  $V(x - \xi)$  is different from zero, the wave function varies slowly and  $\psi(x\xi)$  and  $\psi(\xi x)$  are nearly equal, unless  $\psi$  has a node at the point  $x = \xi$  which, however, for the nuclei up to  $He^4$  will not occur.

‡ 'Nuovo Cim.', vol. 11, p. 227 (1934).

§ The very short range of the interaction forces between neutron and proton leads to consequences for the scattering of slow neutrons by protons which have been discussed by Wigner (*loc. cit.*) and Wick ('Z. Physik,' vol. 84, p. 799 (1934)) and *loc. cit.* These authors have pointed out that observations of very fast neutrons would afford a means for both estimating the range of the forces and deciding directly between equations (1A, B, C). The mathematical treatment for high energies given in these papers is, however, not quite complete and we hope to come back to this question in a later publication.

V at small distances (these two conditions are equivalent) one can replace the solution of (3)

for  $l \neq 0$  by wave functions of free particles,  
for  $l = 0$  by solutions of

$$\frac{\hbar^2}{M} \frac{1}{r} \frac{d^2}{dr^2} (ru) + Eu = 0, \quad (4)$$

with the boundary condition

$$\left( -\frac{1}{ru} \frac{d(ru)}{dr} \right)_{r=0} = \alpha. \quad (5)$$

Under these assumptions, there is only one discrete state with  $l = 0$  and the wave function

$$u_0 = \frac{C}{r} e^{-\alpha r}. \quad (6)$$

The condition of normalization determines C :

$$C = \sqrt{\alpha/2\pi}. \quad (7)$$

The corresponding energy is

$$-\varepsilon = -\frac{\hbar^2}{M} \alpha^2, \quad (8)$$

and  $\alpha$  has to be so determined as to make  $\varepsilon$  equal to the binding energy of  $H^2$ . This energy has recently been determined fairly accurately by Chadwick† and was found to be

$$\varepsilon = 2.1 \cdot 10^6 \text{ volts},$$

which gives

$$\alpha = 2.2 \cdot 10^{12} \text{ cm}^{-1},$$

$1/\alpha$  is indeed much larger than the assumed range of the interaction forces.

### 3—ABSORPTION OF $\gamma$ -RAYS BY DIPLONS

If a  $\gamma$ -ray of energy  $h\nu$  falls on a diplon it may be absorbed, producing a proton and a neutron. This process is analogous to the photo-electric effect of an atom and the effective cross section is given by the well-known formula‡

$$\sigma = \frac{8\pi^3 e^2 \nu}{c} |z_{0E}|^2, \quad (9)$$

† Chadwick and Goldhaber, 'Nature,' vol. 134, p. 237 (1934).

‡ Cf., e.g., Kemble and Hill ('Rev. mod. Phys.,' vol. 2, p. 11 (1930)) formula (28). This formula differs from (9) by a factor  $\hbar^2 \nu / c$ , because the authors calculate the ratio of transition probability and radiation density per unit frequency instead of the cross-section per unit energy.



where  $e$  is the protonic charge,  $c$  the velocity of light and

$$z_{0E} = \int u_0 \frac{1}{2} z \phi_E d\tau. \quad (10)$$

Here  $u_0$  is given by (6),  $z$  is the  $z$ -component of  $\mathbf{r}$ , and  $\frac{1}{2}z$  consequently the  $z$ -co-ordinate of the proton with respect to the centre of gravity.  $\phi_E$  is the wave function of a state with the energy  $E = h\nu - \epsilon$  normalized in energy scale. Of all possible states with this energy, only that belonging to  $l = 1$  gives a non-vanishing matrix element (10). The corresponding wave function is then, as we have seen, identical with the wave function for free motion. If, on the other hand, we insert in (10) all wave functions of free motion with energy  $E$ , again only that with  $l = 1$  gives a non-vanishing integral. We are therefore allowed to use wave functions for free motion in Cartesian co-ordinates :

$$\phi_k = (2\pi)^{-3/2} e^{i(\mathbf{k} \cdot \mathbf{r})} \quad (11)$$

(11) is normalized in wave vector scale. In order to obtain energy normalization we have to add a factor

$$k^2 \frac{dk}{dE} d\Omega = \frac{Mk}{2\hbar^2} d\Omega,$$

where  $d\Omega$  is the element of the direction of motion, and the reduced mass  $M/2$  enters instead of  $M$  as we are concerned with the relative motion. Thus in (9) we may write

$$|z_{0E}|^2 = \frac{1}{(2\pi)^3} \frac{Mk}{2\hbar^2} \int d\Omega |z_{0k}|^2, \quad (12)$$

where

$$z_{0k} = \int d\tau u_0 \frac{1}{2} z e^{i(\mathbf{k} \cdot \mathbf{r})}. \quad (13)$$

The length of  $k$  is defined by

$$\hbar^2 k^2 = p^2 = M(h\nu - \epsilon). \quad (14)$$

Inserting (6) the matrix element becomes

$$z_{0k} = \frac{1}{2} \sqrt{\frac{\alpha}{2\pi}} \frac{1}{i} \frac{\partial}{\partial k_z} \int d\tau \frac{1}{r} e^{-\alpha r + ikr \cos \theta},$$

where  $\theta$  is the angle between  $k$  and  $r$ .

$$z_{0k} = -4\pi i \sqrt{\frac{\alpha}{8\pi}} \frac{\partial}{\partial k_z} \left( \frac{1}{\alpha^2 + k^2} \right) = i \sqrt{2\pi\alpha} \frac{2k_z}{(\alpha^2 + k^2)^2}$$

Inserting this into (12) we obtain

$$|z_{0E}|^2 = \frac{2}{3\pi} \frac{M}{\hbar^2} \frac{\alpha k^3}{(\alpha^2 + k^2)^4}, \quad (15)$$

and from (9):

$$\sigma = \frac{16\pi^2}{3} \frac{Me^2 \nu \alpha k^3}{\hbar^2 c (\alpha^2 + k^2)^4}.$$

Expressing now  $\alpha$  and  $k$  from (8) and (14) :

$$\sigma = \frac{16\pi^2}{3} \frac{\hbar^2 e^2 \nu (h\nu - \varepsilon)^{3/2} \varepsilon^{1/2}}{Mc (h\nu)^4},$$

$$\sigma = \frac{8\pi}{3} \frac{e^2}{\hbar c} \frac{1}{\alpha^2} \frac{(\gamma - 1)^{3/2}}{\gamma^3}, \quad (16)$$

where

$$\gamma = h\nu/\varepsilon. \quad (17)$$

With the estimated value of  $\alpha$  given above†

$$\sigma \approx 1.25 \cdot 10^{-26} (\gamma - 1)^{3/2} \gamma^{-3} \text{ cm}^2.$$

This becomes a maximum for  $\gamma = 2$  where  $\sigma = 1.6 \cdot 10^{-27} \text{ cm}^2$ .

#### 4—CAPTURE OF NEUTRONS BY PROTONS

The inverse process to the one considered in the last section is the formation of a diplon from the separate particles under emission of  $\gamma$ -rays. The probability  $P_{12}$  of this process is connected with that  $P_{21}$  for the inverse in the following way:

$$P_{12}/g_2 = P_{21}/g_1,$$

where  $g_1$  and  $g_2$  are the statistical weights of the final states of the system.

The number of states per unit momentum vector is the same for both particles and light quanta. The number of states for the relative motion per unit energy is proportional to

$$\frac{p^2 dp}{dE} = \frac{Mp}{2},$$

† The experimental value given by Chadwick ('Nature,' vol. 134, p. 237, August 18, 1934) is only  $10^{-28} \text{ cm}^2$ . This would agree with (16) only if  $\gamma - 1 = 0.04$ , i.e., if by accident the energy of the  $\gamma$ -rays employed was only just above the limit of absorption. Whether this is actually so or whether (16) is actually too large can only be decided by further experiments.



where  $p = \frac{1}{2}Mv$  is the momentum in the co-ordinate system where the centre of gravity is at rest. For the light quantum we have instead

$$\frac{(h\nu/c)^2 d(h\nu/c)}{d(h\nu)} = \frac{(h\nu)^2}{c^3},$$

and additional factor 2 for polarization.

Therefore the ratio of the probabilities for capture and absorption will be†

$$\frac{4(h\nu)^2}{c^3Mp}$$

if the density of the incident particles is the same. In order to obtain the cross-section we must compare the yield not for equal density but for equal current and so must add a factor  $c/v$ , where  $v$  is the velocity of the incident neutron. Thus if  $\sigma$  is the cross-section (16) the cross-section for capture becomes

$$\begin{aligned}\sigma' &= 2 \frac{(h\nu)^2}{c^2 p^2} \sigma = 2 \frac{(h\nu)^2}{Mc^2(h\nu - \varepsilon)} \sigma \\ \sigma' &= 2 \frac{\varepsilon}{Mc^2} \frac{\gamma^2}{\gamma - 1} \sigma\end{aligned}\tag{18}$$

or

$$\sigma' = \frac{16\pi}{3} \frac{e^2}{\hbar c} \left( \frac{\hbar}{Mc} \right)^2 \frac{(\gamma - 1)}{\gamma},\tag{18A}$$

(18A) is again a maximum for  $\gamma = 2$ , *i.e.*, if the energy of the incident neutron is  $\varepsilon$  in the relative co-ordinate system, *i.e.*,  $2\varepsilon$  in the system where the proton is at rest. Then

$$\sigma' = 2.70 \cdot 10^{-29} \text{ cm}^2.$$

The capture therefore seems hardly observable.

Instead of being captured, the neutron passing through hydrogen may, of course, radiate only part of its energy, but the cross-section for this process is only of the same order of magnitude.

† This ratio involves the assumption that proton and neutron can be bound together also if their spins are antiparallel and that the probability of capture does not depend on the relative spin directions. The assumption is correct if the model of either Wigner or Majorana is assumed and the magnetic force between the spins is considered as a small perturbation. In Heisenberg's model there would be no bound singlet state, the cross-section for capture would then be reduced by a factor  $\frac{3}{4}$ .

# 5—SCATTERING OF $\gamma$ -RAYS BY DIPLONS

$\gamma$ -rays affect both the relative co-ordinate and the centre of gravity of the diplon. The latter gives the classical scattering according to Thomson's formula, while the inner degree of freedom gives an additional scattered wave which may easily be calculated from the Heisenberg-Kramers formula, using the matrix elements (15).†

The total nuclear scattering per unit solid angle under an angle  $\theta$  then becomes :

$$\left\{ \frac{1}{3} \frac{e^2}{Mc^2} \frac{1}{\gamma^2} (4 + 3\gamma^2 - 2(1 + \gamma)^{3/2} - 2(1 - \gamma)^{3/2}) \right\}^2 \frac{1}{2} (1 + \cos^2 \theta),$$

if  $\gamma < 1$ , and

$$\left\{ \frac{1}{3} \frac{e^2}{Mc^2} \frac{1}{\gamma^2} (4 + 3\gamma^2 - 2(1 + \gamma)^{3/2}) \right\}^2 \frac{1}{2} (1 + \cos^2 \theta) \quad (19)$$

if  $\gamma > 1$ , where again  $\gamma = h\nu/\varepsilon$ .

The scattering is always smaller than that of a free proton and becomes equal to the latter only for  $\gamma \gg 1$ . The order of magnitude of this effect is therefore extremely small, corresponding to a total cross-section of about  $2 \cdot 10^{-31}$  cm<sup>2</sup>, so that it is practically impossible to observe it.

# 6—DISINTEGRATION OF DIPLONS UNDER ELECTRON BOMBARDMENT

If fast electrons of momentum  $\mathbf{P}$  and energy  $W$  strike a diplon, they may be scattered through an angle  $\theta$  and afterwards have a smaller energy  $W'$  (momentum  $\mathbf{P}'$ ), while the energy difference serves to disintegrate the diplon. The calculation of the probability is very similar to the disintegration by a light wave (*cf.* section 3). Instead of the light wave one has to insert the electromagnetic field belonging to the transition of the electron from  $\mathbf{P}$  to  $\mathbf{P}'$ .‡ If  $\Psi_P$  and  $\Psi_{P'}$  are initial and final wave function of the electron, the field derives from the four-potential :

$$\begin{aligned} \Phi_0 &= \frac{4\pi e\hbar^2 (\Psi_P^* \Psi_{P'})}{(\mathbf{P} - \mathbf{P}')^2 - (W - W')^2/c^2} = 4\pi e\hbar^2 c^2 a_0 \frac{\exp(i(\mathbf{q}\mathbf{r})/\hbar c)}{q^2 - (W - W')^2} \\ \Phi_k &= \frac{4\pi e\hbar^2 (\Psi_P^* \alpha_k \Psi_P)}{(\mathbf{P} - \mathbf{P}')^2 - (W - W')^2/c^2} = 4\pi e\hbar^2 c^2 a_k \frac{\exp(i(\mathbf{q}\mathbf{r})/\hbar c)}{q^2 - (W - W')^2} \quad (20) \\ &\quad (k = 1, 2, 3). \end{aligned}$$

† *Cf. e.g.,* Kemble and Hill, *loc. cit.*, p. 22, equation (61).

‡ Chr. Möller, 'Z. Physik,' vol. 70, p. 786 (1931).



Here  $\alpha_k$  are Dirac's matrices,  $a_0$  and  $a_k$  are constants arising from the ratios of the components of the Dirac wave functions  $\Psi_p$  and  $\Psi_{p'}$ . They depend upon  $\mathbf{P}$  and  $\mathbf{P}'$  and upon the spin directions.  $\mathbf{q}/c = \mathbf{P} - \mathbf{P}'$  is the momentum difference between initial and final electronic state. If we consider (20) as a perturbation acting on the dipion, the transition probability is proportional to the square of the matrix element

$$\int u_0 \exp(\frac{1}{2}i(\mathbf{q}\mathbf{r})/\hbar c) (a_0 - \sum_k a_k v_k/c) \phi_E d\tau. \quad (21)$$

Here  $v_k = \frac{\hbar}{iM} \frac{\partial}{\partial r_k}$  is the operator of the proton velocity and  $\frac{1}{2}\mathbf{r}$  in the exponent is again its distance from the centre of gravity.  $\mathbf{q}\mathbf{r}/\hbar c$  is small compared with unity over the region where  $\phi_0 \neq 0$  unless  $2W \sin \frac{1}{2}\theta$  becomes of the order  $4 \cdot 10^7$  volts. Usually, therefore, it is sufficient to expand the exponential, retaining only the constant term where it is multiplied by  $v$  and the linear, multiplied by  $a_0$ . After averaging over the spin directions of the electron the differential cross-section becomes :

$$\begin{aligned} \sigma_{0E} dE d\Omega = 4 \left(\frac{e^2}{\hbar c}\right)^2 \frac{P'}{P} \frac{|z_{0E}|^2}{[q^2 - (W - W')^2]^2} [(q^2 - (W - W')^2)(W^2 + W'^2) \\ - \frac{1}{2}(q^2 - (W - W')^2)^2 - \frac{1}{2}m^2c^4(W - W')^2] dE d\Omega, \\ W' = W - (\varepsilon + E). \end{aligned} \quad (22)$$

Integration with respect to all directions of the scattered electron gives

$$\sigma_{0E} dE = 8\pi \left(\frac{e^2}{\hbar c}\right)^2 |z_{0E}|^2 \left\{ \frac{W^2 + W'^2}{c^2 P^2} \log \frac{WW' + c^2 PP' - m^2 c^4}{(W - W')mc^2} - \frac{3}{2} \frac{P'}{P} \right\} dE, \quad (23)$$

$z_{0E}$  is the matrix element (15).

The total cross-section  $\sigma_D$  is given by the integral of (23) over the range  $0 < E < W - mc^2 - \varepsilon$ .  $|z_{0E}|^2$  decreases rapidly at high energies  $E$  and we may therefore assume  $W' = W$  except in the denominator  $W - W' = E + \varepsilon$ , if  $W$  is large compared with  $\varepsilon$ . We may then also write  $W$  instead of  $cP$  and integrate from 0 to  $\infty$ . Then the total cross-section will be

$$\sigma_D = 16\pi \left(\frac{e^2}{\hbar c}\right)^2 \int_0^\infty dE |z_{0E}|^2 \left\{ \log \frac{2W^2}{(E + \varepsilon)mc^2} - \frac{3}{4} \right\}. \quad (24)$$

We may define an average excitation energy  $A$  by putting (cf. (15))

$$\log \frac{A}{\varepsilon} = \frac{\int_0^\infty dE |z_{0E}|^2 \log(E + \varepsilon/\varepsilon)}{\int_0^\infty dE |z_{0E}|^2} = \frac{\int_1^\infty d\gamma (\gamma - 1)^{3/2} \gamma^{-4} \log \gamma}{\int_1^\infty d\gamma (\gamma - 1)^{3/2} \gamma^{-4}}, \quad (25)$$

where

$$\gamma = (E + \varepsilon)/h\nu,$$

and with this notation  $\sigma_D$  becomes:

$$\begin{aligned}\sigma_D &= 16\pi \left(\frac{e^2}{\hbar c}\right)^2 \int_0^\infty |z_{0E}|^2 dE \left\{ \log \frac{2W^2}{A mc^2} - \frac{3}{4} \right\}, \\ &= 16\pi \left(\frac{e^2}{\hbar c}\right)^2 (z^2)_0 \left\{ \log \frac{2W^2}{mc^2 A} - \frac{3}{4} \right\}.\end{aligned}\quad (26)$$

Here we have used the fact that  $\int_0^\infty dE |z_{0E}|^2$  is the diagonal matrix element of  $z^2$  for the ground state, which is

$$(z^2)_0 = \int u_0^2 \left(\frac{z}{2}\right)^2 d\tau = \frac{1}{1^2} \int u_0^2 r^2 d\tau = \frac{1}{24\alpha^2} \quad (27)$$

(cf. (10), (6)). The numerical evaluation of (25) yields

$$\log \frac{A}{\varepsilon} = 1.375,$$

and therefore

$$\sigma_D = \frac{2\pi}{3\alpha^2} \left(\frac{e^2}{\hbar c}\right)^2 \left\{ \log \frac{W^2}{\varepsilon mc^2} - 1.432 \right\}. \quad (28)$$

This, assuming the value for  $\alpha$  given in section 2, yields

$$\sigma_D = 2.3 \cdot 10^{-29} \left\{ \log \frac{W^2}{\varepsilon mc^2} - 1.432 \right\} \text{ cm}^2.$$

The probability of the process per centimetre path in heavy water will then be

$$1.5 \cdot 10^{-6} \left\{ \log \frac{W^2}{\varepsilon mc^2} - 1.432 \right\}$$

for  $W = 10^3$  volts,  $\log (W^2/\varepsilon mc^2) \sim 9.2$ , the process will therefore occur once in 0.8 km of path. This seems difficult, but not absolutely impossible to observe. The produced neutrons will have kinetic energies of the order  $3\varepsilon$ .

At lower energies, the integral of (23) decreases rapidly, and if the available energy  $W - \varepsilon - mc^2$  is small, the total cross-section becomes proportional to  $(W - \varepsilon - mc^2)^3$ .

## 7—LIMITATIONS OF THE EMPLOYED MODEL

The approximations we have made consisted in assuming (i) that  $u_0$  has the form (6) throughout, while it actually will differ from (6) for



radii of the order  $a \sim 10^{-13}$  cm, and reach a finite value for  $r = 0$ ; (ii) that the functions for  $l \neq 0$  are solutions of the field-free equation while actually at radii smaller than  $a$  they are subject to a strong attracting (for even  $l$ ) or repelling (odd  $l$ ) force.

The actual wave functions will, therefore, give values for  $z_{0E}$  which for small  $E$  differ from our approximation by a relative amount of the order  $\alpha a$ , while for  $E \sim 10^8$  volts,  $|z_{0E}|^2$  may differ appreciably from our result. Since, however, in sections 3 and 4  $|z_{0E}|^2$  enters only for small  $E$ , and in 5 and 6 the large values of  $E$  give no appreciable contribution to the integrals, our results can only be wrong by  $\alpha a$  which is about 15%. An estimate of the influence of the neglect shows that for small energies the deviations for  $l = 1$  have no importance, while that for the ground state tends to increase the matrix element slightly. This latter fact shows that the results are even to a higher approximation independent of whether one assumes equation (1A) or (1B) instead of (1C).

Equations (1A) and (1C) lead to a solution for the ground state whether the spins of proton and neutron are parallel or antiparallel. By taking into account magnetic interaction forces one could obtain a splitting into two levels, which would give rise to a discrete  $\gamma$ -absorption line. It seems impossible to treat this problem with the present theoretical means.

The authors wish to express their thanks to Dr. J. Chadwick for discussion of his experiments which led to these considerations. Their thanks are also due to Manchester University and in particular to Professor W. L. Bragg for their hospitality.

#### SUMMARY

It is shown that our present knowledge of the intra-nuclear forces allows us to make definite predictions for the behaviour of the diplon. According to these calculations the cross-section for disintegration of a diplon by absorption of  $\gamma$ -rays is of the order  $10^{-27}$  cm<sup>2</sup>, § 3. In addition the cross-sections for capture of neutrons by protons, § 4, for scattering of  $\gamma$ -rays, § 5, and disintegration by electron bombardment, § 6, have been calculated.

---