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On the Method of the Theory of Nuclear Forces

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From the standpoint of the meson theory of nuclear force, results of current analysis on phenomenological nuclear potentials are reexamined. A new method of the theory of nuclear forces is proposed. We employed meson potentials for the outside region, and phenomenological parameters for the inside region of the nuclear forces.

§ 1. Problem in the nuclear forces

The phenomenological treatments of the nuclear forces have long been investgated since last ten years, but no definite results were obtained yet. The main reason for it was the scarcity of experimental results to be fitted to and there remained diversity of possibilities. However, as the problem must be treated meson theoretically in its final stage, these possibilities, which do not suggest the connection to or contradict with it, must be put aside. Here we shall investigate some features which the present meson theory imposes to the problem of nuclear forces.

A. Nuclear force range

Recent developments have introduced two restrictions in the theory of nuclear forces. The one concerns with the force range and the other with the neutron-proton and the proton-proton scattering at high energy region.

The former restriction comes from the discovery of two mesons, of which π-meson interacts strongly with nucleons, while the μ-meson has nothing to do with the nuclear forces. The mass of π-meson is observed to be about 275 m_e (m_e being electron mass) and is heavier than that, assumed hitherto for nuclear forces. This fact indicates that the force range must be taken shorter, i.e., about 1.3×10^{-13} cm. Although this alteration introduces only small effects in the phenomena of neutron-proton system, the quadrupole moment of deuterons is an exception, because it behaves as the inverse square of meson mass and the heavier meson mass reduces its value considerably. To investigate the reduction, we made the calculations, with zero cut-off, assuming the general form of nuclear potential:

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$$V(x) \sim \left\{a+b(\sigma_1\cdot\sigma_2)+fS_{12}\left(\frac{1}{x^2}+\frac{1}{x}+\frac{1}{3}\right)\right\}\frac{e^{-x}}{x},$$

where $x=\varkappa r$, $1/\varkappa$ being the Compton wave length for meson; and

$$S_{12}=\left\{\frac{3(\sigma_1\cdot x)(\sigma_2\cdot x)}{x^2}-\sigma_1\cdot\sigma_2\right\}.$$

The table shows that the quadrupole moment is still too small, unless we take unreasonably large cut-off radius. The best value of the quadrupole moment, 2.55×10^{-27} cm^2, is obtained for the coupling constants, $g=1.111$ and $f=0.982$: or to obtain the values in the usual form $(G^2/\hbar c)$ we have only to multiply them by (μ/M) and they are 0.17 and 0.15 respectively. We observe, however, that the cut-off radius is somewhat too large. From this fact we can infer that single range force does not give the correct value of quadrupole moment.[1),2)]

ordinary force $a+b=g$	tensor force f	quadrupole moment (in 10^{-27} cm^2)	cut-off radius (in 1.3×10^{-13} cm)
−0.307	0.491	1.086	0.32
0.307	0.645	2.097	0.37
1.111	0.982	2.55	0.69
1.228	1.474	2.92	0.82
1.842	1.965	3.34	1.10
2.456	2.456	3.95	1.28

Table I. The quadrupole moment of deuterons

While quadrupole moment depends strongly on the meson mass as mentioned above, it depends also on the shape of nuclear potential, though its amount is not so large. The shape of nuclear potential may be divided into two parts: (*a*) the "tail" and (*b*) the concentration of force near origin.[3)] Here (*b*) contributes mainly to the interaction energy, while (*a*) serves to change the tail part of the wave function and consequently to increase the quadrupole moment. The $1/r^3$ term in the tensor force has the effect of (*b*) and reduces quadrupole moment. However, this term must not be omitted, although it has singularity at origin, for no meson theory gives the tensor force without $1/r^3$ term, and the omission of it should give considerable alteration.

Christian and Hart,[4)] for instance, have employed the Yukawa type potential only both in central and tensor forces. This type of potential has long tail and considerably weak concentration, compared with the above case. As the result, they obtained fairly large value of quadrupole moment even for the range of 1.18×10^{-13} cm. But this tendency is special to Yukawa type potential and the situation is not so favourable for the square potential well, which has no tail. In this case, no sufficient amount of quadrupole moment can be obtained for the range shorter than 2.6×10^{-13} cm. Therefore, their results have nothing to suggest the relation to the meson theory. In this substitution of Yukawa type potential the range had to be taken shorter.

B. High energy neutron-proton scattering

90-MeV neutron-proton scattering provides with another criticism, that is, the angular distribution. The scattering at this energy is symmetrical with respect to 90° and Serber proposed the potential which gives no contribution to the wave of odd parity, or meson theoretically the mixture of symmetrical (or charged) and neutral mesons in the same amounts. However, his proposal is too crude and it is possible only phenomenologically, for the mixture of mesons cannot give the exact cancellation of the potential for the odd parity wave. Of course, this cancellation may not be strictly needed. Indeed, the close examination shows that the angular distribution curve has the minimum at 80° or less and not at 90°.

However, the Serber potential imposes restriction to the meson theory of nuclear forces, in that the type of symmetrical (or charged) and neutral mesons must be different, because, otherwise, the cancellation will be too good and sometimes the potential for even parity wave will vanish at the same time. Therefore, the Serber potential can not be taken too seriously in the meson theory of nuclear forces.

§ 2. The proposition of the new method

1. After the π-meson was produced artificially, the experimental evidences concerning the meson theory have been accumulated by these artificial mesons. It has been revealed from these experiments that the interactions between π mesons and nucleons are strong enough and so there will be no doubt that most part of nuclear forces are due to π mesons. Therefore, the study on the nuclear forces must be based on the meson theory.

2. The conventional methods to obtain the nuclear forces from the meson theory are divided into two; i.e., the original Yukawa theory which assumes the single meson-interaction between mesons and nucleons, and pair theory. However, according to the experimental results on π and μ mesons, the meson pair theory has been excluded.[2),5)]

3. The experimental results on nuclear forces have been investigated by several authors. They, however, employed there mainly the phenomenological potentials such as square well, exponential well, Yukawa well, and so on, and the meson theoretical potentials have scarcely been treated. Moreover, in the conventional studies the same simple phenomenological potentials have been applied to all the phenomena, regardless of the energies of the systems.

4. The analysis by Christian and Noyes[6)] on the proton-proton scattering at high energies has revealed that the tensor type forces must be highly singular, which fact indicates that the potential due to the meson theory is superior to the

conventional, phenomenological potentials.

5. We have insisted previously[7] that in treating the nuclear forces one must separate the region near and further than the force range and that part in the neighbourhood of the nucleon. In the region near the nucleon, the potentials involve the effects due to the higher order perturbations, non-static forces, heavy mesons, relativistic parts, strong couplings and so on and thus the problem will become necessarily complicated.

6. On the contrary, in that region arround and further than the force range, the potential is considered to be due to lower order terms of the perturbation expansions. This part of potential plays the important role in the nuclear scattering at low energies and the quadrupole moments of deuterons.

Here we could consider that the meson theory does not break down and the potentials due to the usual second and fourth order perturbation will give the correct results, i.e., the relativistic effect and thus non-static forces will not be important. Even if the excited states of the nucleons should exist, little influences will be given on the potentials for this region.

7. The fourth order perturbation will give the deviation from the second order calculations. Bethe has taken, in his "meson theory of nuclear forces",[8] for the cut-off radius the range where the magnitude of the second and the fourth perturbation will become comparable, i.e., the one-third of the nuclear force range.

Machida[9] has performed the calculations of the fourth order perturbation on the potential due to the pseudo-scalar meson field, and obtained the results that the fourth order perturbation gives strong enough contribution even in the region outside the range of the second order forces where only the static potential is important.

8. From these considerations we could conclude that in the investigation of the nuclear forces it will be meaningless to take the same simple potential from the neighbourhood of the nucleon to the outside region throughout. Therefore, we propose here for the analysis of nuclear forces that the problem is to be treated substantialisticaly in the outside region, while in the inside region phenomenologically, i.e., in the outside region we employ the second order and non-static fourth order potential, which is cut-off where it breaks down and substituted by the phenomenological potential, say square well potential, for the inside region. The depth and the width of the potential is so adjusted to fit the experimental results according as each process and according as each energy region. The spin- and charge-dependence of the potential is also chosen suitably according to the phenomena. This treatment corresponds to the statement in the previous section that more than two ranges are necessary for the nuclear forces.

9. For the outside region, the combinations of the four types of meson field are taken and the parameters for the potential in the inside region are adjusted. If the parameters cannot be chosen suitably for certain combination in the outside region, this type of the meson potential should be excluded.

10. If we could obtain the agreements with experiment by the suitable choice of parameters, the set of the parameters will give the data to lead to the future correct theory.

11. The static potential due to the fourth order calculation has not always meanings, since in the region where the fourth order calculation has sufficient enough magnitude, other non static effects will generally become strong too. However, in the outer region where the static potential due to the second order perturbation is influenced by the fourth order calculations, the spin charge and radial dependence of the latter will be taken into consideration.[10]

12. On the type of meson we have attained to the conclusions as the results of analysis on the several phenomena that the pseudo-scalar meson will be best fitted.[11]

Onuma and Kôide[12] have performed the calculations, employing the hard core for the inside potential as the simplification of our considerations, and the symmetrical pseudo-scalar meson potential for the substantialistical part of the potential, and obtained the results that if one takes 0.45 of the force range for the radius of hard core, the reasonable value for the quadrupole moments of deuterons will be obtained.

13. In the symmetrical vector meson theory, the quadrupole moment of deuterons becomes negative and it seems impossible to give the positive sign whatever characters one may give to the inside potential, because its range is too short. This fact is contrary to the expectation of Heisenberg.[13]

14. Heisenberg[13] has emphasized to introduce vector π-meson to have the strong L–S coupling in the nuclear forces. However, it seems probable that the fourth order perturbations will give L–S coupling from the pseudo-scalar meson theory as its non-static part, and therefore we could not conclude that vector meson is indispensable.

15. Jastrow[14] has employed the exponential potential with hard core in its centre to explain the proton-proton scattering at high energies by the charge independent potential. The radius of the hard core has been taken as 0.6×10^{-13} cm. This is considered to be the simplified case of our method and from these investigations we could expect that our method will give good results so long as it concerned with the proton-proton scattering. However, Jastrow has employed phenomenological potential also for the outside region and thus, his method has no meanings for the investigation of the meson theory.

References

1) The similar calculations have previously been performed,[2] but at that time the meson mass was taken larger and, as the result, the cut-off radius had to be taken larger than the force range itself to obtain the sufficient quadrupole moment. We have first proposed in that paper that more than two force ranges are to be needed. Afterwards the meson mass has been observed to be somewhat smaller and the cut-off radius comes inside the force ranges. However, it is still too large and yet more than two ranges are needed.
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