Introduction

Nuclear Forces and Meson Theory

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During a period in the postwar time the theory of elementary particles had made amazing progress, but later seemed to be stagnant. Recently many people have come to believe that the bright prospect of the theory has been opened step by step. However even in that stagnant time, under the ground, buds waiting for spring were nourished and prepared for producing flowers.

As soon as the experimental works were reopened in many countries after the war, Tomonaga's super-many-time theory, Sakata's theory of two-meson in cosmic rays, which were all accomplished during the war, were verified and those theories themselves were also developed to great extent. Calculations on the two-meson theory and the decay of the neutral meson were done by the newly developed Tomonaga's method and many contributions to the meson theory were accumulated. In fact, since Yukawa's prediction of the existence of the meson in 1935, we had been informed many properties of the meson during the postwar period. However, if we reminded carefully the situation at that time, we did not know anything quantitative with respect to the properties of the meson, though the qualitative behaviours were known fairly well.

We might conclude that the only established fact up to 1950 is that the meson is not the Fermi particle as Marshak insisted upon, but the Bose particle as Sakata predicted. The fact was clarified by the analysis of the pion, which was discovered by Powell and his collaborators (1947) with the nuclear plate. As it is well known, there were four possibilities with respect to the types of the meson field; vector, pseudovector, scalar, pseudoscalar. We had to first determine the type. As it is shown in Progress of Theoretical Physics issued in Commemoration of the Fifteenth Anniversary of the Discovery of Meson Theory (1950), the vector type was promising for the meson field. (Later it was found that the meson field is the pseudoscalar type.) However it might be natural that the knowledge of pions was not developed, because the mesons which were observed up to that time were muons.

Although everybody expected that, if the meson could be created artificially and any desirable experiments would become possible, the meson theory might be established immediately, it was unfortunately not the case.

This is due to the fact that the interaction between the meson and the nucleon is not so simple as the interaction between the electromagnetic field and the electron. The latter was successfully treated by Tomonaga's theory of quantum electrodynamics. However, if we applied the same method to the meson field, we had to encounter many difficulties. First, since the coupling constant of the interaction between the meson and the nucleon ($g^2/\hbar c \cong 1/10$) is much stronger than that of the interaction between the electromagnetic field and the electron ($e^2/\hbar c = 1/137$), the convergency of the perturbation expansion could not be expected, contrary to the latter case. Secondly, the type of the interaction between the meson and the nucleon is rather complicated. It shows not only the very strong coupling but the divergent character. In order to overcome these difficulties the strong coupling theory and Tomonaga's intermediate coupling theory were proposed, but much success was not brought in.

In such a situation we purposed to attack this problem from the side of nuclear forces, i. e., the interaction between nucleons. The reasons why we chose the nuclear forces, where the meson is invisible, while the real meson can be produced artificially and available for any experiment, were based on the following considerations.

To construct the nuclear forces in the meson theory, as Yukawa proposed in his first paper, we calculated the process of the meson-exchange between two nucleons. In this case the meson is not observed as a *real* particle but only as a *virtual* one in the intermediate state. If the real meson is absorbed or scattered by a nucleon, the heavy particle which mass is 273 times the electron mass must be absorbed or scattered. Such a process might cause a drastic change. It seems far from the classical picture of a field as compared with the case of photons which rest mass is zero. On the contrary, in the case of nuclear forces we may expect rather classical and simpler character than the drastic change in the case of the scattering or the production of the real meson, because in the former case mesons are exchanged without appearing as real mesons.

Secondly, if the inter-nucleon distance is very large the nuclear forces should be rather simple, because the meson field in such a case is necessarily weak. The interaction between the meson field and the nucleon is certainly strong as pointed out in the strong coupling theory or in the intermediate coupling theory, and very hard to be treated. However, if the meson field itself is weak at large distances, interaction will be small effectively even as the coupling itself is large and the conventional perturbation method may be useful as in the electromagnetic field.

Furthermore there is one more special reason favourable to the nuclear force problem. The force ranges of the nuclear forces depend on the number of mesons to be exchanged at the intermediate state resulting in the nuclear forces. In fact, at very large distances the one-meson-exchange potential is most effective, and in the next region the two-meson-exchange potential becomes dominant, etc. Therefore we may be able to separate successfully the effect of the meson-exchange process according to the number of exchanging mesons, and examine each term calculated with the meson theory being compared with experimental data step by step. On the contrary in other phenomena the higher terms mix in all together and there is no evidence for the fact that the higher terms are small, so that it seems very difficult to obtain the meaningful result.

In addition to these considerations, we tried to find the important instruction in the transition case from the classical theory to the quantum mechanics. It is the Rutherford scattering. Rutherford calculated the scattering of the charged particle by the Coulomb field in the classical electromagnetic theory. However, if we calculate the same phenomenon in the quantum mechanics, we obtain the very same formula as Rutherford's. The formula does not contain the Planck constant h, though the formula is derived quantum-mechanically. For the most part quantum mechanics is separated by a crevasse from the classical theory. However, since there exists fortunately a continuous way at the place of the Rutherford scattering, going through this way, Rutherford succeeded in obtaining some important atomic information.

The quantum mesodynamics might be much more difficult than the quantum electrodynamics. However, as in the case of the Rutherford scattering, we might expect a fairly simple problem in which we could obtain plausible results even by applying the method of the quantum electrodynamics. Then we might be able to reveal the substantialistic properties of the meson theory in the region of such a problem, and on the assurance we should proceed to the mesodynamics. Otherwise we could not know whether our knowledge of the meson field was wrong or the mesodynamics was incomplete. In other words we did not know which part of our knowledge of the meson theory was incorrect and to what extent the meson theory had been established.

Furthermore, at that time it was not trusted by many people that one should apply the meson theory to the nuclear force problem. In such a situation we proposed the following definite methodology to attack the nuclear force problem. First, we have to make a substantialistic treatment for the nuclear forces when the two nucleons separate very widely each other. In other words in this outer region the meson-theoretical calculation seems to be useful. Secondly, if the two nucleons approach very near each other, the higher-order effects of the meson theory are very complicated and the concept of potential becomes doubtful. Besides, there appear not only the static part but the dynamical part, the relativistic effect, the effect of heavy mesons, etc. Consequently we can not carry out any theoretical treatment of the nuclear forces in the inner region. That is, the region is far beyond the applicability of the present meson theory so that we should determine rather phenomenologically parameters to fit with experiments.

Composing the above considerations, we proposed a new methodology for the nuclear force research (Taketani, Nakamura, and Sasaki 1951; this paper* will be referred to TNS). The first definite result obtained by proceeding the research under this methodology was that the type of the meson was determined not as a vector but as a pseudoscalar for the first time (1951). Later we were informed that about the same time the similar result concerning the type of the meson were obtained by the experiment on the real meson in the United States.

Then, using this method we calculated the fourth-order potential which gives fairly good fit with the experimental results in the main features. This potential will be referred to the TMO potential (Taketani, Machida, and Onuma 1951).

Since we had written mainly the fundamental planning and the many theoretical outlooks in TNS, it was very necessary to calculate practically each problem and to verify these outlooks.

In those days we were informed that the International Conference on Theoretical Physics would be held in Japan in the fall of 1953. Then Japanese workers made the plan that they would organize the research group in order to attack each problem systematically and let the representative of each group read the review of their results. In such a way the Japan nuclear force group was organized from the workers in every place in Japan. Now the problems mentioned above were ready to be solved one by one by the nuclear force group.

At that time against our method there were strong criticism and also apathy in many people. Especially the intermediate coupling school critisized our method from the beginning. The nuclear force group was accumulating the convincing answers to these questions.

The TMO potential was derived from the perturbation method. However, in the case of the *ps*-meson wtih *pv*-coupling it was shown that the fourth-order potential exceeds much more the second-order potential even near the pion Compton wave length where the one-pion-exchange potential seemed to be most effective. We called this fact Machida's effect. It is due to the special character of the *ps*-meson. However the fact that the fourth-order effect is larger than the second-order one near the range means

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* The reprint of this paper is included in the present supplement as Part V for convenience of readers (see p. 169).

that the perturbation approximation may be doubtful. Therefore we had to investigate the following points: Where from the sixth-order potential becomes appreciable and what is the order of the magnitude? What is the behaviour of the higher-than-sixth-order terms? Is the radiative correction appreciable near the force range?

The calculation of the sixth-order potential was performed by Machida and his collaborator, who showed that the effect can be neglected outside $0.6\kappa^{-1}$ or $0.7\kappa^{-1}$. (κ^{-1} is the pion Compton wave length*).

Although the TMO potential is a static potential, according to our opinion the dynamical effect may be very small outside the neighbourhood of the force range except in the triplet odd state, where the dynamical effect and the relativistic effect are very sensitive to the nuclear force, so that much more careful calculation may be needed. However, in general our static potential is valid outside the force range. In this point the TMO potential shows a remarkable contrast with Lévy's nonadiabatic calculation which appeared in the next year to the TMO potential. Lévy's calculation was getting into very popular in the United States, while the TMO potential was ignored in the United States. The non-static calculation had been proceeding in the hands of I. Sato in Japan before Lévy's calculation. Then the non-static calculation was developed by Sato and it was shown that the static potential is good enough to explain the experiments outside about the force range.

Machida's effect compelled us to adopt some method other than the perturbation calculation. Besides, since the isobar state of a nucleon was suggested by Fujimoto and Miyazawa, it was needed to take account of the effect into our calculation of nuclear forces. My anticipation of this problem at that time was as follows: The isobar effect would become considerably large in the nucleon-nucleon scattering over 100 Mev. Below the energy the effect would not be appreciable but slightly change the coupling constant. However, the isobar effect might be different in various states of the two-nucleon system. Therefore, taking into account the qualitative considerations we had only to modify a few points.

Concerning such an anticipation Kikuta estimated the effect by the strong coupling method and showed that the effect is not so strong as to change the character of the nuclear potential in the outer region. Furthermore it seemed necessary to calculate the nuclear potential of the two-nucleon system by the method of Tomonaga's intermediate coupling approximation. The meson clothes of the two-nucleon system are not the simple sum of the meson clothes of each nucleon but we should introduce the gourd-shaped deformation from first. We called this approximation the

* Since κ^{-1} is about 10^{-13}cm, we would like to introduce a new unit of length, the **yukawa** (1 yukawa=1y=10^{-13}cm).

"Hyotan approximation," where Hyotan means a gourd in Japanese and a dumb-bell like wine bottle. This attempt was also to answer to the criticism of the intermediate coupling school against our method. Using this approximation, Hasegawa and his collaborator made the calculation of nu- effective clear forces.

On the other hand the comparison of the meson potential with experiments was pushed on by Tani, Otsuki, and their collaborators. After the war in the United States analyses of high energy nucleon-nucleon scatterings were performed by the use of phenomenological potentials, e.g., Yukawa, Gauss, and exponential. However, since the characters of these potentials are not so distinctive, Matsumoto and Watari, assuming the combination of a shallow and wide square-well potential and a deep and narrow square-well potential, made clear what part of the nuclear forces is effective to each phenomenon.

In foreign countries the D-state probability of about 4%, which was derived from the deuteron magnetic moment, prevailed as the most reliable value and it gave the too strong restriction to the nuclear force problem. For several reasons we did not believe the value from first. It was strongly longed for to derive the D-state probability from the more reliable method. Machida satisfied our demand, determining the value as 5%~10%. (Recently, using new data, Machida has recalculated it as 6%~10%.)

Most parts of the above results were reported by the present author at the International Conference on Theoretical Physics, Kyoto, in the fall of 1953. Unfortunately foreign workers on nuclear forces did not seem to understand thoroughly our method.

Later the most general method to construct the nuclear potential was proposed by Fukuda, Sawada, and Taketani, which will be referred to FST's method. Then, by my group in Rikkyo University the concrete shape of the meson potential derived by FST's method was given explicitly. Comparing with the result calculated by the intermediate coupling method, Machida revealed the reliable region of the meson potential. It has been shown by Machida and his collaborators that the one-pion-exchange potential is not changed but only the unrenormalized coupling constant is replaced by the renormalized one in the outer region, even if we include the radiative effects.

On the other hand Otsuki and his collaborators compared the meson potential with experimental results very in detail, and showed that (1) the meson potential is indispensable outside the force range, and (2) the coupling constant can be determined uniquely as $g^2/\hbar c \sim 0.08$ from only the outer part of the nuclear forces.

Considering these many results mentioned above, now we may be allowed to conclude that the meson potential as the outer part of the nuclear forces has been established and also that we have succeeded in establishing the meson theory in the region of the classical theory.

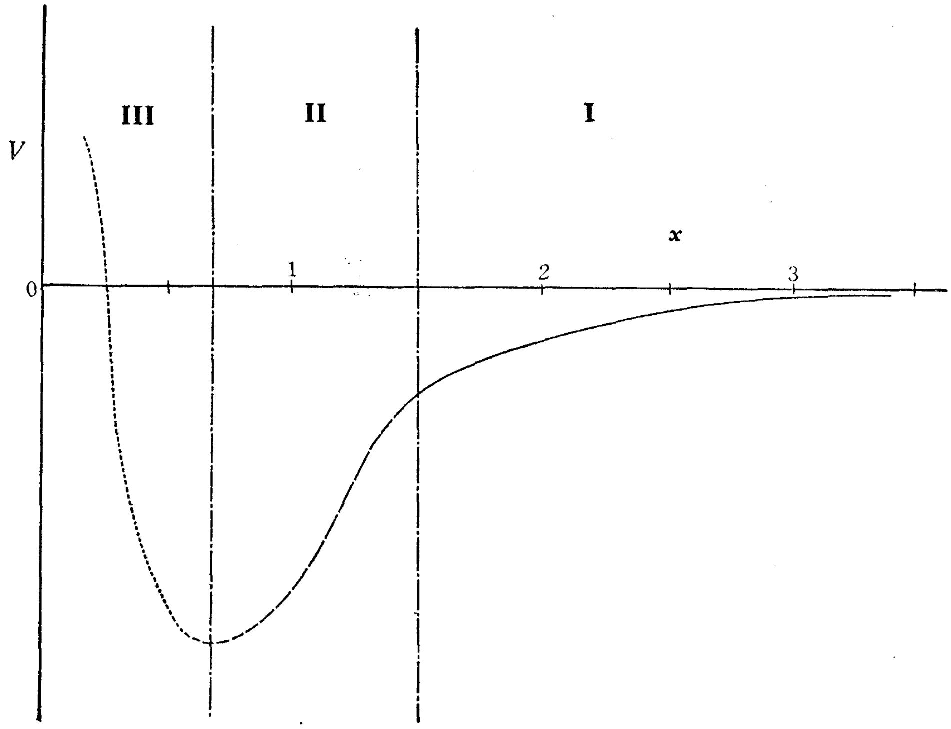

Fig. I. Three Regions of Nuclear Potential.*

Region I. *Classical region*, $r \lesssim 1.5\kappa^{-1}$, ($\kappa^{-1}$ is the pion Compton wave length) where the one-pion-exchange potential dominates and the quantitative behavior of the potential has been established.

Region II. *Dynamical region*, $0.7\kappa^{-1} \lesssim r < 1.5\kappa^{-1}$, where the two-pion-exchange potential competes with and exceeds the one-pion-exchange potential. The recoil effect is also appreciable in this region. The qualitative behavior, however, has been clarified.

Region III. *Phenomenological region*, $r \lesssim 0.7\kappa^{-1}$, where exist so many complicated effects, e.g., the relativistic effect, the isobar effect, the effect of new particles, etc., that at present we may have no means but some phenomenological treatment to fit with experiments.

* x is the inter-nucleon distance in the unit of the pion Compton wave length, $\kappa^{-1} \cong 1.4 \times 10^{-13}$ cm (see p. 35).

Figs. II—V. Pion Theoretical Potential*

in the triplet even state: Fig. II,
in the singlet even state: Fig. III,
in the triplet odd state: Fig. IV,
in the singlet odd state: Fig. V.

For derivation methods refer Part III.

OPEP: The one-pion-exchange potential with $g_e^2/4\pi=0.08$.

FST: The potential constructed by Fukuda, Sawada, and Taketani's method, with $g_e^2/4\pi=0.08$. The probability that the nucleons are to be bare is properly taken into account. The high frequency part of the pion field is cut off by the gaussian factor with the cut-off momentum $k_c=6\mu c$, μ being the pion mass (Part III, **3** and **4**).

TMO: The second- plus fourth-order potential in the perturbation expansion calculated by Taketani, Machida, and Onuma, with $g^2/4\pi=0.08$. The probability part is also expanded in powers of the coupling constant (Part III, **1** and **3**).

BW: The second- plus fourth-order potential calculated by Brueckner and Watson with $g^2/4\pi=0.08$. The probability part is approximated by unity (Part III, **2** and **3**).

IC: The one- and two-pion-exchange potential by the intermediate coupling method with $g_e^2/4\pi=0.08$. The cut-off momentum of the gaussian cut-off factor is $k_c=4.1\mu c$ (Part III, **6**).

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* x is the inter-nucleon distance in the unit of the pion Compton wave length, $\kappa^{-1}\cong 1.4\times 10^{-13}$ cm (see p. 35).

Triplet Even

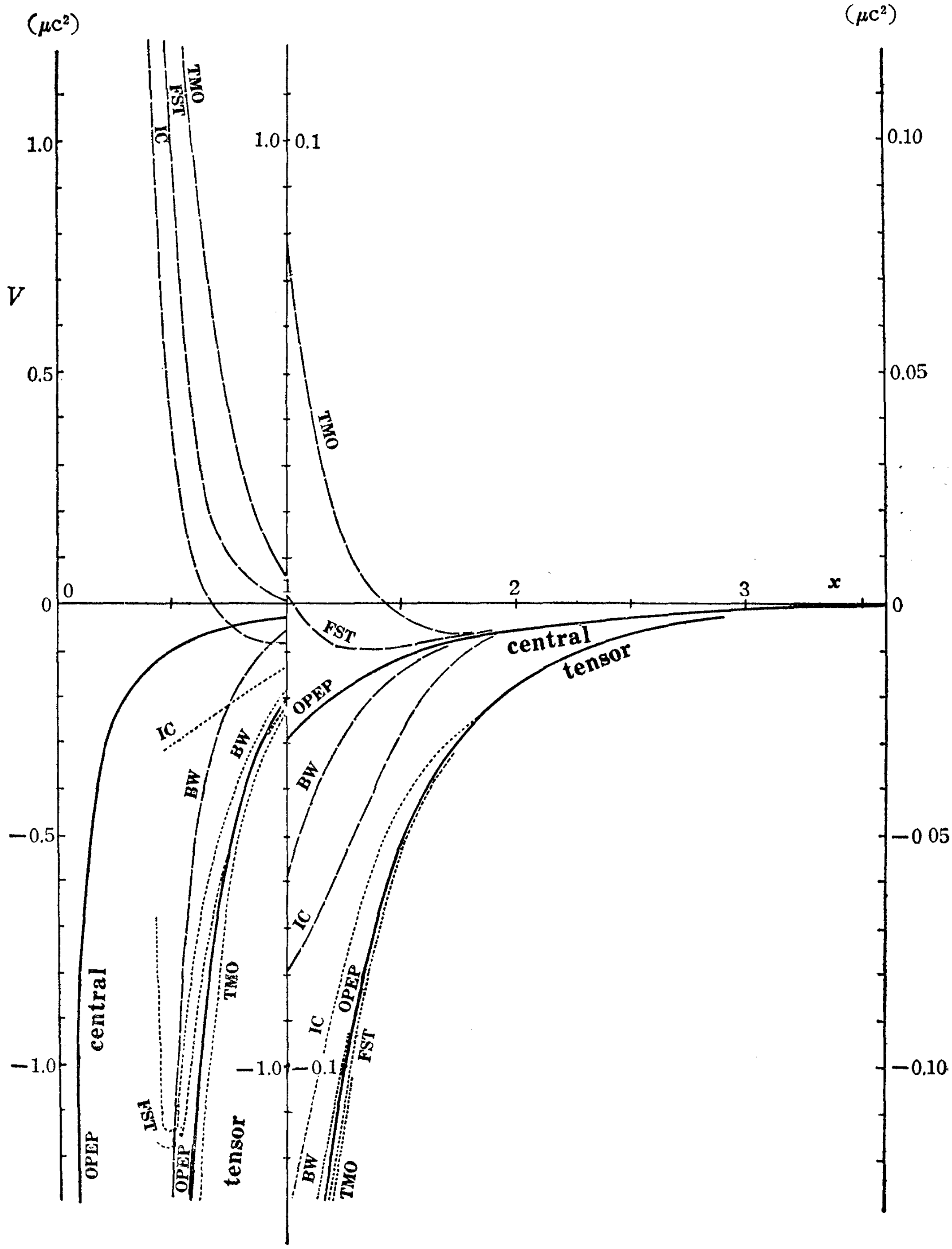

Fig. II

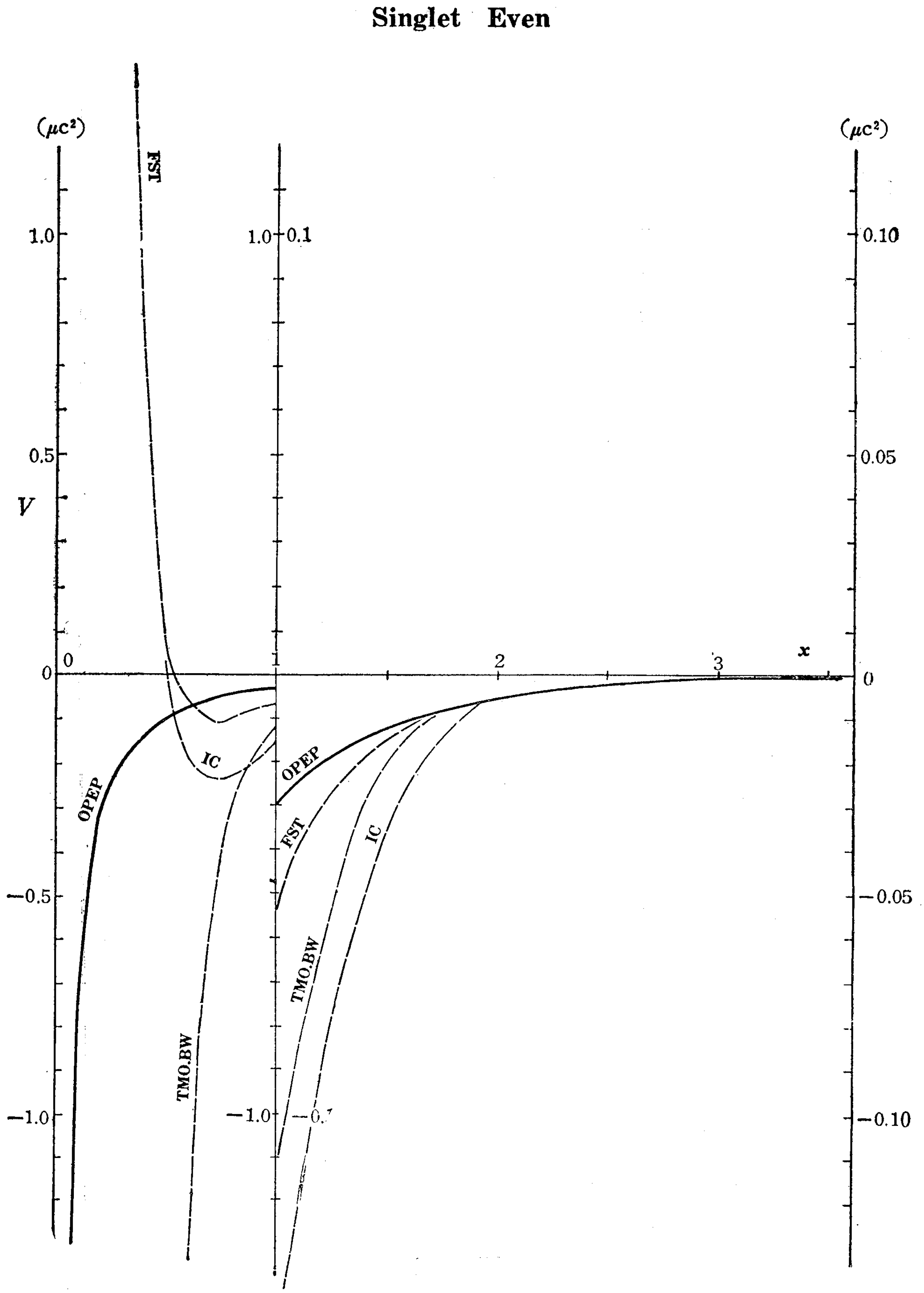
Singlet Even
(μc²)
(μc²)
FST
1.0
1.0
0.1
0.10
0.5
0.05
V
0
0
1
2
3
x
0
OPEP
IC
OPEP
FST
IC
TMO.BW
TMO.BW
−0.5
−0.05
−1.0
−1.0
−0.10

Fig. III

Triplet Odd

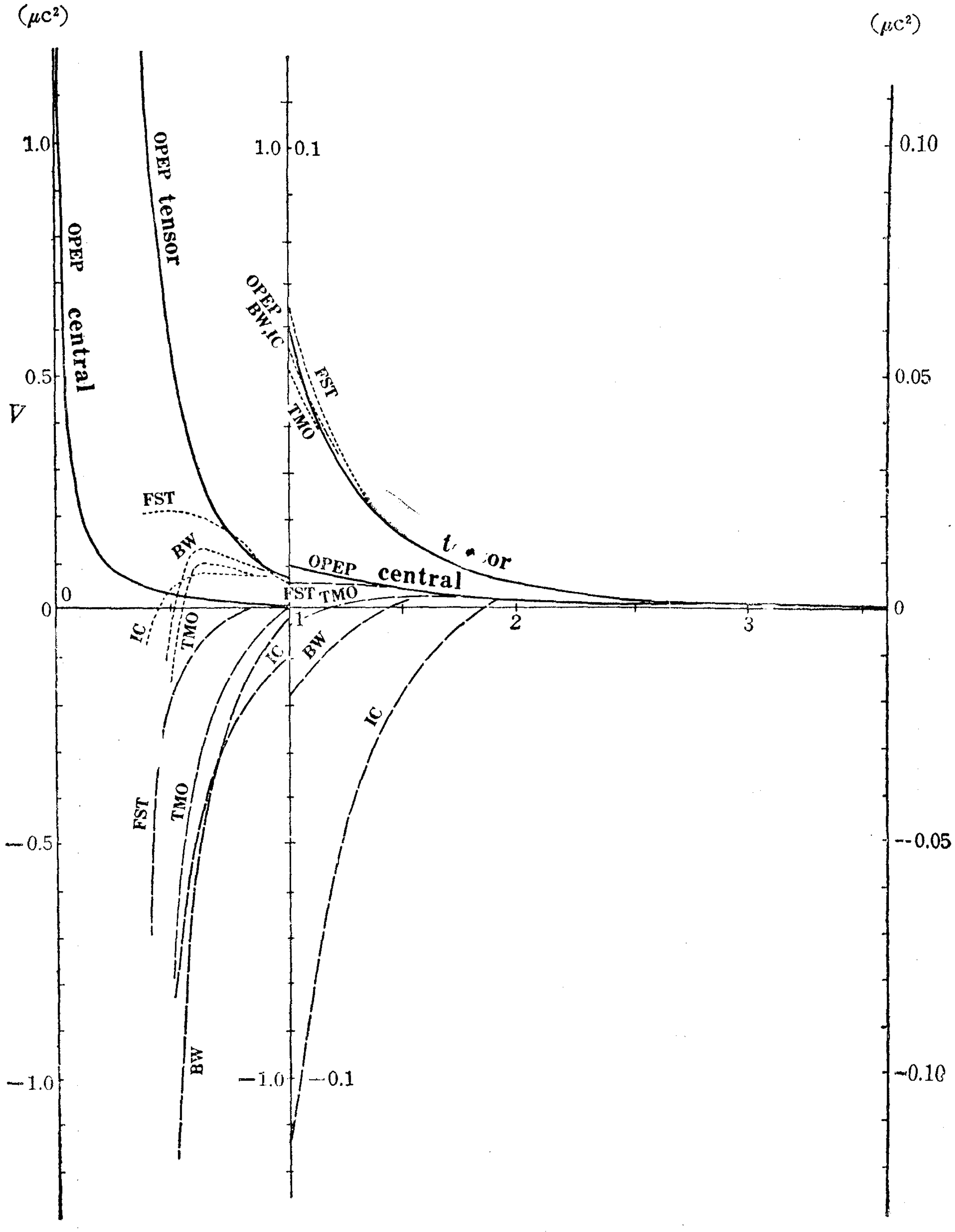

Fig. IV

Singlet Odd

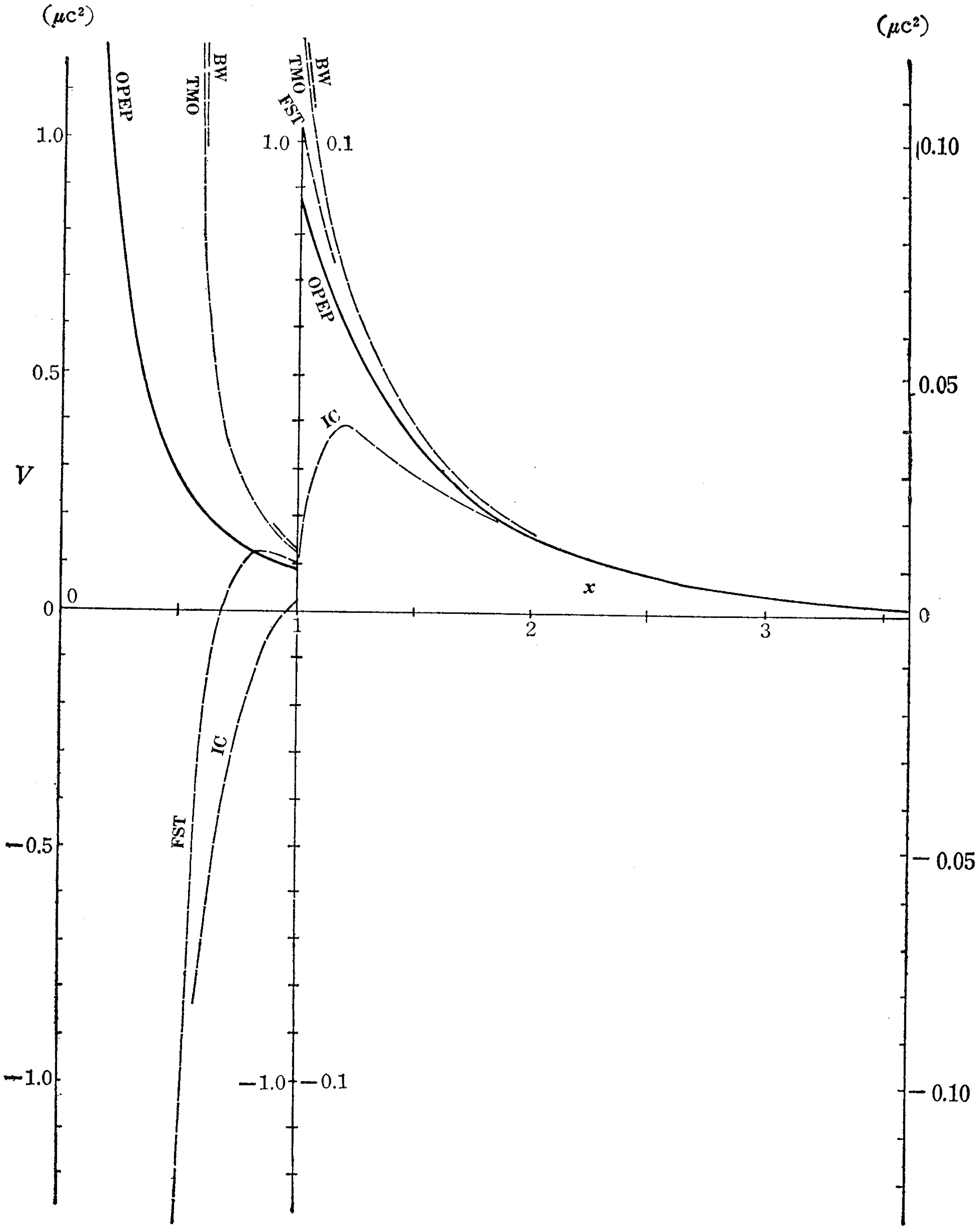

Fig. V