Systematics from Correction Factors

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Starting from the experimental cross section,

$$\sigma^{exp} = \sigma^{exp}_{uncorr} \cdot f_1 \cdot f_2 ... \cdot f_n \tag{1}$$

where σ^{exp}_{uncorr} is the uncorrected Yield divided by the SIMC phase space, and $f_n's$ are the correction factors,

$$
\begin{aligned}
f_1 &= \frac{1}{1 - m \cdot I_{avg}} && \text{target boiling factor, where } m \text{ is the slope} \\
f_2 &= \frac{1}{p_T} && \text{proton transmission factor} \\
f_3 &= \frac{1}{\epsilon_{eTrk}} && \text{e- tracking efficiency} \\
f_4 &= \frac{1}{\epsilon_{hTrk}} && \text{h tracking efficiency} \\
f_5 &= \frac{1}{\epsilon_{tLT}} && \text{total live time} \\
f_6 &= \frac{1}{Q_{tot}} && \text{total charge} \\
f_7 &= f_{rad} && \text{radiative correction factor} \\
f_8 &= f_{bc} && \text{bin-centering correction factor}
\end{aligned}
$$

The systematic uncertainty on the cross section due to the uncertainty in each of these correction factors is

$$(d\sigma^{exp}_{syst})^2 = \sum_{i=1}^{8} \left(\frac{\partial \sigma^{exp}}{\partial f_i} \right)^2 df_i^2 \tag{2}$$

If the derivative with respect to factor f_i is

$$\frac{\partial \sigma^{exp}}{\partial f_i} = \frac{\sigma^{exp}}{f_i} \tag{3}$$

Substituting (3) in (2), one obtains

$$(d\sigma^{exp}_{syst})^2 = (\sigma^{exp})^2 \sum_{i=1}^{8} \left(\frac{df_i}{f_i} \right)^2 \tag{4}$$

For a given data set with multiple runs, the correction factor might differ from run to run, for example, the tracking efficiencies or total live time might be slightly different. In this case, the correction factor and its uncertainty is calculated for each run. The weighted average is then determined as follows:
Define a weight, w,

$$w_i \equiv \frac{1}{df_i^2} \tag{5}$$

where the sum is over all run numbers, and σ_i is the uncertainty in the correction factor f_i for the i run. The weighted average and its uncertainty on the correction factor can then be expressed as

$$f_w = \frac{\sum_i f_i w_i}{\sum_i w_i} \tag{6}$$

$$d_{fw} = \frac{1}{\sqrt{\sum_i w_i}} \tag{7}$$

where (6) and (7) are the correction factor and its uncertainty for a given experimental data set. Substituting (6) and (7) in (4) would then give the total systematic uncertainty for that particular data set. Each of the (P_m, θ_{nq}) for that data set would have a specific cross section, σ_{exp}, but all bins would have the same systematic error, d_{fw}.