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The Meson Theory of Nuclear Forces, I*

—— The Deuteron Ground State and Low Energy Neutron-Proton Scattering ——

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Adopting the new method of the theory of nuclear forces proposed by one of us (M.T) and others, the deuteron ground state and low energy neutron-proton scattering have been investigated. The symmetrical pseudoscalar π-meson potentials including second- and fourth-order terms are assumed in the outside region, while in the inside region where the static meson potential becomes meaningless, phenomenological potentials represented by square well are adopted. It is shown that we can then account for the experimental data, if we take the value $0.09 \sim 0.10$ as the magnitude of the coupling constant between π-meson and nucleon. Saturation does not result from these potentials in the static approximation.

§ 1. Introduction and summary

Although it is well known that the problem of nuclear forces is one of the most important in nuclear theory, little is known with certainty of their detailed natures. This problem has so far been attacked essentially from two different quarters. On the one hand simple assumptions about the forces are made which account for as many facts as possible, such as binding energies, stability rules and various data from low and high energy scattering experiments. This is the so called " phenomenological " theory of nuclear forces. On the other hand it has been attempted to derive the nuclear forces from field theory and this is the meson theory of nuclear forces firstly suggested by Yukawa.

Since π-meson were produced artificially, many important natures of mesons have been clarified. Especially, it has been made clear that π-mesons interact strongly with nucleons and most part of the nuclear forces are due to π-mesons. In spite of the fact that each phenomenological nuclear potentials proposed so far can explain the experimental results in each energy region within which they are assumed, one know little about the relations between these phenomenological natures of nuclear forces and the properties of π-meson which would be essential to these forces. Therefore, in the present stage, the analysis of nuclear forces from the point of view of meson theory is much preferable to the phenomenological method and it is necessary to make clear how far we can explain experimental results about nuclear forces assuming the present meson theory.

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* Preliminary reports have been published in this journal.[1)–3)]

In this paper we shall treat the deuteron ground state and low energy neutron-proton scattering. Many investigations have been made on the same problem assuming various forms of interactions between neutron and proton.

A new method proposed by one of us (M.T) and others[4] for attacking the problem of nuclear forces is employed in the present paper. Briefly speaking, the static π-meson potential is assumed in the outside region, and in the inside region where the static meson potential becomes meaningless, phenomenological potential represented, for simplicity, by the square well is adopted. The latter stands for various effects which become important at small distances of the nucleon, namely the effects of non-static forces, higher-order terms, strong coupling and heavier mesons, etc., and is so adjusted to fit the experimental results according as each process and according as each energy region.*

As the potential assumed outside we have adopted the symmetrical *ps*** meson potential with both *ps* and *pv* couplings in the static approximation.*** In fact, recent experiments[6),7)] on the process $\pi^+ + d \rightleftarrows p + p$ and π^--capture by deuterium definitely show that charged π-mesons are pseudoscalar. Though the type of neutral π-meson is not yet well known beyond the fact that they have the spin zero, it is much plausible that both charged and neutral π-mesons have the same type, viz., pseudoscalar. Taking these facts into consideration, besides the charge independent nature of nuclear forces established at least in the low energy region,[8)-10)] symmetrical theory was assumed. We have, therefore, calculated neutron-proton scattering only.

Methods of calculation used throughout this paper are developed in § 2. They are direct numerical integrations of the wave equations using the Störmer-Levy method, essentially same as that used by Bethe.[11)]

Using these methods, we shall at first investigate the deuteron ground state and low energy neutron-proton scattering under the assumption of second- order meson potential outside in § 3. From the results which are obtained there, we suggest strongly spin dependent nature of the inside potentials.

The largest correction to these second order meson potential assumed in § 2 is probably the effect due to fourth-order potential. Higher-order terms will be negligible in the outside regin, because the range of $2n$-th order meson potential is $1/n$ times of that of the second-order.[4),12)]

The effects of the fourth-order potentials will be discussed in § 4. Deriving the fourth-order *ps* meson potential in Part (A) of that section by the S-matrix method firstly developed by Nambu[13)], some qualitative but important features of the *ps* meson potential including second- and fourth-order terms are given in Part (B), and we have confirmed the inference about the inside potential as has been expected in § 2.

Results of calculations assuming these *ps* meson potential are given in Part (C). We also consider the saturation property of nuclear forces in that section.

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* Estimated value of the critical distance where static approximation breaks down, is about half or one third of force range.[5)]

** Hereafter we shall use the following abbreviations : *ps* pseudoscalar, *pv* pseudovector.

*** Concerning the validity of weak coupling theory, see references 4), 5) and 12).

The analysis of high energy nucleon-nucleon scattering will be developed in Part II of our paper using the same mothod as that of the present Part I.

§ 2. Computational methods

Various experimental results of the low energy region (including some derived quantities) employed in this paper are es follows:—

a) binding energy of the deuteron: $|\epsilon| = 2.227(1 \pm 0.0015)$ Mev.[14]

b) electric quadrupole moment of the deuteron: $Q = (2.738 \pm 0.016) \times 10^{-27}$cm².[15]*

c) singlet effective range for neutron-proton scattering: ${}^1r = (2.7 \pm 0.5) \times 10^{-13}$cm.[14]**

d) triplet effective range for neutron-proton scattering:
${}^3r = (1.704 \pm 0.030) \times 10^{-13}$cm.[14]

e) singlet scattering length for neutron-proton scattering:
${}^1a = -23.68(1 \pm 0.0025) \times 10^{-13}$cm.[14]

f) triplet scattering length for neutron-proton scattering:
${}^3a = 5.388(1 \pm 0.0045) \times 10^{-13}$cm.[14]

g) radius of the deuteron: $r_d = 4.314 \cdot (1 \pm 0.0008) \times 10^{-13}$cm.[14]

h) mass of charged π-meson: $m_\pi = (276 \pm 6) m_e$.[17]
The value chosen here gives for the π-meson Compton wave length $\varkappa^{-1} = \hbar / m_\pi c$ $= (1.40 \pm 0.03) \times 10^{-13}$cm which is also the range of the π-meson potential.

The general form of the static interaction potential adopted in this paper is

$$V(\boldsymbol{x}) = V_c(\boldsymbol{x}) + V_t(\boldsymbol{x}) \cdot S_{12}, \tag{1}$$

where $\boldsymbol{x}$ is the relative position vector for the neutron-proton system measured in units of $\varkappa^{-1}$, while S_{12} is the usual tensor operator

$$S_{12} = [3(\boldsymbol{\sigma}_1 \cdot \boldsymbol{x}) \cdot (\boldsymbol{\sigma}_2 \cdot \boldsymbol{x}) / x^2] - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2). \tag{2}$$

(A) *Low energy singlet S scattering*

The radial wave function u of the singlet S state for zero energy obeys the wave equation

$$d^2u(x)/dx^2 - W_c(x)u(x) = 0, \tag{3}$$

where the notation $W_c(x) = (M/\varkappa^2) \cdot V_c(x)$ is introduced and M is the nucleon mass.***

As has been stated in § 1, we consider the two alternative ways of cutting-off:

a) "zero cut-off"[11]

$$W_c(x) = 0 \quad \text{for } x < x_0, \tag{4a}$$

b) infinite repulsive well inside (hard core model)[9),18]

$$W_c(x) = +\infty \quad \text{for } x < x_0. \tag{4b}$$

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* Newell has obtained a different value for the quadrupole moment of deuteron, see ref. 16).

** Singlet effective range for proton-proton scattering coincides with that for neutron-proton scattering within the experimental error.

*** We adopt the natural units $\hbar = c = 1$ throughout this paper.

x_0 is the cut-off radius where the meson potential assumed in the outside region breaks down.[4]

The above wave equation is to be solved subject to the boundary conditions

$$u(x=0)=0. \tag{5}$$

$$u(x \to \infty) \sim 1-({}^1a)^{-1}r = 1-({}^1a)^{-1}\varkappa^{-1}\cdot x\,. \tag{6}$$

The integration for $x < x_0$ being elementary, we have for zero cut-off

$$u=cx \tag{7a}$$

with an arbitrary constant c and for hard core inside

$$u=0. \tag{7b}$$

For $x > x_0$, direct numerical integrations (Stormer-Levy method) have been carried out from the outside in, starting from the boundary condition (6) at large x. Since the potential $V(x)$ increases rapidly with decreasing x, the transformations

$$y=\log x, \quad U=u\cdot x^{-1/2} \tag{8}$$

are very convenient for such calculations.[11]

Then the wave equation (3) reduces to the form

$$d^2U/dy^2 = (x^2 W_c + 1/4)\cdot U. \tag{3'}$$

The starting point for numerical integration was in general $y=1.65$, corresponding to $x=5.207$, while the interval was chosen equal to 0.15 units of the natural logarithm.

Cut off radius x_0 is determined by the continuity condition, a smooth join of wave function and its derivative to the interior solution (7a) or (7b). This condition is for zero cut-off

$$(1/U)\cdot(dU/dy) = 1/2 \ \text{at} \ x=x_0 \tag{9a}$$

and for hard core inside

$$U=0 \ \text{at} \ x=x_0\,. \tag{9b}$$

Singlet effective range ${}^1r$ is expressed in terms of the wave function $u(x)$ by the expression[10]

$${}^1r = 2\int_0^\infty \{[1-({}^1a\varkappa)^{-1}x]^2 - u^2(x)\}\,dx \tag{10}$$

where $u(x)$ is normalized so that it approaches the asymptotic form (6) at large distances.

(B) *The deuteron ground state and low energy triplet S scattering*

The wave function for the deuteron ground state may be written as a linear combination of S and D states

$$\psi = (1/x)[u(x) + (1/8^{1/2})\cdot S_{12}\cdot w(x)]\,\chi_m \tag{11}$$

where χ_m is the spin function with magnetic quantum number m. Then, if we make the following abbreviations,

$$\gamma^2 = (M/\varkappa^2)\cdot|\epsilon|, \quad A=\gamma^2 + (M/\varkappa^2)\cdot V_c(x),$$
$$B=-2(M/\varkappa^2)\,V_t(x), \tag{12}$$

$u(x)$ and $w(x)$ satisfy the two simultaneous differential equations

$$d^2u(x)/dx^2 = Au(x) - \sqrt{2}Bw(x) \tag{13a}$$

$$d^2w(x)/dx^2 = (A+B+6/x_2)\cdot w(x) - \sqrt{2}Bu(x). \tag{13b}$$

We consider again the two alternatives for cutting-off:

a) zero cut-off

$$A=B=0 \quad \text{for } x< x_0, \tag{14a}$$

b) infinite repulsive well inside

$$A=B=+\infty \quad \text{for } x< x_0. \tag{14b}$$

Numerical integrations in the region from $x=5.207$ to $x=x_0$ were the same as that of (A). The wave functions $u(x)$ and $w(x)$ thus obtained were normalized according to

$$\int_0^\infty \{u^2(x)+w^2(x)\}\cdot dx=1. \tag{15}$$

Then the quadrupole moment

$$Q=(\sqrt{2}/10\varkappa^2)\int_0^\infty x^2[uw-(1/2\sqrt{2})w^2]\,dx \tag{16}$$

and D-state probability p_D

$$p_D=\int_0^\infty w^2(x)\cdot dx \tag{17}$$

are readily calculated using the above wave functions. We can also calculate the triplet effective range ${}^3r$ using the expression[20]

$${}^3r=2\int_0^\infty [\exp(-2\gamma x)-(u^2+w^2)]\,dx. \tag{18}$$

Here the normalization of (u^2+w^2) must be chosen so that it approaches $\exp(-2\gamma x)$ as $x\rightarrow\infty$.

§ 3. Results of the calculations using the second-order ps meson potential

The second-order ps meson potential with ps coupling does not appear in the static approximation. So we have the pv coupling term only,

$$V_c^{(2)}=-\varkappa\cdot g^2/4\pi\cdot\exp(-x)/x, \tag{19}$$

$$V_t^{(2)}=-\varkappa\cdot g^2/4\pi\cdot(1+3/x+3/x^2)\cdot\exp(-x)/x. \tag{20}$$

where the prime " 2 " indicates that $V^{(2)}$ is the second-order potential.

For $x<x_0$, we adopted for convenience' sake, the hard core*

$$V_c=V_t=+\infty. \quad \text{for } x<x_0. \tag{21}$$

* Ferretti has also treated the same problem assuming repulsive potential inside.[21] We are indebted to Prof. Ferretti for sending us a reprint of his paper.

The results are given in Table 1.

Table 1. Second order π-meson potential outside and hard core inside

$g^2/4\pi$	x_0	Q (in 10^{-27} cm^2)	p_D (%)
0.025	0.122	1.07	4.2
0.050	0.228	1.877	6.3
0.075	0.387	2.542	7.4
0.084*	0.458	2.766	7.54

* These values were obtained by extrapolation.

As is readily seen from these results, the correct value of the quadrupole moment is obtained for $g^2/4\pi=0.084$ by extrapolation. If, however, the quadrupole moment is abjusted to the experimental data, the amount of D-state admixture is much larger than the usually adopted value 4%. This is because the range of the π-meson potential is somewhat shorter than that of phenomenological potentials with short tail.[4] The quadrupole moment decreases rapidly with decreasing force range. Therefore, to get the correct magnitude of quadrupole moment, we must assume a large value for coupling constant and, as a necessary consequence, the D-state probability increases together.[20] Concerning the relation between the magnetic moment of deuteron and the D-state probability, we shall discuss further in next section.

The repulsive potential assumed in the inside region plays an important role. That is, the wave function is pushed out, so the quadrupole moment and triplet effective range increase in comparison with the other cutting-off method.[1),22]

From these results, we can infer that the potential at small distances where the second-order meson potential breaks down may be fairly repulsive for triplet S state.

On the other hand, it is impossible to account for the experimental value of singlet scattering length provided that the value of the coupling constant is so chosen as to fit the correct value for the deuteron quadrupole moment. From our standpoint, we must adopt the same potential with the same value of coupling constant in the outside region for all states. Therefore, the inside potential for singlet even state must necessarily attractive in contrast to the strong repulsion required in the triplet even state. This inference is confirmed by the calculation of fourth-order meson potential in next section.

§ 4. Fourth-order meson potential

(A). *Derivation of fourth-order meson potential**

Fourth-order terms of the static *ps* meson theory have been calculated by the method of S-matrix.[13] From the Feynman diagrams (Fig. 1), we get the results up to first order of the expansion in terms of (m_π/M) as follows:[2]

$$V_{ps}{}^{(4)}=\varkappa\cdot\left(\frac{f^2}{4\pi}\right)^2\frac{1}{8\pi}\left(\frac{m_\pi}{2M}\right)^2\cdot\sum_{k,l=1}^{3}[\rho_2\tau_k,\ \rho_2\tau_l]_+^{(1)}[\rho_2\tau_k,\ \rho_2\tau_l]_+^{(2)}\ \frac{K_1(2x)}{x^2}$$

$$=\varkappa\cdot\left(\frac{f^2}{4\pi}\right)^2\frac{3}{8\pi}\left(\frac{m_\pi}{M}\right)^2\frac{K_1(2x)}{x^2}, \qquad (22)$$

* K. Nishijima has obtained the same results for *pv* coupling, using a different method of calculation (canonical transformations). See, reference[12].

$$V_{pv}^{(4)}=\varkappa\cdot\left(\frac{g^2}{4\pi}\right)^2\cdot\sum_{k,l=1}^{3}[(\sigma\nabla)\tau_k,\ (\sigma\nabla)\tau_l]_-^{(1)}[(\sigma\nabla)\tau_k,\ (\sigma\nabla)\tau_l]_-^{(2)}\frac{K_0(2x)}{x}$$
$$=\varkappa\cdot\left(\frac{g^2}{4\pi}\right)^2[(\tau^{(1)}\tau^{(2)})U_\tau(x)+(\sigma^{(1)}\sigma^{(2)})U_\sigma(x)+S_{12}U_T(x)]. \tag{23}$$

If one attempts to carry out this calculation relativistically, unrenormalizable divergences appear in the pv coupling case. In the static approximation, however, these divergences disappear fortunately. Furthermore, we can reduce the divergences resulting from the diagrams shown in Fig. 2 to renormalizable terms and to contact interaction terms, provided that the static approximation is allowed.[12)]

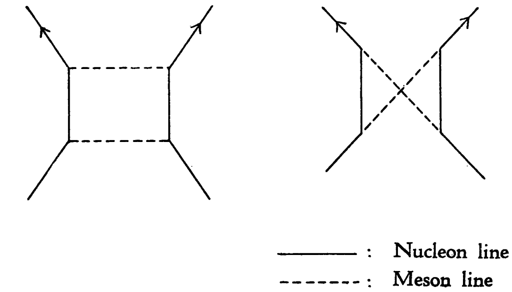

Fig. 1.

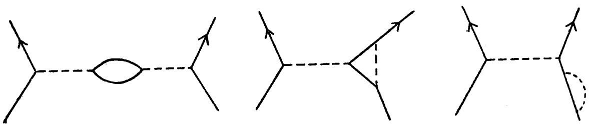
Fig. 2.

Under these inevitable circumstances arising from the incompleteness of the present field theory, we could not help confining ourselves to the static approximation. Important terms such as spin-orbit coupling will probably result from non-static part of the fourth-order potentials, but unambiguous calculation is very difficult in the present stage of the field theory.

As can be seen from the commutator in the expression of $V^{(4)}$, fourth-order meson potentials are purely quantum mechanical effects.[5)]

(B). *Qualitative natures of the meson potential including fourth-order terms*

Assuming that the value of the ps coupling constant $f^2/4\pi$ is not much larger than that of the pv coupling, $g^2/4\pi$, we confine ourselves to the pv coupling only.* Then the fourth-order meson potentials (23) reduces to the following form in each state:

a) triplet even state

$$V_c^{(4)}=\varkappa(g^2/4\pi)^2\cdot(-3U_\tau+U_\sigma), \tag{24a}$$
$$V_t^{(4)}=\varkappa(g^2/4\pi)^2\cdot U_T, \tag{24b}$$

b) singlet even state

$$V_c^{(4)}=\varkappa(g^2/4\pi)^2\cdot(U_\tau-3U_\sigma), \tag{25}$$

c) triplet odd state

$$V_c^{(4)}=\varkappa(g^2/4\pi)^2\cdot(U_\tau+U_\sigma), \tag{26a}$$
$$V_t^{(4)}=\varkappa(g^2/4\pi)^2\cdot U_T, \tag{26b}$$

* Since $V_{ps}^{(4)}$ is an ordinary (Wigner) type, neglect of this term does not alter the following results essentially.

d) singlet odd state

$$V_c^{(4)} = -x(g^2/4\pi)(3U_\tau + 3U_\sigma). \tag{27}$$

Here U_σ, U_τ and U_T are defind using the modified Bessel functions K_0 and K_1*,

$$U_\sigma = 8/\pi\{(3/x^2)K_0(2x) + (2/x^2+3/x^4)K_1(2x)\}, \tag{28}$$

$$U_\tau = -8/\pi\{(1/x+23/4x^3)K_0(2x) + (2/x^2+23/4x^4)K_1(2x)\}, \tag{29}$$

$$U_T = -8/\pi\{(3/x^3)K_0(2x) + (1/x^2+15/4x^4)K_1(2x)\}. \tag{30}$$

These potentials are shown in Figs. 3, 4, 5 and 6 respectively.

In the same figures, the second-order potentials $V^{(2)}$ and the resultant potentials $V = V^{(2)} + V^{(4)}$ are illustrated together.

From these figures, one can see some qualitative but very important features of the *ps* meson potential.

1). In the triplet even state, central force is strongly repulsive and tensor force is strongly attractive, being similar to that of Bethe's neutral vector meson theory.[11] This repulsive potential was represented in last section by the infinite repulsive well assumed in the inside region.

2). Strong attractive force in the singlet even state may account for the experimental value for the singlet scattering length. We have suggested this attractive force in last

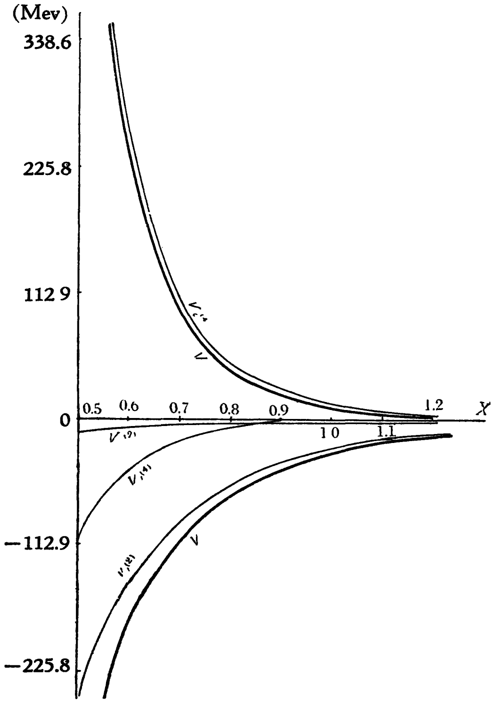

Fig. 3 Triplet even stats $g^2/4\pi = 0.08$

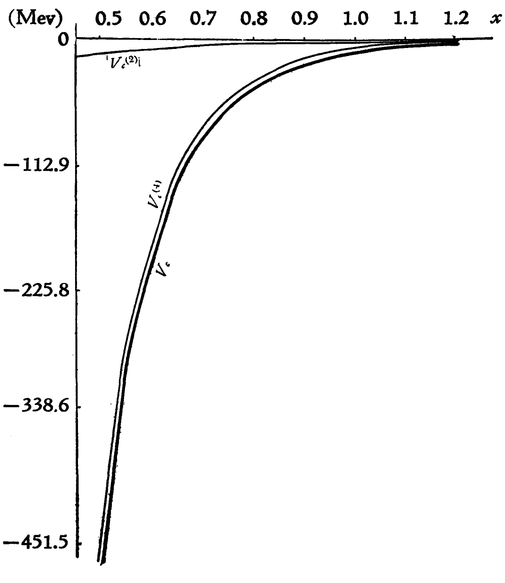

Fig. 4 Singlet even state $g^2/4\pi = 0.08$

* U_τ and U_σ are negative while U_T is positive. Roughly speaking, $|U_\sigma| \sim U_T \sim 1/2|U_\tau|$.

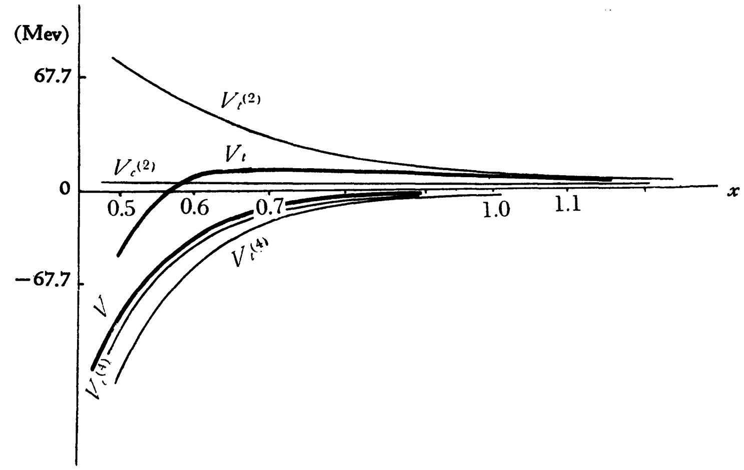

Fig. 5 Triplet odd state $g^2/4\pi=0.08$

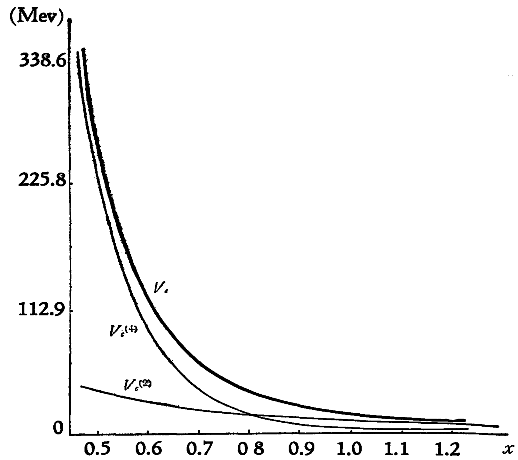

Fig. 6 Singlet odd state $g^2/4\pi=0.08$

section.

3). Low and high energy proton-proton scattering may be explained fairly well if a hard core is assumed inside, since these potentials are similar to those adopted by Jastrow.[18]

4). The fact that odd state interactions are much weaker than those of the even state is just what is required for accounting for the experimental angular distribution of high energy neutron-proton scattering.

We shall discuss the high energy neutron-proton scattering in the subsequent paper (Part II).

(C). *Results of calculations and discussions*

Tables 2 and 3 show the results which have been obtained using the above potentials $V=V^{(2)}+V^{(4)}$.

Table 2a. Singlet neutron-proton scattering; zero cut-off

$g^2/4\pi$	${}^1r$ (in 10^{-13} cm)	x_0
0.075	1.925	0.5886
0.090	2.457	0.665

Table 2b. Singlet neutron-proton scattering; infinite repulsive well inside

$g^2/4\pi$	${}^1r$	x_0
0.075	2.104	0.3285
0.090	2.585	0.384

Table 3. The deuteron ground state and triplet neutron-proton scattering; zero cut-off *

$g^2/4\pi$	x_0	Q (in 10^{-27}cm^2)	^3r (in 10^{-13}cm)	p_D (%)
0.05	0.16	1.85	1.325	7.42
0.08	0.232	2.37	1.582	8.65
0.10	0.303	3.25	1.75	11.3

* Infinite repulsive potential assumed inside does not give a correct binding energy for deuteron.

As is well known, the percenrage of D-state wave function in the ground state of deuteron is closely related to the magnetic moment, and the value 4% obtained from this relation has been adopted so far. This relation forms, however, an unreliable restriction, not only because of the uncertain relativistic corrections[23] but also of a dependence of the magnetic moment of one nucleon on the proximity of another. Recently, the latter effect has been confirmed by the experimental ratio of hyperfine structure in deuterium and hydrogen.[24] Taking these facts into consideration, it seems meaningless to adhere to the value 4%, although the correct value is not known.** Therefore, the D-state probability obtained here, viz., $9\sim10\%$, is not necessarily incompatible with the present experimental data.

Whether this potential satisfies the saturation requirements or not has also been investigated. Rough estimation shows that conditions for saturation[25] break down in several respects. For example, spin saturated neutron cluster may be stable. Even if we take into account the term of ps coupling, not all saturation requirements cannot be satisfied.

§ 5. Conclusions

From the above results, it is possible to conclude that the ps meson potentials with pv coupling including second- and fourth-order terms may well account for the experimental data for the deuteron ground state and low energy neutron-proton scattering, provided that suitable potentials are assumed in the inside region. The best value of the coupling constant between π-meson and nucleon is $0.09\sim0.10$.

Saturation does not result on the assumption of the ps meson potential only in the static approximation.

** **Note added in proof:** Recently, H. Miyazawa has estimated the magnitude of D-state admixture. The value obtained here is not in contradictory to his results. See H. Miyazawa, Prog. Theor. Phys. **7** (1952), No. 2.

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