Part I

Development of Pion Theory of Nuclear Forces

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1. Introduction

Since Yukawa (35) proposed the meson theory of nuclear forces in 1935, the theory has been developed extensively, especially in Japan. In order to clarify the present situation we shall survey briefly the historical background of the meson theory of nuclear forces. It may be reasonable to divide the period into two parts: One from 1935 to 1947 when the properties of the pion were established experimentally, and the other from 1947 to date. In this section we shall review important results and the prevailing thinking in the first period.

During this period many authors calculated potentials assuming various meson theories, while phenomenological analyses of experimental data with respect to nuclear forces were performed elaborately (see, for example, Supplement of the Progress of Theoretccal Physics, No. 1 (1955); No. 2 (1956); Rosenfeld 48). One of the most important results in the beginning of the period is that the scalar meson theory was excluded due to the following reason (Yukawa 38). At that time it was well known that the spin dependent nuclear force is necessary to explain the experiments. Therefore, the scalar meson theory which can not give the spin dependent force should be excluded.

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On the other hand the contact interaction and the so-called $1/r^3$ singularity which appeared in the meson potential were considered very important problems. It is absolutely meaningless to discuss these problems in the second-order potential, because they may be altered drastically by the higher order quantum effects. However, at that time many authors were thinking consciously or unconsciously as follows: The effect of the field reaction has no meaning at all, and the field reaction may disappear automatically in the coming genuine field theory. Consequently they believed that the problems mentioned above should have important meanings. After the war the relativistic treatment of the field theory was developed initially by Tomonaga and Schwinger. Being stimulated by the development, several authors (Araki 49, 51, Nambu 49, Toyoda 50) tried to reexamine these problems in the second-order potential, and showed that the singularity of the potential at the origin is changed if we take into account the relativistic effect.

The calculation in the strong coupling approximation (Pauli 43) was also done. According to the results the asymptotic form of the potential at large inter-nucleon distances is the same as the second-order potential in the weak coupling approximation multiplied by 1/9. Therefore, if we change the magnitude of the coupling constant, there is no difference between the two approximations. However, it was concluded that at small inter-nucleon distances the strong coupling effect appears and freedoms of the spin and the isotopic spin are frozen. Then the nuclear forces at small inter-nucleon distances become independent of the isotopic spin. It was thought that if one would take such a potential at small inter-nucleon distances, one could not explain the quadrupole moment of the deuteron.

2. Discovery of Pion and Problem of Nuclear Forces

In 1947 the Rome group (Conversi 47) and the Bristol group (Lattes 47) discovered the existence of the nuclear-force meson (pion) and the cosmic-ray meson (muon), which had been predicted by Sakata and Inoue (Sakata 42, 46, Tanikawa 46), and established that the nucleon interacts strongly with the pion and interacts weakly with the muon, as suggested in Sakata and Inoue's two meson theory. This problem became clearer when the pion was created artificially with the Berkeley cyclotron (Gardner 48) and the properties of the pion were investigated.

Taketani, Nakamura, Ono, and Sasaki (49) analysed possible models of the two meson theory, and came to a conclusion that the pion would be a Boson. Assuming the vector and pseudoscalar types for the pion, they calculated nuclear forces of the lowest order in the perturbation approximation. Since the reaction and damping of the pion field might possibly change the strength of the potential, they left the strength as phenomenological parameters, keeping only the transformation properties of the meson potential. Namely, they assumed the potential of the shape

$$V(x)=(1/3)(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\{a+b(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)+fS_{12}(1+3/x+3/x^2)\}e^{-x}/x \quad \text{for } x>x_0,$$
$$=0 \qquad \text{for } x<x_0,$$

where $x=(\mu c/\hbar)r$, the pion mass μ being identified to the observed value in those days, 286 m_e, and S_{12} is the operator of the tensor force. Varying the values of the parameters a, b, and f of the potential, they analysed the deuteron problem extensively. They used the zero cut-off of the potential near the origin to avoid the $1/r^3$ singularity, and fixed the binding energy of the deuteron to be 2.17 Mev (the experimental value at that time) and its D-state probability to be 4%. Then they had to take the cut-off radius x_0 as large as 1.28 in order to fit the quadrupole moment Q. Such too large a cut-off radius compelled them to think that there should be another potential with a different range, in addition to the above potential. Even if we assume the more correct value of the pion mass, 276 m_e, the situation is not altered (Taketani 51).

In this way, the research on nuclear forces in Japan started from its beginning along the line that the nuclear forces should be interpreted in terms of the pion field. Although such an idea is now familiar to all, the majority of people at that time, having given up this line of approach, sticked to phenomenological interpretations of nuclear forces.

3. Effective Range Theory (Shape Independent Approximation)

The phenomenological theory of nuclear forces made a considerable technical advance around 1949. It had been demonstrated earlier by Breit and his collaborators (Breit 39, Hoisington 39) that the scattering problem of two nucleons at low energies seems to be independent of the detailed shape of the assumed potential. Landau and Smorodinsky (Landau 44, Smorodinsky 44, 47) found that the energy dependence of the n-p scattering phase shift δ is well approximated by the first two terms in its expansion in powers of the energy as follows,

$$k\cot\delta=-(1/a)+(1/2)r_ek^2,$$

where k is the wave number. Thus two parameters are introduced, scattering length a and effective range r_e, which can be adjusted to reproduce experimental data by varying the depth and range parameters of the potential irrespective of its assumed shape. They were named as shape independent parameters. Later, Schwinger (47) gave a general proof for this theory by using his variation formalism. It was shown that all scattering problems at energies lower than about 10 Mev can be described in a consistent way with the shape independent parameters. This theory was called the effective range theory, and reformulated and applied extensively by Bethe, Blatt, Jackson, and others (Bethe 49, Blatt 48, 49, Jackson 50, Chew 49, Chiristian 49).

The effective range theory was found very powerful to understand the experimental data in the low energy scattering phenomenologically, so that some people inclined to think that there were nothing important left at low energies, and that further studies should be done on high energy phenomena which were expected to determine the shape of the potential. It was only a few researchers who noticed that it was the ultimate purpose of the theory to explain the simplified data at low energies in a more realistic way, that is, in terms of the pion theory.

4. High Energy Nucleon-Nucleon Scattering*

When the Berkeley cyclotron was accomplished in the fall of 1947, experiments of the nucleon-nucleon scattering in the high energy region were started. By that time, to compare the theory of nuclear forces with experiments there had been only the binding energy of the deuteron in the ground state, the electric quadrupole moment and the magnetic moment of the deuteron, the radiative capture from the continuum to the ground state of the deuteron, the photodisintegration of the deuteron, and the cross section of the nucleon-nucleon scattering in the low and intermediate energy regions ($\lesssim 20$ Mev). Although there are, besides, several phenomena related with nuclear forces, as the binding energies of light nuclei, or the saturation phenomena of heavy nuclei, it does not seem easy to deduce any information on the nuclear potential from these phenomena, because various factors which are difficult to predict theoretically may appear.

Owing to the effective range theory it became clear that the knowledge obtained from the low energy experiments is rather limited and the analysis of the high energy experiments is necessary to obtain the more information regarding nuclear forces.

In spite of the fact that the nuclear force problem was the embryo of the meson theory, the essential progress had been stopped for a long time in the meson theory of nuclear forces, as described above. Therefore, the starting of the experiments of nucleon-nucleon scattering in the high energy

* In this report: The low energy region: $E_{lab} \lesssim 5$ Mev,
The intermediate energy region: 5 Mev $\lesssim E_{lab} \lesssim 20$ Mev,
The high energy region: 20 Mev $\lesssim E_{lab} \lesssim 100$ Mev,
The very high energy region: 100 Mev $\lesssim E_{lab}$.

region at Berkeley was welcome and expected to break the long time stagnation. Unfortunately, contrary to the expectation, the experimental results could not be related simply with the properties of the pion, but seemed to contradict the knowledge of those days. Then theoretical physicists were perplexed.

Why were they perplexed so much? We shall sketch briefly the situation of those days. First, the n-p scattering experiments in the high energy region (40, 90, and 260 Mev) performed in 1949 gave the following results (see Segrè 52):

(1) The total cross section of the n-p scattering is rather small, i.e., it is proportional to the inverse of the energy up to 200 Mev.

(2) The angular distribution is approximately symmetrical around about 90° in the center of mass system. (According to recent experiments the backward scattering becomes larger than the forward scattering as the energy goes high. See Part II, **4. 3.**) This implies that the scattering in the even states of the angular momentum is predominant. In other words the interaction may be small in the odd states.

In order to explain the experiments Serber, who was a staff member of the theoretical group at Berkeley, proposed a potential which has the following factor:

$$a+bP_x, \quad a\sim b\sim 1/2,$$

where P_x is the exchange operator of coordinates of two nucleons.

At Berkeley the inverse experiment was also done. An about 400 Mev proton beam was injected into target nuclei, and it was observed that a narrow beam of neutrons with an average energy of about 350 Mev was emitted in the forward direction. It was also observed that neutrons with lower energies were knocked out at right angles.

However, if we want to explain the saturation property of nuclear forces using such a kind of two-nucleon force and the individual-particle nuclear model, we need to assume much more exchange force. Since the saturation in heavy nuclei is a property as an ensemble of many-nucleon system, it does not seem to us a serious inconvenience even if we could not succeed in explaining the saturation property with the two-nucleon potential and the Fermi gas model. Nevertheless, it was a tradition since Heisenberg's first trying (Heisenberg 32) on the nuclear binding energies to derive the saturation property from the exchange two-nucleon potential. Therefore, it was rather surprising that the high energy experiments show about 1:1 ratio of the exchange force to the non-exchange force.

At the same time, p-p scattering experiments at 32 Mev and 340 Mev were done also at Berkeley (see Segrè 52), and the results were completely different from the n-p scattering. Many experiments at various energies were done later. The result was as follows (see Part II 4.3):

(1) The total cross section is approximately proportional to inverse of the energy through 5 Mev to 150 Mev, and almost constant between 150 Mev and 450 Mev.

(2) The angular distribution is nearly isotropic if we subtract the effect of the Coulomb scattering.

This is quite contrary to the *n-p* scattering.

Christian and others (Christian 50, 50a), who analysed these experiments, abandoned the charge independence of nuclear forces, and assumed that there is no singularity in the *n-p* force while the *p-p* force has a strong singularity. Using this assumption, they tried to explain the large magnitude of the cross section at high energies and the nearly isotropic angular distribution in the *p-p* scattering. Afterward Jastrow (51) proposed a potential which is charge independent and fits fairly well the experiments. The potential gives a very strong repulsive force inside a certain range in the singlet even state. Case and Pais (50) did the same kind of approach using a singular spin-orbit coupling force.

Although they were able to find a certain type of potential which has a good fit in the scattering experiments at high energies, they paid little attention to relate it to the pion theory of nuclear forces. Particularly, an unexpected behavior of the *p-p* scattering at high energies brought them to a considerable confusion in understanding the problem. Here it may be sufficient only to point out that the validity of the conventional concept of potential should have been reexamined in such a high energy region before detailed analyses were made.

5. Taketani's Theory*

As was described in the preceeding sections, a kind of apathy for the meson theory began to prevail in the world. For example, Fermi and Yang (49) proposed a theory that the pion is not an elementary particle, but a composite particle. In such a situation Taketani was sunk in the thought "On the Method of the Theory of Elementary Particles," which was published in Japanese in 1949 (Taketani 49a) and was also contributed to the special issue of Progress of Teoretical Physics for commemoration of the fifteenth anniversary of the discovery of meson theory (Taketani 50). In the paper

* The proposal mentioned in this section used to be called Taketani's method or Taketani's methodology. However, as it will be understood later, it is not simply a method of calculation nor a philosophy. In fact, it has been an indispensable guiding principle to push the nuclear force research, and it has produced many meaningful results, as Taketani expected in his first proposal. Therefore, we would like to call it Taketani's theory of nuclear forces.

Taketani, Nakamura, and Sasaki pointed out that the Compton wave length has a significant role to play in the theory of elementary particles, and the length will have an important influence in the transition phase of the theories. They applied their analysis to the nuclear force problem, and gave a very powerful tool to attack the problem.

The above consideration on the one hand and the detailed analyses of the deuteron problem made just after the discovery of pions (described in 2) on the other hand were the main motive forces which drove Taketani to make a proposal in the paper entitled "On the Method in Theory of Nuclear Forces" (Taketani 51, see Part V). The proposal is described as follows:

(1) Since it had been revealed by the experiments of artificial pions that the main part of nuclear forces should be attributed to the pion-nucleon interaction, the theory of nuclear forces should be based on the pion theory instead of conventional arbitrary potentials.

(2) Many complicated effects, e. g., the higher order terms in the perturbation method, relativistic corrections, the strong coupling effect, the existence of heavy particles, etc., appear in the neighbourhood of a nucleon. Therefore, at present stage we should take some phenomenological treatment in such a neighbourhood of a nucleon.

(3) On the contrary, these effects may be neglected outside about the Compton wave length of the pion, and only the low order perturbation potentials may be sufficient to be considered. Furthermore, the outer potentials are expected to play an important rôle in the analyses of the low energy scattering and the deuteron problem, especially the quadrupole moment of the deuteron. If there might exist the excited states of a nucleon, their influence would be small in such an outer region.

(4) From the above considerations it seems quite plausible to divide the region of the potential into two parts, i. e., the inner region and the outer region. In the outer region we may be able to take a substantialistic method and adopt the pion theoretical potential, while in the inner region we can not do better than assume some phenomenological potential, e. g., a square well with a hard core. The depth and the range of the inner phenomenological potential should be determined to fit each experimental result, and the spin and the charge exchange characters also be chosen adequately. Jastrow's hard core is simply one example of the phenomenological potential.

(5) If we assume a certain type of the pion theoretical potential in the outer region and can not succeed in determining parameters of the inner phenomenological potential to fit the experimental results, this type of the pion theoretical potential should be excluded.

(6) If we obtain a set of the parameters to fit the experimental results, the set will serve valuable data for a future sound theory.

6. Type of Pion Field

After the proposal, Taketani and his collaborators started a systematic study along their idea. Taketani, Onuma, and Koide (51a) examined the deuteron problem with the vector and pseudoscalar meson potentials calculated in the lowest order approximation of the perturbation theory. At that time, some people (e.g., Heisenberg 50) supposed that the vector meson potential was more promising because it gave a strong spin-orbit coupling with the sign favorable to the shell model of nuclei. However, their analysis showed even if we assume any type of potential in the inner region, the contribution from the inner region to the value of the deuteron quadrupole moment is very small, so that the value can be determined uniquely by the type of the field producing the outer region of the nuclear potential. Then they found that the vector type is excluded and the pseudoscalar potential can give the correct value. Consequently they succeeded in obtaining good values for the quadrupole moment, the triplet scattering length, and the effective range, adopting the coupling constant $g^2/4\pi=0.084$ for the p-wave coupling together with a hard core of the radius $x_0=0.458$. These results gave us the convince that the nuclear forces could not be explained if we assumed only the vector pion. On the other hand at the same time the experimental investigations of the pion reactions were done extensively in the United States and it was confirmed that the pion is pseudoscalar as follows (see for example Bethe 55):

(1) The spin of the charged pion was determined to be zero by the detailed balancing consideration;

$$\pi^+ + d \rightleftarrows p + p.$$

(2) The charged scalar pion was excluded by the neutron absorption of the negative pion in deuterium;

$$\pi^- + d \rightarrow n + n.$$

Thus we could conclude that the charged pions are pseudoscalar.

(3) The neutral pion cannot have spin 1 since it decays into two photons;

$$\pi^0 \rightarrow 2\gamma.$$

(4) By the mesic absorption of the negative pions in hydrogen, it is found that the parity of the neutral pion is very likely the same as that of the charged pions;

$$\pi^- + p \rightarrow \pi^0 + n.$$

7. Taketani-Machida-Onuma Potential

The first success of Taketani's theory has been brought by Taketani, Machida, and Onuma's calculation (Taketani 51b). In order to investigate out of what distance the second-order perturbation potential may be valid, they calculated the fourth-order adiabatic potential assuming the symmetrical pseudoscalar pion theory with the pseudovector coupling in 1951.* The potential is called the TMO potential. With respect to the calculation of the fourth order potential Taketani, Machida, and Onuma used the method which had been presented in Nambu's paper (Nambu 50).

Nishijima (51) made also the same calculation using the canonical transformation and obtained the same potential as that of Taketani et al. Although the calculation itself was very standard at that time, it should be kept in mind that no author had ever attempted to calculate the fourth-order potential for lack of the positive understanding of the validity of the pion theoretical potential as was discussed in **5**.

The outstanding result of the TMO potential is that the fourth-order potential is much larger than expected earlier, and the effect spreads upto the force range. In consequence, even the qualitative behavior of the second-order potential is drastically changed around the force range. The qualitative character of the TMO potential can be summarized as follows:**

state	central force	tensor force
triplet even	strong repulsion	strong attraction
singlet even	strong attraction	....
triplet odd	weak attraction	weak repulsion
singlet odd	repulsion	....

At that time it was considered very interesting that the odd state forces were smaller than the even state ones, because such a potential as the Serber potential was derived theoretically from the symmetrical pion theory. Hence the symmetrical pseudoscalar meson theory gained its reliability, although such an argument is not convincing at present as will be shown later.

Machida, Onuma, and Taketani (51) examined their potential applying it to the low energy phenomena, and found that all the properties of the deuteron and the low energy scattering can be explained quantitatively by the potential with the coupling constant $g^2/4\pi=0.09\sim0.10$ if some inner potentials e.g., the zero cut-off and the infinite repulsive core are properly assumed. The probability of the D-state of the deuteron was found to amount to $9\sim10\%$, which was considerably larger than the value of about 4% prevailing at that time. However, they did not regard this discrepancy as a serious one, because not only ambiguous relativistic corrections but also the change of the magnetic moment of one nucleon due to the proximity of

* In the case of the pseudoscalar coupling their results had one trivial error in the numerical factor as corrected later (Machida 53).

** Shape of the TMO potential as well as those of other potentials are shown in Figs. II～V in p. 9～12.

the other can alter the value of the D-state probability. They considered that the agreement of the quadrupole moment with experiments is significant since the quadrupole moment is determined mainly by the outer potential. All of these results were summarized in the article of Taketani, Machida, and Onuma (52) in 1951. Later Miyazawa (52) made an estimate of the mesonic correction to the magnetic moment of the deuteron and concluded that the D-state probability could be as high as 10% due to the correction. On the other hand Machida (53) examined the D-state probability by using the data of the hyperfine structures of the deuterium and the hydrogen, and found it to be between 5 and 15%. Comparing these two results we can not fix the magnitude of the D-state probability near 4% but should consider it 5~10%. Consequently the result obtained above pion theoretically does not contradict with the experiments but is rather favored.

In 1953, Fujii, Iwadare, Otsuki, Taketani, Tani, and Watari (53, 54) applied the TMO potential to the high energy n-p and p-p scatterings and showed that the TMO potential with the corrected inner potential* can give a qualitative fit to the angular distributions of nucleon-nucleon scattering at 40 and 90 Mev. There the calculated total cross sections were found a little larger than the experimental ones, but this small discrepancy can be considered to be attributed to the inner phenomenological potential as was argumented by Matsumoto and Watari (54, 54a) (see **9**).

Later in Brasil, Lopes and Feynman (52) tried to explain the properties of the deuteron by using the second-order potential of the symmetrical pseudoscalar meson theory outside a certain distance and a phenomenological potential inside the distance. They adopted, however, a special type of the wave function for the inside region, and concluded that such an attempt might not be hopeful because of the disagreement with experimental results. In addition, they interpreted unfortunately the result by Taketani, Machida and Onuma as follows: The fact that the fourth-order potential is larger than the second-order one even at the force range might be an evidence that the perturbational method is meaningless for the nuclear force problem. Due to this unjustified conclusion Lopes and Feynman gave up their attempt.

8. Non-Adiabatic Potential

As was shown in the previous section, Taketani, Machida, and Onuma

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* It was pointed out by Fujii *et al.* that a bound deuteron wave function could not be obtained unless some attractive force was added at small distances, because the fourth-order TMO potential has the strongly repulsive central force in the triplet even state. Thus the straight cut-off in Taketani, Machida, and Onuma's work had to be replaced by some attractive potential. Fujii *et al.* assumed for it a square well potential of the range $x_0=0.6$ with the depth -125 Mev.

calculated the fourth order potential in the adiabatic approximation and pointed out that the two-pion-exchange effect is rather predominant at the distance of the Compton wave length of the pion contrary to expectation. This result gives rise to the following problems:

(1) The fact that the two-pion-exchange potential surpasses the one-pion-exchange potential up to the distance of the force range suggests that the conventional perturbation method may not be applicable in the inner region of the force range.

(2) Generally the recoil correction may be appreciable in the region where the two-pion-exchange potential is predominant. Furthermore, it seems desirable to derive the potential not in the adiabatic approximation but by using some non-adiabatic treatment.

(3) At the present stage of the meson theory is there any region where we can obtain unambiguously the meaningful potential? If there is such a region, is it possible to find experiments which can be compared with the meson theory by considering only the region?

First, the calculations in non-adiabatic methods were done by many authors. Levy (52) calculated the fourth-order potential from the symmetrical pseudoscalar meson theory with the pseudoscalar coupling using a non-adiabatic method, and showed that the fourth-order potential is also large and that the deuteron properties and the low energy scattering can be explained fairly well. Although this work gave a strong stimulation to the American researchers, Lévy could not reach the meaningful result because he had not the clear method to analyse the problem of nuclear forces nor the correct recognition of the region where his calculation loses its reliability. For example he insisted that something like Jastrow's hard core (Jastrow 51) could be derived in his calculation. Although his contribution was highly estimated in the very point (Bethe 55, Marshak 53), it is absolutely meaningless to discuss the hard core region from his calculation, because there appear many factors which may drastically change his result.

On the other hand Lévy, Marshak, Bethe, and most American researchers regarded the renormalizability as a guiding principle, and excluded the pseudovector coupling due to its unrenormalizability (see for example Bethe 55). However, such an opinion is based simply on that the method which is temporarily successful in the quantum electrodynamics may be also applicable directly to all kinds of elementary particles. Therefore there is no strong evidence to support the opinion. In fact there is nothing to support that the discernment by the renormalizability based on the perturbation method is correct, especially in the meson theory.*

* Recently Arnowitt and Deser (55), and Cooper (55) discussed the renormalizability of the interaction which includes derivatives and can not be renormalized perturbation-theoretically.

Lévy (52) calculated the potential using the pseudoscalar coupling, but made some mistakes. Correcting his mistakes and improving the approximation, Klein (53) obtained the same result in the outer region ($x>0.6$) as the TMO potential derived from the pseudovector coupling. Consequently we can conclude that it is impossible to discriminate between the pseudoscalar coupling and the pseudovector coupling from the low energy experiments of nuclear forces.

Brueckner and Watson (53) proposed a general algebraic method to construct a non-adiabatic potential in the quantum field theory. However, they did not normalize correctly the wave function as Sawada (53) pointed out. Hence Fukuda, Sawada, and Taketani (54) took account of the point and gave a general method to construct a potential. This method is called the FST method and may be regarded as the most reliable one at present.

Brueckner and Watson (53a) made an estimate under their formalism and insisted upon that a part of the fourth-order potential given by Taketani, Machida, and Onuma in the pseudovector coupling was a kind of non-adiabatic correction and was small. Then they dropped the part and obtained a different potential from the TMO potential in the triplet even state. However the part is not a non-adiabatic correction nor small (Inoue 56).

Inoue, Machida, Taketani, and Toyoda (56) calculated the static potential using the FST method and showed that the behavior of the potential inside the force range depends strongly on the extension of sources, but outside the range the latter influence is small.

Recently Gartenhaus (55) calculated the potential up to the fourth-order term with the BW prescription (Brueckner 53) assuming an extended source function. However, as described previously, the prescription has misgivings. Besides, Gartenhaus applied the potential obtained by the prescription to the zero distance between two nucleons. Since higher order effects, relativistic effects, and new-particle effects can not be neglected inside about a half of the pion Compton wave length, it is very hard to find what part of the calculated result is determined by the outer potential if the inner potential is treated similarly to the outer one. Therefore we have nothing to discriminate what part of his potential is reliable.

9. Impact Parameter

Matsumoto and Watari (54) applied Taketani's idea to the phenomenological studies on the nucleon-nucleon scattering and calculated the phase shifts of the nucleon-nucleon scattering assuming various central potentials. They showed that the phase shift for the partial wave with a given orbital angular momentum $L\hbar$ is dependent only slightly on the behavior of the potential at distances smaller than one half of the impact parameter b

$$b=\sqrt{L(L+1)}\,\hbar/p_\infty\ ,$$

where p_∞ is the relative momentum at infinitely large separations. This finding was very valuable for phenomenological researches in which one can see whether a certain phenomenon depends mainly on the potential at large distances or on the potential at small distances.

For example, it is shown that in the nucleon-nucleon scattering with the energy lower than about 100 Mev, the phase shifts of G- or higher partial waves are negligible and all phase shifts but those of the S- and P-waves are mostly dependent on the features of the potential at distances larger than one half of the pion range. It means that the magnitude of the total cross section depends strongly on the potential of phenomenological nature at very small distances, while the angular distribution is governed mainly by the pion theoretical potential at large distances. In this way the too large total cross sections calculated with the TMO potential described in 7 are attributed to a wrong choice of the phenomenological inner potential.

With full utilization of the concept of impact parameter, some analyses of experimental data were made. Otsuki and Fujii (54) investigated the properties of the potential in the triplet odd state by analysing the data of 18 Mev p-p scattering (Yntema 54). Matsumoto and Watari (54a) examined the effect of the phenomenological inner potential in the triplet even state varying the parameters of the potential in a wide region. They showed that some phenomenological potentials added to the pion theoretical potential can give a good fit to the deuteron parameters and the n-p scattering data at 90 Mev including the total cross section, if an infinitely repulsive hard core with the radius of about 0.4 times the pion range is assumed.

10. One-Pion-Exchange Potential

In 1954, there appeared a strong evidence for the pion theoretical potential. Onuma (53) pointed out in 1953 that an information on the potential in the triplet odd state could be obtained from the analysis of experiments of p-p scattering at low energies. For this purpose very accurate data were needed, which were found in a careful experiment at Wisconsin (Worthington 53) in the energy region 1~4 Mev. Onuma analysed the experiment assuming a purely phenomenological potential.

Later in 1954, Otsuki and Tamagaki (54a) again took up the same problem in the spirit of the pion theoretical approach on nuclear forces. The experimental value of the averaged phase shift in the triplet P-state was known to be negative. They showed that it corresponds to the weakly repulsive character of the pion theoretical potential at large distances $x\gtrsim1.5$. The fact can be considered as one of the most striking successes of the pion theory of nuclear forces. The weak repulsive force at large distances $x\gtrsim1.5$

originates from the weak second-order potential, whereas the fourth-order potential is overwhelmingly attractive only inside the pion range, so that the resultant potential changes its sign near the pion range. Such a non-monotonous behavior of the potential is a natural consequence of the pion theory, but has no bearing on the conventional phenomenological point of view.

Therefore, the negative P-wave phase shift at low energies is considered as the first strong evidence for the existence of the second-order potential. After this success Japanese physicists were no longer content with the qualitative discussions. They began to make more quantitative discussions, paying special attention to the second-order potential.

In order to find up to what distance the three-pion-exchange force and higher ones are appreciable Machida and Senba (55) calculated the sixth-order potential corresponding to the three-pion-exchange effect and showed that the potential is negligibly small outside $x \sim 0.7$. On the other hand Brueckner and Watson (53a) estimated the effect of the multiple scattering of virtual pions, and found that the effect is small outside $x \sim 0.6$.

With regard to the recoil effect of nucleons some calculations were done by several authors (Sato, S. 55, Sato, I. 55, Iwadare 55). Their results shows that the recoil effect can not be neglected usually in the region where the two-pion-exchange potential is appreciable.

The radiative correction of the pion field is equivalent simply to the replacement of the coupling constant by the renormalized one for the one-pion-exchange potential where two nucleons are at rest (Hiida 56). Therefore, if two nucleons lie widely apart and no recoil effect nor two-pion-exchange effect is appreciable, the second-order potential holds rigorously.

The radiative correction to the two-pion-exchange potential was estimated by Brueckner and Watson (53a) and Henley and Ruderman (53) using the result calculated with the Tamm-Dancoff method for the p-wave pion-nucleon scattering. This correction can be also neglected outside $x \sim 0.6$.

Recently Miyazawa (56) tried to construct the two-pion-exchange static potential only from the experimental results of the pion-nucleon scattering by using the dispersion formula. Concerning the p-wave contribution his method may give probably the same result as Henley and Ruderman's one. However in Miyazawa's method the s-wave contribution of pions can be treated without difficulties. According to his result the interaction between s-wave pions and nucleons becomes about 1/10 times the magnitude of the lowest term in the perturbation method if we insert the experimental data of the s-wave pion-nucleon scattering in his formula. Hence the s-wave contribution to the two-pion-exchange potential may be samll.

Hasegawa and Nogami (Hasegawa 55, Nogami 56) applied Tomonaga's intermediate coupling approximation (Maki 53) to the calculation of the potential. Their result is roughly in agreement with that of Inoue *et al.* using the FST method (Inoue 56), if the same source function is assumed.

In conclusion of this section we can divide the region where the nuclear potential acts into three parts as follows:

The region I, $x \gtrsim 1.5$, where the second-order static potential in the perturbation method holds exactly, although the renormalized coupling constant should be used there. Since the second-order potential can be derived without the field quantization, the region I may be called the classical region.

The region II, $1.5 \gtrsim x \gtrsim 0.7$, where the quantum theoretical effects as the two-pion-exchange potential and the recoil effect are appreciable and the result depends strongly on the shape of the source function. Hence it is very difficult to obtain the quantitative potential at the present stage, while the qualitative discussion of the potential may be possible to some extent.

The region III, $x \lesssim 0.7$, where the more-than-two-pion-exchange effect, the relativistic effect, the radiative effects, the new particle effect, etc., give the very important contribution to the potential. Therefore we have nothing for the region but to treat it phenomenologically at the present stage of the theory. In fact in this region the potential concept seems entirely useless (see Fig. I in p. 7).

11. Verification of Pion Theory of Nuclear Forces

The achievements described in the previous section with repect to the analyses of the experimental data and the derivation of the potential made it necessary to analyse all experimental facts in a new point of view, i.e. as links of the chain of the detailed investigation on the potential due to the one-pion-exchange process and on the interaction in the region II. This task was undertaken by Iwadare, Otsuki, Tamagaki, and Watari placing right reliance on the one-pion-exchange potential at large distances.

The first work was to determine the value of the effective coupling constant $g_e^2/4\pi$ of the one-pion-exchange potential. They noticed that the value of the quadrupole moment of the deuteron depends mainly on the detailed features of the potential at large distances if the wave function of the deuteron is assumed to give the correct values for the binding energy and the effective range (Iwadare 56, 56a). They used the one-pion-exchange potential for the outer region at distances larger than the pion range. They found that the value of the quadrupole moment can be obtained correctly, if and only if one takes the effective coupling constant for the one-pion-exchange potential as $g_e^2/4\pi = 0.075 \pm 0.015$, irrespective of the inner interactions. They also showed that, by taking account of the effective range in the singlet even state, $g_e^2/4\pi$ is larger than 0.07 (Iwadare 56, 56b). Such a value of the coupling constant agrees well with that obtained from the analyses of pion reactions $g_e^2/4\pi \sim 0.08$ (Chew 56).

Assuming the established one-pion-exchange potential with $g_e^2/4\pi=0.080 \pm 0.010$ in the outer region, many phenomena in the low energy region were analysed; the deuteron data (Iwadare 56a), the singlet low energy data (Iwadare 56b) the triplet P-wave phase shift at low energies (Otsuki 54a, 55), the p-p scattering at 18 Mev (Iwadare 56c). The detailed results will be discussed in Part II. It is found that all the above phenomena are explicable quantitatively in terms of the one-pion-exchange potential. At the same time the properties of the inner potential in the region II inferred from the analyses are found in good agreement with those of the two-pion-exchange potential. Furthermore other phenomena at higher energies such as the photodisintegration of the deuteron at the photon energy of about 20 Mev (Iwadare 56d) and the nucleon-nucleon scattering up to 100 Mev can be reproduced qualitatively by the pion theoretical potential (see Part II, 4.3).

Blatt and Kalos (53) assumed the second-order potential added by some spin and isotopic spin independent attractive central potentials in the inner region and examined the deuteron problem, varying extensively the value of the constant $g^2/4\pi$ of the second-order potential and the depth and shape of the inner central potentials which have a hard core cut-off, with use of an electronic computor. They regarded the force of this type as a reasonable generalization of Levy's potential (Lévy 52). Their computational result rather surprised them in that the interaction of the above type did not give a satisfactory fit to the low energy data in the triplet and singlet states simultaneously. Although they knew that a good fit could be obtained if the added central potential was spin dependent, they thought that such a fit meant very little because the number of theoretical parameters exceeded the number of experimetal data. Their thought would have been natural if they had had no theories to rely upon and been obliged to confine themselves within a phenomenological approach. However, it seems that they should have duly appreciated consequences of the pion theory, not only of Lévy's work but also of the others. At that time already, there was a strong indication that the nuclear forces of the higher-order coming from the p-wave coupling which are to be added to the second-order potential are spin and isotopic spin dependent as the TMO potential. If they had taken this indication into account and made a computing plan with the powerful machine, they would have obtained another result which might be useful for the comparison of the pion theory of nuclear forces with experimental data. Although they noticed that the value of the quadrupole moment determined the value of $g^2/4\pi$ in the first appoximation, they could not attempt to generalize this conclusion on a more general basis.

As mentioned before, however, all low and intermediate energy phenomena can be explained and we may conclude that the essential validity of the one-pion-exchange potential in the outer region and the qualitative plausibility of the interaction due to the two-pion-exchange processes in the inner region have been successfully established.

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