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MESON EXCHANGE MODEL FOR THE NUCLEON-NUCLEON INTERACTION††

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Abstract: Nucleon-nucleon interactions obtained from several models for the field theoretic scattering amplitude are studied. The interaction includes contributions from one-pion and one-omega exchange and from two-pion exchange as calculated in a dispersion theory framework. The resulting interaction is regularized by a cut-off factor obtained by the eikonal approximation to multiple vector meson exchange processes. The Blankenbecler-Sugar equation is solved with the interaction and nucleon-nucleon scattering phase parameters are computed. For the best model good agreement with phenomenological phase parameters is achieved for physically reasonable values of the meson-nucleon coupling constants and the spectral functions needed for the evaluation of the two-pion exchange effects. The deuteron wave function is computed as are the deuteron charge and quadrupole form factors. The interaction is shown to have significantly weaker short-range repulsion than commonly found in local phenomenological potentials and in one-boson exchange models.

1. Introduction

During the last few years there have been several promising attempts to use field theoretic techniques to construct models of the nucleon-nucleon interaction which are capable of reproducing the NN scattering amplitude in the elastic region [1–5]). The most fruitful description of the basic NN interaction includes one-boson exchange and irreducible two-pion exchange (TPE) processes constructed along conventional lines with the aid of Lagrangian models of the boson-nucleon interactions and the best available experimental estimates for the boson masses and coupling constants involved. Using dispersion theory techniques, the two-pion exchange contributions to the NN amplitude may be related to the amplitude for the reaction $N\bar{N} \to \pi\pi$ which is connected to the empirical πN scattering amplitude by crossing symmetry. A marriage of dispersion techniques and Lagrangian models is thus flexible enough to allow the inclusion of the effects of $\pi\pi$ and πN rescattering in the intermediate state of the TPE contribution to the NN interaction. Such rescattering effects are known to be of particular importance in low $\pi\pi$ partial waves (e.g., in S-waves they are frequently roughly approximated in terms of a fictitious scalar, isoscalar boson

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of low mass), but cannot be described in a fourth-order perturbation theory based on a specific Lagrangian model for the πN interaction involving only the pion and nucleon fields. The most important contributions to the NN interaction are due to one-pion (OPE) and two-pion exchange and to ω-meson exchange which is largely responsible for the short range repulsion in the interaction. Note that the effects of the exchange of a finite width ρ-meson are included naturally in the dispersion theoretic description of rescattering effects in $\pi\pi$ P-waves. Other mesons may also contribute to the interaction, but there is no evidence that they are quantitatively significant.

The efforts of refs. [1-5]) were aimed specifically at constructing local approximations to the resulting NN potential from the adiabatic limit of the field theoretic amplitude. Depending on the details of the models used, the components of the resulting potentials calculated in refs. [2,3]) are seen to be in qualitative, if not quantitative, agreement with the corresponding components of the Hamada-Johnston (HJ) potential [6]) for intermediate and large internucleon separations. Since the HJ potential agrees by construction with empirical NN scattering parameters, it is clear that the approach to the construction of the NN interaction used in these references and in the present work should be at least qualitatively reliable in peripheral partial waves.

In order to calculate the NN scattering amplitude in lower partial waves it is necessary to regard our interaction as an adequate approximation to the sum of all irreducible processes which may be iterated through a suitable integral equation (i.e., the Bethe-Salpeter equation) to obtain the sum of all processes, both reducible and irreducible. To the extent that the parameters in the basic interaction are established independent of low energy NN scattering data, we are entitled to replace the numerically complicated four-dimensional Bethe-Salpeter equation by one of the many approximate three-dimensional scattering equations only if such a replacement is numerically reliable. Such replacements are likely to be reliable only when the approximate equation satisfies the same relativistic elastic unitary relation as the field theoretic amplitude and the solution to the Bethe-Salpeter equation [2,7]). In such cases, the TPE interaction is guaranteed to be real in the low energy, elastic scattering region and is found to have a relatively weak energy dependence. If one considers a fixed NN interaction due only to one boson exchanges (i.e., first-order contributions), different three-dimensional approximations to the Bethe-Salpeter equation lead to strikingly different results in low partial waves†. If "higher order contributions" (e.g., two-pion exchange) are included in the interaction and if the ω-meson coupling constant is allowed to vary within its experimental uncertainty; the differences between various approximate unitarizing equations become much less important [8]). Thus, in the present work we shall consider only

† Because of this fact it is not always possible to identify "effective" meson-nucleon coupling constants obtained in phenomenological OBE potentials with pole-term residues in physical amplitudes.

the Blankenbecler-Sugar equation (BBSE)[9]). This equation satisfies relativistic elastic unitarity and has a form satisfyingly similar to the non-relativistic Lippmann-Schwinger equation. Since the TPE interaction is obtained as the full two-pion exchange contribution to the scattering amplitude less the iterated OPE interaction, we note that the explicit form of the TPE interaction depends on the specific choice of an approximate unitarizing equation.

The one-boson exchange and TPE interactions constructed as above are not suitable for use in the BBSE due, in a momentum space representation, to a failure of the interactions to fall off for large momenta. In coordinate space, these divergences are related to terms involving δ-functions which are sometimes ignored in the process of obtaining a local potential from the field theoretic interaction. In order that the kernel of the BBSE be compact, the meson exchange potentials must be damped by a cut-off factor at high momenta (i.e., short distance). Damping factors are normally selected in an *ad hoc* manner with the unfounded hope that calculated phase shifts will be insensitive to the choice. This damping factor can be interpreted in terms of higher-order meson exchange contributions (e.g., vertex functions) not included in our one- and two-boson exchange picture of the interaction. Woloshyn and Jackson[10]) exploited the fact that processes involving all orders of soft neutral vector mesons may be summed analytically with the aid of the relativistic eikonal approximation[11]) to provide a microscopic description of damping functions. Similar arguments lead to a model of the nucleon electromagnetic form factor which is in excellent agreement with experiment[12]). The eikonal approach may be thought of as providing a theoretical framework for determining the nucleon strong form factors from the experimentally known nucleon electromagnetic form factors in a manner which should be reliable in the limit of large momentum transfer (i.e., the region which forces the inclusion of damping functions). We consider this close and intuitively satisfying relation between strong and electromagnetic form factors of the nucleon to be the primary virtue of this model. Using meson exchange interactions regularized in this manner, the BBSE can be solved numerically using standard techniques. The features of the solution of the BBSE obtained with this non-local and energy-dependent interaction differ substantially from those of the solution of the Schrödinger equation obtained with the local approximation to the interaction augmented by a hard core. As one would expect, differences occur primarily in the lowest partial waves.

This paper has two aims. The first is to establish the necessary formalism for obtaining a regularized potential in momentum space starting from the field theoretic amplitude. In this context we discuss the off-energy-shell extrapolation of the interaction and note the important fact that the off-shell interaction is described by six independent spin or helicity components, whereas the on-shell field theoretic amplitude is described by at most five helicity amplitudes. The spirit and notation of this paper follows that of Chemtob, Durso and Riska[2]) closely. This paper may be viewed as a logical continuation of ref.[2]). The main novelty is the development

of the formalism pertinent to the use of the complete non-local and energy-dependent meson exchange potential in a momentum space representation and the results obtained with that potential.

The second object of this paper is to demonstrate that it is possible to construct an interaction leading to NN phase shifts in good agreement with experiment using experimentally acceptable values for the parameters in the OPE, ω-exchange and TPE interactions. As a consequence of the ω-meson coupling constant, we find that the short-range repulsion of the resulting NN interaction is significantly weaker than that of familiar phenomenological potentials [e.g., the Reid [13]) and HJ potentials].

While the forms of the OPE and ω-exchange interactions are relatively unambiguous, the form of the TPE potential cannot be determined definitely as it depends on the incompletely known $N\bar{N}$ amplitude below threshold. At present, therefore, one is forced to rely on models for this amplitude. The most crucial quantities in the TPE contribution are the S- and P-wave partial wave amplitudes for the reaction $N\bar{N} \to \pi\pi$ [refs. [14–16])]. We shall study the change in the TPE potential when varying these amplitudes within reasonable limits. We find a particular set of values for these amplitudes which, when combined with the OPE and ω-exchange potentials, provides a fairly good description of NN scattering parameters. We emphasize, however, that no formal fit to the experimental data has been performed. This interaction model has many features which are not sensitive to the details of the $N\bar{N} \to \pi\pi$ amplitude used (e.g., the relatively weak short-range repulsion). Thus, the fact that it corresponds to our own "guess" for the S- and P-wave amplitudes for $N\bar{N} \to \pi\pi$ is not as serious a fault as might be supposed. Until consistent results for $N\bar{N} \to \pi\pi$ amplitudes can be obtained from the elastic πN scattering data the best one can do is make reasonable models for these amplitudes.

We have made no attempt to suppress the non-local and energy-dependent features of this interaction. In fact, these features are among those which distinguish the present model from the myriad of local and energy-independent phenomenological potentials. Although our model is not unique, we believe it to be of sufficient interest to suggest its use in nuclear structure calculations as an alternative to the usual local phenomenological potentials. Since most nuclear structure calculations are solved in terms of G-matrices, which are both non-local and energy dependent, these features of our interaction would appear to cause minimal additional complications in nuclear structure calculations. Neither should the use of the BBSE present insurmountable obstacles in nuclear many-body problems in view of its striking similarity to the Lippmann-Schwinger equation. Using our best interaction we report the results of calculations of deuteron properties and the binding energy of nuclear matter.

We have divided this paper into six sections. In sect. 2 we review the method of constructing the potential given in ref. [2]), and discuss the spin structure of the NN amplitude. In that section we also present the partial wave decomposition of the interaction. In sect. 3 we write the BBSE in the form of a Fredholm equation and

discuss the method of solving for the deuteron wave function. In that section we also discuss the high momentum damping of the potential.

Sect. 4 is devoted to a review of the framework for calculating the TPE amplitude presented in refs. [14, 2]). The investigation of specific models for the potential is presented in sect. 5. Here we also discuss the input parameters and functions. Sect. 6 contains a summarizing discussion of the results obtained.

2. Spin structure of the NN interaction

2.1. DEFINITION OF THE INTERACTION

In this section we review the method of constructing the nucleon-nucleon (NN) potential from the field theoretical elastic nucleon-nucleon scattering amplitude given by Chemtob, Durso and Riska [2]). We shall use the same notation as in ref. [2]) as far as possible. The basic idea in this method is to identify the quantum mechanical amplitude $\mathscr{T}$ satisfying the Blankenbecler-Sugar equation (BBSE) with the field theoretical elastic NN scattering amplitude. We write the BBSE in the c.m. system in the form

$$\mathscr{T}(\boldsymbol{p}', \boldsymbol{p}; W) = \mathscr{V}(\boldsymbol{p}', \boldsymbol{p}; W) + \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \mathscr{V}(\boldsymbol{p}, \boldsymbol{k}; W) \times \frac{m^2 \Lambda_+^{\mathrm{p}}(\boldsymbol{k}) \Lambda_+^{\mathrm{n}}(-\boldsymbol{k})}{E_k (E_k^2 - W^2 - i\varepsilon)} \mathscr{T}(\boldsymbol{k}, \boldsymbol{p}; W). \tag{2.1.1}$$

Here $\mathscr{V}$ is the BBSE interaction. The two nucleons are given identification indices n, p. The initial, intermediate and final relative momenta (and also the respective momenta of nucleon p) are denoted $\boldsymbol{p}$, $\boldsymbol{k}$, and $\boldsymbol{p}'$. The positive energy projection operators for the nucleons are

$$\Lambda_+^{\mathrm{i}}(\boldsymbol{k}) = \frac{\gamma_4 E_k - i\boldsymbol{\gamma} \cdot \boldsymbol{k} + m}{2m}. \tag{2.1.2}$$

In (2.1.1) W is the energy of one incoming nucleon in the c.m. system and $E_k = \sqrt{\boldsymbol{k}^2 + m^2}$, with m being the nucleon mass.

To construct the potential from the field theoretic scattering amplitude M one considers a separation of M in terms associated with singularities in the $\mathrm{N\bar{N}} \to \mathrm{N\bar{N}}$ channel:

$$M = M^{(1)} + M^{(2)} + \ldots. \tag{2.1.3}$$

Here $M^{(1)}$ represents terms of second order in the pion-nucleon coupling constant (OPE), $M^{(2)}$ represents all fourth-order terms and so on. Expanding the interaction $\mathscr{V}$ as

$$\mathscr{V} = \mathscr{V}^{(1)} + \mathscr{V}^{(2)} + \ldots, \tag{2.1.4}$$

we chose $\mathscr{V}^{(m)}$ such that the mth order iterative solution of eq. (2.1.1) agrees with the first m terms of eq. (2.1.3). Writing the BBSE in the compact form $\mathscr{T} = \mathscr{V} + \mathscr{V} g \mathscr{T}$, we obtain the following expressions for the potential terms:

$$\begin{aligned} \mathscr{V}^{(1)} &= M^{(1)}, \\ \mathscr{V}^{(2)} &= M^{(2)} - \mathscr{V}^{(1)} g \mathscr{V}^{(1)}, \\ \mathscr{V}^{(3)} &= M^{(3)} - \mathscr{V}^{(1)} g \mathscr{V}^{(1)} g \mathscr{V}^{(1)} - \mathscr{V}^{(2)} g \mathscr{V}^{(1)} - \mathscr{V}^{(1)} g \mathscr{V}^{(2)}, \end{aligned} \tag{2.1.5}$$

and so forth. The OPE interaction is thus simply given by the OPE amplitude while the TPE interaction is given by the difference between the full fourth-order contribution to the scattering amplitude and the iteration of the OPE interaction. We shall assume the three-pion exchange interaction to be dominated by the narrow (9.8 MeV) ω-meson resonance with quantum numbers (T, J^{π}) equal to $(0, 1^{-})$. Due to the small width of the ω-resonance, it is justified to include this contribution as an additional pole term in $M^{(1)}$.

There are other three-pion resonances which we do not include in the interaction. The η-meson is an isoscalar, pseudoscalar meson of mass 549 MeV. Since it is significantly heavier than the pion and has a coupling constant which is estimated to be less than the pion coupling constant, its inclusion would have very little effect on calculated NN phase shifts. This expectation has been verified numerically. The ϕ-meson is an isoscalar, vector three-pion resonance with mass 1019 MeV which, in most models, is assumed not to couple to the nucleon. Since the ϕ- and ω-mesons have identical quantum numbers, the effects of possible ϕN coupling are readily incorporated in and indistinguishable from adjustments in the ωN coupling constant. Since the ω-meson is largely responsible for the short-range repulsion in the NN interaction† and since calculated phase shifts at low energies are relatively insensitive to the amount of short-range repulsion, we neglect the effects of the exchange of higher mass (and hence shorter range) mesons and higher-order terms in eq. (2.1.5) on the grounds that they should be effectively masked by ω-exchange.

From eq. (2.1.5) it is clear that the specific form of $\mathscr{V}^{(2)}$ and higher-order terms depends explicitly on the Green function of the approximate unitarizing equation. Retaining terms through $\mathscr{V}^{(m)}$, however, ensures by construction that the mth order iterative solution to eq. (2.1.1) agrees with the first m-terms of eq. (2.1.3) independent of the choice of the approximate Green function. Studies by Woloshyn and Jackson [8]) using a model in which, following partial wave projection, all nucleons and exchanged mesons are treated as scalar particles suggest that calculated NN phase shifts are insensitive to the specific choice of an approximate unitarizing equation

† This is readily seen by analogy with the Coulomb interaction. Here the baryon number assumes the rôle of the strong charge.

providing that the TPE contribution, $\mathscr{V}^{(2)}$, is retained and the ω-meson coupling constant is allowed to vary within its experimental error. This result may be taken over to the present problem with the important provision that the positive energy nucleon projection operators of eq. (2.1.1) be retained in any equation considered. This is a consequence of the fact that pions are pseudoscalar particles so that matrix elements leading from positive energy to negative energy nucleon states are roughly a factor of $(m/\mu_\pi)^2$ larger than those connecting positive energy to positive energy states. There is a substantial cancellation in $M^{(2)}$ (and hence in $\mathscr{V}^{(2)}$) from the individually large terms involving nucleons in negative energy intermediate states. (This cancellation is frequently referred to as "pair suppression".) The inclusion of even relatively small "impurities" of negative energy projection operators in the approximate Green function would upset this balance in $\mathscr{V}^{(2)}$ and would needlessly worsen the convergence rate of eq. (2.1.4).

It is convenient to note that the BBSE is easily recast in the form of the more familiar Lippmann-Schwinger equation

$$T(\boldsymbol{p}', \boldsymbol{p}; W) = V(\boldsymbol{p}', \boldsymbol{p}) - \int \frac{\mathrm{d}^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m}{E_k^2 - W^2 - i\varepsilon} T(\boldsymbol{k}, \boldsymbol{p}; W), \tag{2.1.6}$$

with the definitions

$$\begin{pmatrix} V(\boldsymbol{p}', \boldsymbol{p}) \\ T(\boldsymbol{p}', \boldsymbol{p}) \end{pmatrix} = -\sqrt{\frac{m}{E_{p'}}}\, \bar{u}^{(\mathrm{p})}(\boldsymbol{p}')\bar{u}^{(\mathrm{n})}(-\boldsymbol{p}') \begin{pmatrix} \mathscr{V}(\boldsymbol{p}', \boldsymbol{p}) \\ \mathscr{T}(\boldsymbol{p}', \boldsymbol{p}) \end{pmatrix} u^{(\mathrm{p})}(\boldsymbol{p})u^{(\mathrm{n})}(-\boldsymbol{p}) \sqrt{\frac{m}{E_p}}\,. \tag{2.1.7}$$

The quantity $T(\boldsymbol{p}', \boldsymbol{p}; W)$ now has the normal non-relativistic relation to phase shifts. Eqs. (2.1.6) and (2.1.7) suggest a "natural" way to extend our results to nuclear many-body problems: construct $V(\boldsymbol{p}', \boldsymbol{p})$ from eq. (2.1.7) and then use normal (non-relativistic) techniques. It is precisely this approach which we shall adopt in calculating the deuteron wave function and in performing nuclar matter calculations below. In spite of this close formal relation to non-relativistic potential theory, it is important to bear in mind that our goal remains the construction of numerically reliable approximations to the field theoretic amplitude M (or, alternatively, to solutions of the Bethe-Salpeter equation for some reliable approximation to the sum of all irreducible diagrams).

2.2. THE NN AMPLITUDE

The on-mass-shell elastic NN scattering amplitude, written for distinguishable nucleons, can be expressed in terms of five spin invariants P_j as [14, 2])

$$M(s, t, u) = \sum_{j=1}^{5} (3p_j^+(s, t, u) + 2p_j^-(s, t, u)\tau^{\mathrm{p}} \cdot \tau^{\mathrm{n}})P_j, \tag{2.2.1}$$

where the kinematical variables s, t, and u are defined as

$$\begin{aligned} s &= -(p_1+n_1)^2, \\ t &= -(p_2-p_1)^2, \\ u &= -(p_2-n_1)^2. \end{aligned} \tag{2.2.2}$$

Here p_1 and p_2 are the initial and final four-momenta of nucleon p and n_1, n_2 the initial and final four-momenta of nucleon n (fig. 2.2.1). The isospin operators for the

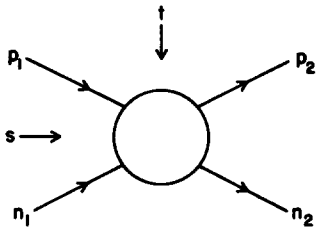

Fig. 2.2.1. Kinematics for NN scattering.

two nucleons are denoted τ^p, τ^n. The spin invariants P_j are defined as [14])

$$\begin{aligned} P_1 &= 1^p 1^n, \qquad P_2 = i(\gamma^p \cdot N)1^n + i(\gamma^n \cdot P)1^p, \\ P_3 &= i(\gamma^p \cdot N)i(\gamma^n \cdot P), \qquad P_4 = \gamma^n \cdot \gamma^p, \qquad P_5 = \gamma_5^p \gamma_5^n. \end{aligned} \tag{2.2.3}$$

The four-vectors N and P are defined to be

$$P = \tfrac{1}{2}(p_1+p_2), \qquad N = \tfrac{1}{2}(n_1+n_2). \tag{2.2.4}$$

The scalar amplitudes $p_j^{\pm}$ satisfy dispersion relations in the momentum transfer variable [14]):

$$p_j^{\pm}(s, t, u) = \frac{1}{\pi}\int_{4\mu^2}^{\infty} dt' \, \frac{\rho_j^{\pm}(s, t') \mp (-)^j \rho_j^{\pm}(u, t')}{t'-t} + \text{pole terms}. \tag{2.2.5}$$

Here μ is the pion mass. The OPE pole term contribution to the amplitudes p_j is

$$p_j^{\pm}(s, t, u) = -\tfrac{1}{2}g^2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{\delta_{j5}}{\mu^2 - t}, \tag{2.2.6}$$

with g being the pseudoscalar πN coupling constant ($g^2/4\pi = 14.48$).

Using a Lagrangian to describe the ωN coupling of the form

$$\mathscr{L}_{\omega NN} = i g_\omega \bar{N} \left\{ (1+\kappa_s)\gamma_\mu + i \frac{\kappa_s}{m} P_\mu \right\} \phi_\omega^\mu N, \tag{2.2.7}$$

with N being the nucleon and ϕ_ω the ω-field operators, and with† $\kappa_s = -0.12$ and P defined as half the sum of the initial and final nucleon four-momenta, we obtain the ω-exchange pole term contribution in the form

$$p_1^+(s, t, u) = -\frac{g_\omega^2 \kappa_s^2 (s-u)}{12m^2(m_\omega^2 - t)},$$

$$p_2^+(s, t, u) = -\frac{g_\omega^2 \kappa_s (1+\kappa_s)}{3m(m_\omega^2 - t)}, \tag{2.2.8}$$

$$p_4^+(s, t, u) = -\frac{g_\omega^2 (1+\kappa_s)^2}{3(m_\omega^2 - t)}.$$

Here m_ω is the mass of the ω-meson. The ω-meson gives no contribution to the other amplitudes p_j.

We give the expressions for the spectral functions ρ_j in (2.2.5) describing the two-pion exchange contribution in sect. 4. Those expressions, based on the t-channel unitarity relation, give the TPE contribution in terms of the $N\bar{N} \to \pi\pi$ reaction amplitude in the pseudophysical region below the $N\bar{N}$ threshold.

2.3. SPIN STRUCTURE OF THE POTENTIAL

Because of the necessity, in the process of solving the BBSE, of treating particles which are off-mass-shell, the spin structure of the NN interaction is more complicated than that of the elastic scattering amplitude. Whereas the elastic NN scattering amplitude can be expressed in terms of at most five linearly independent spin invariants or, equivalently, five independent helicity amplitudes, the maximum number of linearly independent helicity amplitudes of the interaction is six††. We shall here define six linearly independent helicity components of the potential, and give the partial wave decomposition in terms of these helicity components.

The BBSE potential defined in eq. (2.1.5) in terms of the field theoretical amplitude M, as given in eq. (2.2.1), can be written in the form

$$\mathscr{V}(\boldsymbol{p}', \boldsymbol{p}; s) = \sum_{j=1}^{5} \{3v_j^+(\boldsymbol{p}', \boldsymbol{p}; s) + 2v_j^-(\boldsymbol{p}', \boldsymbol{p}; s)\boldsymbol{\tau}^p \cdot \boldsymbol{\tau}^n\} P_j. \tag{2.3.1}$$

The relation between the momentum variables $\boldsymbol{p}'$ and $\boldsymbol{p}$ and the kinematical variables t and u is

† This value is assigned on the basis of vector dominance models of nucleon electromagnetic form factors in which κ_s for the ω-meson can be related to the sum of the neutron and proton anomolous magnetic moments.

†† In the general case time-reversal invariance does not reduce the number of independent amplitudes as it does for elastic scattering. This point was observed in ref. [17]), where however an incorrect relation between two of the helicity components (eq. (2.7) of ref. [17])) is given.

$$\begin{aligned} t &= -\boldsymbol{p}'^2 - \boldsymbol{p}^2 + 2\boldsymbol{p}' \cdot \boldsymbol{p}, \\ u &= -\boldsymbol{p}'^2 - \boldsymbol{p}^2 - 2\boldsymbol{p}' \cdot \boldsymbol{p}. \end{aligned} \tag{2.3.2}$$

The potential components for the OPE interaction are, according to eq. (2.2.6),

$$v_j^-(\boldsymbol{p}', \boldsymbol{p}; s) = -\tfrac{1}{2}g^2 \frac{\delta_{j5}}{\mu^2 + p^2 + p'^2 - 2pp'z}, \tag{2.3.3}$$

with $z = \cos\theta$, θ being the c.m. scattering angle between the momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$. The potential components for one ω-meson exchange are

$$\begin{aligned} v_1^+(\boldsymbol{p}', \boldsymbol{p}; s) &= -\frac{g_\omega^2 \kappa_s^2 (s + p^2 + p'^2 + 2pp'z)}{12m^2(m^2 + p^2 + p'^2 - 2pp'z)}, \\ v_2^+(\boldsymbol{p}', \boldsymbol{p}; s) &= -\frac{g_\omega^2 \kappa_s (1 + \kappa_s)}{3m\,(m_\omega^2 + p^2 + p'^2 - 2pp'z)}, \\ v_4^+(\boldsymbol{p}', \boldsymbol{p}; s) &= -\frac{g_\omega^2 (1 + \kappa_s)^2}{3(m_\omega^2 + p^2 + p'^2 - 2pp'z)}. \end{aligned} \tag{2.3.4}$$

As stated above, we always use the value $\kappa_s = -0.12$. The explicit forms for the TPE potential components are given in sect. 4.

The fact that only five spin invariants occur in the BBSE potential expression (2.3.1) is due to our construction of the potential from the on-mass-shell field theoretical elastic scattering amplitude. In general the TPE contribution off-shell is not reducible to only five spin components, but, as a dispersion theory framework is employed for the calculation of the TPE amplitude, no off-shell extrapolation is possible at this stage.

In a specific Lagrangian perturbation theory framework it is possible to avoid use of the Dirac equation and to retain the complete off-shell spin structure. Effects due to different ways of extrapolating off-shell the TPE contribution are however only seen in potential matrix elements away from the main diagonal. Small differences in such matrix elements are probably not very important, since those elements are strongly damped in the properly regularized potential. We shall illuminate this point by means of a numerical example in sect. 5.

We define the following six independent helicity components of the potential that are to be used in the BBSE:

$$\begin{aligned} \varphi_1(p', p, z; s) &= N\langle + + |\mathcal{V}| + + \rangle, \\ \varphi_2(p', p, z; s) &= N\langle + + |\mathcal{V}| - - \rangle, \\ \varphi_3(p', p, z; s) &= N\langle + - |\mathcal{V}| + - \rangle, \\ \varphi_4(p', p, z; s) &= N\langle + - |\mathcal{V}| - + \rangle, \\ \varphi_5(p', p, z; s) &= N\langle + + |\mathcal{V}| + - \rangle, \\ \varphi_6(p', p, z; s) &= N\langle + - |\mathcal{V}| + + \rangle. \end{aligned} \tag{2.3.5}$$

This choice of helicity components is suggested by the original definitions in ref. [18]). In eq. (2.3.5) we have used the notation

$$N = -m/4\pi. \tag{2.3.6}$$

Here $E = \sqrt{p^2+m^2}$ and $E' = \sqrt{p'^2+m^2}$. On the energy shell E equals E' and one has $\varphi_5 = -\varphi_6$ as a consequence of time-reversal invariance. For the partial wave decomposition of the potential it is convenient to define the following combinations of helicity components [18]):

$$\begin{aligned} f_1 &= \varphi_1 - \varphi_2, \\ f_2 &= \varphi_1 + \varphi_2, \\ f_3 &= \varphi_3/(1+z) - \varphi_4/(1-z), \\ f_4 &= \varphi_3/(1+z) + \varphi_4/(1-z), \\ f_5 &= 2\varphi_5/y, \\ f_6 &= 2\varphi_6/y. \end{aligned} \tag{2.3.7}$$

Here $y = \sqrt{1-z^2}$.

The isospin structure of the potential components f is the same as that of the potential components v_j in (2.3.1), i.e.,

$$f_k = 3f_k^+ + 2f_k^- \boldsymbol{\tau}^p \cdot \boldsymbol{\tau}^n. \tag{2.3.8}$$

For states of definite isospin we have

$$\begin{aligned} f_k^{(T=1)} &= 3f_k^+ + 2f_k^-, \\ f_k^{(T=0)} &= 3f_k^+ - 6f_k^-. \end{aligned} \tag{2.3.9}$$

The relation between the components f_k and v_j in (2.3.1) can be written in the form

$$f_k^{\pm}(p', p, z; s) = \frac{1}{8\pi} \sum_{j=1}^{5} a_{kj}(p', p, z; s) v_j^{\pm}(p', p, z; s). \tag{2.3.10}$$

The elements of the 6×5 matrix a are given in table 2.3.1. The matrix elements a_{kj} are calculated without any on-shell approximation. Six helicity amplitudes are generated from the five linearly independent potential components.

We introduce the following partial wave decompositions of the components f:

$$f_0^J(p', p; s) = \frac{1}{2} \int_{-1}^{1} \mathrm{d}z\, P_J(z) f_1,$$

$$f_{11}^J(p', p; s) = \frac{1}{2} \int_{-1}^{1} \mathrm{d}z\, P_J(z) f_2,$$

$$
\begin{aligned}
f_{12}^{J}(p', p; s) &= \frac{1}{2}\int_{-1}^{1} dz \frac{\sqrt{J(J+1)}}{2J+1}[P_{J+1}(z)-P_{J-1}(z)]f_5, \\
f_{21}^{J}(p', p; s) &= -\frac{1}{2}\int_{-1}^{1} dz \frac{\sqrt{J(J+1)}}{2J+1}[P_{J+1}(z)-P_{J-1}(z)]f_6, \\
f_{22}^{J}(p', p; s) &= \frac{1}{2}\int_{-1}^{1} dz \left\{P_J(z)f_3 + \frac{JP_{J+1}(z)+(J+1)P_{J-1}(z)}{2J+1} f_4\right\}, \\
f_{1}^{J}(p', p; s) &= \frac{1}{2}\int_{-1}^{1} dz \left\{P_J(z)f_4 + \frac{JP_{J+1}(z)+(J+1)P_{J-1}(z)}{2J+1} f_3\right\}.
\end{aligned}
\tag{2.3.11}
$$

TABLE 2.3.1

Elements of the matrix a connecting the six helicity components of the potential with the five spin components

$a \equiv E'E+m^2$, $b \equiv E'E-m^2$, $c \equiv p'p$, $d \equiv p'^2+p^2$, $e \equiv E'+E$, $f \equiv p^2E'+p'^2E$

$a_{11} = a-cz$	$a_{41} = a$
$a_{12} = -m[d+e\sqrt{s}+2cz]$	$a_{42} = -m[\sqrt{s}e+d]$
$a_{13} = \frac{1}{4}\{d(b+cz)+2c(c+bz)+2\sqrt{s}[f+cez+s(a+cz)]\}$	$a_{43} = \frac{1}{4}\{sa+(db+2c^2)+2\sqrt{s}f'\}$
$a_{14} = 2(EE'+b)$	$a_{44} = a+b$
$a_{15} = b-cz$	$a_{45} = -b$
$a_{21} = -c+az$	$a_{51} = -me$
$a_{22} = -m\{2c+z[d+e\sqrt{s}]\}$	$a_{52} = a\sqrt{s}+f$
$a_{23} = \frac{1}{4}\{d(bz+c)+2c(cz+b)+2\sqrt{s}[fz+ce]+s[az+c]\}$	$a_{53} = \frac{1}{4}m\{(E'-E)(p^2-p'^2)-se-2\sqrt{s}d\}$
$a_{24} = 4c+2m^2z$	$a_{54} = -2mE$
$a_{25} = -c+bz$	$a_{55} = -m(E'-E)$
$a_{31} = -c$	$a_{61} = -a_{51}$
$a_{32} = -2mc$	$a_{62} = -a_{52}$
$a_{33} = \frac{1}{4}\{c(2b+d)+2\sqrt{s}ce+sc\}$	$a_{63} = -a_{53}$
$a_{34} = 2c$	$a_{64} = 2mE'$
$a_{35} = c$	$a_{65} = -m(E'-E)$

The potential matrix elements between states of definite angular momentum $\langle J'L'S'|\mathscr{V}|JLS\rangle$ can be expressed in terms of the partial wave components f^J in the following way:

$$
\begin{aligned}
\langle p', J, J, 0|\mathscr{V}(s)|p, J, J, 0\rangle &= f_0^J, \\
\langle p', J, J, 1|\mathscr{V}(s)|p, J, J, 1\rangle &= f_1^J, \\
\langle p', J, J+1, 1|\mathscr{V}(s)|p, J, J+1, 1\rangle &= \frac{1}{2J+1}[(J+1)f_{11}^J + Jf_{22}^J - \sqrt{J(J+1)}(f_{21}^J + f_{12}^J)], \\
\langle p', J, J+1, 1|\mathscr{V}(s)|p, J, J-1, 1\rangle &= -\frac{1}{2J+1}[\sqrt{J(J+1)}(f_{22}^J - f_{11}^J) \\
&\qquad -(J+1)f_{12}^J + Jf_{21}^J],
\end{aligned}
$$

$$\langle p', J, J-1, 1|\mathscr{V}(s)|p, J, J+1, 1\rangle = -\frac{1}{2J+1}\,[\sqrt{J(J+1)}(f^J_{22}-f^J_{11})+Jf^J_{12} -(J+1)f^J_{21}],$$

$$\langle p', J, J-1, 1|\mathscr{V}(s)|p, J, J-1, 1\rangle = \frac{1}{2J+1}\,[Jf^J_{11}+(J+1)f^J_{22}+\sqrt{J(J+1)}(f^J_{21}+f^J_{12})]. \tag{2.3.12}$$

An overall sign in the expressions for the off-diagonal potential matrix elements $\mathscr{V}_{J-1,J+1}$ and $\mathscr{V}_{J+1,J-1}$ in (2.3.12) has been introduced so that the phase parameter representation of the S-matrix will be the conventional one. These relations establish the connection between the potential in states of definite angular momentum and the general potential expression in terms of spin invariants (2.3.1). It is to be noted that once the potential expression (2.3.1) is given the off-mass-shell extrapolation is unambiguous.

3. Solution of the Lippmann-Schwinger equation

3.1. THE FORM OF THE LSE

The LSE as written in eq. (2.1.6) connecting the T-matrix (scattering amplitude) with the potential is inconvenient to solve numerically because of the complex propagator. We shall therefore introduce the "R-matrix" (often called "K-matrix") defined as [7])

$$R = (i\pi)^{-1}\frac{1-S}{1+S}, \qquad S = 1-2\pi iT. \tag{3.1.1}$$

The R-matrix satisfies a LSE with a real propagator:

$$R(\boldsymbol{p}', \boldsymbol{p}; W) = V(\boldsymbol{p}', \boldsymbol{p}) - \mathrm{P}\int \frac{\mathrm{d}^3k}{(2\pi)^3}\, V(\boldsymbol{p}', \boldsymbol{k})\frac{m}{E_k^2 - W^2}\, R(\boldsymbol{k}, \boldsymbol{p}; W). \tag{3.1.2}$$

The partial wave decomposition of this equation takes the form

$$R_{L'L}(p', p; W) = V_{L'L}(p', p) - \frac{2m}{\pi}\sum_{\lambda}\mathrm{P}\int_0^\infty \mathrm{d}k \frac{k^2}{k^2-p^2}\, V_{L'\lambda}(p', k)R_{\lambda L}(k, p; W). \tag{3.1.3}$$

This equation can be written in a non-singular Fredholm form by the well-known method of Kowalski [19]) and Noyes [20]), which can be generalized in a straightforward way to the case of coupled partial waves [21]). To write the coupled channel LSE in a non-singular form, define a new matrix f in LL' space as

$$f(p'; W) = R(p', p; W)R^{-1}(p, p; W). \tag{3.1.4}$$

Clearly, $f(p, W)$ is the identity matrix in LL' space. By subtracting appropriate multiples of the fully on-shell version of (3.1.3), we obtain the following non-singular

integral equation for f (which is to be regarded as a matrix equation in LL' space):

$$f(p', W) = V(p', p)V^{-1}(p, p) - \frac{2m}{\pi}\int \mathrm{d}k \frac{k^2}{k^2-p^2}\{V(p', k) - V(p', p)V^{-1}(p, p)V(p, k)\}f(k, W). \quad (3.1.5)$$

Note that the matrix in brackets vanishes for k equal to p, so that the integrand is explicitly not singular at this point. The on-shell R-matrix is given in terms of f by

$$R(p, p; W) = \left\{1 - \frac{2m}{\pi}\mathrm{P}\int_0^\infty \mathrm{d}k \frac{k^2}{k^2-p^2} V(p, k)f(k; W)\right\}^{-1} V(p, p). \quad (3.1.6)$$

Eq. (3.1.5) is only one of many non-singular integral equations that can be derived from the LSE, but it represents the simplest generalization of the Kowaslki-Noyes method to the case of coupled partial waves.

We note that it is not necessary to use a non-singular form of the integral equation to obtain numerical solutions. We have found that solving the singular equation (3.1.3) by the method of ref. [7]) leads to the same results as obtained solving the non-singular version (3.1.5). The non-singular equation is at least aesthetically preferable and has been used in this work.

3.2. PHASE PARAMETERS AND THE DEUTERON WAVE FUNCTION

The relation between scattering phase parameters and on-shell R-matrix elements for an uncoupled partial wave is [22]):

$$\delta_{JJS}(k) = \arctan\{-kmR_{JJ}(k, k)\}, \quad (3.2.1)$$

and for coupled partial waves

$$\begin{aligned}
\delta_{J,J-1,1}(k) &= \arctan\left\{-\frac{km}{2}\left[R_{11}(k, k)+R_{22}(k, k)+\frac{R_{11}(k, k)-R_{22}(k, k)}{\cos 2\varepsilon_J}\right]\right\},\\
\delta_{J,J+1,1}(k) &= \arctan\left\{-\frac{km}{2}\left[R_{11}(k, k)+R_{22}(k, k)-\frac{R_{11}(k, k)-R_{22}(k, k)}{\cos 2\varepsilon_J}\right]\right\}, \quad (3.2.2)\\
\varepsilon_J(k) &= \tfrac{1}{2}\arctan\left\{\frac{2R_{12}(k, k)}{R_{11}(k, k)-R_{22}(k, k)}\right\}.
\end{aligned}$$

Here the general notation for the eigenphase parameters [23]) is δ_{JLS} and ε_J. In (3.2.2) we have abbreviated $R_{J-1,J+1}$ by R_{12}.

The relations between the eigenphase shifts and the nuclear bar phase parameters $\bar{\delta}_{JLS}$ and $\bar{\varepsilon}_J$ [ref. [24])] are

$$\begin{aligned}
\bar{\delta}_{JJS} &= \delta_{JJS},\\
\bar{\delta}_{J,J-1,1}+\bar{\delta}_{J,J+1,1} &= \delta_{J,J-1,1}+\delta_{J,J+1,1}, \quad (3.2.3)
\end{aligned}$$

$$\sin(\bar{\delta}_{J,J-1,1}-\bar{\delta}_{J,J+1,1}) = \tan 2\bar{\varepsilon}_J/\tan 2\varepsilon_J,$$

$$\sin(\delta_{J,J-1,1}-\delta_{J,J+1,1}) = \sin 2\bar{\varepsilon}_J/\sin 2\varepsilon_J.$$

To find the deuteron binding energy numerically one can solve the LSE (3.1.3) for negative values of $p^2 = mE_B$ and determine the binding energy E_B as the energy for which the determinant of the system of linear equations used in the numerical solution vanishes. Noting that

$$\langle \boldsymbol{q}|\psi_d\rangle = m(mE_B - q^2)^{-1}\langle \boldsymbol{q}|V|\psi_d\rangle, \tag{3.2.4}$$

it is clear that the deuteron wave function is determined by the residue at the bound state pole $\langle \boldsymbol{q}|V|\psi_d\rangle$, a quantity which satisfies the homogeneous form of the LSE in (3.1.3). The normalization of the wave function is fixed by the condition †

$$\int_0^\infty \mathrm{d}q\, q^2 |\langle \boldsymbol{q}|\psi_d\rangle|^2 = \tfrac{1}{2}\pi. \tag{3.2.5}$$

Finally the deuteron wave function in configuration space is given by the Hankel transform of the momentum space wave function:

$$\psi_L(r) = \frac{2}{\pi}\int_0^\infty \mathrm{d}q\, q^2 j_L(qr)\psi_L(q). \tag{3.2.6}$$

Here the subscript L denotes the orbital angular momentum component of the wave function.

3.3. REGULARIZATION OF THE POTENTIAL

The meson exchange potentials derived in sect. 2 do not satisfy the requirement that the kernel of the BBSE be compact. This is most readily seen by the fact that some potential components diverge in the large momentum limit in both the forward and backward directions. The potential therefore has to be regularized (i.e., damped) in the high momentum limit before it can be used in the solution of the BBSE.

It has been pointed out by Woloshyn and Jackson [10]) that sufficient damping factors for the potential can be provided by certain multiple neutral vector meson exchange processes treated in the relativistic eikonal approximation [11]). In this

† It is easily seen, using the techniques of ref. [25]), that eq. (3.2.5) is the correct normalization condition for the BBSE using energy-independent interactions. For energy-dependent interactions the correct normalization condition is

$$\langle \psi_d|\delta[G_0^{-1} - V]/\delta E|\psi_d\rangle = 1,$$

where G_0 is the free Green function. In the special case where the energy dependence of V is due solely to energy denominators in the treatment of virtual excitations of inelastic channels, the term $\langle \psi_d|\delta V/\delta E|\psi_d\rangle$ is readily seen to represent the probability that the deuteron is not simply one proton plus one neutron.

approach one considers a typical "hard" meson exchange Feynman diagram with a fixed number of additional neutral vector meson exchange lines. The leading contribution of such a Feynman diagram is extracted by treating the neutral vector mesons as soft [10, 11]). The technique is the same as that employed to extract the infrared divergent part of a multiple photon exchange diagram. After obtaining the soft vector meson limit of the Feynman diagram one sums over the number of exchanged vector mesons. The result is a high momentum damping factor multiplying the "hard" meson exchange amplitude. Jackson and Woloshyn pointed out the usefulness of retaining this damping factor when interpreting the amplitude as a potential [10]). For the regularized potential the following expression results:

$$V_R(p', p) = e^{2i[\chi(t)+\chi(u)]} V_0(p', p), \tag{3.3.1}$$

where V_0 is the asymptotically ill-behaved "bare" meson exchange potential. The "eikonal" function χ can be written in a one-parameter form as

$$i\chi(t) = -\gamma(2m^2 - t) \int_{4m^2}^{\infty} dt' \frac{1}{(t'-t)\sqrt{t'(t'-4m^2)}}. \tag{3.3.2}$$

The parameter γ is in fact formally a divergent integral which we treat as a parameter. The integral in (3.3.2) is readily evaluated and seen to have the form, valid for $t < 0$,

$$i\chi(t) = \frac{-2\gamma(2m^2 - t)}{\sqrt{-t(4m^2 - t)}} \log\left[\left(-\frac{t}{4m^2}\right)^{\frac{1}{2}} + \left(1 - \frac{t}{4m^2}\right)^{\frac{1}{2}}\right]. \tag{3.3.3}$$

In order that the meson exchange pole residues and TPE cut discontinuity not be changed when modifying the potential with the eikonal cut-off factor, we shall normalize the damping function in the following way:

$$V_R(p', p) = \frac{e^{2i[\chi(t)+\chi(u)]}}{e^{2i[\chi(-m_v^2)+\chi(4m^2-s-m_v^2)]}} V_0(p', p). \tag{3.3.4}$$

Here m_v is the mass of the exchanged meson (in the case of the TPE contribution m_v^2 is set equal to the integration variable t').

Other types of damping factors have been employed in the literature, in particular dipole cut-off factors depending on the momentum transfer variable t, which have been interpreted as a product of two monopole type meson-nucleon vertex form factors [26]). While regularizing the interaction in the backward direction, such form factors cannot cure the forward divergence of the interaction. Thus, such damping factors are not sufficient for the solution of the BBSE. The eikonal cut-off factor has symmetric dependence on the variables u and t and thus provides sufficient damping.

Identical arguments may be used to construct nucleon electromagnetic form factors in terms of vector dominance models supplemented by vertex functions obtained as the sum of soft neutral vector meson processes [11, 12]). The vertex function in this case is $\exp[i\chi(t)]$ where $\chi(t)$ is again given by eq. (3.3.2). The quantitative success of this model of nucleon electromagnetic form factors provides a direct experimental

measure of the validity of the eikonal damping functions to be used in the construction of the NN interaction which we consider to be the primary virtue of this approach over more arbitrary choices of the damping function. The analysis of nucleon electromagnetic form factor data [12]) allows the value of the parameter γ in eq. (3.3.2) to be set in the range 1.25 to 1.50 and, with the aid of data regarding the electromagnetic decay of the ω-meson, also yields the value $g_\omega^2/4\pi = 6.4 \pm 2.6$ for the ω-nucleon coupling constant under the assumption that the ϕ-nucleon coupling constant is zero [27]).

The scale of the damping in the eikonal cut-off factor is $2m$ corresponding to the lowest mass intermediate state in vector meson vertex modifications. This scale mass may be somewhat too large for the OPE contribution since the OPE contribution has a modification due to $\rho\pi$N triangle vertex diagrams which have a damping scale of roughly $\mu + m_\rho$ which is closer to m than $2m$. For simplicity, however, we shall use the eikonal cut-off factor for all meson exchange contributions in this paper.

4. The TPE potential

4.1. GENERAL FORMALISM

The OPE and ω-exchange potentials were given in subsect. 2.3. We shall here proceed to give the explicit form for the TPE potential components based on the formalism of ref. [2]). According to eq. (2.1.5), the TPE potential appears as the difference between the TPE contribution to the field theoretical elastic NN scattering amplitude and the BBSE iteration of the OPE potential. Consequently, we can write the potential components $v_j^\pm$ defined in eq. (2.3.1) in the form

$$v_j^\pm(\boldsymbol{p}', \boldsymbol{p}; s) = p_j^\pm(s, t, u) - r_j^\pm(s, t), \tag{4.1.1}$$

where $p_j^\pm$ are the TPE amplitudes defined in (2.2.1) and $r_j^\pm$ are the contributions from the iteration of the OPE potential. The relations between the invariant variables t and u and the c.m. momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ are given in eq. (2.3.2).

The scalar amplitudes p_j satisfy the following dispersion relations:

$$p_j^\pm(s, t, u) = \frac{1}{\pi} \int_{4\mu^2}^\infty \mathrm{d}t' \, \frac{\rho_j(s, t) \mp (-)^j \rho_j(u, t)}{t' - t}. \tag{4.1.2}$$

The spectral functions ρ_j can be determined by using the unitarity relation in the t-channel. Note that for the purpose of constructing a potential, the nucleons can be treated as if they were distinguishable.

It was found in ref. [2]) that the potential components r_j have a similar spectral representation:

$$r_j^\pm(s, t) = \frac{1}{\pi} \int_{4\mu^2}^\infty \mathrm{d}t' \, \frac{q_j^\pm(s, t')}{t' - t}. \tag{4.1.3}$$

The explicit expressions for the spectral functions q_j obtained with pseudoscalar πN coupling are given in terms of the η_i of ref. [2]) by the inverse of transformation (5.9) of ref. [2]).

By means of the t-channel unitarity relation the TPE spectral functions ρ_j can be written in terms of the $N\bar{N} \to \pi\pi$ reaction amplitude in the pseudophysical energy region below the physical $N\bar{N}$ threshold and above the $\pi\pi$ threshold. This reaction amplitude then constitutes the main input to the TPE potential calculations.

The $N\bar{N} \to \pi\pi$ amplitude is related by crossing symmetry to the elastic πN scattering amplitude. To go from the πN scattering amplitude to the $N\bar{N} \to \pi\pi$ amplitude requires an analytic continuation in both the invariant energy and invariant momentum transfer (t') variables. The extrapolation in the energy variable can be performed by means of the fixed-t' dispersion relation for the πN amplitude [28]). The $N\bar{N} \to \pi\pi$ amplitude is then given as the sum of the nucleon pole term and a dispersion integral over the imaginary part of the πN amplitude (s-channel discontinuity) which still has to be extrapolated to positive values of t'. In the original work of Amati, Leader and Vitale [29]) and in the subsequent applications in refs. [30, 31]), this extrapolation was done by assuming the πN amplitude to be dominated by a few low-lying πN resonances which were treated in the sharp resonance approximation. In this approximation one neglects the important feature of $\pi\pi$ interactions in the intermediate states. Since $\pi\pi$ interactions are important at low $\pi\pi$ energies (small t') only in S- and P-waves, it was suggested that the two lowest partial waves in the crossed ($N\bar{N}$) channel be treated separately [14]). For that purpose the spectral functions ρ_j can be written in the following way [2]):

$$\rho_j^{\pm} = d_j^{\pm} + b_j^{\pm} + c_j^{\pm}. \tag{4.1.4}$$

The spectral functions d_j represent the result obtained by approximating the πN amplitude by πN resonances (or more generally, the result obtained by describing the πN amplitude by a fixed-t dispersion relation) and assuming the imaginary part of the πN amplitude to be analytic in the finite t-plane [15]). The functions c_j represent the total contribution to ρ_j from S- and P-waves in the t-channel. The functions b_j are the negative t-channel S- and P-wave projections of the dispersion relation contributions d_j, introduced to avoid double counting. The explicit forms for these functions are given in ref. [2]).

4.2. MODELS FOR THE $N\bar{N} \to \pi\pi$ AMPLITUDE

The simplest model that can be used to compute the spectral functions d_j in (4.1.4) is to assume that the πN elastic scattering amplitude contains only the "nucleon pole term", i.e. that the only intermediate state is the nucleon. This is not a good model for physical πN scattering which is characterized by the multitude of πN resonances. The model used in ref. [2]) therefore included the three lowest-lying πN resonances N′ (1236 MeV), N′(1470 MeV) and N′(1520 MeV) in addition to the nucleon as intermediate states in the sharp resonance approximation. The more ambitious model

of ref. [3]) used the complete πN amplitude as known in terms of phase shifts. However, all models including πN rescattering resonances with spin $> \frac{1}{2}$ [ref. [2])] or equivalently partial waves with $L \geqq 1$ [ref. [3])] give divergent contributions to the NN scattering amplitude. Such divergences are generally treated surgically by cutting-off the dispersion integral of eq. (4.1.2) at t' on the order of $50\mu_\pi^2$. Although calculated NN phase shifts in high partial waves are not sensitive to moderate changes in this cut-off (providing that it is chosen large enough), the cut-off is a crucial parameter for S- and P-wave NN scattering. Generally, models including the cut-off divergent πN rescattering effects lead to too much attraction in the NN interaction unless the cut-off is chosen to be artificially small. There is, therefore, an obvious need to regularize the rescattering contributions, e.g., by introducing form factors at each vertex in the spirit of subsect. 3.3. We shall, in the following, mainly consider models in which the rescattering contributions have been omitted in the spectral functions d_j under the assumption that, once regularized, the rescattering contributions will be of less importance than the nucleon pole term in low energy elastic NN scattering. We shall illustrate the cut-off dependence of the divergent rescattering contributions by a numerical example and discuss those physical processes responsible for their effective cut-off in sect. 6.

The explicit expressions for the spectral functions d_j and their S- and P-wave projections in the t-channel (b_j) are taken from ref. [2]).

The S- and P-wave contributions in the t-channel to the spectral functions ρ_j, denoted c_j in eq. (4.1.4), have the following expressions [2,13]):

$$\begin{aligned} c_1^+(s,t) &= \pi N|\lambda_0|^2, \\ c_1^-(s,t) &= -\tfrac{1}{24}\pi N s(t-4\mu^2)|\lambda_1|^2, \\ c_2^-(s,t) &= \tfrac{1}{12}\pi N(t-4\mu^2)\,\mathrm{Re}\,\{\eta^*\lambda_1\}, \\ c_4^-(s,t) &= -\tfrac{1}{12}\pi N(t-4\mu^2)|\eta|^2. \end{aligned} \tag{4.2.1}$$

Here we have used the notation $N = ((t-4\mu^2)/t)^{\frac{1}{2}}/64\pi^2$. The quantities λ and η in (4.2.1) can be expressed in terms of helicity amplitudes for $N\bar{N} \to \pi\pi$ scattering [26,14]) as

$$\begin{aligned} \lambda_0(t) &= -4\pi f_+^{(+)0}(t)/p^2, \\ \lambda_1(t) &= 12\pi\{m f_-^{(-)1}(t)/\sqrt{2} - f_+^{(-)1}(t)\}/p^2, \\ \eta(t) &= 6\pi\sqrt{2} f_-^{(-)1}(t), \end{aligned} \tag{4.2.2}$$

with $p^2 = \frac{1}{4}t - m^2$. The S- and P-wave helicity amplitudes f_+^0 and $f_\pm^1$ are the most important input quantities in the calculation of the TPE NN potential [2,16]).

The helicity amplitude f_+^0 which represents the S-wave contribution to the reaction $N\bar{N} \to \pi\pi$ is not well known. It can only be evaluated by a complicated extrapolation

of πN scattering data to the pseudophysical region. Several somewhat conflicting results for this amplitude have been published in the recent literature [4, 5, 32-34]). For $t \leqq 16\mu^2$ the argument of f_+^0 is equal to the isospin-0 S-wave $\pi\pi$ scattering phase shift. That phase shift has been extracted from peripheral π-production data for $t \gtrsim 13\mu^2$ [ref. [35])]. The f_+^0 amplitude therefore contains information about both $N\bar{N} \to \pi\pi$ and $\pi\pi$ elastic scattering. The fact that the NN potential is sensitive to the variations in f_+^0 found in these data analyses may be viewed as silver lining as well as cloud: Given the reliability of our description of the NN interaction, NN scattering data is seen to provide additional constraints on f_+^0 independent of $\pi\pi$ and πN scattering results. In sect. 5 we illustrate the sensitivity of the NN interaction to this quantity.

The helicity amplitudes $f_\pm^1$ which describe the P-wave contribution to the reaction $N\bar{N} \to \pi\pi$ are somewhat better known than the S-wave amplitude f_+^0 because additional information on $f_\pm^1$ is provided by the isovector nucleon form factors. The results in the literature for $f_\pm^1$ are, therefore, more consistent [32, 33]). In sect. 5 we investigate the importance of this amplitude for the NN potential.

We find that presently available results for f_+^0 and $f_\pm^1$ obtained from πN scattering data do not lead to a quantitative description of NN scattering in the lowest partial waves within the framework of the present calculation. This is due, in part, to the neglect of relevant processes in our model of the interaction. We are thus led to manufacture a combination of helicity amplitudes which leads to a potential which does reproduce elastic NN scattering data fairly well in the low energy region. In sect. 5 we discuss this model and study the sensitivity of the results to the input parameters. We note, however, that our constructed amplitudes lie well within the spread of amplitudes coming from the above data analyses.

5. Models for the NN interaction

5.1. OPE AND SIMPLE TPE MODELS

In this and the following subsection we shall calculate the NN scattering phase parameters with specific models for the NN interaction. Before taking into account the full complexity of the TPE interaction it is instructive to consider some simple models. We first consider the OPE potential as given in eq. (2.3.3) and regularized by the eikonal damping factor according to eq. (3.3.3). For the πN pseudoscalar coupling constant we use the value $g^2/4\pi = 14.48$ throughout this paper. The value of the parameter γ in the eikonal damping factor is taken to be 1.25, which is consistent with the recent fit to the nucleon electromagnetic form factors by Iachello, Jackson and Lande [12]).

The nucleon-nucleon scattering phase parameters obtained by solving the BBSE with the regularized OPE interaction are shown in figs. 5.1 as functions of laboratory kinetic energy. The phase shifts in partial waves of large angular momentum are in

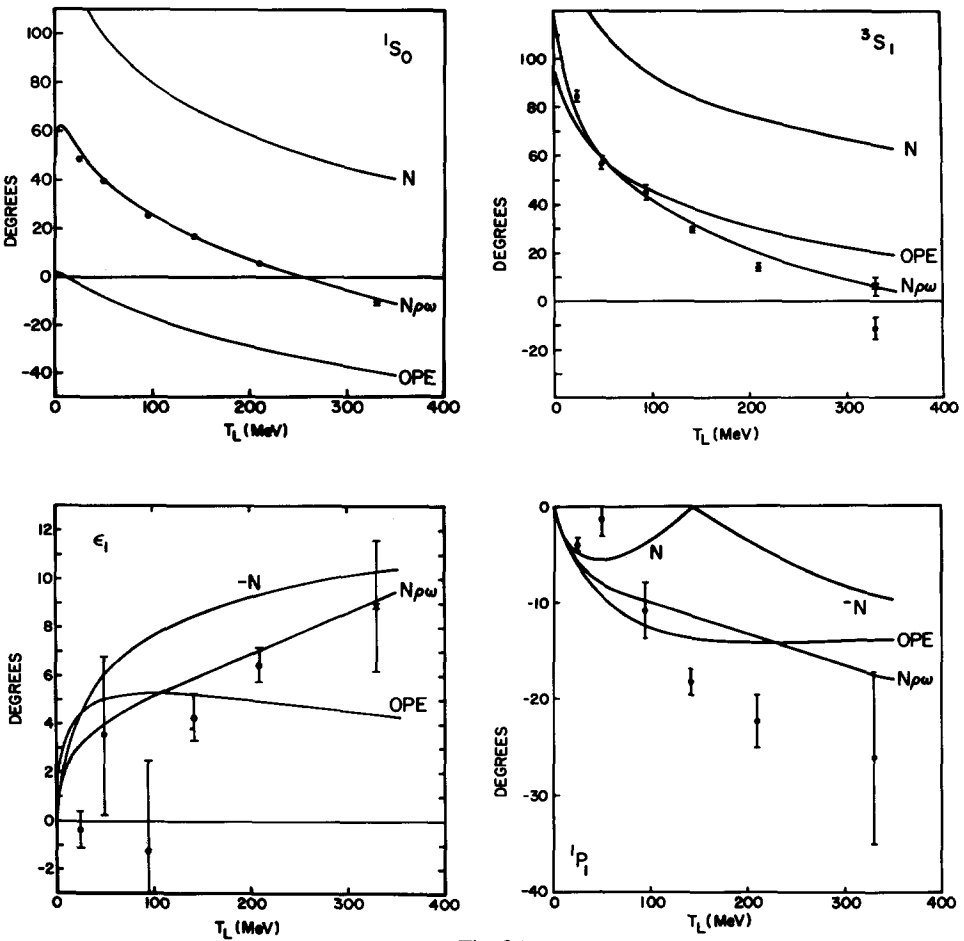

Fig. 5.1a.

good agreement with those obtained by direct unitarization of the field theoretical amplitude without the use of a potential [28]). This fact is necessary for the present method of constructing the interaction to be meaningful. The phenomenological phase shift "data" in figs. 5.1 are taken from the energy-independent analyses of refs. [36,37]).

In figs. 5.1 we also display the NN scattering parameters obtained by solving the BBSE with an interaction model containing in addition to the OPE contribution a simple TPE contribution constructed from the fourth-order Feynman diagram calculated with pseudoscalar πN coupling theory. This potential is obtained by retaining only the spectral functions d_j on the r.h.s. of eq. (4.1.4) and evaluating the nucleon pole term contribution to the functions d_j. This model for the TPE potential is the one proposed by Partovi and Lomon [1]).

The main feature of this model is that it is too attractive, and well illustrates the

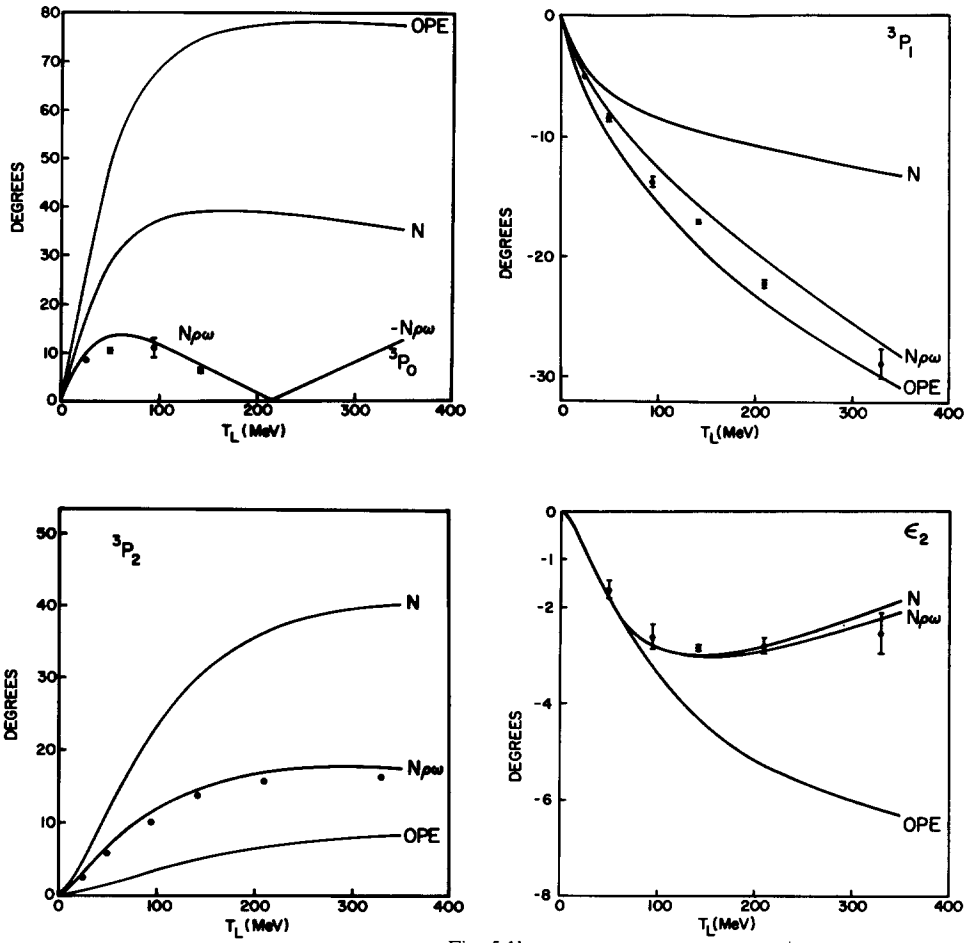

Fig. 5.1b.

need for additional repulsive contributions due to vector mesons. By adding a ρ- and ω-meson exchange contribution to this interaction model it is possible to obtain scattering phase parameters in significantly better agreement with the phenomenological results. This is illustrated by the last set of phase parameters shown in figs. 5.1. These results were obtained by adding an omega exchange contribution with an ωN coupling constant $g_\omega^2/4\pi = 6.24$ and a finite width ρ-meson contribution with a ρN coupling constant $g_\rho^2/4\pi = 0.594$ and a ρ-meson width of 125 MeV (the finite width ρ-meson contribution is discussed in subsect. 5.2). In this model and in the preceding one we have cut off the dispersion integral in eq. (4.1.2) for the TPE contribution. This cut-off is introduced only to facilitate comparison with the results obtained with the more elaborate models in the subsequent section, and not because of a diverging integrand.

Although the simple fourth-order TPE contribution amended with vector meson

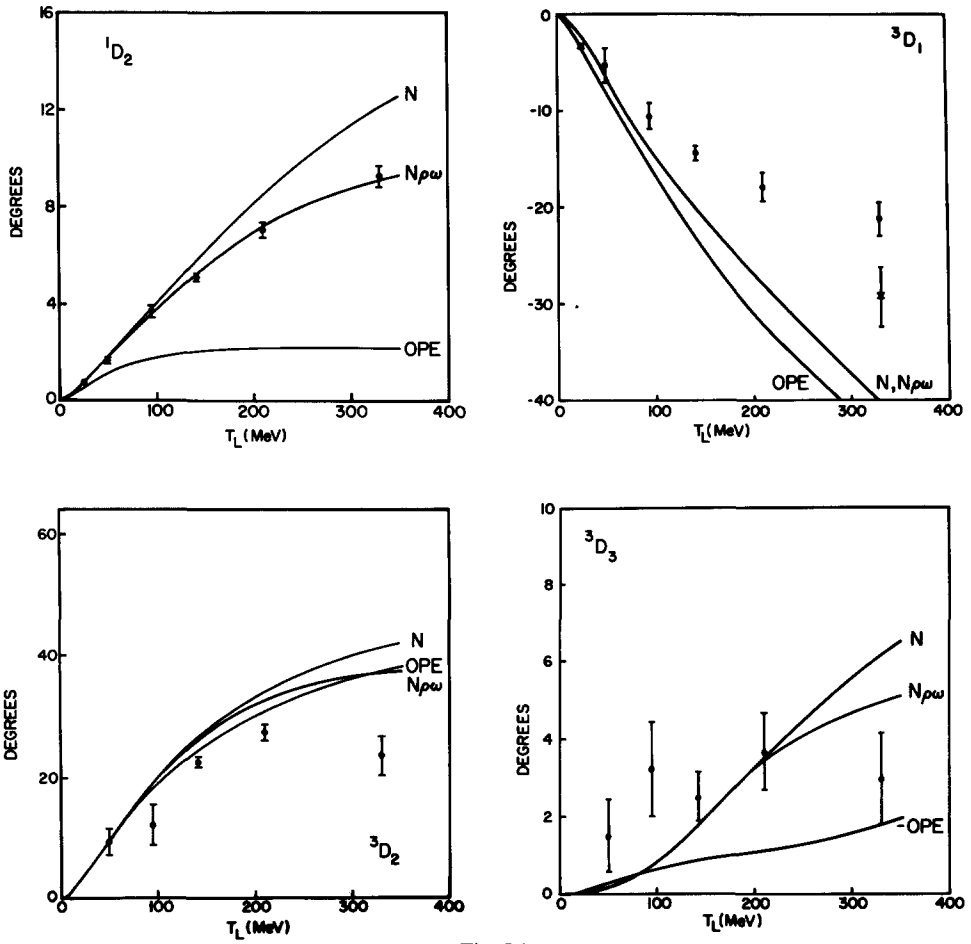

Fig. 5.1c.

exchange contributions may appear to provide a reasonable model for the NN interaction on basis of the phase parameter results in figs. 5.1, it is theoretically unsatisfactory because the nucleon pole term calculated with pseudoscalar πN coupling theory used as a model for the input $N\bar{N} \to \pi\pi$ reaction is not consistent with the low energy πN scattering amplitude. The fact that the $T = \frac{1}{2}\pi$N S-wave scattering length is positive rules out the unmodified pseudoscalar Born term as a valid first approximation to the low energy πN scattering amplitude [38]). The reason that the NN phase shifts obtained using a TPE interaction calculated with pseudoscalar πN coupling are not in gross disagreement with the phenomenological results can be found in the fact that the TPE contribution depends only on the magnitude and not on the sign of the $N\bar{N} \to \pi\pi$ reaction amplitude. In order to achieve consistency between empirical knowledge on πN and $\pi\pi$ interactions and the input

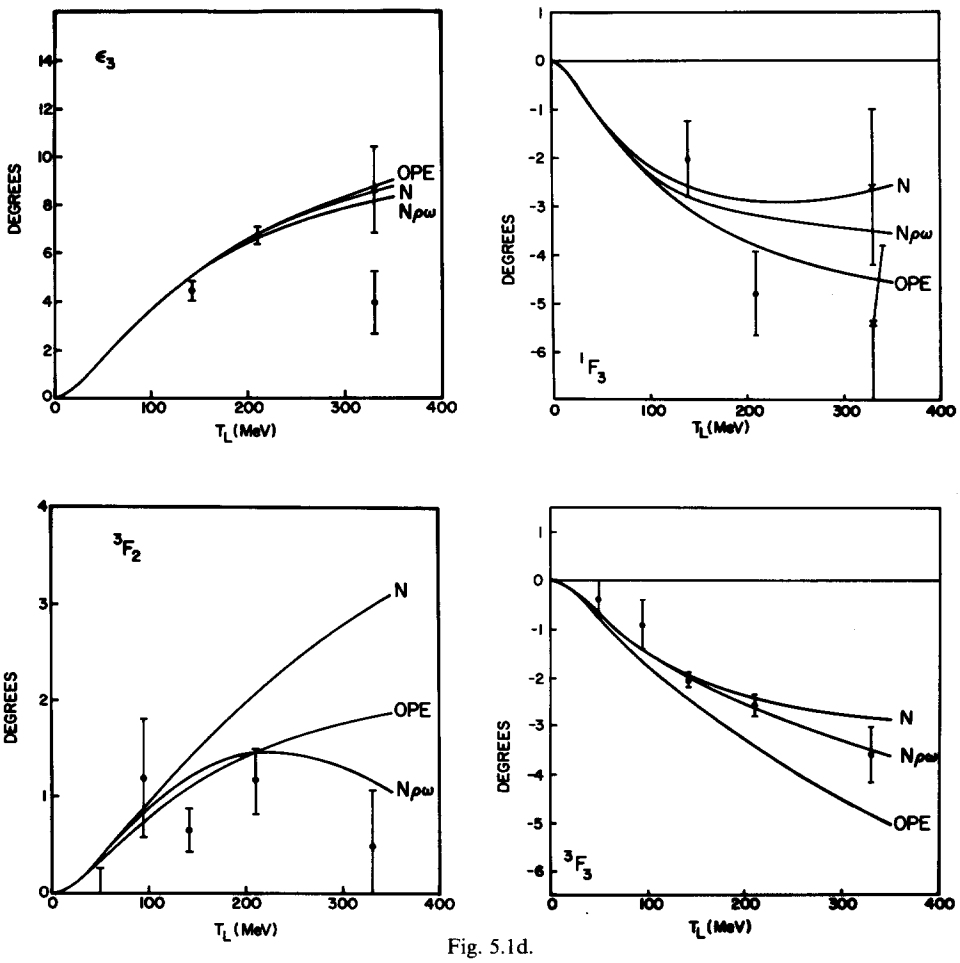

Fig. 5.1d.

model for the $N\bar{N} \to \pi\pi$ amplitude we therefore abandon the simple fourth-order TPE model, and turn to the more "realistic" models in the following subsection.

5.2. REALISTIC MODELS AND SOME NUMERICAL RESULTS

In this section we amend the simple TPE potential presented in the previous section to include the contributions of $\pi\pi$ interactions in the intermediate state. We construct the TPE potential using the complete expression (4.1.4) with models for the S- and P-wave $\pi\pi$ amplitudes f_+^0 and $f_\pm^1$ which determine the functions c_j. Although many different results for these helicity amplitudes have been obtained in the literature [4, 5, 32–34]) the most convincing analysis seems to be that of Nielsen and Oades (NO) [33]). In fig. 5.2.1 we display the NN phase shifts obtained with a potential

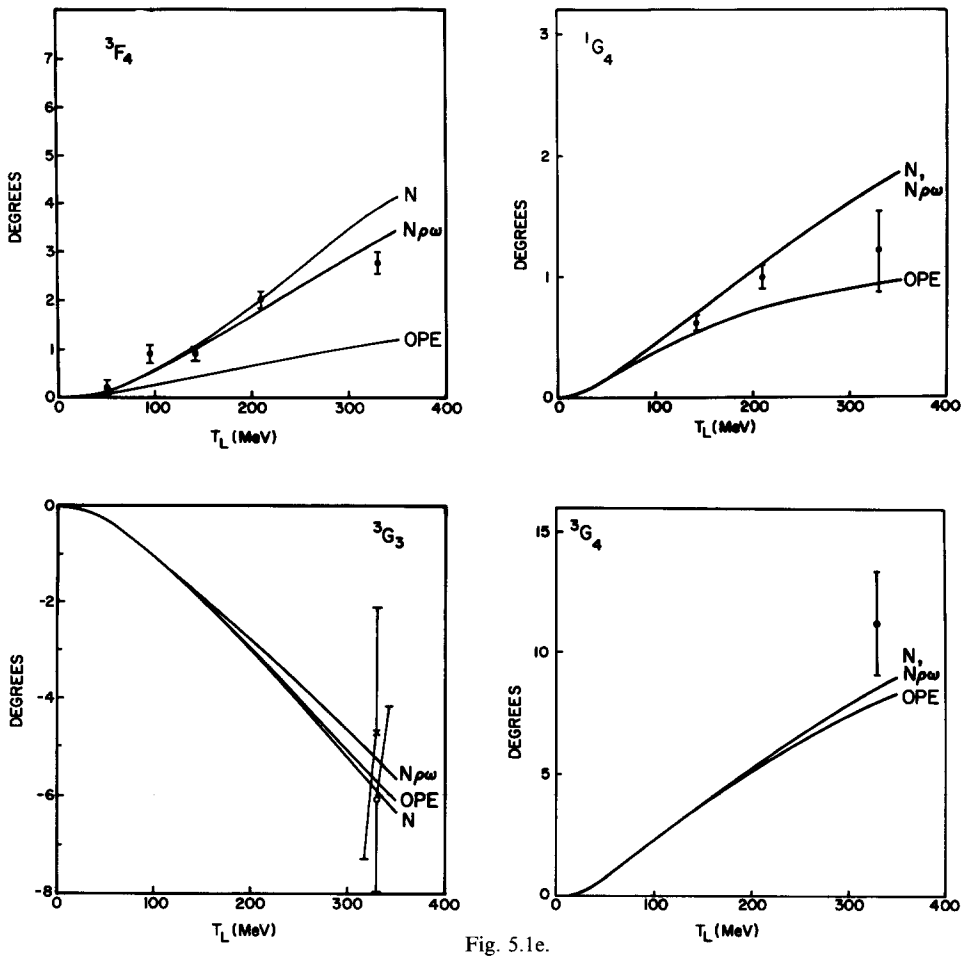

Fig. 5.1e.

Fig. 5.1. Nucleon-nucleon scattering phase shifts obtained by solving the BBSE with the one-pion exchange potential (OPE), the one-pion plus two-pion exchange potential (N), and the one-pion + two-pion + rho + omega exchange potential (N$\rho\omega$). The TPE contribution for the potential N is calculated with the nucleon pole term contributions only in the $N\bar{N} \to \pi\pi$ amplitudes and corresponds to the model of ref. [1]). The potential model N$\rho\omega$ is the potential model N plus a finite width ρ-meson (model F_1') and an ω-meson with $g_\omega^2/4\pi = 6.24$. The phase shift "data" are taken from ref. [36]) (dots: energy independent, plusses: energy dependent) and ref. [37]) (crosses).

containing the OPE, TPE and ω-exchange contributions in which the TPE potential is constructed using the nucleon pole term contribution to the functions d_j in eq. (4.1.4) and the NO results [33]) for the helicity amplitudes f_+^0 and $f_\pm^1$ (curve NO). The ωN coupling constant g_ω was adjusted to an unreasonably large value so that the results agree with the phenomenological result for the $^1S_0$ phase shift at intermediate energies ($g_\omega^2/4\pi = 13.0$). The cut-off in the dispersion integral in eq. (4.1.2) was taken

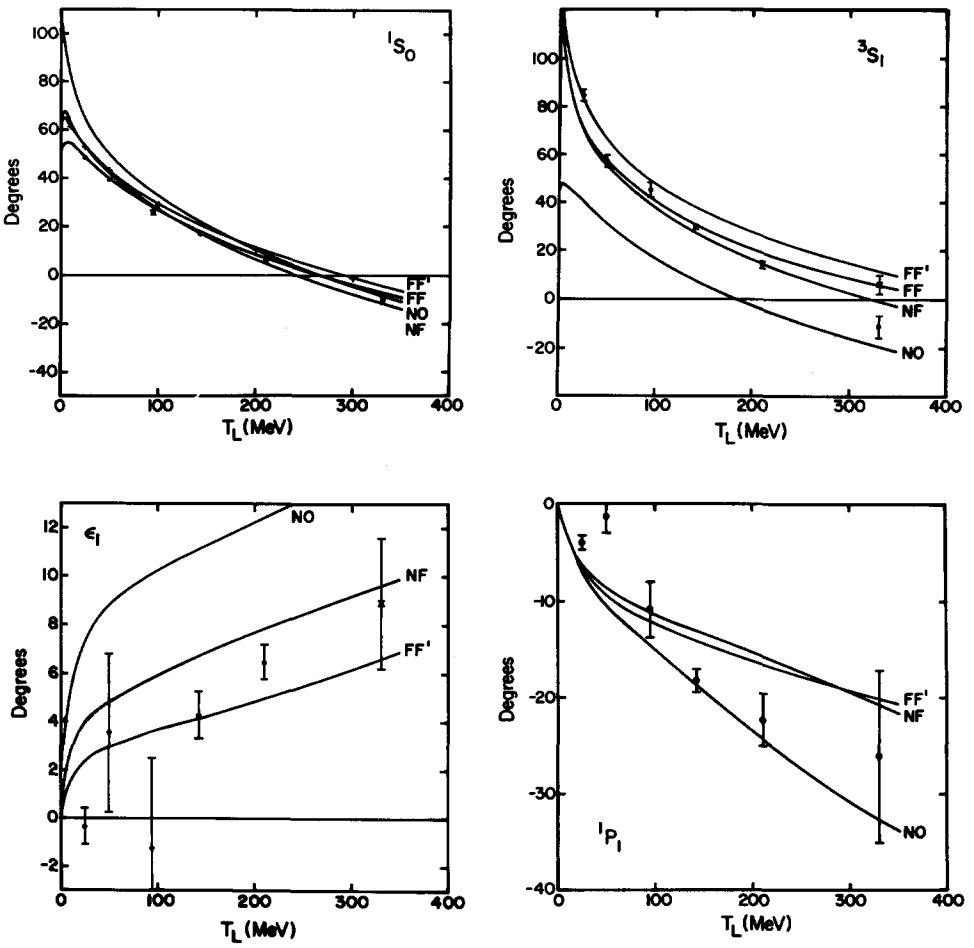

Fig. 5.2.1a.

to be $50\mu^2$ which is the largest t-value to which the NO analysis extends. The phase shift results obtained with this model are clearly far from being in satisfactory agreement with the phenomenological phase shift results. The large slope of the $^1S_0$ phase shift indicates that the intermediate range attraction due to the isospin symmetric potential component v^+ (mainly determined by f_+^0 at low t-values) is overestimated. The fact that the $^1S_0$ interaction is too attractive whereas the $^3S_1$ interaction is too repulsive also indicates that the potential component v^- is overestimated. The origin of this may be made more transparent by interpreting the NO amplitudes $f_\pm^1$ as made up of a smooth background term due to the nucleon pole contribution and a ρ-meson resonance described by a Breit-Wigner form. Such an analysis suggests $g_\rho^2/4\pi = 0.68$ and $\kappa_v = 4.33$ which are to be compared with the con-

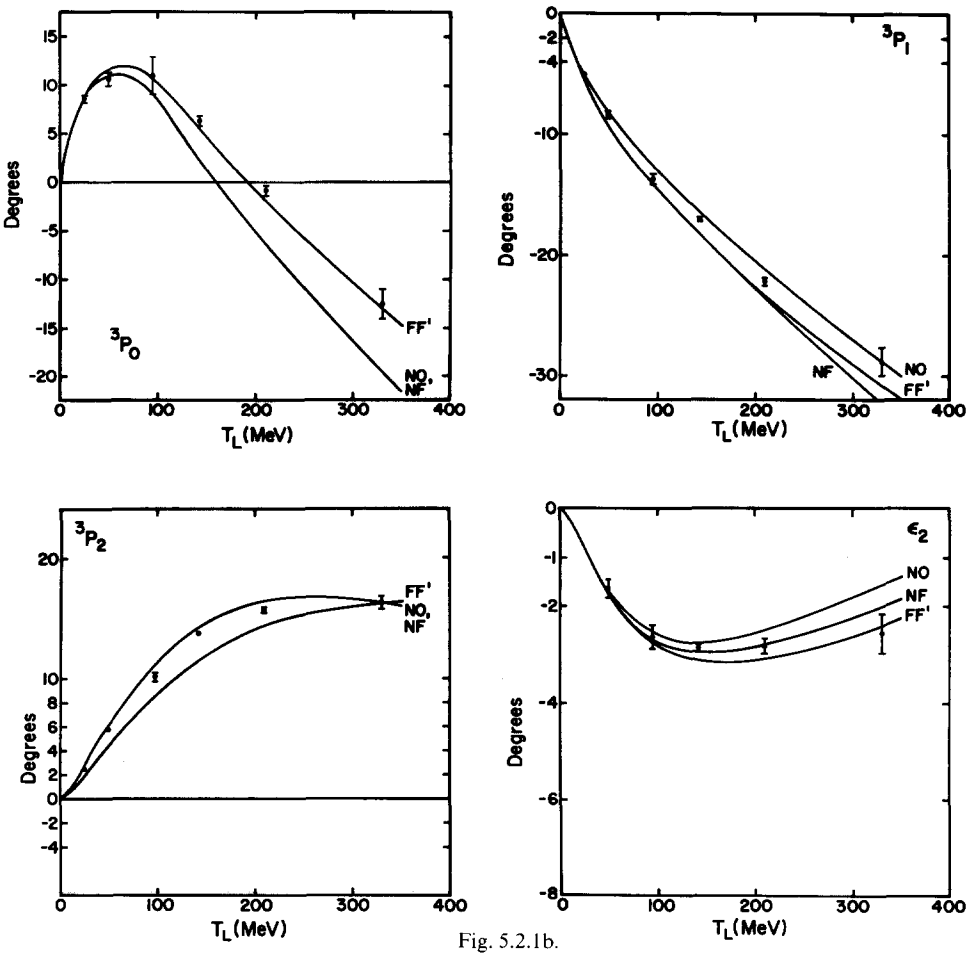

Fig. 5.2.1b.

ventional values of $g_\rho^2/4\pi = 0.52$ and $\kappa_v = 3.71$†. (Here we have used the notation of eq. (2.2.7).) We therefore draw the conclusion that the NO results overestimate both the f_+^0 and the $f_\pm^1$ amplitudes. This conclusion is indirectly supported by the fact that most other evaluations of these amplitudes have led to results of smaller magnitude [5, 32, 33]).

In fig. 5.2.2 we have plotted the squared magnitude of the amplitude λ_0^+ defined in eq. (4.2.2) for both the NO result and the result of Brown and Durso [34]), and in

† The $\rho\gamma$ coupling constant may be determined from the experimental partial widths for the (e^+e^-) and $(\mu^+\mu^-)$ decays of the ρ-meson. The contribution of the ρ-meson to the nucleon electromagnetic form factor of the nucleon is proportional to the product of the known $\rho\gamma$ coupling constant and the ρNN coupling constant which we wish to determine. With the assumption of vector meson dominance, this product equals one. Using 1965 vintage data for the leptonic decays of the ρ-meson, one obtains the familiar value of $g_\rho^2 = 0.52$. More recent data gives 0.43 ± 0.10. The parameter κ_v is determined, again assuming vector meson dominance, from the nucleon isovector anomalous magnetic moment.

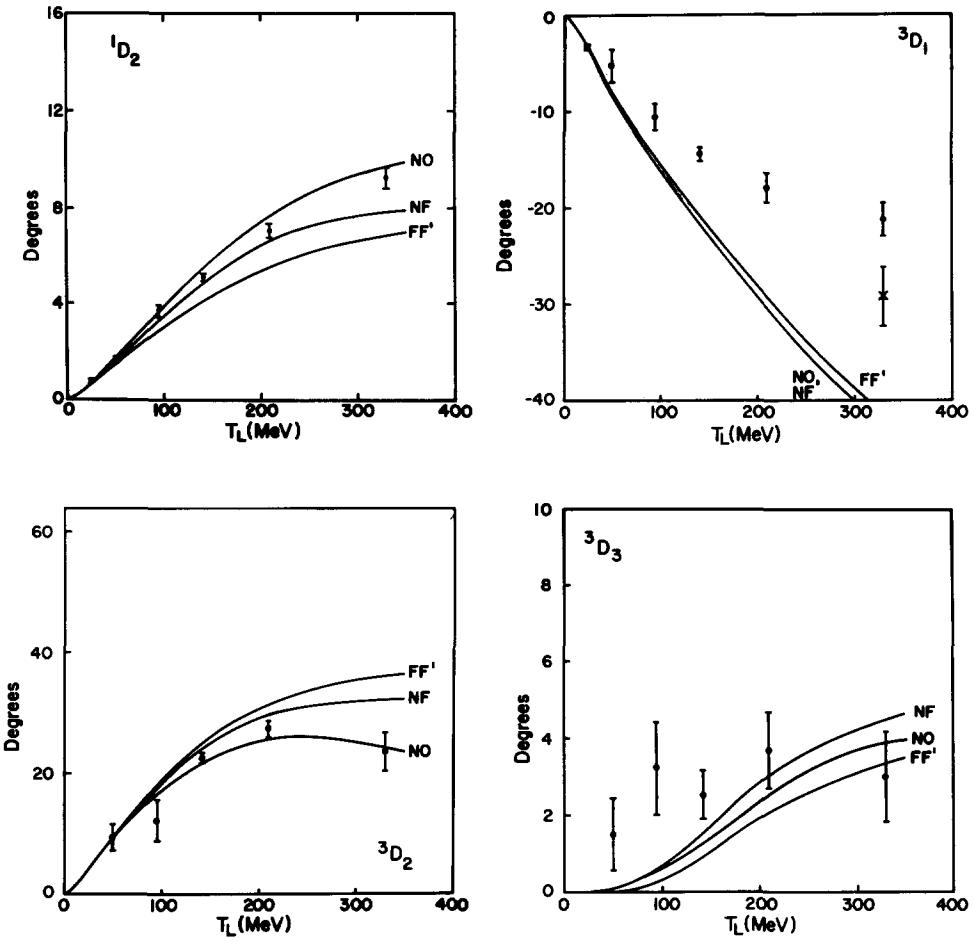

Fig. 5.2.1c.

fig. 5.2.3 we have plotted the combinations λ and η of the amplitudes $f^1_\pm$ defined in eq. (4.2.2) for the NO results. In view of the large discrepancies between these results obtained from πN scattering data, we feel entitled to construct models for these amplitudes within these limits which lead to better agreement with NN scattering data.

To obtain a model for the $f^1_\pm$ amplitudes in which the ρ-meson is coupled to the nucleon with coupling constants closer to experimental values, we have used a simpler model comprised of a nucleon pole term and a Breit-Wigner form describing a ρ-meson resonance of width 125 MeV [refs. [2,29,31])]. The explicit forms for the amplitudes $f^1_\pm$ in this model are given in sect. 6 of ref. [2])†. In addition to terms corre-

† In applying the results of ref. [2]) it is useful to note the following definitions which are not stated explicitly there: $t_r = (m_\rho^2 - \frac{1}{4}\Gamma_\rho^2)$, $\gamma q_r^3 = m_\rho \Gamma_\rho$, $N_1 = -4\kappa_v g^2_{\rho NN}/m$, and $N_2 = 4g^2_{\rho NN}(1+\kappa_v)$ where Γ_ρ is the width of the ρ-meson.

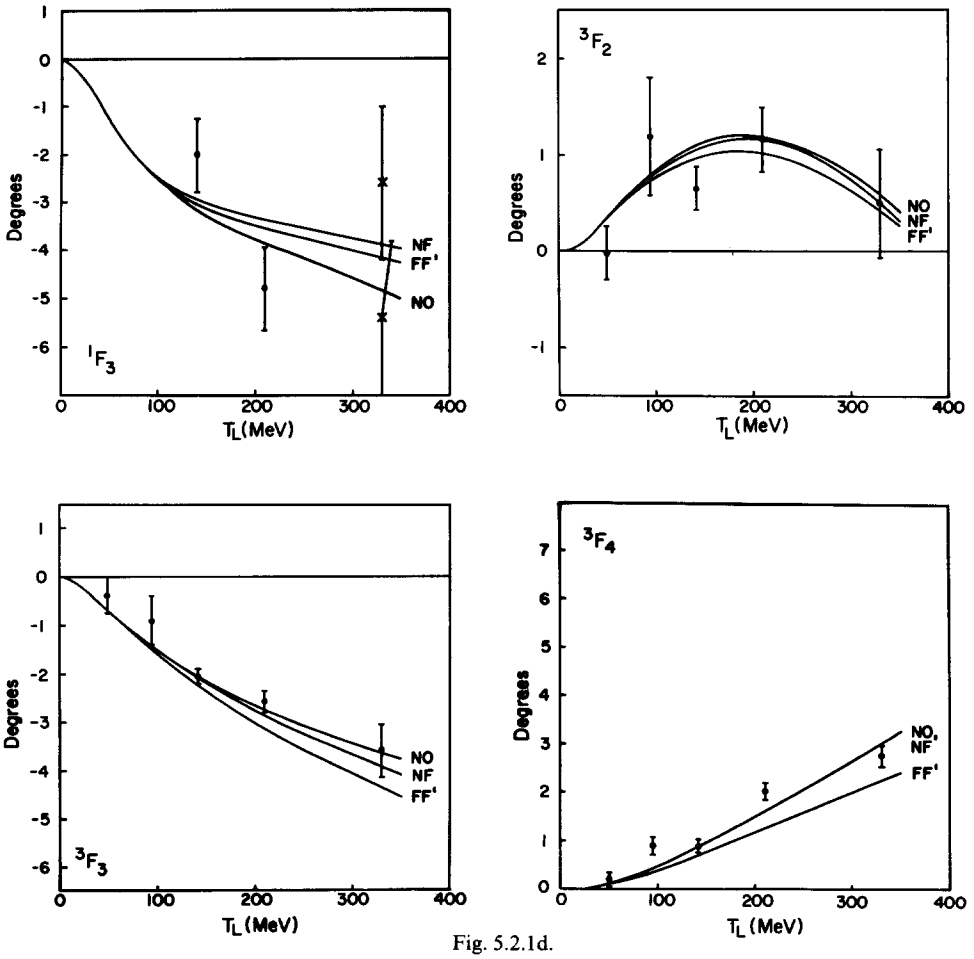

Fig. 5.2.1d.

Fig. 5.2.1. Nucleon-nucleon phase shifts obtained by solving the BBSE with several interaction models. The curves NO correspond to using the Nielsen-Oades [33]) results for f_+^0 and $f_\pm^1$ in the TPE contribution and $g_\omega^2/4\pi = 13.0$. The curves NF represent the result when the NO model for f_+^0 and the F_1 model for $f_\pm^1$ are used and $g_\omega^2/4\pi = 13.0$. The curves FF represent the result obtained with the model F_0 for f_+^0 and F_1 for $f_\pm^1$ and $g_\omega^2/4\pi = 7.0$. The curves FF′ stand for the result obtained with the models F_0 and F_1' and $g_\omega^2/4\pi = 6.24$. The phase shift "data" are as labeled in fig. 5.2.1.

sponding to the uncorrelated exchange of two pions and to the exchange of a ρ-meson of distributed mass, this model includes the effects of the "decay in flight" of the ρ-meson into two pions which are absorbed by the second nucleon. The evaluation of this process demands knowledge of the $\rho\pi\pi$ coupling which, following the postulate of universality, is set equal to $2g_{\rho NN}$. In this picture the width of the ρ-meson (which we determine empirically) should also be related to $g_{\rho\pi\pi}$. Thus, in the limit of zero resonance width, the contribution of such "decay in flight" processes to the NN

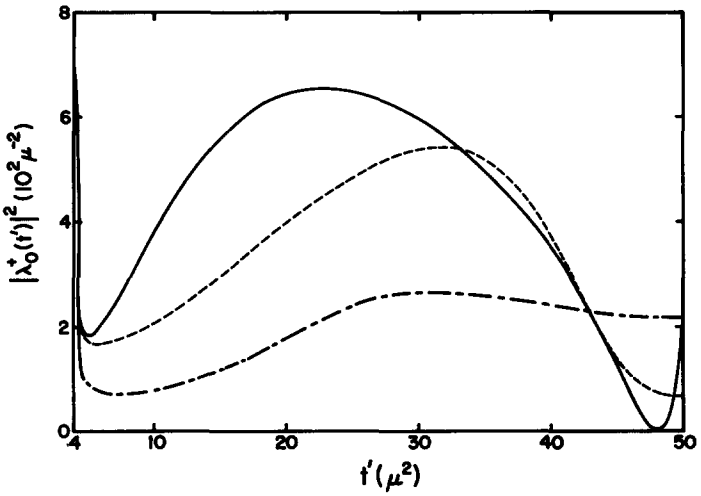

Fig. 5.2.2. The magnitude of λ_0^+ in several models: (i) Nielsen-Oades [33]) (solid curve), (ii) Brown-Durso [34]) (dot-dash curve), and (iii) the model F_0 constructed for the purpose of obtaining a good NN potential (dashed curve).

interaction will vanish. In fig. 5.2.3 we include these model amplitudes designated F_1 calculated with $g_\rho^2/4\pi = 0.52$ and $\kappa_v = 3.7$. This change in the $\pi\pi$ P-wave amplitudes is not a large one and is within the uncertainty in the Nielsen-Oades amplitudes estimated on the basis of the experimental uncertainties in the physical πN scattering amplitude.

We emphasize that, although physically appealing, this model is used only to provide reasonable variations in $f_\pm^1$ within the limits established by the analysis of πN scattering data. It is nonetheless satisfying to note that the fit to experimental NN phase shifts is greatly improved by replacing the NO $f_\pm^1$ amplitudes by the amplitude model F_1 obtained with physically reasonable values of the ρ-meson coupling constants. Phase shifts calculations using the model F_1, the NO amplitude f_+^0, and the same anomalously large value $g_\omega^2/4\pi = 13.0$ are shown in fig. 5.2.1. The improvement in the agreement between calculated and phenomenological phase shifts due to this replacement is dramatic and supports the conclusion that the NO amplitudes overestimate the ρ-meson contribution.

In order to reduce the excessive intermediate range attraction in the NN interaction implied by the NO result for f_+^0 and best demonstrated by the very steep slope of the $^1S_0$ phase shift, we have made a model for f_+^0, denoted F_0, which is in agreement with the NO result both at threshold and for large values of t, but which has a smaller magnitude in the intermediate region $5\mu^2 \leqq t \leqq 30\mu^2$. The model F^0 for f_+^0 is also shown in fig. 5.2.2. This change in the $\pi\pi$ S-wave amplitude is a significant one, and the amplitude F_0 does not lie within the uncertainty in the NO amplitude estimated on the basis of the experimental uncertainty in the physical πN amplitude only. This

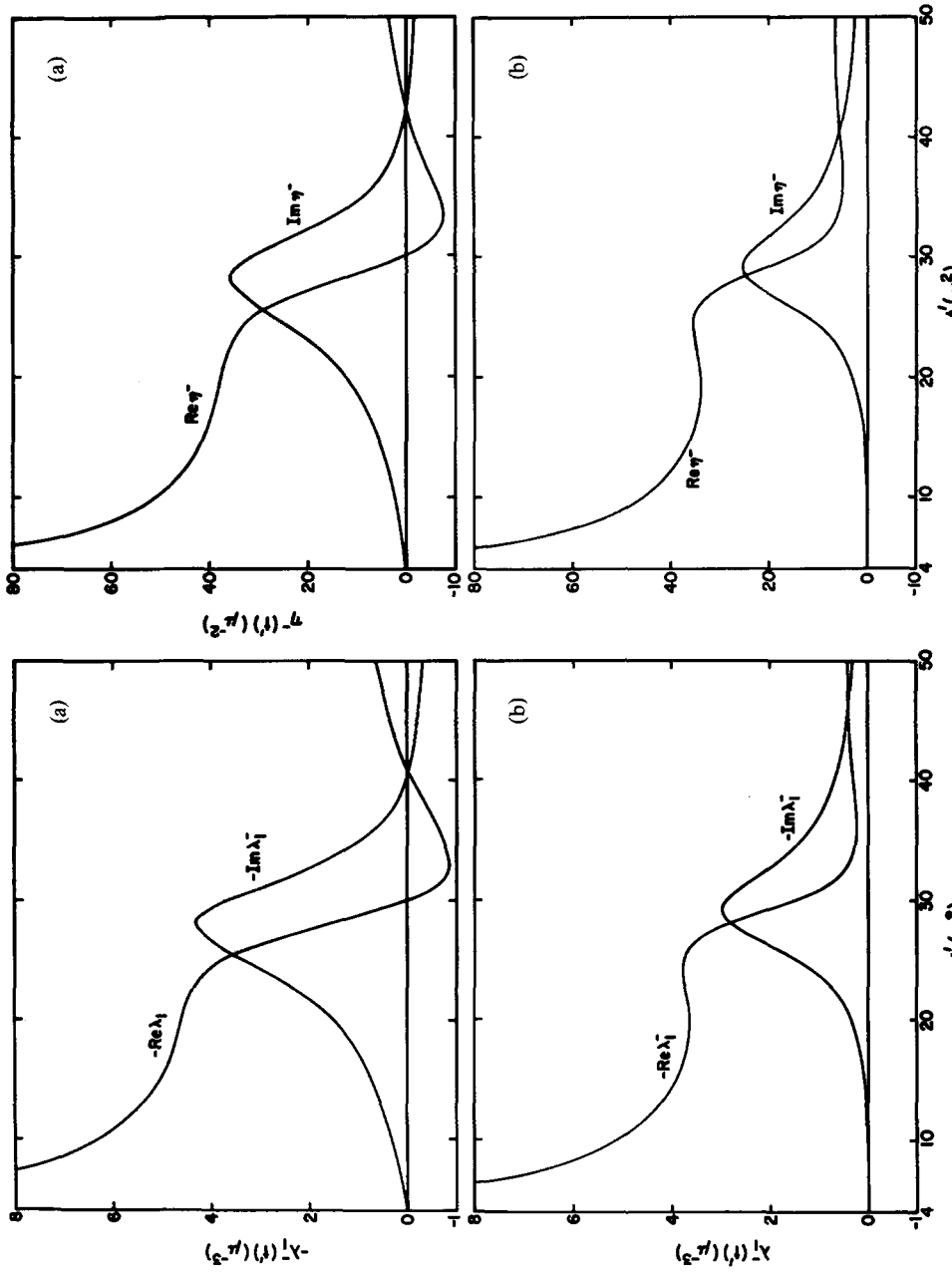

Fig. 5.2.3. The P-wave amplitudes λ and η for $N\bar{N} \to \pi\pi$: (a) The Nielsen-Oades result and (b) nucleon pole terms + Breit-Wigner term describing the ρ-meson resonance with $a_1^2/4\pi = 0.52$.

disagreement is not necessarily alarming since there are other sources of uncertainty in the NO amplitudes. (E.g., the sensitivity of f_+^0 to the form chosen for extrapolating the πN amplitude.)†

In fig. 5.2.1 we also show the S-wave phase shifts obtained with the interaction containing OPE, TPE with the model amplitudes F_0 and F_1 and ω-exchange. In this case the reduced intermediate range attraction in the amplitude F_0 permits the use of the smaller, and more physically reasonable, ωN coupling constant $g_\omega^2/4\pi = 7.0$. With this model we obtain quite satisfactory agreement with phenomenological phase shifts in essentially all partial wave channels including S-waves. (We shall return to the problem of the fairly large discrepancy in the $^3D_1$ phase shifts where the calculated phase shifts are consistently more repulsive than empirical phase shifts independent of the model of the TPE interaction.) In particular, we note that they are improved by replacing the NO result for f_+^0 by our model F_0. We list the effective range parameters obtained with the two interaction models NO and F_0F_1 in table 5.2.1. The values for the effective ranges are in good agreement with empirical values in all cases. The scattering lengths, hich are very sensitive functions of the potential parameters in the case when the T-matrix has a pole close to zero energy, are not well reproduced by either model. In order to study the deuteron wave function it is essential that the potential model lead to the correct scattering length and hence the correct asymptotic form of the bound state wave function.

For this purpose we have thus modified the F_1 model for the helicity amplitudes $f_\pm^1$ by using the ρ-meson coupling constants $g_\rho^2/4\pi = 0.594$ and $\kappa_v = 3.24$. These ρN coupling constants were chosen to leave the amplitude λ unchanged. In calculating phase shifts with this F_0F_1' model of the S- and P-wave $N\bar{N} \to \pi\pi$ helicity amplitudes, the ωN coupling constant was taken to be $g_\omega^2/4\pi = 6.24$. The resulting phase shifts are again shown in fig. 5.2.1 where it is seen that the calculated phase shifts differ

TABLE 5.2.1

Nucleon-nucleon effective range parameters from data analysis and potentials models

		a_s (fm)	r_{0s} (fm)	a_t (fm)	r_{0t} (fm)
MAW X[a]):	pp	−7.815	2.795		
	np	−23.679	2.51	5.397	1.727
Potential models:	F_0F_1	−10.5	2.64	9.76	1.87
	NF_1	−33.0	2.49	9.24	1.96
	F_0F_1'	−22.9	2.47	5.58	1.62
	NO	18.8	2.14	−5.85	1.72

[a]) Ref. [36]).

† Following completion of this manuscript we were informed by H. Nielsen of a more recent extraction of the $\pi\pi \to N\bar{N}$ amplitudes. The new results indicate only slight changes in $f_\pm^1$ from the previous NO results shown in fig. 5.2.3 but substantial changes from the NO f_+^0 amplitude shown in fig. 5.2.2. Our constructed amplitude F_0 is in markedly better agreement with this new f_+^0 than with the previous NO values shown in fig. 5.2.2.

only slightly in most partial wave channels from those obtained with the model F_0F_1. The only significant difference is in the S-waves where agreement with empirical np scattering lengths has been enforced (see table 5.2.1) at the cost of a small increase in the $^3S_1$ phase shift at higher energies.

We have solved the LSE obtained from the BBSE by the substitutions eqs. (2.1.6) and (2.1.7) to obtain the deuteron wave function with the potential F_0F_1'. In fig. 5.2.4

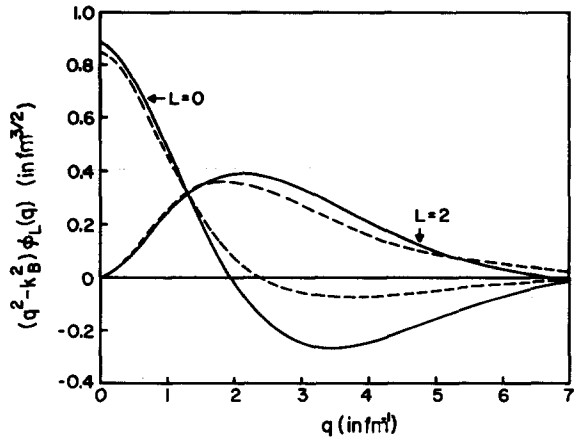

Fig. 5.2.4. The residue of the deuteron wave functions, $(E_B-q^2)\langle q|\psi_d\rangle$. The solid curves correspond to the results obtained with the Reid soft core potential [12]) and the dashed curves are the results obtained with the potential model FF'.

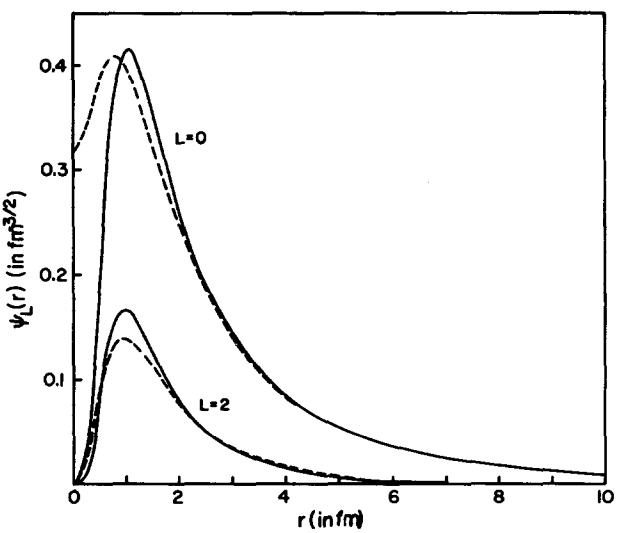

Fig. 5.2.5. Deuteron wave functions as functions of nucleon separation. The solid curves are the wave functions obtained with the Reid soft-core potential [12]) and the dashed curves are the results obtained with the potential model FF'.

we plot the quantity $\langle q|V|\psi_d\rangle$ which is related to the deuteron wave function in momentum space by eq. (3.2.4). For comparison we also show the corresponding quantity obtained with the Reid soft-core potential [12]). The deuteron binding energy obtained with the model F_0F_1' is 2.227 MeV. The good agreement with the experimental value is, of course, expected since this interaction reproduces the scattering length and effective range in the $^3S_1$ channel by construction. The calculated electric quadrupole moment is 3.09 mb in quite good agreement with the recent experimental value of 2.86 mb [ref. [39])]. Although the electric quadrupole moment may be related using effective range arguments to the behaviour of ε_1 near threshold, this mixing parameter is particularly poorly determined at low energies and, thus, this agreement represents a real test of the interaction model. The D-state probability of this wave functions is 6.17 % which may be compared with 6.43 % for the Reid soft-core potential. In fig. 5.2.5 we have plotted the same deuteron wave functions in a configuration space representation as a function of internucleon separation using eq. (3.2.6). The most striking difference in this figure is the weaker short-range repulsion in the interaction model F_0F_1' indicated by the short distance behaviour of the S-state wave functions. This relatively weak short-range repulsion is a general feature of our approach and is due to the fact that we have insisted on the use of vector meson-nucleon coupling constants which are consistent with data, regarding the leptonic decays of the vector mesons and the notions of vector meson dominance. This is not a general feature of OBE models of the NN interaction. For example, Erkelenz *et al.* [40]) use an isoscalar coupling constant of $(g_\omega^2+g_\phi^2)/4\pi = 57.2$ which is approximately nine times larger than the values adopted in this work. The strength of the ρN coupling constant is similarly increased in their work. We shall discuss this point in somewhat greater detail in the following section.

Finally, we have computed those matrix elements of the deuteron wave function which would correspond to the charge and quadrupole form factors of the deuteron in the absence of mesonic corrections and with the assumption of point nucleons. (The neglect of mesonic corrections is roughly consistent with and closely related to the neglect of terms $\langle\psi_d|\delta V/\delta E|\psi_d\rangle$ in the normalization condition for the bound state wave function.) In this case we define

$$F_C(q^2) = \int_0^\infty \mathrm{d}r\, r^2[\psi_S^2(r)+\psi_D^2(r)]j_0(\tfrac{1}{2}qr), \tag{5.2.1}$$

$$F_Q(q^2) = \int_0^\infty \mathrm{d}r\, r^2[2\psi_S(r)\psi_D(r)-\sqrt{\tfrac{1}{2}}\psi_D^2(r)]j_2(\tfrac{1}{2}qr), \tag{5.2.2}$$

in terms of the S- and D-state radial wave functions. In practice, the integrals in eqs. (5.2.1) and (5.2.2) are numerically unwieldy, and it is more convenient to calculate these form factors in momentum space using the inverse of eq. (3.2.6). The resulting integral over three spherical Bessel functions is readily performed and leaves integrals in momentum space with non-oscillating integrands which are

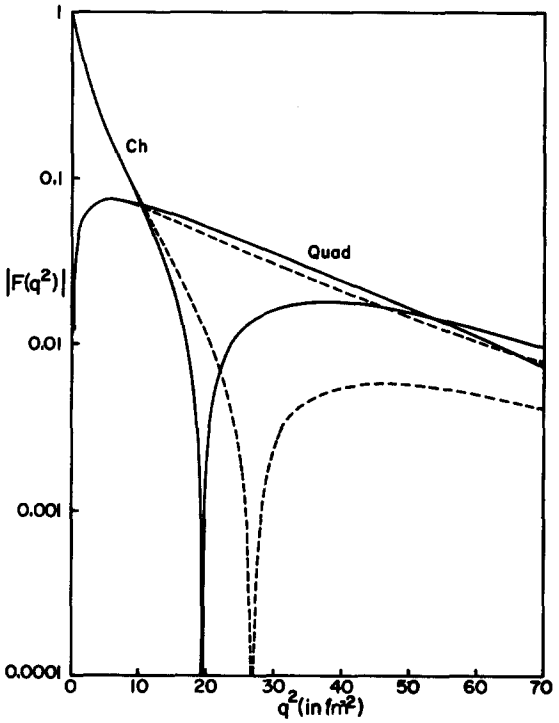

Fig. 5.2.6. Deuteron charge (Ch) and quadrupole (Quad) form factors calculated without nucleon form factors or mesonic corrections. The solid curves are the results obtained with the Reid soft-core potential [12]) and the dashed curves the results obtained with the potential model *FF'*.

numerically convenient [41]). The results of such a calculation are shown in fig. 5.2.6 along with the corresponding results for the Reid deuteron wave function. As would be expected from fig. 5.2.5 and eqs. (5.2.1) and (5.2.2), the greatest difference is found in $F_C(q^2)$ for relatively large momentum transfers. Unfortunately, in the absence of deuteron polarization measurements, elastic electron-deuteron scattering yields only the deuteron electric form factor

$$F_E^2(q^2) = F_C^2(q^2) + F_Q^2(q^2),$$

which is much less sensitive to these interesting differences in the charge form factor for momentum transfers $20\ \text{fm}^{-2} < q^2 < 30\ \text{fm}^{-2}$.

We find the results for elastic NN scattering obtained with the interaction model $F_0 F_1'$ to be sufficiently encouraging to suggest its use as a meaningful approximation to the NN interaction. This model has some limitations at energies near the pion production threshold because of the neglect of the πN rescattering contributions in the spectral functions d_j (4.1.4) in the TPE contribution. Since rescattering effects give rise to highly energy-dependent terms in the TPE potential near and above the

pion production threshold, it is possible that our weakly energy-dependent potential model will not be satisfactory in that region.

We have also computed the binding energy per particle of nuclear matter for the interaction F_0F_1' under the assumption that the "potential" of eq. (2.1.7) which is designed for the calculation of phase shifts using the LSE is also appropriate for use in a normal (non-relativistic) Brueckner reaction matrix calculation of the nuclear matter binding energy. Thus, we have solved the reaction matrix equation with an angle-averaged Pauli operator using single particle energies determined self-consistently using an effective mass approximation. Throughout this calculation the energy-dependence of the interaction was artificially suppressed and s was fixed at $4m^2$. No higher-order effects (e.g., three-body clusters) were included. The resulting maximum nuclear matter binding energy of 8.4 MeV per particle at $k_F = 1.38$ fm^{-1} is roughly consistent with the value of 9.9 MeV per particle at $k_F = 1.36$ fm^{-1} obtained in an essentially equivalent calculation of Haftel and Tabakin [42]) using the Reid soft-core potential. The close agreement masks the fact that, in our calculation, low partial waves contribute somewhat greater attraction and high partial waves contribute greater repulsion to the net binding energy. These differences probably have the origins directly in systematic differences in calculated phase shifts and not in the vagaries of the interaction off-shell. It is perhaps of greater interest to note that the total "wound" is approximately half that obtained with the Reid potential (i.e., $K = 0.082$ for $k_F = 1.5$ fm^{-1} and $k_0 = 0.9$ fm^{-1} in the notation of ref. [42])). This is another manifestation of the relatively weak short-range repulsion in our model of the interaction. Thus, our interaction is not consistent with the frequently observed proportionality between nuclear matter binding energy and the total wound [43]).

5.3. DISCUSSION OF OTHER INPUT PARAMETERS

It is important to separate the essential dynamic features of the models of the NN interaction discussed in the previous section from those parameters in the interaction which do not permit independent experimental verification. Essential parameters include the meson-nucleon coupling constants and the $N\bar{N} \to \pi\pi$ reaction amplitudes which may, at least in principle, be determined independent of NN phase shifts. Also, in our approach, the strong form factors needed to obtain a well-behaved interaction have a strong but somewhat model-dependent connection to nucleon electromagnetic form factors. More arbitrary "parameters" include the cut-off value in the dispersion integral eq. (4.1.2), the value used for γ in the eikonal cut-off function eq. (3.3.2) and, indeed, the functional form of this cut-off, and the on-shell assumptions made in the construction of the TPE interaction. It is only to the extent that calculated phase shifts are relatively insensitive to these otherwise inaccessible parameters that the present work represents anything more than very complicated phenomenology. Thus, in this section we shall study the sensitivity of

the results obtained in the previous section to these more arbitrary ingredients in the interaction.

We first study the cut-off value in the dispersion integral in eqs. (4.1.2) and (4.1.3). In the previous models we used a cut-off value of $50\mu^2$, mainly motivated by the fact that no results for the helicity amplitudes f_+^0 and $f_\pm^0$ are available at higher t-values. The NN scattering phase shift results have been obtained for the potential model F_0F_1' with the cut-off chosen at $40\mu^2$ and also at $60\mu^2$ instead of $50\mu^2$. To obtain the results at $60\mu^2$ we simply extrapolated the results for the helicity amplitudes smoothly. At 150 MeV the resulting $^1S_0$ phase shifts were about 4° more repulsive for the cut-off at $40\mu^2$ and 2.5° more attractive for the cut-off at $60\mu^2$ relative to the results for the cut-off at $50\mu^2$. The $^3S_1$ phase shifts at the same energy were 2° more attractive and 1° more repulsive for the cut-off at $40\mu^2$ and $60\mu^2$ respectively relative to the results for $50\mu^2$. These changes in the phase shifts were approximately uniform shifts which were independent of the energy. All other phase shifts are practically unchanged. This is a very satisfactory result. The stability of the P-wave phase shifts shows that the model for intermediate range interaction (i.e., the part of the interaction one hopes to explain in terms of simple meson exchange mechanisms) is not sensitive to the particular choice of cut-off value. Although it is clear that S-wave phase shifts can depend on the nature of the interaction at short distances (e.g., $t > 50\mu^2$), it is satisfying to note that the degree of sensitivity is not large and may, in fact, be compensated by a variation of $g_\omega^2/4\pi$ within its experimental uncertainty.

The parameter γ which determines the shape of the eikonal cut-off factor (3.3.2) was chosen to be 1.25 on the basis of a recent fit with a modified vector dominance model to the nucleon electromagnetic form factors [12]). An equivalent fit to the electromagnetic form factors was obtained in ref. [12]) in a slightly different model in which γ was chosen to be 1.4. In order to establish the relative insensitivity of our results to the parameter γ we show in fig. 5.3.1 the NN phase shifts obtained with the interaction model F_0F_1' modified only by the change of γ from 1.25 to 1.4. The change in S-wave phase shifts, shown in the figure, is slight and again may be compensated by small changes in the vector meson coupling constants. Changes in higher partial wave phase shifts are virtually undetectable.

As a measure of the sensitivity of the results to the functional form of the cut-off, we consider the replacement of the eikonal form, $\exp[i\chi(t)-i\chi(\mu_B^2)]$, by a monopole form, $[(\Lambda^2-\mu_B^2)/(\Lambda^2-t)]$. The value of Λ is again established by a fit to nucleon electromagnetic form factors as $\Lambda = 1.54\ m$. This leads to the replacement of eq. (3.3.4) by

$$V_R(\boldsymbol{p}', \boldsymbol{p}) = \left(\frac{\Lambda-m_v^2}{\Lambda^2-t}\right)^2 \left(\frac{\Lambda^2-4m^2+m_v^2+s}{\Lambda^2-u}\right)^2 V_0(\boldsymbol{p}', \boldsymbol{p}). \tag{5.3.1}$$

This "dipole" form of the cut-off is a familiar ingredient in more *ad hoc* treatments (see, for example, ref. [40])) where Λ is usually constrained only as being several

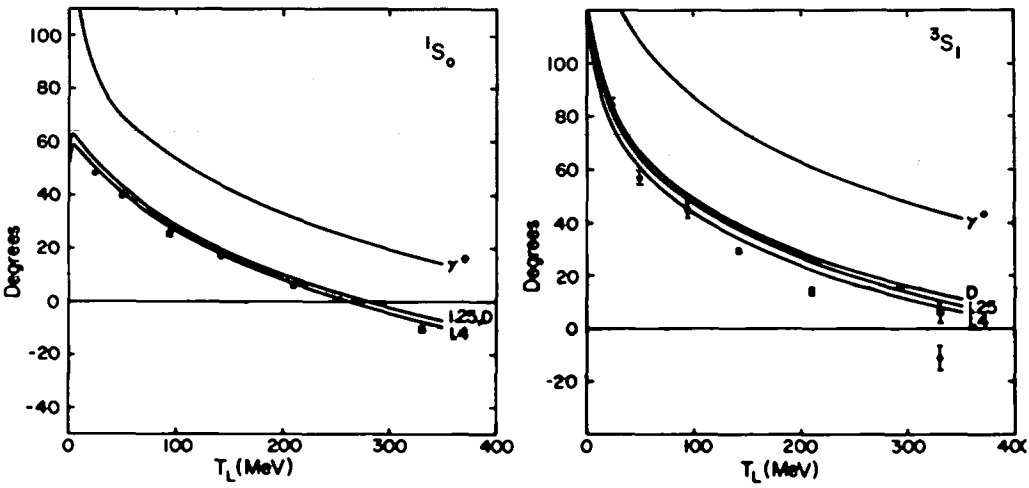

Fig. 5.3.1. Sensitivity of the phase shift results to the parameter γ in the eikonal cut-off function ($\gamma = 1.25$ and $\gamma = 1.4$). The curves D stand for the results when a dipole type regularization factor is used instead of the eikonal damping factor. The curves γ^* correspond to the replacement $m \to 2m$ in eq. (3.3.3) and is the only case which is in gross disagreement with nucleon electromagnetic form factors. All other parameters are those of the model *FF'*.

nucleon masses. In many OBE models sufficiently strong adiabatic approximations are enforced in a manner which makes u-channel cut-offs unnecessary, and such cut-offs are generally not included. In fig. 5.3.2 we show NN phase shifts obtained with the model F_0F_1' regularized by the dipole cut-off, eq. (5.3.1), rather than the eikonal cut-off, eq. (3.3.3). We see that phase shifts are not sensitive to the functional form of the cut-off providing that the form used is in satisfactory agreement with nucleon electromagnetic form factor data.

As an indication of the large uncertainties in phase shifts which can result when the cut-off factor is freed from the very strong constraint of fitting nucleon electromagnetic form factors, we make the arbitrary replacement $m \to 2m$ in eq. (3.3.2). From the asymptotic behaviour of eq. (3.3.2) for large negative t,

$$\exp\left[i\chi(t)\right] \sim (-t/4m^2)^{-\gamma}, \tag{5.3.2}$$

we see that this replacement yields a slower damping in t without changing the functional form of the cut-off. Phase shifts calculated with the interaction F_0F_1' and this replacement are shown in fig. 5.3.2. In this case the effects on phase shifts are significant and would require a substantial increase in vector meson coupling constants in order to restore agreement with experiment (e.g., $g_\omega^2/4\pi \approx 11.5$). This sensitivity emphasizes the importance of the constraint provided by the empirical nucleon electromagnetic form factors. The fact that many OBE models employ "conservative" cut-offs which damp the amplitude more slowly than electromagnetic form factors would suggest can thus result in the need for vector meson coupling

constants which are too large. [For example, the OBE interaction of ref. [17]) employs a dipole cut-off of the form of eq. (5.3.1) with $\Lambda \approx 2m$ which is somewhat larger than the value of 1.54m needed to describe electromagnetic form factors. The common neglect of u-channel cut-offs also reduces the rate of damping.]

Let us now turn to the question of the validity of the on-shell assumptions made in constructing the TPE potential from the field theoretical scattering amplitude. We have made two on-shell assumptions, the first being the assumption that the BBSE potential can be expressed in terms of only five spin invariants (P_j), and the second being the assumption that the external particles can be taken to be on the mass shell in deriving the analytic expressions for the scalar amplitudes p_j defined in eq. (2.2.1).

The first of these assumptions is implicit in using the dispersion theory framework for relating the TPE contribution to the NN scattering amplitude to the $N\bar{N} \to \pi\pi$ reaction amplitudes. The on-shell assumption allows the use of the Dirac equation external particles to reduce the very complicated spin structure of the TPE amplitude. By means of the Dirac equation the TPE amplitude can be expressed in terms of any set of five linearly independent spin amplitudes equivalent on-shell, but having different off-shell extrapolations. There thus remains an ambiguity in the TPE potential. This ambiguity turns out to be relevant only in the lowest partial waves reflecting the fact that only off-diagonal potential matrix elements can have different forms. To illustrate the numerical importance of this ambiguity we have used an alternative off-shell reduction of the p_j amplitudes to helicity components by first expressing the P-invariants in terms of the non-invariant spin amplitudes Q_j introduced in ref. [2]):

$$\begin{aligned} Q_1 &= 1^p 1^n, \qquad Q_2 = \gamma_4^p \gamma_4^n, \qquad Q_3 = \gamma_4^p + \gamma_4^n, \\ Q_4 &= \gamma^p \cdot \gamma^n, \qquad Q_5 = \gamma_5^p \gamma_5^n. \end{aligned} \tag{5.3.1}$$

To express the P-amplitudes in terms of the Q-amplitudes the Dirac equation has to be used, and these amplitudes therefore have inequivalent off-shell behaviour. In fig. 5.3.2 we compare the NN phase shifts obtained with the potential model $F_0 F_1'$ as obtained with the previous off-shell extrapolation (from the amplitudes P) with the same phase shifts obtained with the off-shell extrapolation from the amplitudes Q. It can be seen that the only phase shift which is appreciably different for the two off-shell extrapolations is the $^3S_1$ phase shift. We thus conclude that this off-shell ambiguity in the potential is not very serious, does not affect the results in a qualitative way, and, in fact, in most cases has no quantitative significance.

The second assumption that the external particles can be taken to have physical four momenta in the derivation of the analytic expressions for the scalar amplitudes p_j is less trivial. Clearly the particles must be allowed unphysical four momenta in the potential. The problem is however that if the external four momentum has a large magnitude – corresponding to a large effective mass – the analytic structure of TPE

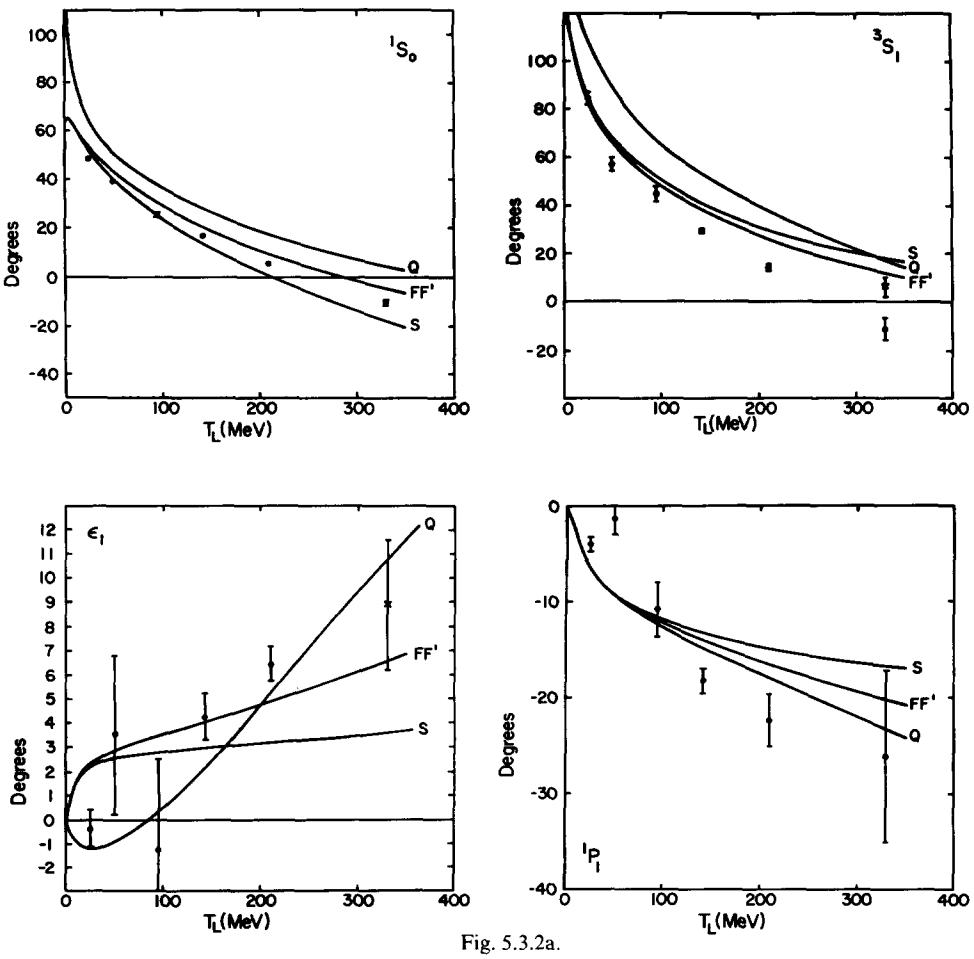

Fig. 5.3.2a.

amplitudes changes – at least in fourth-order perturbation theory – and the double spectral representation breaks down [45]). In that case it is not clear that the potential interpretation of the amplitude has much meaning, since the simple cut structure in the variables s and t is essential for the interpretation of the potential [46]). Note however that in the case of the adiabatic limit ($s = 4m^2$) the double spectral representation is valid for all off-shell four momenta occurring in the intermediate states in the BBSE framework.

The energy dependence of the potential. The meson theoretical NN potential is explicitly energy dependent, since it is constructed from the energy-dependent scattering amplitude. This energy dependence enters partly through the spin factor structure and partly through the presence of distant cuts along the real s-axis in the amplitude. The energy dependence of the single meson exchange potential (e.g.,

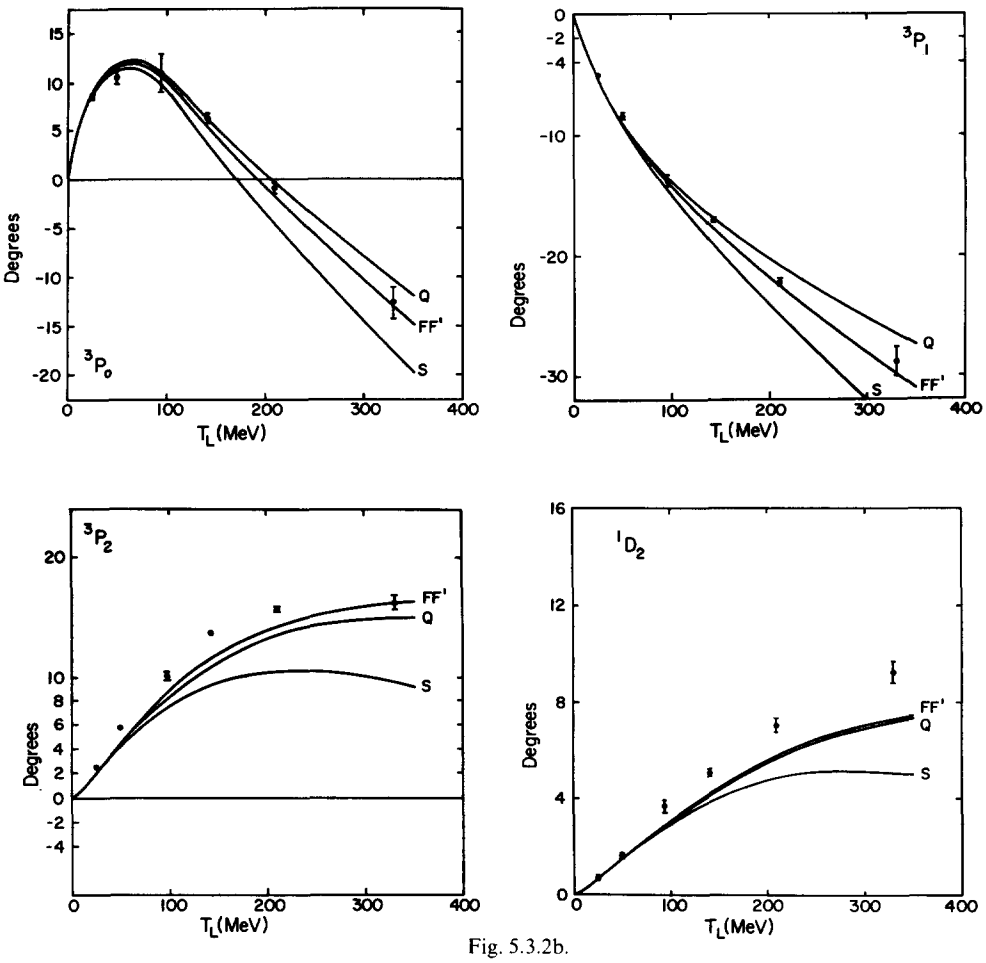

Fig. 5.3.2b.

Fig. 5.3.2. Nucleon-nucleon phase shift results obtained with the full potential model *FF'* (curves FF') compared with the results where all energy dependencies are set at $s = 4m^2$ (curves S). The curves Q show the results with the *FF'* model if a different off-shell extra-polation of the potential is used (i.e. using the Q instead of the P spin amplitudes to describe the potential).

ω-meson exchange) is purely due to the spin structure of the amplitude which leads to residue functions of the variable $s-u$ at the t-channel meson pole (c.f. eq. (2.3.4)). The TPE potential also contains explicit energy dependence in the spectral functions ρ_j and q_j defined in eqs. (4.1.2) and (4.1.3). The explicit energy dependence of the potential is relatively weak. It is either polynomial in nature or due to distant cuts [2]). This fact has justified the consideration of only the adiabatic limit of the potential in refs. [1-5]). We find that this limit is adequate for P and higher partial waves but that it may be somewhat misleading for the S-wave NN interaction. In fig. 5.3.2 we compare the results for the NN phase shifts obtained with the adiabatic limit ($s = 4m^2$)

of the potential model F_0F_1' and the full energy-dependent potential. The use of the adiabatic limit increases the slope of the $^1S_0$ and decreases the slope of the $^3S_1$ phase shift while leaving most other phase shifts practically unchanged. In taking the adiabatic limit we do not set $s = 4m^2$ in the eikonal regularization factor as this would lead to improper normalization of the damping factor. These results show that one should not employ the adiabatic limit of the potential in calculations aimed at quantitatively accurate results †.

6. Discussion and conclusions

Two main conclusions can be drawn on the basis of the results presented in the preceding section. First, it is possible to construct a field theoretic model of the NN interaction which is consistent with low energy scattering data and such that all essential parameters in the model are related to and consistent with the results of independent experiments. We note, however, that NN phase shifts are relatively sensitive to variations of the $\pi\pi$ S- and P-wave amplitudes within their quoted uncertainties. Second, whatever future, and more definite, results for the $N\bar{N} \to \pi\pi$ amplitude may be, we expect the general features of the NN interaction to be similar to the models discussed in this paper. Specifically, the interaction should be non-local and moderately energy dependent and should have short-range repulsion which is significantly weaker than that commonly found in one-boson-exchange and local phenomenological models of the NN interaction.

We have constructed one interaction model, labelled F_0F_1' in sect. 5, which describes deuteron observables adequately and yields NN phase shifts in fairly good agreement with the results of phenomenological analyses. The formalism and parameters necessary to compute the TPE interaction spectral functions ρ_j and q_j (defined in eqs. (4.1.2) and (4.1.3)) appearing in this model are given in ref. [2])††. The non-locality and energy dependence of the interaction make simple parametrization impractical.

It is clear that our interaction does not afford as good a fit to empirical phase shifts as the phenomenological Hamada-Johnston [6]) and Reid [13]) potentials or the semiphenomenological one-boson exchange potentials which contain large numbers of parameters [47, 48]). A phase shift fit good in the χ^2 sense was not a primary goal of the present work. Our interaction contains an absolute minimum of adjustable parameters (i.e., vector meson coupling constants, the $N\bar{N} \to \pi\pi$ amplitude in S-waves, and the parameter γ in the eikonal function) all of which are prohibited from assuming physically unreasonable values in an attempt to mock up the effects

† The moderate energy dependence of the interaction indicated by fig. 5.3.2 may be misleading. As noted by Vinh Mau (private communication), the net energy dependence probably arises as a result of a more marked increase in both short-range repulsion and intermediate-range attraction with increasing energy.

†† Matrix elements of this interaction in momentum space can be obtained from the authors upon request.

of other processes not included in the model. For example, πN rescattering effects in $\pi\pi$ partial waves with $L \geqq 2$ are missing in our model. While such effects are not expected to play a dominant rôle when included correctly, there is no reason to expect their contribution to be negligible. It is also worth noting that present NN phase shift analyses contain substantial ambiguities. The most striking of these ambiguities are the strongly correlated errors on ε_1 and $\delta(^1P_1)$ at low energy (which prohibit even the determination of the sign of ε_1 at low energies) [36]) and the existence of two distinct local χ^2 minima in the energy-independent data analysis at 330 MeV [ref. [37])]. (The phase shift ambiguity at 330 MeV is shown in the figures.) Such experimental ambiguities dampen our enthusiasm for a more detailed phase shift fit within the framework of the present model.

Let us now turn our attention to those πN rescattering effects in the TPE potential which have not been included in the present model. As noted in subsect. 4.2, the contribution of πN rescattering to the spectral functions d_j (defined in eq. (4.1.4)) is divergent in contrast to the convergent nucleon pole term contribution. If such contributions are evaluated either in terms of πN resonances or as an integral over the imaginary part of the physical πN scattering amplitude in the fixed-t dispersion relation for the $N\bar{N} \to \pi\pi$ amplitude, the cut-off value in the dispersion integral eq. (4.1.3) becomes a sensitive parameter. This is clearly undesirable. The source of this sensitivity is the divergence of the πN rescattering effects due to the unjustified extrapolation of the partial wave expansion of the imaginary part of the πN amplitude into the pseudophysical region for the $N\bar{N} \to \pi\pi$ channel ($t \geqq 4\mu^2$). The use of this extrapolation has been justified in refs. [1-3]) for weakly t-dependent terms (i.e., contributions from low partial waves for πN scattering). For purposes of considering NN scattering in high partial waves or the long range tail of the TPE potential this approach has been proven successful [2, 31]). Clearly, this method is not adequate for the description of NN scattering in low partial waves which is sensitive to the $N\bar{N} \to \pi\pi$ amplitudes for large positive t-values where the partial wave series is certainly invalid.

Similar divergences are encountered when these rescattering effects are described using nucleon isobars in intermediate states as indicated in fig. 6.1. To illustrate this point we make a model of the interaction in which $\pi\pi$ S- and P-waves are described by the helicity amplitudes F_0 and F_1 respectively. In addition to the nucleon pole terms in higher $\pi\pi$ partial waves we include the πN rescattering corrections of fig. 6.1 evaluated as in ref. [2]) with the three lightest nucleons isobars: Δ(1236 MeV) $J^\pi = (\frac{3}{2})^+$ $T = \frac{3}{2}$, N(1470 MeV) $J^\pi = (\frac{1}{2})^+$ $T = \frac{1}{2}$, and N(1518 MeV) $J^\pi = (\frac{3}{2})^-$ $T = \frac{1}{2}$. The rescattering effects were calculated in the sharp resonance approximation, and the NN*π coupling constants were obtained from the πN partial decay widths of the isobars. In this model $g_\omega^2/4\pi$ was set equal to 7.0. The high sensitivity of the resulting phase shifts (in low partial waves) to the choice of the cut-off in eq. (4.1.2) is shown in fig. 6.2 for the $^3P_0$ and $^3P_1$ channels. This figure indicates that the effects of πN rescattering cannot be calculated reliably within the present framework.

Formally, these divergences could be removed by the introduction of strong form factors at each of the pion vertices in fig. 6.1 in the spirit of subsect. 3.3. While mathematically adequate, this approach misses a crucially important physical effect: processes (such as those shown in fig. 6.3) in which one or both of the pions in fig. 6.1 is replaced by a ρ-meson have a strong tendency to cancel the TPE diagrams of fig. 6.1 at short and intermediate distances.

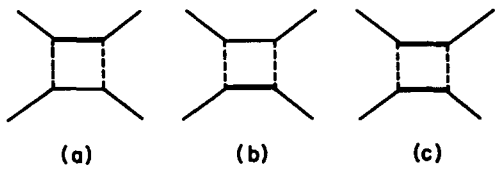

Fig. 6.1. Significant corrections to πN rescattering effects not included in the present model.

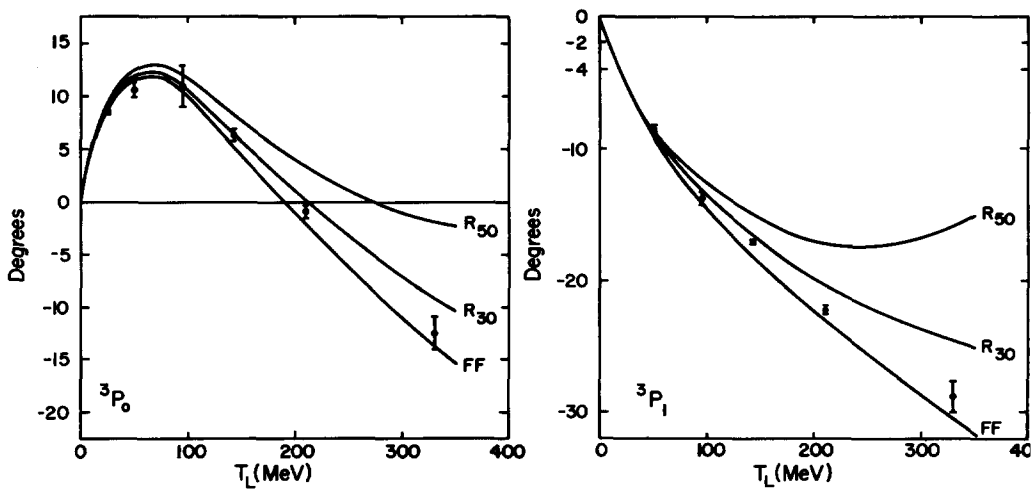

Fig. 6.2. Nucleon-nucleon phase shifts obtained with the potential model which includes the πN rescattering effects in the TPE contributions using the isobar model. The divergent nature of the rescattering effects is illustrated by the sensitivity of the results to the cut-off value in the dispersion integral. The curves FF represent the results obtained with the model *FF* without rescattering effects. The curves R_{30} and R_{50} represent the results obtained from the complete isobar model using cut-off values of $50\mu^2$ for the nucleon terms and $30\mu^2$ and $50\mu^2$ respectively for the isobar terms. Note that the phase shifts obtained with the cut-off at $30\mu^2$ provide probably the most reliable estimate of rescattering effects within the framework of the present model.

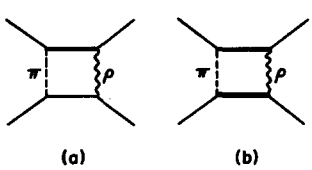

Fig. 6.3. Important corrections to the πN rescattering diagrams of fig. 6.1.

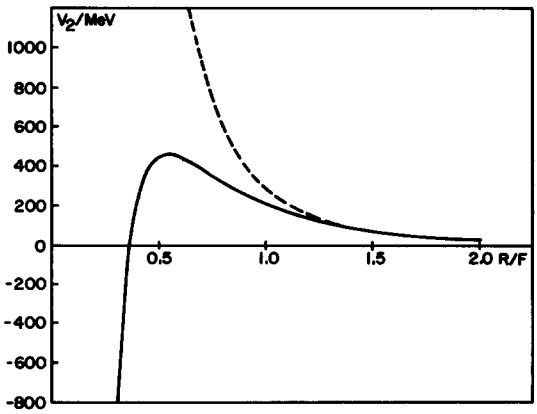

Fig. 6.4. The transition potential V_2 of ref. [50]) resulting from one-pion exchange (dashed curve) and one-pion plus rho exchange (solid curve).

Working within a coupled channels framework, Green and Haapakoski [49, 50]) have constructed local transition potentials which take a state of two nucleons into a state containing one nucleon and one $\Delta(1236)$ isobar. This transition potential includes both the effects of π- and ρ-meson exchange with $\pi N\Delta$ and $\rho N\Delta$ coupling constants taken form the quark model†. The dominant feature of the transition potential is its strong tensor component:

$$[3(\boldsymbol{\sigma}_N \cdot \hat{\boldsymbol{r}})(\boldsymbol{S}_\Delta \cdot \hat{\boldsymbol{r}}) - \boldsymbol{\sigma}_N \cdot \boldsymbol{S}_\Delta] V_2(r),$$

where $\boldsymbol{\sigma}_N$ and $\boldsymbol{S}_\Delta$ are nucleon and isobar spin operators. The potential $V_2(r)$ is shown in fig. 6.4 which also indicates the dramatic cancellation between the π- and ρ-meson exchange contributions. These results suggest that the inclusion of such ρ-meson exchange processes is imperative for a realistic description of the effects of πN rescattering. Clearly, their inclusion would greatly reduce the spectral functions in the vicinity of the ρ-mass and would thus reduce the sensitivity of calculated phase shifts to the details of any additional strong form factors needed to ensure convergence. In the absence of detailed calculations, we expect that taking the cut-off at $t' \approx m_\rho^2 \approx 30\mu^2$ in fig. 6.2 would provide the best estimate of πN rescattering within the framework of the present model.

To carry this point farther we note that the effects of the processes of fig. 6.3 are certainly not included in the empirically determined TPE helicity amplitudes for $N\bar{N}$ S- and P-waves. The process of fig. 6.3a involving the $\Delta(1236)$ isobar is operative only in NN channels with $T = 1$, and simple estimates suggest that it will contribute

† In this regard we note that the quark model estimate of the $\pi N\Delta$ coupling constant is only 15% lower than the value obtained directly from the πN partial decay width of the $\Delta(1236)$. Also, the success of the quark model in describing $\Delta(1236)$ photo-production suggests that quark model estimates of the $\rho N\Delta$ coupling constant should also be reliable.

substantial short-range repulsion through the $N\bar{N}$ P-wave channel. This can be simulated by an increase in the ωNN coupling constant in $T = 1$ NN channels only. With this in mind, we return to the phase shifts NO of fig. 5.2.1 calculated with the Nielsen-Oades amplitudes and the increased value $g_\omega^2/4\pi = 13.0$. The $T = 1$ phase shifts are in good agreement with experiment. A similar calculation using the Nielsen-Oades helicity amplitudes and $g_\omega^2/4\pi = 8.0$ (i.e., closer to the physical value) yields $T = 0$ phase shifts of quality comparable to those of the model *FF*. This suggests that the inclusion of the processes of fig. 6.3 might allow the use of helicity amplitudes much closer to those of Nielsen and Oades. Further work on this problem is in progress.

We conclude with some general comments on the phase shifts obtained with the model $F_0 F_1'$ (fig. 5.2.1). The main difficulty in obtaining good results for the $^1S_0$ phase shift in a model with so few parameters is to obtain the correct slope. We find this slope to be incompatible with the magnitude of the f_+^0 helicity amplitude obtained by Nielsen and Oades [33]) in the intermediate t-region. It is hope that improved experimental results for the (isoscalar) S-wave $\pi\pi$ phase shifts will reduce this discrepancy. Once the magnitude of f_+^0 in the intermediate region is reduced, the slope of the $^1S_0$ phase shift is obtained correctly. The magnitude of $g_\omega^2/4\pi$ has very little effect on this slope. Acceptable values for the $^3S_1$ phase shift can be obtained by modest adjustments of the effect ρN coupling constants. The fact that this phase shift is not well determined at large energies (330 MeV) [36, 37]) makes it difficult to decide what its correct behaviour should be. Large uncertainties in $^1P_1$ phase shifts also make model discrimination difficult since almost any reasonable model agrees with phenomenological phase shifts. The remaining P-waves usually turn out to be reasonable once the S-waves have been fit. We note that the dominant effect of the missing πN rescattering near threshold will come from processes (a) and (b) in fig. 6.1 where the isobar is the $\Delta(1236$ MeV) with $J = \frac{3}{2}$ and $T = \frac{3}{2}$. Since the $^1D_2$ channel is the channel of lowest angular momentum in which a $\Delta(1236)$ can be produced in an S-state relative to the other nucleon, we would expect the effects of rescattering to be concentrated in this channel near threshold. All other phase shifts are in reasonable agreement with data for any reasonable model of the interaction with the exception of the $^3D_1$ channel.

All models considered lead to $^3D_1$ phase shifts which are significantly more repulsive than empirical ones. As mentioned above, there is an ambiguity in phenomenological phase shifts at 330 MeV, and the correct $^3D_1$ phase shift may be more repulsive than is usually assumed [37]). Nonetheless, our models result in too much repulsion. This phase shift is particularly sensitive to the details of the tensor force which are in turn dominated by OPE terms in the interaction. It is possible that contributions from $N\rho\pi$ and $N\omega\pi$ triangle vertex corrections shown in fig. 6.5 (which are manifestly not included in our eikonal approximation to vertex functions) could alter the structure of the tensor force at intermediate distances. Such processes bear a striking resemblance to the $\pi\pi$ S- and P-wave processes included in our TPE

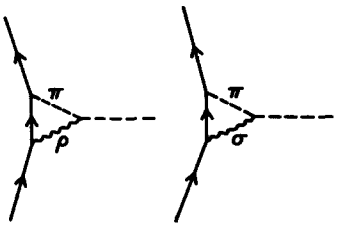

Fig. 6.5. Contributions to the πNN vertex not included in the eikonal form factors.

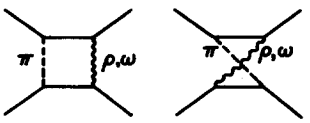

Fig. 6.6. Two-boson exchange processes involving one pion and one vector meson.

interaction. In this case one of the pions is absorbed by the initial nucleon. Irreducible two-boson exchange processes involving one pion and one vector meson, as shown in fig. 6.6, might lead to similar modifications. Such processes are of only marginally shorter range than ω-exchange and, although their influence will be decreased by short-range repulsion, it is not clear that it is negligible. Such effects might be simulated by allowing different values of γ or m in the eikonal function eq. (3.3.2) for use in damping the OPE amplitude. In the absence of *a priori* estimates of the nature of such changes, it is contrary to the spirit of this paper to simply turn these parameters loose. The question is nonetheless interesting and merits further investigation.

Throughout the course of this work we have enjoyed many constructive and also critical discussions with Professor G. E. Brown. We thank Dr. W. Weng for letting us use his computer code for solving the reaction matrix equation for nuclear matter.

Note added in proof: The *tu* symmetric form of the regularized interaction (3.3.4) involves an approximation in the evaluation of the u-channel processes. In the evaluation of the fourth-order crossedbox diagram involving one hard meson and one soft vector meson for fixed total momentum transfer, it is assumed that the hard amplitude does not vary for internal momenta such that the vector meson is soft. This approximation, which is adequate for heavy hard mesons, is suspect when the hard meson is a pion. A more careful analysis suggests that it is better to use $\gamma \approx 0.25$ in (3.3.3) in constructing $\chi(u)$ and $\chi(4m^2 - s - m_\pi^2)$ for the u-channel regularization of the one-pion exchange potential only via (3.3.4). This replacement is found to resolve the qualitative problems found in $^3D_1$ phase shifts as shown in figs. 5.1 and 5.2.1. Note that this approximation has no effect on t-channel regularizations.

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