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OFF-SHELL ELECTRON-NUCLEON CROSS SECTIONS
The impulse approximation

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Abstract: With the goal of improving the analysis of quasi-elastic (e, e') and (e, e'N) experiments, ambiguities in the impulse approximation are investigated by comparing various off-shell extrapolations of the Rosenbluth cross section. It is found that these ambiguities are especially large for low incident electron energies and momentum transfers and that they can be reduced by applying various physical constraints, such as those due to Lorentz invariance and current conservation. In particular the latter is important: most off-shell cross sections that have been used do not treat this correctly and, as a consequence, in the limit of the photon point give infinite photoabsorption cross sections.

1. Introduction

The two main reasons usually given for using electromagnetic interactions to probe nuclear structure are that they are relatively weak and well known [1]). Due to the former, true two-step processes (called dispersion corrections in electron scattering), which considerably complicate the extraction of the desired nuclear structure, are minimized. For light nuclei and high-energy electrons, the situation is particularly simple. In this case the Born approximation is very good, and therefore the momentum transferred to the nucleus is known. Thus one can essentially just divide out the terms arising from the known electromagnetic interaction and extract the information about the nuclear wave function directly from the cross section.

To state that the electromagnetic interaction is well known, however, requires some qualification. For electron scattering, part of the interaction, namely that involving the electron and the photon, is known, to all intents and purposes, perfectly from QED. The remaining part, i.e. the nuclear current, is much more poorly understood. The primary difficulty is that the current depends upon the dynamical structure of the nucleons and their interactions. This is most obvious when relativistic effects are included. For example, vector and scalar meson exchange interactions reduce to the same form of non-relativistic potential, but affect the nucleon spinors and therefore the current differently. Thus a more fundamental understanding, such as would be provided by a relativistic field theory, is required. The problem of incorporating such relativistic effects is not an easy one. There have, however, been several investigations of such effects, mainly in the framework of the σ, ω model [2]).

For these reasons, one often makes the assumption that the current is given by the sum of the currents of the individual nucleons, treated as free particles [1]). This is usually referred to as the impulse approximation. Since the nucleus is a low-density system, this would seem to be a reasonable assumption. Also there is some empirical support: calculations of inclusive quasi-elastic electron scattering, based on the impulse approximation, usually predict the magnitude of the experimental cross sections quite accurately [3,4]). On the other hand, one knows that at some point this approximation must break down: for example, in order to maintain current conservation, exchange currents must be added. Their contribution is generally quite small. (In the absolute sense, the relative contribution can be very large when the nucleon contribution is small.) We shall not consider exchange currents in this paper, but instead concentrate on a related difficulty: since the nucleons in the nucleus are manifestly off-shell, what is meant by "treated as free particles" is not well defined. Thus the basic assumption one is making is ambiguous, and therefore so are the results that one predicts. In this paper we investigate this ambiguity for the case of high-energy nucleon knock-out reactions [5]), the goal being to improve the analysis of quasi-elastic (e, e'N) and (e, e') experiments.

The approach we take in studying the off-shell ambiguities in this paper is to compare various different off-shell extrapolations and investigate to what extent imposing certain physical requirements, such as current conservation, can reduce the ambiguity. One such requirement, which we shall always impose, is that for free kinematics the fully relativistic on-shell current is obtained. We thus do not consider approaches that use expansions in $1/m$, treating relativistic effects as corrections [3,6]). The rationale for this approach is that in the non-relativistic limit the impulse approximation gives a unique prediction for the current and thus the ambiguities are relegated to the relativistic corrections, which are hopefully small. However, that the results are unique in the non-relativistic limit does not imply that one is making a good approximation. Rather it is a consequence of the fact that to lowest order in $1/m$ the current has a very simple form. One is thus not forced to resolve the contradiction between on- and off-shell kinematics. In situations where one is almost on-shell, the use of such approximations does not resolve, but rather introduces ambiguities.

The paper is divided into three main sections. In the following section we present the general formalism for (e, e'X) in the Born approximation for the electromagnetic interaction. The result is analogous to the more commonly known result for the cross section for (e, e') given in ref. [1]). We then specialize to (e, e'N) in the plane wave impulse approximation (PWIA). Next we derive and discuss an off-shell cross section σ_1^{cc}. This cross section was actually first presented in ref. [7]), and it has been used in various calculations of inclusive quasi-elastic scattering [8]). However, the discussion in ref. [7]) of how this cross section was obtained is rather cryptic. We also present a similar cross section, σ_2^{cc}, obtained by a slightly different off-shell extrapolation. In the last part we compare these cross sections with various other cross sections that have been proposed, under a variety of kinematic conditions. The goal is to determine to what extent the off-shell ambiguities can be reduced by imposing physically reasonable requirements and by optimizing the kinematics.

2. Formalism for (e, e'N): the plane wave impulse approximation

In order to develop the formalism for (e, e'N) reactions, we start from the general form of the cross section for (e, e'X) in the one-photon-exchange approximation. Following ref. [9])† one has

$$\frac{d^4\sigma}{d\Omega_{k_2}\, d\varepsilon_2\, d\Omega_{p'}\, dE'} = \frac{2\alpha^2}{q_\mu^4}\frac{k_2}{k_1} p'E'\eta_{\mu\nu}W_{\mu\nu}\,, \tag{1}$$

with

$$\eta_{\mu\nu} = k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu} + \tfrac{1}{2}q_\mu^2\delta_{\mu\nu}\,, \tag{2}$$

$$W_{\mu\nu} = \overline{\sum_{\mathrm{i}}}\sum_{\mathrm{f}} \delta(E' + E_{\mathrm{f}} - \omega - E_{\mathrm{i}})\langle f, \boldsymbol{p}'|J_\mu(q)|i\rangle\langle f, \boldsymbol{p}'|J_\nu(q)|i\rangle^*\,. \tag{3}$$

Here $k_{1,2\mu} = (\boldsymbol{k}_{1,2}, i\varepsilon_{1,2})$ is the initial (final) electron four-momentum and $p'_\mu = (\boldsymbol{p}', iE')$ that of the knocked-out particle. The four-momentum transfer is denoted by $q_\mu = (\boldsymbol{q}, i\omega) = k_{1\mu} - k_{2\mu}$. E_i is the energy of the initial nucleus and E_{f} that of the final (residual) nucleus. For a given separation energy E_{s}, these energies are related by $E_{\mathrm{i}} - E_{\mathrm{f}} = m - E_{\mathrm{s}} - E_{\mathrm{rec}}$ where m is the mass of the knocked-out particle and E_{rec} the recoil energy of the residual nucleus. We neglect the mass of the electron.

The tensor $W_{\mu\nu}$ which depends on the matrix elements of the nuclear current, $J_\mu(q)$, is the only unknown. On general grounds (Lorentz invariance, gauge invariance), however, we know that $W_{\mu\nu}$ only depends on four independent structure

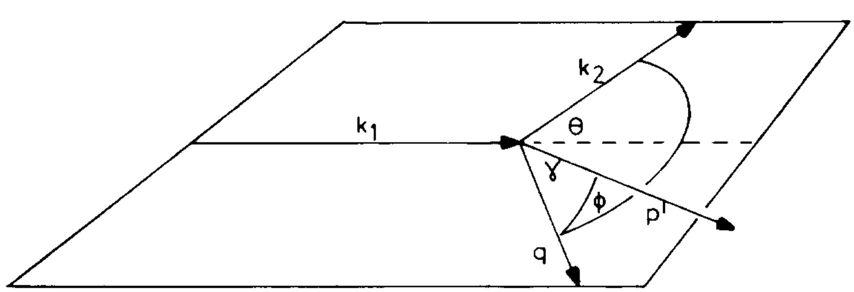

Fig. 1. Kinematics of the (e, e'X) reaction.

† For simplicity we have dropped the factor 'E' (the total energy of the initial nucleus) in $W_{\mu\nu}$, and we work in the lab frame.

functions [9,10]) and the cross section can be written as [9])

$$\frac{d^4\sigma}{d\Omega_{k_2}\,d\varepsilon_2\,d\Omega_{p'}\,dE'}=p'E'\sigma_M\left\{\frac{q_\mu^4}{q^4}W_C+\left(\frac{q_\mu^2}{2q^2}+\tan^2\tfrac{1}{2}\theta\right)W_T\right.$$
$$\left.+\frac{q_\mu^2}{q^2}\left(\frac{q_\mu^2}{q^2}+\tan^2\tfrac{1}{2}\theta\right)^{1/2}W_I\cos\phi+\left(\frac{q_\mu^2}{q^2}\cos^2\phi+\tan^2\tfrac{1}{2}\theta\right)W_S\right\}, \tag{4}$$

where $\sigma_M=4\alpha^2\varepsilon_2^2\cos^2(\frac{1}{2}\theta)/q_\mu^4$ is the Mott cross section, θ is the scattering angle of the electron and ϕ is the angle between the scattering plane and the plane defined by the vectors $\boldsymbol{p}'$ and $\boldsymbol{q}$ (see fig. 1). The structure functions can be expressed as

$$\begin{aligned}W_C&=\langle\rho^2\rangle\,,\\ W_T&=2\langle J_\perp^2\rangle\,,\\ W_S&=\langle J_\parallel^2\rangle-\langle J_\perp^2\rangle\,,\\ W_I&=-\langle\rho J_\parallel\rangle-\langle J_\parallel\rho\rangle\,,\end{aligned}\tag{5}$$

where we have used the short-hand notation

$$W_{\mu\nu}=\langle J_\mu J_\nu\rangle\,, \tag{6}$$

with

$$J_\mu=(\boldsymbol{J},i\rho)\,, \tag{7}$$

$$\begin{aligned}J_\perp&=\boldsymbol{J}\cdot\hat{\boldsymbol{n}}_\perp, &\quad \hat{\boldsymbol{n}}_\perp&=(\boldsymbol{q}\times\boldsymbol{p}')/|\boldsymbol{q}\times\boldsymbol{p}'|\,,\\ J_\parallel&=\boldsymbol{J}\cdot\hat{\boldsymbol{n}}_\parallel, &\quad \hat{\boldsymbol{n}}_\parallel&=(\hat{\boldsymbol{n}}_\perp\times\boldsymbol{q})/|\hat{\boldsymbol{n}}_\perp\times\boldsymbol{q}|\,,\end{aligned}\tag{8}$$

where $J_\perp$ and $J_\parallel$ are the transverse (perpendicular to $\boldsymbol{q}$) components of $\boldsymbol{J}$ which are perpendicular to and in the plane defined by $\boldsymbol{p}'$ and $\boldsymbol{q}$ respectively.

In obtaining these results, current conservation, that is the relation

$$\boldsymbol{q}\cdot\boldsymbol{J}=\omega\rho\,, \tag{9}$$

has been used to eliminate the explicit dependence on the longitudinal current in the current–current interaction. This results in the explicit factors of q_μ^2/q^2 associated with the Coulomb interaction in eq. (4). These factors can be interpreted as converting the retarded photon propagators $1/q_\mu^2$ into instantaneous ones, $1/q^2$. We have thus, in essence, used current conservation to convert from the current–current interaction of the Lorentz gauge to the Coulomb gauge.

If, however, one uses a model in which current is not conserved (which is more the rule than the exception) this gauge invariance is broken. In such cases one generally assumes that the predictions for the charge density are more reliable than for the current. It is thus more reasonable to use the results given in the Coulomb gauge, or equivalently use eq. (9) rather than the model to determine the longitudinal current. This is the same procedure that is normally used in calculations of inclusive, (e, e′), electron scattering [1]). It is also the basis of the Siegert theorem [11]). Straightforward application of the current–current interaction would not give rise to the factors of q_μ^2/q^2 mentioned above with dire results in the limit $q_\mu^2 \to 0$, namely the photoabsorption cross section would be infinite. This effect, which is present in many off-shell extrapolations of the Rosenbluth cross section, is readily apparent in fig. 7 of the last section.

Now that the general formalism of (e, e′X) has been established we specialize to (e, e′N) in the PWIA. In this approximation we include the assumption that the detected nucleon is the one that was struck by the electron. This is necessary since the plane wave is not orthogonal to the bound single-particle wave functions; thus one could have apparent "knock-out" of a particle which in no way interacted with the electron.

Due to the assumption of plane waves, the initial momentum of the knocked-out nucleon has a known, fixed, value $\boldsymbol{p} = \boldsymbol{p}' - \boldsymbol{q}$, and aside from a sum over spins, s_i, one can factorize the matrix element in (3) into two terms: the nucleon current and the Fourier transform of the overlap integral. Eq. (3) can thus be written

$$W_{\mu\nu} = \overline{\sum_{\mathrm{i}}}\sum_{\mathrm{f}}\sum_{s_is_js_f}\langle \boldsymbol{p}', s_f|\Gamma_\mu(q)|\boldsymbol{p}, s_i\rangle\langle \boldsymbol{p}', s_f|\Gamma_\nu(q)|\boldsymbol{p}, s_j\rangle^*$$

$$\delta(E' + E_f - \omega - E_i)\langle \mathrm{f}, \boldsymbol{p}'|a_{\boldsymbol{p},s_i}|\mathrm{i}\rangle\langle \mathrm{f}, \boldsymbol{p}'|a_{\boldsymbol{p},s_j}|\mathrm{i}\rangle^*$$

$$= \sum_{s_is_js_f}\langle \boldsymbol{p}', s_f|\Gamma_\mu(q)|\boldsymbol{p}, s_i\rangle\langle \boldsymbol{p}', s_f|\Gamma_\nu(q)|\boldsymbol{p}, s_j\rangle^* S_{s_is_j}(\boldsymbol{p}, E)\,, \tag{10}$$

where

$$S_{s_i,s_j}(\boldsymbol{p}, E) = \overline{\sum_{\mathrm{i}}}\langle \mathrm{i}|a^+_{\boldsymbol{p},s_j}\delta(E' + H - \omega - E_i)a_{\boldsymbol{p},s_i}|\mathrm{i}\rangle\,. \tag{11}$$

Here $j_\mu^{\mathrm{if}} = \langle \boldsymbol{p}', s_f|\Gamma_\mu(q)|\boldsymbol{p}, s_i\rangle$ is the current for a single nucleon and S is the single-nucleon spectral function. The latter represents the probability of finding a nucleon of momentum p and energy $E = m - E_s$ in the initial nucleus [5]). In general, for example in distorted wave impulse approximation (DWIA), an off-diagonal spectral function is involved, since both the spins and momenta of the annihilation and creation operators can be different. In PWIA, however, there is only one momentum, and as a consequence S is also diagonal in the spin, $S_{s_is_f} = \frac{1}{2}\delta_{s_is_f}S$. This can be seen by noting that the only spin dependence that can be formed with one momentum is $S \sim \boldsymbol{\sigma}\cdot\boldsymbol{p}$, but this is a pseudoscalar and thus not allowed unless there is parity violation [12]). As a consequence the terms involving the electromagnetic interaction can be factorized from the spectral function and expressed as an off-shell electron–nucleon cross section:

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}\Omega_{k_2}\,\mathrm{d}\varepsilon_2\,\mathrm{d}\Omega_{p'}\,\mathrm{d}E'} = p'E'\sigma_{\mathrm{eN}}S(p, E)\,, \tag{12}$$

$$\sigma_{\mathrm{eN}}=\frac{2\alpha^2}{q_\mu^4}\frac{k_2}{k_1}\eta_{\mu\nu}\overline{\sum_{s_\mathrm{i}}}\sum_{s_\mathrm{f}} j_\mu^{\mathrm{if}} j_\nu^{\mathrm{if}*}$$

$$=\sigma_\mathrm{M}\left\{\frac{q_\mu^4}{q^4}w_\mathrm{C}+\left(\frac{q_\mu^2}{2q^2}+\tan^2\tfrac{1}{2}\theta\right)W_\mathrm{T}\right.$$

$$\left.+\frac{q_\mu^2}{q^2}\left(\frac{q_\mu^2}{q^2}+\tan^2\tfrac{1}{2}\theta\right)^{1/2}W_\mathrm{I}\cos\phi+\left(\frac{q_\mu^2}{q^2}\cos^2\phi+\tan^2\tfrac{1}{2}\theta\right)W_\mathrm{S}\right\}, \tag{13}$$

where the W's can be expressed in terms of j_μ in complete analogy with (5). For free nucleons (13) reduces to the cross section for scattering on a moving nucleon, except that there is no recoil or flux term. The former arises from the delta function of energy conservation which in (12) has been incorporated in the spectral function S, and the flux is defined with respect to the target which here is the nucleus, not the nucleon.

3. Off-shell extrapolation of the Rosenbluth cross section

Within the general framework for (e, e'N) in the PWIA, off-shell extrapolation of the electron–nucleon cross section amounts to choosing a method to calculate the off-shell nucleon current (we suppress the spin indices)

$$j_\mu=\langle \boldsymbol{p}'|\Gamma_\mu(q)|\boldsymbol{p}\rangle\,. \tag{14}$$

On-shell one has

$$j_\mu=i\bar{U}(\boldsymbol{p}')\left(\gamma_\mu F_1(q_\mu^2)-\sigma_{\mu\nu}q_\nu\frac{\kappa}{2m}F_2(q_\mu^2)\right)U(\boldsymbol{p})\,, \tag{15}$$

where the fact that j_μ only depends on two independent form factors is a consequence of the fact that all other permissible terms can be reduced to this form by using the (free) Dirac equation. In fact, in deriving the Rosenbluth cross section it is convenient to use such a relation, the Gordon reduction [13] (in reverse), to reduce the number of γ-matrices. One then obtains

$$j_\mu=i\bar{U}(\boldsymbol{p}')\left(\gamma_\mu(F_1(q_\mu^2)+\kappa F_2(q_\mu^2))+i(p'+p)_\mu\frac{\kappa}{2m}F_2(q_\mu^2)\right)U(\boldsymbol{p})\,. \tag{16}$$

When one goes off-shell one must decide which form factors, operator and spinors to use in (14). In the spirit of trying to treat the nucleons as free it is natural to use the free form factors and spinors (as determined by the three-momentum). The choice of which operator to use is not so obvious, for one is no longer restricted to only two types of terms. Some argument can be made for keeping the term $\gamma_\mu F_1$ as it is: one expects such a term to be present from minimal coupling. How one should treat the term $\sigma_{\mu\nu}q_\nu F_2$ associated with the anomalous magnetic moment, however, is not at all clear: for this one really needs a theory which describes the origin of this term [2]. Without this information, one must hope that the different off-shell extrapolations do not differ very much. In this paper we consider two such extrapolations, namely (15) and (16) above, for off-shell kinematics. The difference between these two currents is that q_μ appears explicitly in (15) but not in (16). Thus the factor ω in (15) is replaced by $\bar{\omega} = E' - \bar{E}$ in (16), where $\bar{E} = (p^2+m^2)^{1/2}$. The degree to which one is off-shell is given by the difference between $\bar{E}$ and $E' - \omega = m - E_s - E_{rec}$. Increasing either p or E_s clearly brings one further off-shell.

Of these two extrapolations, the one obtained from eq. (16) gives the simplest results: the currents, have exactly the same form as the free currents only one must use $\bar{q}_\mu = (\boldsymbol{q}, i\bar{\omega})$ and $\bar{E}$ instead of q_μ and E whenever the latter appear explicitly. This gives

$$w_C = \frac{1}{4\bar{E}E'}\left\{(\bar{E}+E')^2\left(F_1^2 + \frac{\bar{q}_\mu^2}{4m^2}\kappa^2 F_2^2\right) - q^2(F_1+\kappa F_2)^2\right\},$$

$$w_T = \frac{\bar{q}_\mu^2}{2\bar{E}E'}(F_1+\kappa F_2)^2,$$

$$w_S = \frac{p'^2 \sin^2\gamma}{\bar{E}E'}\left(F_1^2 + \frac{\bar{q}_\mu^2}{4m^2}\kappa^2 F_2^2\right),$$

$$w_I = -\frac{p' \sin\gamma}{\bar{E}E'}(\bar{E}+E')\left(F_1^2 + \frac{\bar{q}_\mu^2}{4m^2}\kappa^2 F_2^2\right), \tag{17}$$

where γ is the angle between $\boldsymbol{p}'$ and $\boldsymbol{q}$. Inserting these results in (13), one obtains the cross section σ_1^{cc} given in ref. [7]†. Eq. (15) leads to the more complicated result (18):

$$\begin{aligned} w_C = \frac{1}{\bar{E}E'}\Big\{&(\bar{E}E' + \tfrac{1}{2}(\bar{p}_\mu p'_\mu + m^2))F_1^2 - \tfrac{1}{2}q^2 F_1 \kappa F_2 \\ &- ((\bar{p}_\mu q_\mu E' + p'_\mu q_\mu \bar{E})\omega - \bar{E}E' q_\mu^2 + \bar{p}_\mu q_\mu p'_\nu q_\nu \\ &- \tfrac{1}{2}(\bar{p}_\mu p'_\mu - m^2)q^2)\frac{\kappa^2}{4m^2}F_2^2\Big\}, \end{aligned}$$

$$\begin{aligned} w_T = \frac{1}{\bar{E}E'}\Big\{&-(\bar{p}_\mu p'_\mu + m^2)F_1^2 + \bar{q}_\mu q_\mu F_1 \kappa F_2 \\ &+ (2\bar{p}_\mu q_\mu p'_\nu q_\nu - (\bar{p}_\mu p'_\mu - m^2) q_\mu^2)\frac{\kappa^2}{4m^2}F_2^2\Big\}, \end{aligned}$$

$$w_S = \frac{p'^2 \sin^2\gamma}{\bar{E}E'}\left(F_1^2 + \frac{q_\mu^2}{4m^2}\kappa^2 F_2^2\right),$$

$$w_I = \frac{p' \sin\gamma}{\bar{E}E'}\left\{-(\bar{E}+E')F_1^2 + ((\bar{p}+p')_\mu q_\mu \omega - (\bar{E}+E')q_\mu^2)\frac{\kappa^2}{4m^2}F_2^2\right\}. \tag{18}$$

We shall refer to the corresponding cross section as σ_2^{cc}.

† Note; there is a misprint in ref. [7]), the factor q_μ^2/q^3 should be q_μ^2/q^2.

Of the other cross sections considered in this paper, ours are most similar to that presented in ref. [14]) in that the off-shell extrapolation is considered at the level of the nucleon currents, rather than the cross section itself. In both cases, the nucleons are initially treated as free particles. This leads to a violation of both energy-momentum conservation and current conservation. Our approaches differ in that in ref. [14]) an attempt is made to restore these conservation laws by modifying the current in an *ad hoc* way. Our philosophy is that unless one understands the dynamics responsible for these conservation laws, it is best not to force them to be satisfied.

This difference is most apparent in the treatment of current conservation. As has already been mentioned, in our approach, current conservation is taken into account by working in the Coulomb gauge, or, equivalently, working with the current–current interaction of the Lorentz gauge but then using a longitudinal nuclear current which is determined, via (9), from the transition charge density, rather than that given directly by the model. In ref. [14]) current conservation is restored by adding a term proportional to q_μ to the current. The conservation law then determines the coefficient of this term. This particular choice for extrapolating the current gives a deceptively simple result: since the electron current is conserved, $q_\mu \eta_{\mu\nu} = 0$, the new term gives no contribution at all (in the Lorentz gauge). However, the modified current is actually very singular in the limit of the photon point, $j_\mu \sim 1/q_\mu^2$. In the Coulomb gauge it can be seen that this behavior leads to a cancellation of all the explicit factors of q_μ^2/q^2 in eq. (4) which were due to current conservation in the first place. Thus current conservation is only obtained by making a very unrealistic extrapolation.

A second, less important, difference concerns the treatment of energy-momentum conservation. In ref. [14]) this is restored by using an effective mass, $m^* = (-p_\mu^2)^{1/2}$ for the initial state spinor. But one could just as well have chosen an effective energy $E^* = (p^2 + m^2)^{1/2}$. Since $E^* = \bar{E}$ is the free, i.e. kinetic, energy this corresponds to exactly the procedure we have followed. Without additional information there is no reason to prefer one procedure to the other. This is an area where relativistic field theories, and in particular the σ, ω model, play an important role. The potentials arising from scalar and vector meson exchanges have the same structure as the mass and energy terms in the Dirac equation, and thus naturally give rise to such effective masses and energies, but then not confined to only the initial state.

4. Comparison of electron–nucleon cross sections

In order to illustrate the importance of the off-shell ambiguities, various formulae for the electron–nucleon cross section which have come to the attention of the author will be compared. It is useful to distinguish between the cross sections on the basis of two criteria: whether or not they are consistent with the general form

(4) required for (e, e'X) cross sections and, if they are, whether they are consistent with current conservation, as discussed in the previous section. We shall classify these cross sections in categories C, B and A in order of increasing restrictiveness.

Examples of cross sections in category C are given by $\sigma_C^{\mathrm{IN,FIN}}$ in eqs. (19), (20). These cross sections were obtained [15] in analogy with a procedure often used in (strong) nuclear reactions: using an on-shell cross section as determined by either the final or the initial state kinematics (which is a quantity that can be measured). We note that eqs. (19), (20) treat the photon propagator correctly, i.e. there is an explicit factor $1/q_\mu^4$. Actually one knows how to treat not only the photon, but also the electron, exactly, thus there is no need to suppress the explicit dependence on $k_{1\mu}$ or $k_{2\mu}$. The same procedure, but keeping the known dependence on the kinematics of the electron, leads to cross sections [16] $\sigma_B^{\mathrm{IN,FIN}}$ in eqs. (21), (22). We note that since the electron and photon are treated exactly, one is guaranteed that the general form of the cross section eq. (4) is satisfied. We also note that in the spirit of PWIA, the choice of final-state kinematics would be more appropriate than initial-state kinematics for (e, e'X).

There also exist (at least) three published cross sections and one unpublished which fall into category B: σ^{S}, which has been extensively used by the Saclay group [14]), σ^{P}, a cross section obtained by Potter based upon the work of Goldberg [17]), σ^{M}, one of several cross sections contained in Mougey's thesis [15]) and σ^{B}, a cross section used in a calculation of inclusive quasi-elastic scattering by Bidasaria *et al.* [18]). These are reproduced in eqs. (23)–(26). The motivation used in the derivation of σ^{S} was described in the previous section, and σ^{M} was essentially obtained in the same way as σ_2^{cc}, but using the formalism given by eqs. (1)–(3) instead of eq. (13). The other two cross sections were published without any derivation or discussion:

$$\sigma_C^{\mathrm{IN}} = \frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{\bar{E}(\bar{E}+\omega)} \times \left\{ (4(k_{1\mu}\bar{p}_\mu)^2 + 2k_{1\mu}\bar{p}_\mu q_\nu^2 - m^2 q_\mu^2)\left(F_1^2 + \frac{q_\mu^2}{4m^2}\kappa^2 F_2^2\right) + \tfrac{1}{2}q_\mu^4 (F_1+\kappa F_2)^2 \right\}, \tag{19}$$

$$\sigma_C^{\mathrm{FIN}} = \frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{(E'-\omega)E'} \times \left\{ (4(k_{2\mu}p'_\mu)^2 + 2k_{2\mu}p'_\mu q_\nu^2 - m^2 q_\mu^2)\left(F_1^2 + \frac{q_\mu^2}{4m^2}\kappa^2 F_2^2\right) + \tfrac{1}{2}q_\mu^4 (F_1+\kappa F_2)^2 \right\}, \tag{20}$$

$$\sigma_B^{\mathrm{IN}} = \frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{\bar{E}(\bar{E}+\omega)} \times \left\{ (4k_{1\mu}\bar{p}_\mu k_{2\nu}\bar{p}_\nu - m^2 q_\mu^2)\left(F_1^2 + \frac{q_\mu^2}{4m^2}\kappa^2 F_2^2\right) + \tfrac{1}{2}q_\mu^4 (F_1+\kappa F_2)^2 \right\}, \tag{21}$$

$$\sigma_{\mathrm{B}}^{\mathrm{FIN}}=\frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{(E'-\omega)E'}$$

$$\times\left\{(4k_{1\mu}p'_\mu k_{2\nu}p'_\nu-m^2q_\mu^2)\left(F_1^2+\frac{q_\mu^2}{4m^2}\kappa^2F_2^2\right)+\tfrac{1}{2}q_\mu^4(F_1+\kappa F_2)^2\right\}, \tag{22}$$

$$\sigma^{\mathrm{S}}=\frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{EE'}\frac{1}{1+\dfrac{q_\mu^2}{(m+m^*)^2}}\left\{(4k_{1\mu}p'_\mu k_{2\nu}p'_\nu-m^2q_\mu^2)\left(F_1-\frac{q_\mu^2}{4m^2}\kappa F_2\right)^2\right.$$

$$\left.+\frac{q_\mu^2}{(m+m^*)^2}(4k_{1\mu}p'_\mu k_{2\nu}p'_\nu+2(q_\mu p'_\mu)^2+m^2q_\mu^2)(F_1+\kappa F_2)^2\right\}, \tag{23}$$

$$\sigma^{\mathrm{P}}=\frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{EE'}\left\{(4k_{1\mu}p_\mu k_{2\nu}p_\nu+2(q_\mu p_\mu)^2-m^2q_\mu^2)F_1^2\right.$$

$$\left.+(4k_{1\mu}p_\mu k_{2\nu}p_\nu+2(q_\mu p_\mu)^2+q_\mu p_\mu q_\nu^2+m^2q_\mu^2)\frac{q_\mu^2}{4m^2}\kappa^2F_2^2+q_\mu^4F_1\kappa F_2\right\}, \tag{24}$$

$$\sigma^{\mathrm{M}}=\frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{\bar{E}E'}\left\{(2k_{1\mu}\bar{p}_\mu k_{2\nu}p'_\nu+2k_{1\mu}p'_\mu k_{2\nu}\bar{p}_\nu+p'_\mu\bar{p}_\mu q_\nu^2)\left(F_1^2+\frac{q_\mu^2}{4m^2}\kappa^2F_2^2\right)\right.$$

$$\left.+\tfrac{1}{2}q_\mu^2\bar{q}_\nu^2F_1^2+\bar{q}_\mu q_\mu q_\nu^2F_1\kappa F_2-(\tfrac{1}{2}(\bar{p}+p')_\mu^2q_\nu^2-2q_\mu\bar{p}_\mu q_\nu p'_\nu)\frac{q_\mu^2}{4m^2}\kappa^2F_2^2\right\}, \tag{25}$$

$$\sigma^{\mathrm{B}}=\frac{\alpha^2}{q_\mu^4}\frac{\varepsilon_2}{\varepsilon_1}\frac{1}{EE'}\left\{(4k_{1\mu}p_\mu k_{2\nu}p_\nu-q_\mu p_\mu q_\nu^2-m^2q_\mu^2)F_1^2+(4k_{1\mu}p_\mu k_{2\nu}p_\nu+2(q_\mu p_\mu)^2\right.$$

$$\left.+q_\mu p_\mu q_\nu^2+m^2q_\mu^2)\frac{q_\mu^2}{4m^2}\kappa^2F_2^2-2(q_\mu p_\mu q_\nu^2+(q_\mu p_\mu)^2-\tfrac{1}{4}q_\mu^4)F_1\kappa F_2\right\}. \tag{26}$$

Two cross sections are considered in category A, σ_1^{cc} and σ_2^{cc}.

The results for the ten different cross sections for proton knock-out are shown in figs. 2–7. To facilitate comparison, the same logarithmic scale is used in each of the figures and, except in fig. 7, the cross sections have been normalized such that $N\sigma_1^{\mathrm{cc}}=1$ at $\gamma=0$. In fig. 7, due to the very strong change in the magnitude of the cross section, we instead plot the ratio of the various cross section to σ_1^{cc}. Due to the recoil energy, the atomic number of the target, A, has a slight influence on the results. Rather than specifying a certain A, we take it to be large and neglect recoil. For all the results shown, the final momentum of the proton has been taken to be $2.3\ \mathrm{fm}^{-1}$, corresponding to 104 MeV, a rather typical value for the experiments that have been performed so far. The form factors of Janssens *et al.* [19]) were used.

Fig. 2 shows the results, for 500 MeV incident electrons and 0 MeV separation energy, using the so-called perpendicular kinematics. With these kinematics, one initially chooses $\boldsymbol{p}'=\boldsymbol{q}$, and then varies the angle between $\boldsymbol{p}'$ and $\boldsymbol{q}$ while keeping the kinematics of the electron fixed. At $\gamma=0$ the cross sections are all equal since,

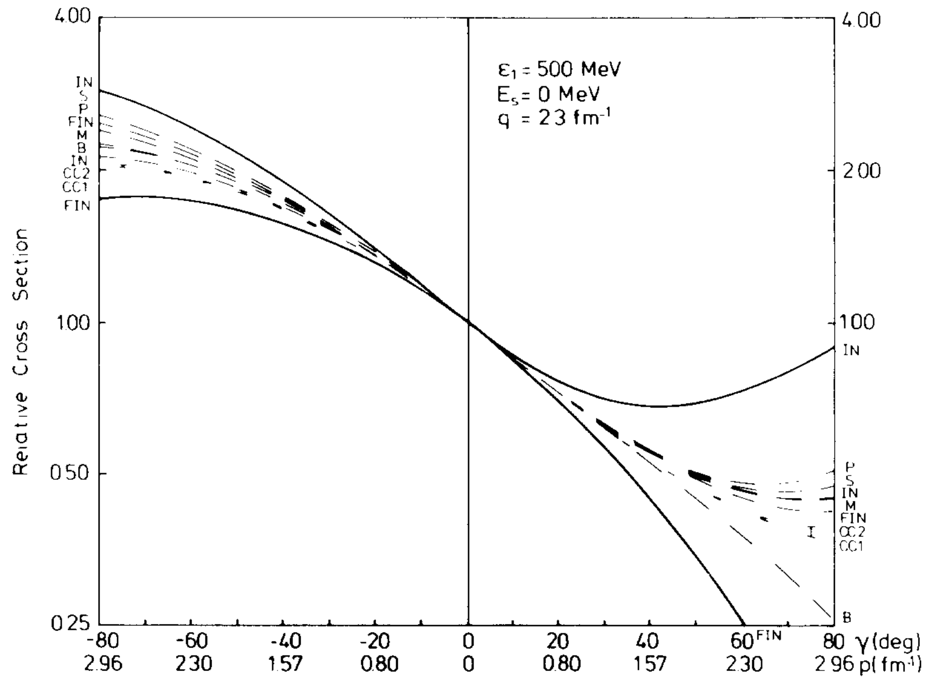

Fig. 2. Off-shell cross sections for scattering of 500 MeV electrons at 2.3 fm^{-1} momentum transfer and 2.3 fm^{-1} momentum for the detected proton, 0 MeV separation energy and in-plane ($\phi = 0$) perpendicular kinematics. γ is the angle between the momentum of the knocked-out proton, $\boldsymbol{p}'$, and the momentum transfer, $\boldsymbol{q}$, with the positive angles being in the forward direction (with respect to the incident beam). The short-dashed lines (connected by vertical lines) give the cross sections in category A, the long-dashed for B and the solid for C. Within a given category the different cross sections are distinguished by the corresponding labels in the margins. Due to space limitations these labels are not always directly adjacent to the curves. They are, however, listed in the same order.

for zero separation energy, this corresponds to on-shell scattering. Fig. 3 is the same, but with a separation energy of 60 MeV. As expected the ambiguities increase both with increasing initial momentum and separation energy. Since it is rather easy to extrapolate from the case of a separation energy of 60 MeV to 0 MeV, in the following we only consider the former.

Figs. 4 and 5 are for the same q as in fig. 3, but for lower and higher electron energies. As the energy increases, there is a tendency of certain cross sections to converge, but only to a limited extent: the results shown in fig. 5 are essentially converged to the asymptotic limit. One consequence of this is that the cross sections in category C are now no worse than those of category B. The discrepancy between the cross sections in categories A and B, however, remains about the same at 15 GeV as it was at 500 MeV. Thus there is really no advantage to be gained by going to very high energies for fixed q. Going to low energies, on the other hand, leads to a definite increase in the ambiguities in the cross sections in categories B and C, and to a lesser extent in category A.

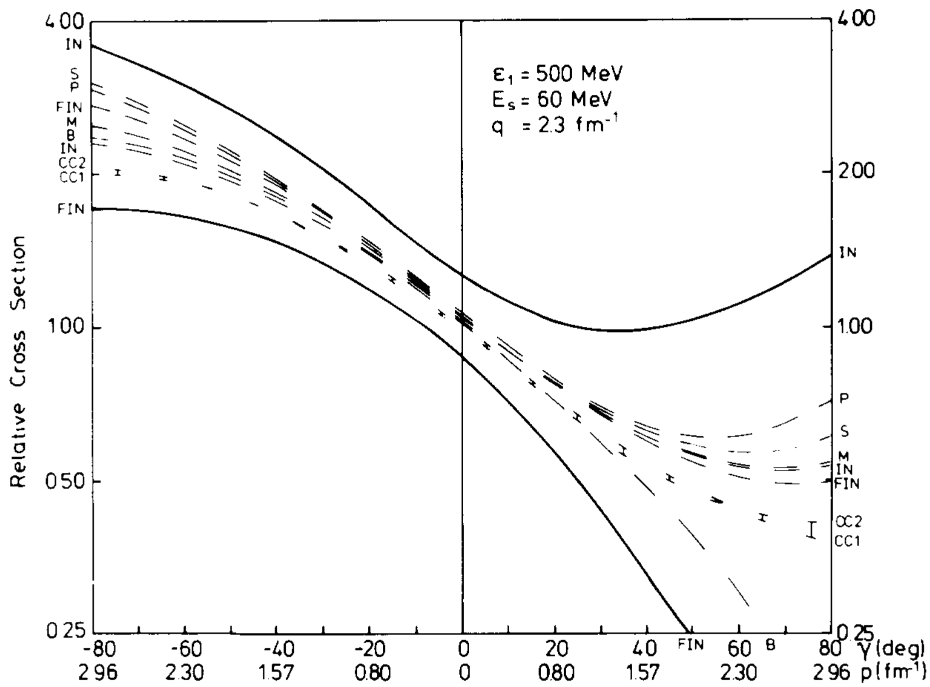

Fig. 3. Same as fig. 2, but for a separation energy of 60 MeV.

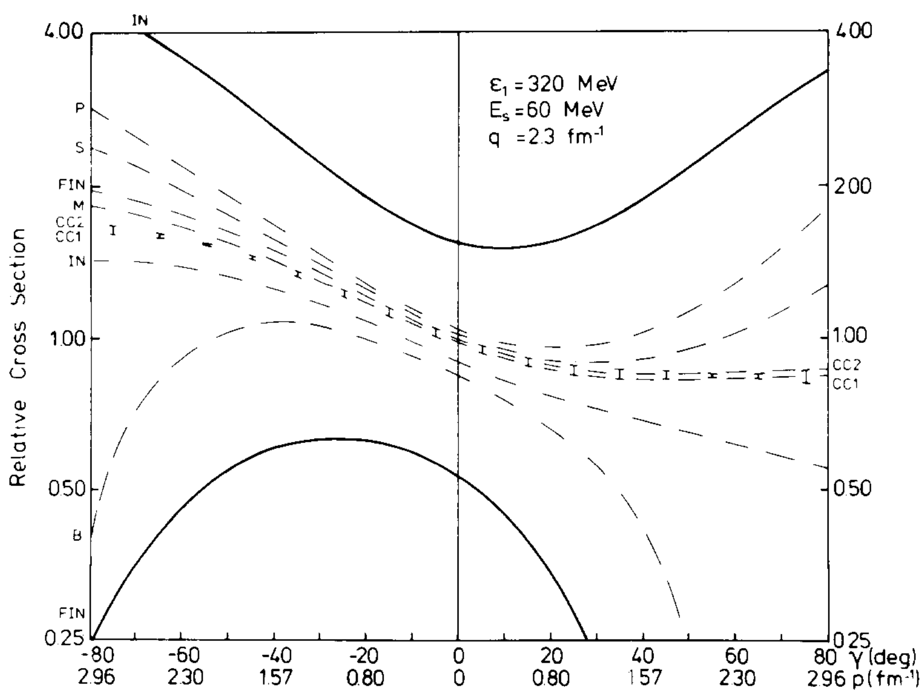

Fig. 4. Same as fig. 3, but for an incident electron energy of 320 MeV.

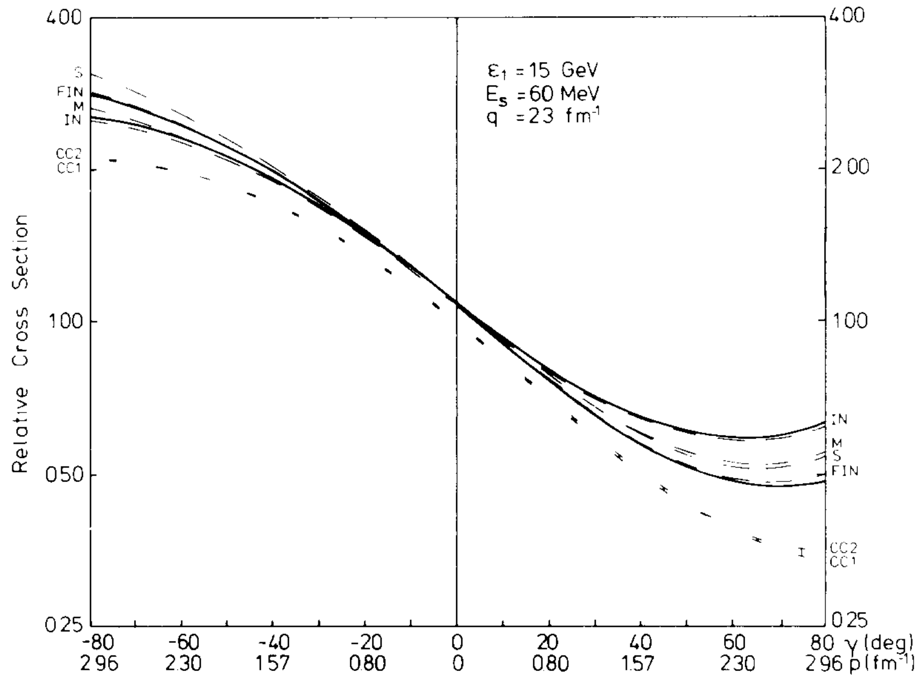

Fig. 5. Same as fig. 3, but with an incident electron energy of 15 GeV. *Note:* the labels IN and FIN in this figure apply to both of the adjacent curves in categories B and C. Also the results for σ^{P} and σ^{B} are virtually identical to those shown for σ_{B}^{FIN}.

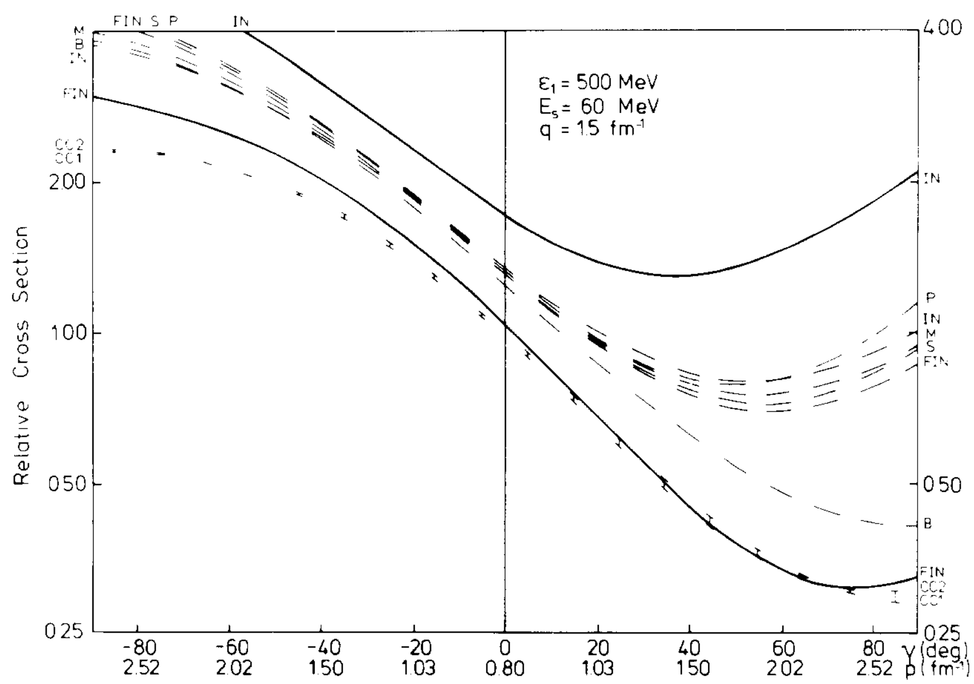

Fig. 6. Same as fig. 3, but for 1.5 fm^{-1} momentum transfer.

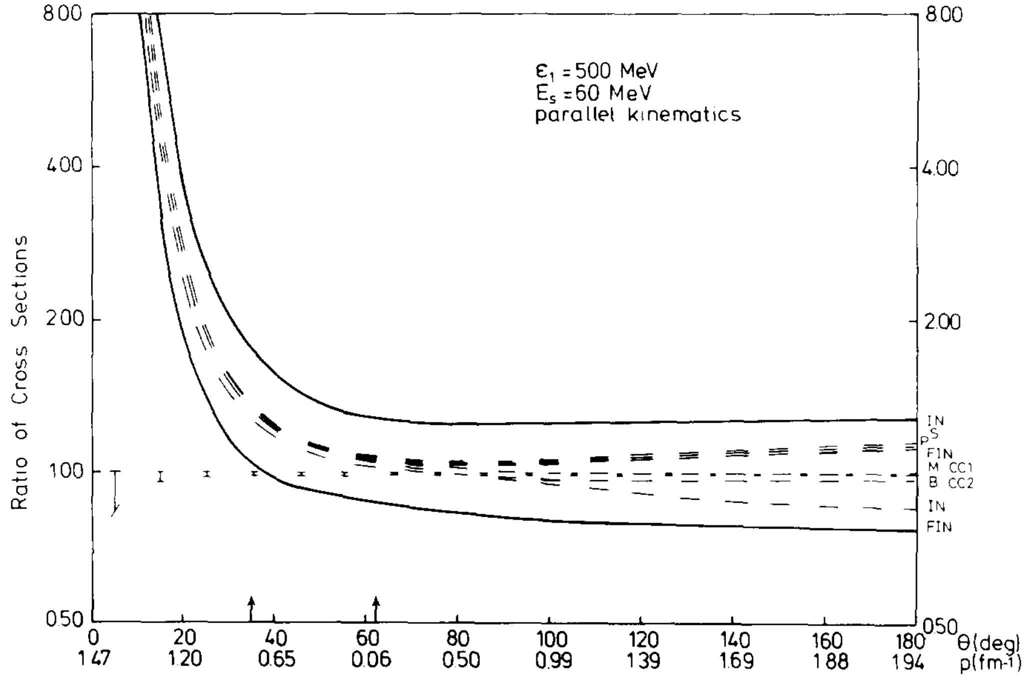

Fig. 7. Same as fig. 3, but for parallel kinematics and the ratio to σ_1^{cc} is plotted. The arrows correspond to the same kinematic conditions as given in figs. 3 and 6 at $\gamma = 0$. *Note:* the angle θ on the horizontal axis is the electron scattering angle.

Fig. 6 is again similar to fig. 3, but using a smaller momentum transfer, q. These kinematics provide a larger electron–proton cross section which is useful for studying the high-momentum components where the spectral function is small. One however misses the $p = 0$ point and therefore these kinematics are used in conjunction with those for fig. 3 [ref. [20])].

Fig. 7 shows the results for the so-called parallel kinematics in which one keeps E' constant and varies the angles of the electron and proton detectors such that $\boldsymbol{q}$ (whose magnitude varies) remains parallel to $\boldsymbol{p}'$. These kinematics were originally proposed to emphasize the effects of the final-state interactions and therefore provide an extra check on the analysis [21]). They intersect with the kinematics of fig. 3 and fig. 6. The results, indicated by small arrows, for $p = 0$ and $p = 0.8\ \text{fm}^{-1}$ are the same as for $\gamma = 0$ in fig. 3 and fig. 6, respectively. The divergence of the cross sections in categories B and C for small q_μ^2, mentioned earlier, is clearly seen. (Note: $q_\mu^2 = 4\varepsilon_1\varepsilon_2 \sin^2 \frac{1}{2}\theta$.) For the kinematics corresponding to fig. 6 this effect has already become important. This explains the large discrepancy between the cross sections of categories A and B in fig. 6. Also within category A, the ambiguity becomes relatively large, but not divergent, in the limit of the photon point. This is not too surprising for it is just in the limit $q_\mu^2 \to 0$ that one expects such effects as the difference between using q_μ or $\bar{q}_\mu$ to be large.

We have also investigated other kinematics, with no particularly striking results. Going from $q = 2.3\,\text{fm}^{-1}$ to $q = 10\,\text{fm}^{-1}$ at 15 GeV leads to a slight reduction in the ambiguity. Detecting the proton out of the scattering plane gives results that are very similar to those for in-plane detection.

We conclude with a few general remarks:

First, for the bulk of quasi-elastic scattering, since one is almost on-shell, it does not make too much difference which cross section one uses. On the other hand, for extracting the smaller, more interesting, parts of the spectral function, the high momentum and large missing energy components, the off-shell ambiguity can be quite large. The total ambiguity, i.e. when all cross sections are considered, is also strongly dependent on the energy of the incident electron. At the lower energies, one finds ambiguities of up to factors of two. For very high energies 10–20% is more typical. Though relatively small, this is still a large uncertainty for a "well-known" interaction. The ambiguity, however, can be minimized by a judicious choice of cross sections. To begin with, there is really no reason to ever use the cross sections $\sigma_{\mathrm{C}}^{\mathrm{IN}}$ or $\sigma_{\mathrm{C}}^{\mathrm{FIN}}$ since one knows that they do not have the correct form. Since the cross sections of category B behave incorrectly in the limit of the photon point, it would seem most reasonable, in general, to use only the cross sections in category A. For the two cross sections in this category that we have considered, at least, the resulting ambiguity is very small.

Several of the individual cross sections deserve special comment. In particular, two of the cross sections, $\sigma_{\mathrm{C}}^{\mathrm{FIN}}$ and σ^{B}, actually become negative in fig. 4. For the latter this can be traced to the complicated form of the cross section: the term $(p_\mu q_\mu)^2$ in the coefficient of $F_1\kappa F_2$ in eq. (26), which in the case of free scattering is cancelled by the term $-\frac{1}{4}q_\mu^2$, gives a negative contribution and dominates the cross section for large γ. For the case of $\sigma_{\mathrm{C}}^{\mathrm{FIN}}$, since it supposedly derives from a free cross section, this behavior is more surprising. The corresponding free cross section, however, turns out to be in an unphysical region. In addition, the cross section σ^{S}, which employs an effective mass, will obviously not be suitable at very high momenta since the effective mass becomes imaginary.

Concerning the choice of kinematics, those corresponding to fig. 3 (and fig. 2), which are typical of experiments that have been performed, are quite reasonable: the ambiguity is relatively small and very high energy electrons are not required. We note that it is preferable to work at negative γ, which is backwards in the lab frame, since one has the correlated advantages of a smaller ambiguity and a larger cross section. Actually, this is the region where the experiments have been performed for a much more practical reason: for positive γ the proton would have to be detected in the region of the outgoing electron beam. An additional advantage of negative γ is that the accidental rate is smaller. For the kinematics corresponding to fig. 4, which have also been used, the situation is not so favorable in that the overall (i.e. including cross sections in both categories A and B) ambiguity is quite large. However, if one confines oneself to using only the cross sections in category A, this is not a problem.

Although the results of sect. 3 have been presented as electron–nucleon cross sections, it is trivial to construct the corresponding nucleon currents. These models can thus also be applied, using eqs. (4), (5), in more sophisticated calculations such as DWIA.

Finally, it should be kept in mind that the ambiguity we have been considering is an ambiguity in an approximation, the impulse approximation. The advantage of having a small ambiguity is thus primarily one of convenience: if the prediction of the approximation were unique, one would not have to argue why a particular cross section was used or, alternatively, present the results as an 'error band'. A small ambiguity, however, does not necessarily imply that the approximation is a good one.

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