THE FORCE BETWEEN NUCLEONS

Even before describing any further experiments to study the force between two nucleons, we can already guess at a few of the properties of the nucleon–nucleon force:

1. At short distances it is stronger than the Coulomb force: the nuclear force can overcome the Coulomb repulsion of protons in the nucleus.
2. At long distances, of the order of atomic sizes, the nuclear force is negligibly feeble; the interactions among nuclei in a molecule can be understood based only on the Coulomb force.
3. Some particles are immune from the nuclear force; there is no evidence from atomic structure, for example, that electrons feel the nuclear force at all.

As we begin to do experiments specifically to explore the properties of the nuclear force, we find several other remarkable properties:

4. The nucleon–nucleon force seems to be nearly independent of whether the nucleons are neutrons or protons. This property is called *charge independence*.
5. The nucleon–nucleon force depends on whether the spins of the nucleons are parallel or antiparallel.
6. The nucleon–nucleon force includes a repulsive term, which keeps the nucleons at a certain average separation.
7. The nucleon–nucleon force has a noncentral or *tensor* component. This part of the force does not conserve orbital angular momentum, which is a constant of the motion under central forces.

In this chapter we explore these properties in detail, discuss how they are measured, and propose some possible forms for the basic nucleon–nucleon interaction.

4.1 THE DEUTERON

A *deuteron* (^{2}H nucleus) consists of a neutron and a proton. (A neutral atom of ^{2}H is called *deuterium*.) It is the simplest bound state of nucleons and therefore gives us an ideal system for studying the nucleon–nucleon interaction. For nuclear physicists, the deuteron should be what the hydrogen atom is for atomic physicists. Just as the measured Balmer series of electromagnetic transitions between the excited states of hydrogen led to an understanding of the structure of hydrogen, so should the electromagnetic transitions between the excited states of the deuteron lead to an understanding of its structure. Unfortunately, there are *no excited states* of the deuteron—it is such a weakly bound system that the only "excited states" are unbound systems consisting of a free proton and neutron.

Binding Energy

The binding energy of the deuteron is a very precisely measured quantity, which can be determined in three different ways. By spectroscopy, we can directly determine the mass of the deuteron, and we can use Equation 3.25 to find the binding energy. Using the mass doublet method described in Section 3.2, the following determinations have been made (we use the symbol D for ^{2}H):

$$m(C_6H_{12}) - m(C_6D_6) = (9.289710 \pm 0.000024) \times 10^{-3}\ \mathrm{u}$$

and

$$m(C_5D_{12}) - m(C_6D_6) = (84.610626 \pm 0.000090) \times 10^{-3}\ \mathrm{u}.$$

From the first difference we find, using 1.007825037 u for the ^{1}H mass,

$$m(^2\mathrm{H}) = 2.014101789 \pm 0.000000021\ \mathrm{u}$$

and from the second,

$$m(^2\mathrm{H}) = 2.014101771 \pm 0.000000015\ \mathrm{u}$$

These precise values are in very good agreement, and using the measured ^{1}H and neutron masses we can find the binding energy

$$B = \left[m(^1\mathrm{H}) + m(\mathrm{n}) - m(^2\mathrm{H})\right]c^2 = 2.22463 \pm 0.00004\ \mathrm{MeV}$$

We can also determine this binding energy directly by bringing a proton and a neutron together to form ^{2}H and measuring the energy of the γ-ray photon that is emitted:

$$^1\mathrm{H} + \mathrm{n} \rightarrow {}^2\mathrm{H} + \gamma$$

The deduced binding energy, which is equal to the observed energy of the photon less a small recoil correction, is 2.224589 ± 0.000002 MeV, in excellent agreement with the mass spectroscopic value. A third method uses the reverse reaction, called *photodissociation*,

$$\gamma + {}^2\mathrm{H} \rightarrow {}^1\mathrm{H} + \mathrm{n}$$

in which a γ-ray photon breaks apart a deuteron. The minimum γ-ray energy that accomplishes this process is equal to the binding energy (again, corrected for the recoil of the final products). The observed value is 2.224 ± 0.002 MeV, in good agreement with the mass spectroscopic value.

As we discussed in Section 3.3, the average binding energy per nucleon is about 8 MeV. The deuteron is therefore very weakly bound compared with typical nuclei. Let's see how we can analyze this result to study the properties of the deuteron.

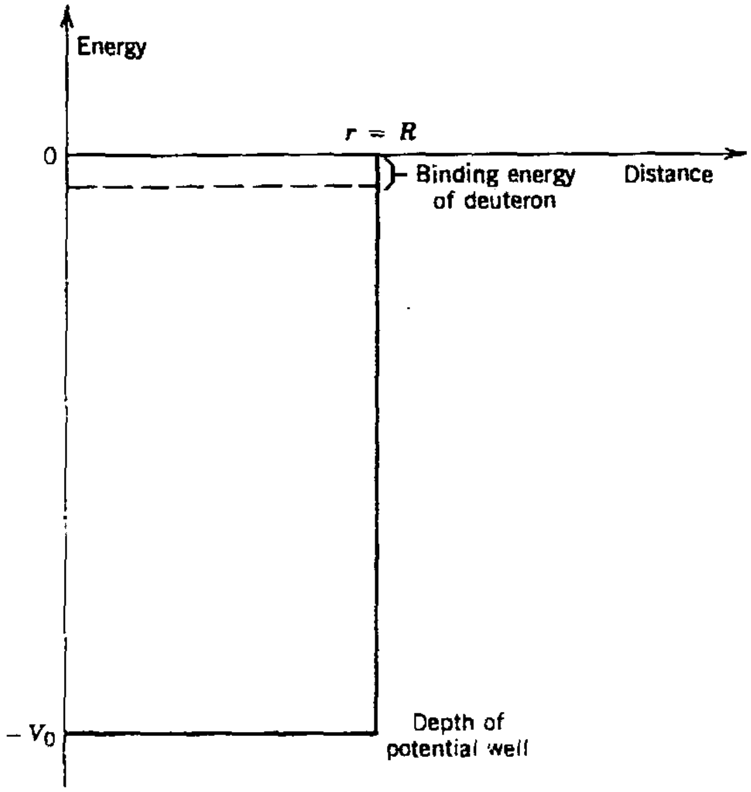

Figure 4.1 The spherical square-well potential is an approximation to the nuclear potential. The depth is $-V_0$, where V_0 is deduced to be about 35 MeV. The bound state of the deuteron, at an energy of about -2 MeV, is very close to the top of the well.

To simplify the analysis of the deuteron, we will assume that we can represent the nucleon–nucleon potential as a three-dimensional square well, as shown in Figure 4.1:

$$\begin{aligned} V(r) &= -V_0 \qquad \text{for } r < R \\ &= 0 \qquad \text{for } r > R \end{aligned} \tag{4.1}$$

This is of course an oversimplification, but it is sufficient for at least some qualitative conclusions. Here r represents the separation between the proton and the neutron, so R is in effect a measure of the *diameter* of the deuteron. Let's assume that the lowest energy state of the deuteron, just like the lowest energy state of the hydrogen atom, has $\ell = 0$. (We justify this assumption later in this section when we discuss the spin of the deuteron.) If we define the radial part of $\psi(r)$ as $u(r)/r$, then we can rewrite Equation 2.60 as

$$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2} + V(r)u(r) = Eu(r) \tag{4.2}$$

This expression looks exactly like the one-dimensional Equation 2.4, and the solutions can be written in analogy with Equations 2.47. For $r < R$,

$$u(r) = A\sin k_1 r + B\cos k_1 r \tag{4.3}$$

with $k_1 = \sqrt{2m(E + V_0)/\hbar^2}$, and for $r > R$,

$$u(r) = Ce^{-k_2 r} + De^{+k_2 r} \tag{4.4}$$

with $k_2 = \sqrt{-2mE/\hbar^2}$. (Remember, $E < 0$ for bound states.) To keep the wave

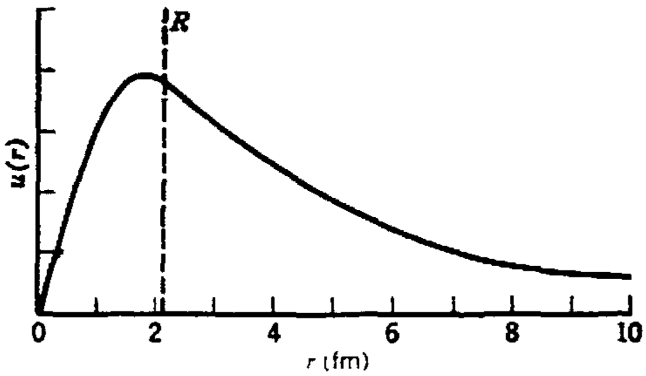

Figure 4.2 The deuteron wave function for $R = 2.1$ fm. Note how the exponential joins smoothly to the sine at $r = R$, so that both $u(r)$ and du/dr are continuous. If the wave function did not "turn over" inside $r = R$, it would not be possible to connect smoothly to a decaying exponential (negative slope) and there would be no bound state.

function finite for $r \to \infty$ we must have $D = 0$, and to keep it finite for $r \to 0$ we must have $B = 0$. (ψ depends on $u(r)/r$; as $r \to 0$, $u(r)$ also must go to zero.) Applying the continuity conditions on u and du/dr at $r = R$, we obtain

$$k_1 \cot k_1 R = -k_2 \tag{4.5}$$

This transcendental equation gives a relationship between V_0 and R. From electron scattering experiments, the rms charge radius of the deuteron is known to be about 2.1 fm, which provides a reasonable first estimate for R. Solving Equation 4.5 numerically (see Problem 6 at the end of this chapter) the result is $V_0 = 35$ MeV. This is actually quite a reasonable estimate of the strength of the nucleon–nucleon potential, even in more complex nuclei. (Note, however, that the proton and neutron are very likely to be found at separations greater than R; see Problem 4.)

We can see from Figure 4.1 how close the deuteron is to the top of the well. If the nucleon–nucleon force were just a bit weaker, the deuteron bound state would not exist (see Problem 3). We are fortunate that it does, however, because the formation of deuterium from hydrogen is the first step not only in the proton-proton cycle of fusion by which our sun makes its energy, but also in the formation of stable matter from the primordial hydrogen that filled the early universe. If no stable two-nucleon bound state existed, we would not be here to discuss it! (For more on the cosmological consequences of the formation of deuterium in the early universe, see Chapter 19.)

The deuteron wave function is shown in Figure 4.2. The weak binding means that $\psi(r)$ is just barely able to "turn over" in the well so as to connect at $r = R$ with the negative slope of the decaying exponential.

Spin and Parity

The total angular momentum I of the deuteron should have three components: the individual spins s_n and s_p of the neutron and proton (each equal to $\frac{1}{2}$), and the orbital angular momentum ℓ of the nucleons as they move about their common center of mass:

$$I = s_n + s_p + \ell \tag{4.6}$$

When we solved the Schrödinger equation for the deuteron, we assumed $\ell = 0$ in analogy with the lowest bound state (the 1s state) in atomic hydrogen. The measured spin of the deuteron is $I = 1$ (how this is measured is discussed in Chapter 16). Since the neutron and proton spins can be either parallel (for a total of 1) or antiparallel (for a total of zero), there are four ways to couple s_n, s_p, and ℓ to get a total I of 1:

(a) s_n and s_p parallel with $\ell = 0$.
(b) s_n and s_p antiparallel with $\ell = 1$.
(c) s_n and s_p parallel with $\ell = 1$.
(d) s_n and s_p parallel with $\ell = 2$.

Another property of the deuteron that we can determine is its *parity* (even or odd), the behavior of its wave function when $r \to -r$ (see Section 2.6). By studying the reactions involving deuterons and the property of the photon emitted during the formation of deuterons, we know that its parity is even. In Section 2.6 we discussed that the parity associated with orbital motion is $(-1)^\ell$, even parity for $\ell = 0$ (s states) and $\ell = 2$ (d states) and odd parity for $\ell = 1$ (p states). The observed even parity allows us to eliminate the combinations of spins that include $\ell = 1$, leaving $\ell = 0$ and $\ell = 2$ as possibilities. The spin and parity of the deuteron are therefore consistent with $\ell = 0$ as we assumed, but of course we cannot yet exclude the possibility of $\ell = 2$.

Magnetic Dipole Moment

In Section 3.5 we discussed the spin and orbital contributions to the magnetic dipole moment. If the $\ell = 0$ assumption is correct, there should be no orbital contribution to the magnetic moment, and we can assume the total magnetic moment to be simply the combination of the neutron and proton magnetic moments:

$$\begin{aligned} \boldsymbol{\mu} &= \boldsymbol{\mu}_n + \boldsymbol{\mu}_p \\ &= \frac{g_{sn}\mu_N}{\hbar}\mathbf{s}_n + \frac{g_{sp}\mu_N}{\hbar}\mathbf{s}_p \end{aligned} \tag{4.7}$$

where $g_{sn} = -3.826084$ and $g_{sp} = 5.585691$. As we did in Section 3.5, we take the observed magnetic moment to be the z component of $\boldsymbol{\mu}$ when the spins have their maximum value $(+\frac{1}{2}\hbar)$:

$$\begin{aligned} \mu &= \tfrac{1}{2}\mu_N(g_{sn} + g_{sp}) \\ &= 0.879804\ \mu_N \end{aligned} \tag{4.8}$$

The observed value is $0.8574376 \pm 0.0000004\ \mu_N$, in good but not quite exact agreement with the calculated value. The small discrepancy can be ascribed to any of a number of factors, such as contributions from the mesons exchanged between the neutron and proton; in the context of the present discussion, we can assume the discrepancy to arise from a small mixture of d state ($\ell = 2$) in the deuteron wave function:

$$\psi = a_s\psi(\ell = 0) + a_d\psi(\ell = 2) \tag{4.9}$$

Calculating the magnetic moment from this wave function gives

$$\mu = a_s^2\,\mu(\ell = 0) + a_d^2\,\mu(\ell = 2) \tag{4.10}$$

where $\mu(\ell = 0)$ is the value calculated in Equation 4.8 and $\mu(\ell = 2) = \frac{1}{4}(3 - g_{sp} - g_{sn})\mu_N$ is the value calculated for a d state. The observed value is consistent with $a_s^2 = 0.96$, $a_d^2 = 0.04$; that is, the deuteron is 96% $\ell = 0$ and only 4% $\ell = 2$. The assumption of the pure $\ell = 0$ state, which we made in calculating the well depth, is thus pretty good but not quite exact.

Electric Quadrupole Moment

The bare neutron and proton have no electric quadrupole moment, and so any measured nonzero value for the quadrupole moment must be due to the orbital motion. Thus the pure $\ell = 0$ wave function would have a vanishing quadrupole moment. The observed quadrupole moment is

$$Q = 0.00288 \pm 0.00002 \text{ b}$$

which, while small by comparison with many other nuclei, is certainly not zero.

The mixed wave function of Equation 4.9, when used as in Equation 3.36 to evaluate Q, gives two contributions, one proportional to a_d^2 and another proportional to the cross-term $a_s a_d$. Performing the calculation we obtain

$$Q = \frac{\sqrt{2}}{10} a_s a_d \langle r^2 \rangle_{sd} - \frac{1}{20} a_d^2 \langle r^2 \rangle_{dd} \tag{4.11}$$

where $\langle r^2 \rangle_{sd} = \int r^2 R_s(r) R_d(r) r^2\,dr$ is the integral of r^2 over the radial wave functions; $\langle r^2 \rangle_{dd}$ is similarly defined. To calculate Q we must know the deuteron d-state wave function, which is not directly measurable. Using the realistic phenomenological potentials discussed later in this chapter gives reasonable values for Q with d-state admixtures of several percent, consistent with the value of 4% deduced from the magnetic moment.

This good agreement between the d-state admixtures deduced from μ and Q should be regarded as a happy accident and not taken too seriously. In the case of the magnetic dipole moment, there is no reason to expect that it is correct to use the free-nucleon magnetic moments in nuclei. (In fact, in the next chapter we see that there is strong evidence to the contrary.) Unfortunately, a nucleon in a deuteron lies somewhere between a free nucleon and a strongly bound nucleon in a nucleus, and we have no firm clues about what values to take for the magnetic moments. Spin-orbit interactions, relativistic effects, and meson exchanges may have greater effects on μ than the d-state admixture (but may cancel one another's effects). For the quadrupole moment, the poor knowledge of the d-state wave function makes the deduced d-state admixture uncertain. (It would probably be more valid to regard the calculation of Q, using a known d-state mixture, as a test of the d-state wave function.) Other experiments, particularly scattering experiments using deuterons as targets, also give d-state admixtures in the range of 4%. Thus our conclusions from the magnetic dipole and electric quadrupole moments may be valid after all!

It is important that we have an accurate knowledge of the d-state wave function because the mixing of ℓ values in the deuteron is the best evidence we have for the noncentral (tensor) character of the nuclear force.

4.2 NUCLEON-NUCLEON SCATTERING

Although the study of the deuteron gives us a number of clues about the nucleon–nucleon interaction, the total amount of information available is limited. Because there are no excited states, we can only study the dynamics of the nucleon–nucleon interaction in the configuration of the deuteron: $\ell = 0$, parallel spins, 2-fm separation. (Excited states, if they were present, might have different ℓ values or spin orientations.) To study the nucleon–nucleon interaction in different configurations, we can perform nucleon–nucleon scattering experiments, in which an incident beam of nucleons is scattered from a target of nucleons. If the target is a nucleus with many nucleons, then there will be several target nucleons within the range of the nuclear potential of the incident nucleon; in this case the observed scattering of a single nucleon will include the complicated effects of multiple encounters, making it very difficult to extract the properties of the interaction between individual nucleons. We therefore select a target of hydrogen so that incident particles can scatter from the individual protons. (It is still possible to have multiple scattering, but in this case it must occur through scattering first from one proton, then from another that is quite far from the first on the scale of nuclear dimensions. If the probability for a single encounter is small, the probability for multiple encounters will be negligible. This is very different from the case of scattering from a heavier nucleus, in which each single encounter with a target nucleus consists of many nucleon–nucleon interactions.)

Before we discuss the nuclear scattering problem, let's look at an analogous problem in optics, the diffraction of waves at a small slit or obstacle, as shown in Figure 4.3. The diffraction pattern produced by an obstacle is very similar to that produced by a slit of the same size. Nuclear scattering more resembles diffraction by the obstacle, so we will concentrate our discussion on it. There are three features of the optical diffraction that are analogous to the scattering of nucleons:

1. The incident wave is represented by a plane wave, while far from the obstacle the scattered wave fronts are spherical. The total energy content of any expanding spherical wave front cannot vary; thus its intensity (per unit area) must decrease like r^{-2} and its amplitude must decrease like r^{-1}.
2. Along the surface of any spherical scattered wave front, the diffraction is responsible for a variation in intensity of the radiation. The intensity thus depends on angular coordinates θ and ϕ.
3. A radiation detector placed at any point far from the obstacle would record both incident and scattered waves.

To solve the nucleon–nucleon scattering problem using quantum mechanics, we will again assume that we can represent the interaction by a square-well potential, as we did in the previous section for the deuteron. In fact, the only difference between this calculation and that of the deuteron is that we are concerned with free incident particles with $E > 0$. We will again simplify the Schrödinger equation by assuming $\ell = 0$. The justification for this assumption has nothing to do with that of the identical assumption made in the calculation for the deuteron. Consider an incident nucleon striking a target nucleon just at its surface; that is, the *impact parameter* (the perpendicular distance from the center of the target nucleon to the line of flight of the incident nucleon) is of the order of $R \simeq 1$ fm. If the incident particle has velocity v, its angular momentum

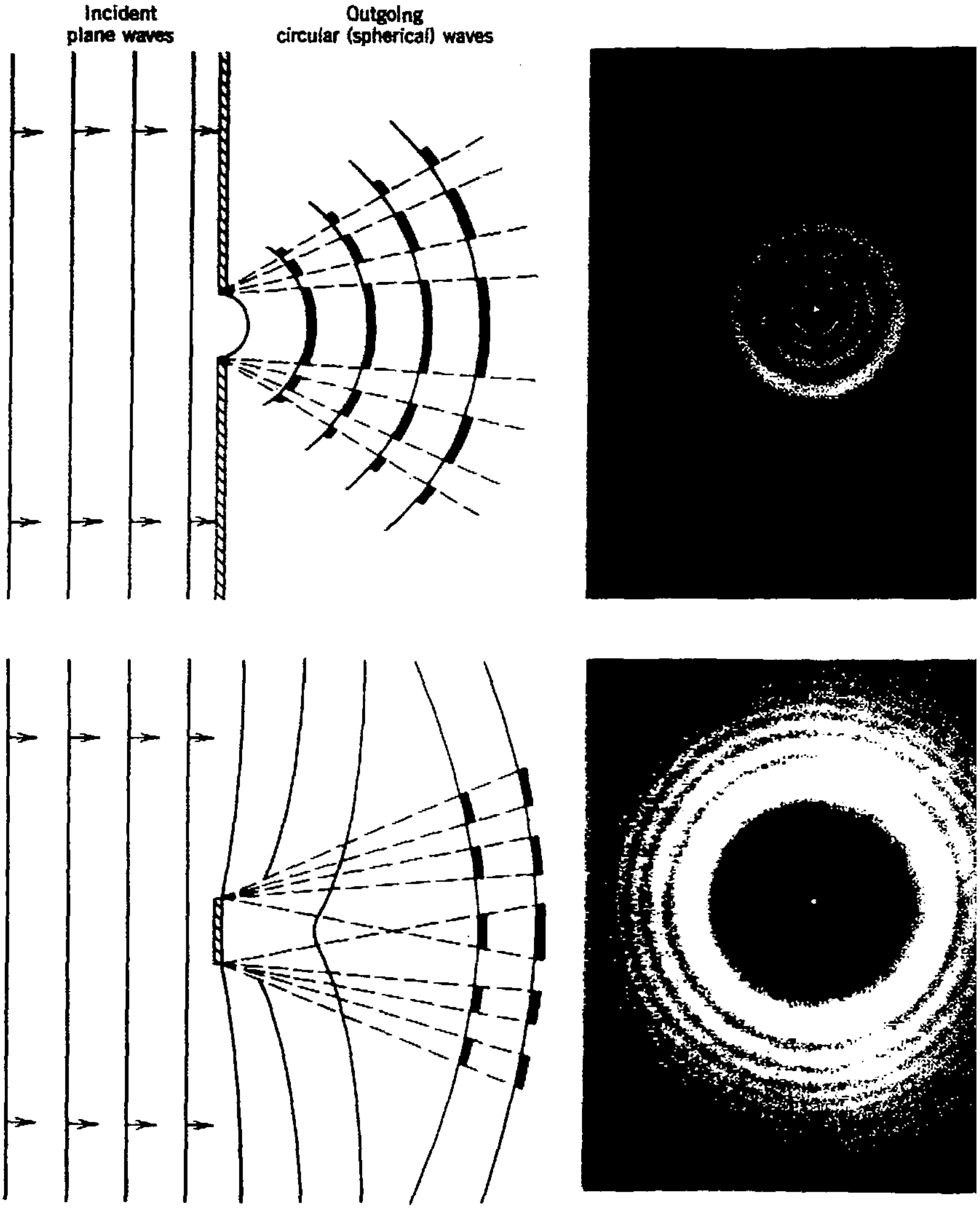

Figure 4.3 Representation of scattering by (top) a small opening and (bottom) a small obstacle. The shading of the wavefronts shows regions of large and small intensity. On the right are shown photographs of diffraction by a circular opening and an opaque circular disk. Source of photographs: M. Cagnet, M. Francon, and J. C. Thrierr, *Atlas of Optical Phenomena* (Berlin: Springer-Verlag, 1962).

relative to the target is mvR. The relative angular momentum between the nucleons must be quantized in units of $\hbar$; that is, $mvR = \ell\hbar$ in semiclassical notation. If $mvR \ll \hbar$, then only $\ell = 0$ interactions are likely to occur. Thus $v \ll \hbar/mR$ and the corresponding kinetic energy is estimated as

$$T = \tfrac{1}{2}mv^2 \ll \frac{\hbar^2}{2mR^2} = \frac{\hbar^2c^2}{2mc^2R^2} = \frac{(200\ \mathrm{MeV}\cdot\mathrm{fm})^2}{2(1000\ \mathrm{MeV})(1\ \mathrm{fm})^2} = 20\ \mathrm{MeV}$$

If the incident energy is far below 20 MeV, the $\ell = 0$ assumption is justified. We will consider only low-energy scattering, for which the $\ell = 0$ assumption is valid.

The nucleon–nucleon scattering problem will be solved in the center-of-mass coordinate system (see Appendix B). The mass appearing in the Schrödinger equation is the reduced mass, which in this case is about half the nucleon mass.

The solution to the square-well problem for $r < R$ is given by Equation 4.3; as before, $B = 0$ in order that $u(r)/r$ remain finite for $r \to 0$. For $r > R$, the wave function is

$$u(r) = C' \sin k_2 r + D' \cos k_2 r \tag{4.12}$$

with $k_2 = \sqrt{2mE/\hbar^2}$. It is convenient to rewrite Equation 4.12 as

$$u(r) = C \sin(k_2 r + \delta) \tag{4.13}$$

where

$$C' = C \cos \delta \quad \text{and} \quad D' = C \sin \delta \tag{4.14}$$

The boundary conditions on u and du/dr at $r = R$ give

$$C \sin(k_2 R + \delta) = A \sin k_1 R \tag{4.15}$$

and

$$k_2 C \cos(k_2 R + \delta) = k_1 A \cos k_1 R \tag{4.16}$$

Dividing,

$$k_2 \cot(k_2 R + \delta) = k_1 \cot k_1 R \tag{4.17}$$

Again, we have a transcendental equation to solve: given E (which we control through the energy of the incident particle), V_0, and R, we can in principle solve for δ.

Before we discuss the methods for extracting the parameter δ from Equation 4.17, we examine how δ enters the solution to the Schrödinger equation. As $V_0 \to 0$ (in which case no scattering occurs), $k_1 \to k_2$ and $\delta \to 0$. This is just the free particle solution. The effect of V_0 on the wave function is indicated in Figure 4.4. The wave function at $r > R$ has the same form as the free particle, but it has experienced a *phase shift* δ. The nodes (zeros) of the wave function are "pulled" toward the origin by the attractive potential. (A repulsive potential would "push" the nodes away from the origin and would give a negative phase shift.) We can analyze the incident waves into components according to their angular momentum relative to the target: $\ell = 0$ (which we have been considering so far), $\ell = 1$, and so on. Associated with each ℓ there will be a different solution to the Schrödinger equation and a different phase shift δ_ℓ.

Let us see how our square-well problem relates to more general scattering theory. The incident wave is (as in the optical analogy) a plane wave traveling in the z direction:

$$\psi_{\text{incident}} = A e^{ikz} \tag{4.18}$$

Let the target be located at the origin. Multiplying by the time-dependent factor gives

$$\psi(z, t) = A e^{i(kz - \omega t)} \tag{4.19}$$

which always moves in the $+z$ direction (toward the target for $z < 0$ and away

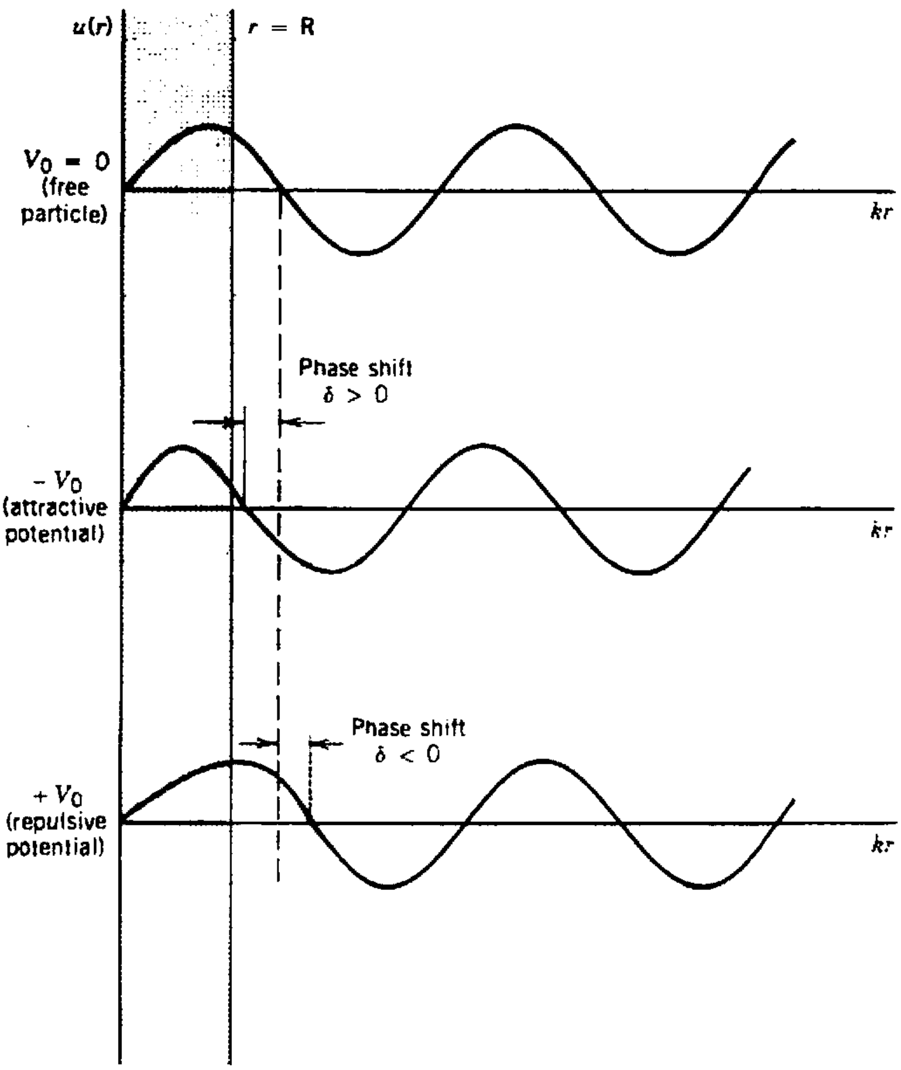

Figure 4.4 The effect of a scattering potential is to shift the phase of the scattered wave at points beyond the scattering regions, where the wave function is that of a free particle.

from it for $z > 0$). It is mathematically easier to work with spherical waves e^{ikr}/r and e^{-ikr}/r, and multiplying with $e^{-i\omega t}$ shows that e^{ikr} gives an outgoing wave and e^{-ikr} gives an incoming wave. (A more rigorous treatment of scattering theory, including terms with $\ell > 0$, is given in Chapter 11.) For $\ell = 0$ we can take

$$\psi_{\text{incident}} = \frac{A}{2ik}\left[\frac{e^{ikr}}{r} - \frac{e^{-ikr}}{r}\right] \tag{4.20}$$

The minus sign between the two terms keeps ψ finite for $r \to 0$, and using the coefficient A for both terms sets the amplitudes of the incoming and outgoing waves to be equal. We assume that the scattering does not create or destroy particles, and thus the scattering cannot change the amplitudes of the e^{ikr} or e^{-ikr} terms (since the squared amplitudes give the probabilities to detect incoming or outgoing particles). *All that can result from the scattering is a change in phase of the outgoing wave*:

$$\psi(r) = \frac{A}{2ik}\left[\frac{e^{i(kr+\beta)}}{r} - \frac{e^{-ikr}}{r}\right] \tag{4.21}$$

where β is the change in phase.

Manipulation of Equation 4.13 gives the relationship between β and δ_0:

$$
\begin{aligned}
\psi(r) &= \frac{C}{r}\sin(kr+\delta_0) \\
&= \frac{C}{r}\,\frac{e^{i(kr+\delta_0)} - e^{-i(kr+\delta_0)}}{2i} \\
&= \frac{C}{2i}e^{-i\delta_0}\left[\frac{e^{i(kr+2\delta_0)}}{r} - \frac{e^{-ikr}}{r}\right]
\end{aligned}
\tag{4.22}
$$

Thus $\beta = 2\delta_0$ and $A = kCe^{-i\delta_0}$.

To evaluate the probability for scattering, we need the amplitude of the scattered wave. The wave function ψ represents all waves in the region $r > R$, and to find the amplitude of only the scattered wave we must subtract away the incident amplitude:

$$
\begin{aligned}
\psi_{\text{scattered}} &= \psi - \psi_{\text{incident}} \\
&= \frac{A}{2ik}(e^{2i\delta_0} - 1)\frac{e^{ikr}}{r}
\end{aligned}
\tag{4.23}
$$

The current of scattered particles per unit area can be found using Equation 2.12 extended to three dimensions:

$$
j_{\text{scattered}} = \frac{\hbar}{2mi}\left(\psi^*\frac{\partial\psi}{\partial r} - \frac{\partial\psi^*}{\partial r}\psi\right) \tag{4.24}
$$

$$
= \frac{\hbar|A|^2}{mkr^2}\sin^2\delta_0 \tag{4.25}
$$

and the incident current is, in analogy with Equation 2.22

$$
j_{\text{incident}} = \frac{\hbar k|A|^2}{m} \tag{4.26}
$$

The scattered current is uniformly distributed over a sphere of radius r. An element of area $r^2\,d\Omega$ on that sphere subtends a solid angle $d\Omega = \sin\theta\,d\theta\,d\phi$ at the scattering center; see Figure 4.5. The *differential cross section* $d\sigma/d\Omega$ is the probability per unit solid angle that an incident particle is scattered into the solid angle $d\Omega$; the probability $d\sigma$ that an incident particle is scattered into $d\Omega$ is the ratio of the scattered current through $d\Omega$ to the incident current:

$$
d\sigma = \frac{(j_{\text{scattered}})(r^2\,d\Omega)}{j_{\text{incident}}} \tag{4.27}
$$

Using Equations 4.25 and 4.26 for the scattered and incident currents, we obtain

$$
\frac{d\sigma}{d\Omega} = \frac{\sin^2\delta_0}{k^2} \tag{4.28}
$$

The *total cross section* σ is the total probability to be scattered in any direction:

$$
\sigma = \int\frac{d\sigma}{d\Omega}\,d\Omega \tag{4.29}
$$

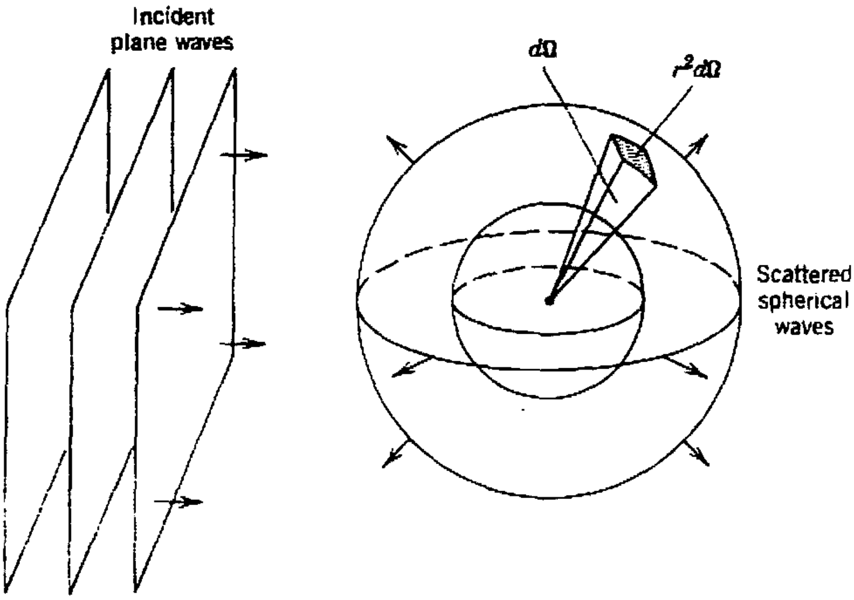

Figure 4.5 The basic geometry of scattering.

In general, $d\sigma/d\Omega$ varies with direction over the surface of the sphere; in the special case of $\ell = 0$ scattering, $d\sigma/d\Omega$ is constant and comes out of the integral:

$$\sigma = 4\pi \frac{d\sigma}{d\Omega}$$

$$= \frac{4\pi \sin^2 \delta_0}{k^2} \tag{4.30}$$

Thus the $\ell = 0$ phase shift is directly related to the probability for scattering to occur. That is, we can evaluate δ_0 from our simple square-well model, Equation 4.17, find the total cross section from Equation 4.30, and compare with the experimental cross section.

We now return to the analysis of Equation 4.17. Let us assume the incident energy is small, say $E \leq 10$ keV. Then $k_1 = \sqrt{2m(V_0 + E)/\hbar^2} = 0.92\ \text{fm}^{-1}$, with $V_0 = 35$ MeV from our analysis of the deuteron bound state, and $k_2 = \sqrt{2mE/\hbar^2} \leq 0.016\ \text{fm}^{-1}$. If we let the right side of Equation 4.17 equal $-\alpha$,

$$\alpha = -k_1 \cot k_1 R \tag{4.31}$$

then a bit of trigonometric manipulation gives

$$\sin^2 \delta_0 = \frac{\cos k_2 R + (\alpha/k_2) \sin k_2 R}{1 + \alpha^2/k_2^2} \tag{4.32}$$

and so

$$\sigma = \frac{4\pi}{k_2^2 + \alpha^2}\left(\cos k_2 R + \frac{\alpha}{k_2} \sin k_2 R\right) \tag{4.33}$$

Using $R \approx 2$ fm from the study of the ^{2}H bound state gives $\alpha = 0.2\ \text{fm}^{-1}$. Thus $k_2^2 \ll \alpha^2$ and $k_2 R \ll 1$, giving

$$\sigma = \frac{4\pi}{\alpha^2}(1 + \alpha R) = 4.6\ \text{b} \tag{4.34}$$

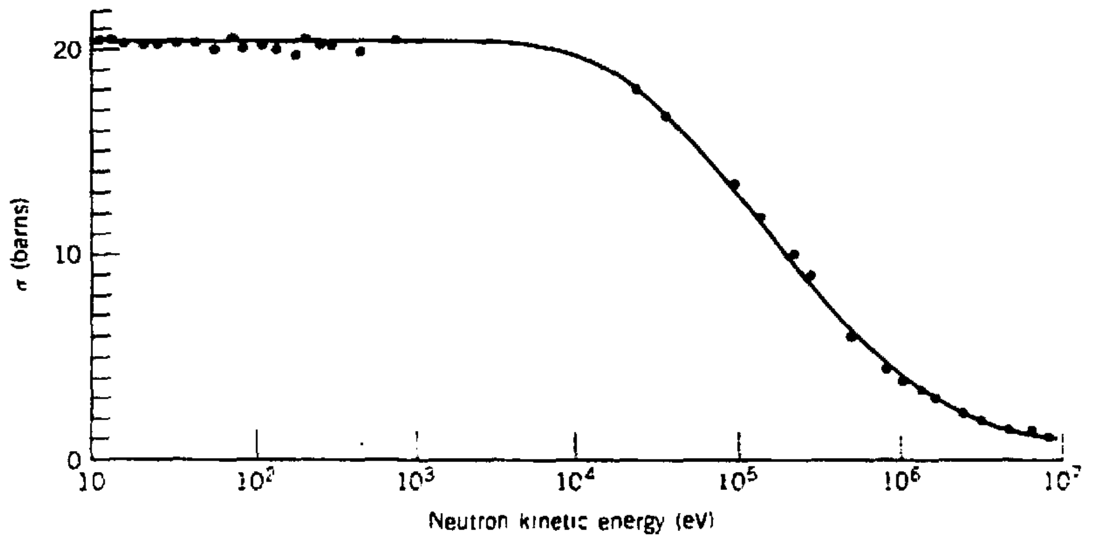

Figure 4.6 The neutron-proton scattering cross section at low energy. Data taken from a review by R. K. Adair, *Rev. Mod. Phys.* **22**, 249 (1950), with additional recent results from T. L. Houk, *Phys. Rev. C* **3**, 1886 (1970).

where 1 barn (b) $= 10^{-28}$ m^2. This result suggests that the cross section should be constant at low energy and should have a value close to 4–5 b.

Figure 4.6 shows the experimental cross sections for scattering of neutrons by protons. The cross section is indeed constant at low energy, and it decreases with E at large energy as Equation 4.33 predicts, but the low-energy cross section, 20.4 b, is not in agreement with our calculated value of 4–5 b.

For the solution to this discrepancy, we must study the relative spins of the incident and scattered nucleons. The proton and neutron spins (each $\frac{1}{2}$) can combine to give a total spin $S = s_p + s_n$ that can have magnitude either 0 or 1. The $S = 1$ combination has three orientations (corresponding to z components $+1, 0, -1$) and the $S = 0$ combination has only a single orientation. For that reason, the $S = 1$ combination is called a *triplet* state and the $S = 0$ combination is called a *singlet* state. Of the four possible relative spin orientations, three are associated with the triplet state and one with the singlet state. As the incident nucleon approaches the target, the probability of being in a triplet state is 3/4 and the probability of being in a singlet state is 1/4. If the scattering cross section is different for the singlet and triplet states, then

$$\sigma = \tfrac{3}{4}\sigma_t + \tfrac{1}{4}\sigma_s \tag{4.35}$$

where σ_t and σ_s are the cross sections for scattering in the triplet and singlet states, respectively. In estimating the cross section in Equation 4.34, we used parameters obtained from the deuteron, which is in a $S = 1$ state. We therefore take $\sigma_t = 4.6$ b and using the measured value of $\sigma = 20.4$ b for the low-energy cross section, we deduce

$$\sigma_s = 67.8 \text{ b}$$

This calculation indicates that there is an enormous difference between the cross sections in the singlet and triplet states—that is, *the nuclear force must be spin dependent.*

Even from our investigation of the deuteron, we should have concluded that the force is spin dependent. If the neutron–proton force did *not* depend on the relative direction of the spins, then we would expect to find deuteron bound states with $S = 0$ and $S = 1$ at essentially the same energy. Because we find no $S = 0$ bound state, we conclude that the force must be spin dependent.

We can verify our conclusions about the singlet and triplet cross sections in a variety of ways. One method is to scatter very low energy neutrons from hydrogen *molecules*. Molecular hydrogen has two forms, known as orthohydrogen and parahydrogen. In orthohydrogen the two proton spins are parallel, while in parahydrogen they are antiparallel. The difference between the neutron scattering cross sections of ortho- and parahydrogen is evidence of the spin-dependent part of the nucleon–nucleon force.

Our discussion of the cross section for neutron–proton scattering is inadequate for analysis of scattering of neutrons from H_2 molecules. Very low energy neutrons ($E < 0.01$ eV) have a de Broglie wavelength larger than 0.05 nm, thus greater than the separation of the two protons in H_2. The uncertainty principle requires that the size of the wave packet that describes a particle be no smaller than its de Broglie wavelength. Thus the wave packet of the incident neutron overlaps simultaneously with both protons in H_2, even though the range of the nuclear force of the individual neutron–proton interactions remains of the order of 1 fm. The scattered neutron waves ψ_1 and ψ_2 from the two protons will therefore combine *coherently*: that is, they will interfere, and the cross section depends on $|\psi_1 + \psi_2|^2$, not $|\psi_1|^2 + |\psi_2|^2$. We cannot therefore simply add the cross sections from the two individual scatterings. (At higher energy, where the de Broglie wavelength would be small compared with the separation of the protons, the scattered waves would not interfere and we could indeed add the cross sections directly. The reason for choosing to work at very low energy is partly to observe the interference effect and partly to prevent the neutron from transferring enough energy to the H_2 molecule to start it rotating, which would complicate the analysis. The minimum rotational energy is about 0.015 eV, and so neutrons with energies in the range of 0.01 eV do not excite rotational states of the molecule.)

To analyze the interference effect in problems of this sort, we introduce the *scattering length a*, defined such that the low-energy cross section is equal to $4\pi a^2$:

$$\lim_{k \to 0} \sigma = 4\pi a^2 \tag{4.36}$$

Comparison with Equation 4.30 shows that

$$a = \pm \lim_{k \to 0} \frac{\sin \delta_0}{k} \tag{4.37}$$

The choice of sign is arbitrary, but it is conventional to choose the minus sign.

Even though the scattering length has the dimension of length, it is a parameter that represents the strength of the scattering, *not* its range. To see this, we note from Equation 4.37 that δ_0 must approach 0 at low energy in order that a remain finite. Equation 4.23 for the scattered wave function can be written for small δ_0 as

$$\psi_{\text{scattered}} \cong A \frac{\delta_0}{k} \frac{e^{ikr}}{r} = -Aa \frac{e^{ikr}}{r} \tag{4.38}$$

Thus a gives in effect the amplitude of the scattered wave.

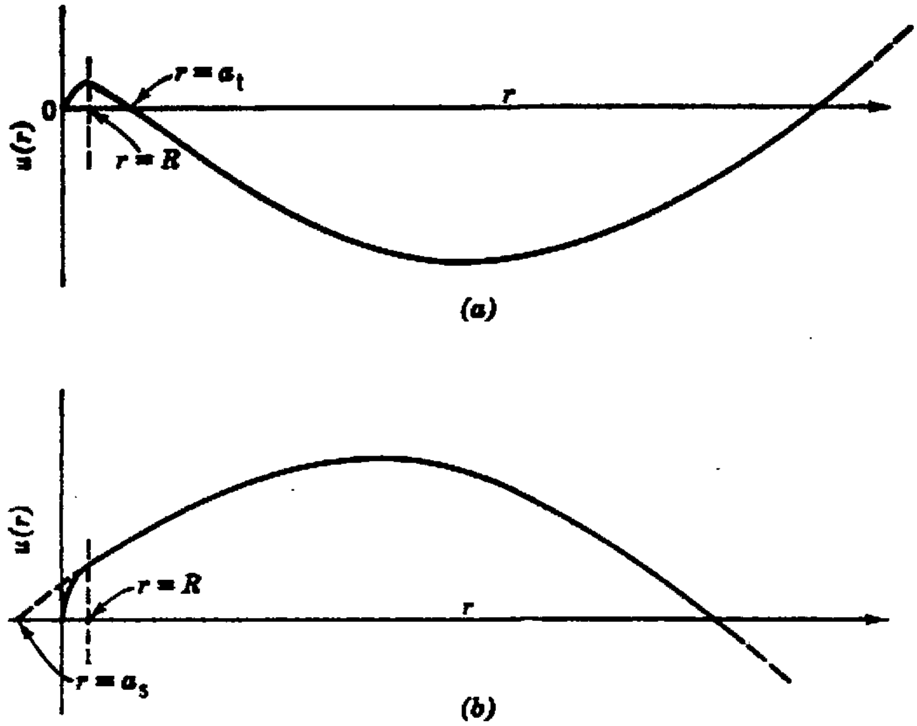

Figure 4.7 (*a*) Wave function for triplet np scattering for a laboratory neutron energy of ~ 200 keV and a well radius of 2.1 fm. Note the positive scattering length. (*b*) Wave function exhibiting a negative scattering length. This happens to be the case for singlet np scattering.

The sign of the scattering length also carries physical information. Figure 4.7 shows representations of the triplet and singlet scattered wave functions $u(r)$. At low energy we can write $a \cong -\delta_0/k$ and the scattered wave function, Equation 4.13, becomes

$$u(r) = C \sin k_2(r - a) \tag{4.39}$$

The value of a is given by the point at which $u(r)$ passes through zero. The triplet wave function for $r < R$ looks just like the bound state wave function for the deuteron: $u(r)$ "turns over" for $r < R$ to form the bound state. The value of a_t is therefore positive. Because there is no singlet bound state, $u(r)$ does not "turn over" for $r < R$, so it reaches the boundary at $r = R$ with positive slope. When we make the smooth connection at $r = R$ to the wave function beyond the potential and extrapolate to $u(r) = 0$, we find that a_s, the singlet scattering length, is negative.

Our estimate $\sigma_t = 4.6$ b from the properties of the deuteron leads to $a_t = +6.1$ fm, and the estimate of $\sigma_s = 67.8$ b needed to reproduce the observed total cross section gives $a_s = -23.2$ fm.

The theory of neutron scattering from ortho- and parahydrogen gives

$$\sigma_{\text{para}} = 5.7(3a_t + a_s)^2 \tag{4.40}$$

$$\sigma_{\text{ortho}} = \sigma_{\text{para}} + 12.9(a_t - a_s)^2 \tag{4.41}$$

where the numerical coefficients depend on the speed of the incident neutron. Equations 4.40 and 4.41 are written for neutrons of about 770 m/s, slower even than "thermal" neutrons (2200 m/s). The measured cross sections, corrected for absorption, for neutrons of this speed are $\sigma_{\text{para}} = 3.2 \pm 0.2$ b and $\sigma_{\text{ortho}} = 108 \pm 1$ b. If the nuclear force were independent of spin, we would have $\sigma_t = \sigma_s$ and $a_t = a_s$; thus σ_{para} and σ_{ortho} would be the same. The great difference between the measured values shows that $a_t \neq a_s$, and it also suggests that a_t and a_s must have different signs, so that $-3a_t = a_s$ in order to make σ_{para} small. Solving Equations 4.40 and 4.41 for a_s and a_t gives

$$a_s = -23.55 \pm 0.12 \text{ fm}$$

$$a_t = +5.35 \pm 0.06 \text{ fm}$$

consistent with the values deduced previously from σ_t and σ_s. A description of these experiments can be found in G. L. Squires and A. T. Stewart, *Proc. Roy. Soc.* (*London*) **A230**, 19 (1955).

There are several other experiments that are sensitive to the singlet and triplet scattering lengths: these include neutron diffraction by crystals that contain hydrogen (such as hydrides) as well as the total reflection of neutron beams at small angles from hydrogen-rich materials (such as hydrocarbons). These techniques give results in good agreement with the above values for a_s and a_t.

The theory we have outlined is valid only for $\ell = 0$ scattering of low-energy incident particles. The $\ell = 0$ restriction required particles of incident energies below 20 MeV, while our other low-energy approximations required eV or keV energies. As we increase the energy of the incident particle, we will violate Equation 4.36 long before we reach energies of 20 MeV. We therefore still have $\ell = 0$ scattering, but at these energies (of order 1 MeV) equations such as 4.38 are not valid. This case is generally treated in the *effective range approximation*, in which we take

$$k \cot \delta_0 = \frac{1}{a} + \frac{1}{2} r_0 k^2 + \cdots \tag{4.42}$$

and where terms in higher powers of k are neglected. The quantity a is the zero-energy scattering length we already defined (and, in fact, this reduces to Equation 4.37 in the $k \to 0$ limit), and the quantity r_0 is a new parameter, the *effective range*. One of the advantages of this representation is that a and r_0 characterize the nuclear potential independent of its shape: that is, we could repeat all of the calculations done in this section with a potential other than the square well, and we would deduce identical values of a and r_0 from analyzing the experimental cross sections. Of course there is an accompanying disadvantage in that we can learn little about the shape of the nuclear potential from an analysis in which calculations with different potentials give identical results!

Like the scattering lengths, the effective range is different for singlet and triplet states. From a variety of scattering experiments we can deduce the best set of $\ell = 0$ parameters for the neutron–proton interaction:

$$a_s = -23.715 \pm 0.015 \text{ fm} \qquad a_t = 5.423 \pm 0.005 \text{ fm}$$

$$r_{0s} = 2.73 \pm 0.03 \text{ fm} \qquad r_{0t} = 1.748 \pm 0.006 \text{ fm}$$

As a final comment regarding the singlet and triplet neutron–proton interactions, we can try to estimate the energy of the singlet n-p state relative to the bound triplet state at -2.22 MeV. Using Equations 4.34, 4.31, and 4.5 we would deduce that the energy of the singlet state is about $+77$ keV. Thus the singlet state is only slightly unbound.

4.3 PROTON – PROTON AND NEUTRON – NEUTRON INTERACTIONS

There is one very important difference between the scattering of identical nucleons (proton–proton and neutron–neutron scattering) and the scattering of different nucleons (neutron–proton scattering). This difference comes about because the identical projectile and target nucleons must be described by a common wave function, as discussed in Section 2.7. Because nucleons have spin $\frac{1}{2}$, their wave functions must be antisymmetric with respect to interchange of the nucleons. If we again consider only low-energy scattering, so that $\ell = 0$, interchanging the spatial coordinates of the two particles gives no change in sign. (This situation is somewhat analogous to the parity operation described in Section 2.6.) Thus the wave function is symmetric with respect to interchange of spatial coordinates and must therefore be antisymmetric with respect to interchange of spin coordinates in order that the total (spatial times spin) wave function be antisymmetric. The antisymmetric spin wave function is of the form of Equation 2.76 and must correspond to a total combined spin of 0; that is, the spin orientations must be different. *Only singlet spin states can thus contribute to the scattering.* (At higher energies, the antisymmetric $\ell = 1$ spatial states can occur, accompanied by only the symmetric triplet spin states.)

The derivation of the differential cross section relies on another feature of quantum physics. Consider Figure 4.8, which represents the scattering of two identical particles in the center of mass reference frame. Since the particles are

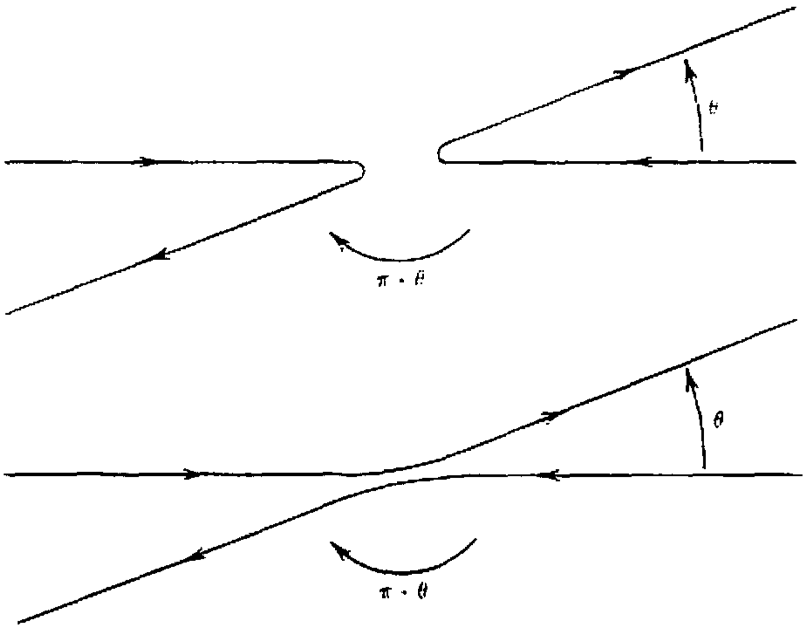

Figure 4.8 Scattering of identical particles in the center-of-mass system. One particle emerges at the angle θ and the other at $\pi - \theta$; because the particles are identical, there is no way to tell which particle emerges at which angle, and therefore we cannot distinguish the two cases shown.

identical, there is no experimental way to distinguish the two situations in the figure. The scattered wave function must therefore include contributions for scattering at θ and at $\pi - \theta$. When we square the scattered wave function to calculate the cross section, there will be a term proportional to the interference between the parts of the wave function that give scattering at θ and at $\pi - \theta$. This interference is a purely quantum effect that has no classical analog.

Let's first consider scattering between two protons; the wave function must describe both Coulomb and nuclear scattering, and there will be an additional Coulomb-nuclear interference term in the cross section. (The scattered wave function must include one term resulting from Coulomb scattering and another resulting from nuclear scattering; the Coulomb term must vanish in the limit $e \to 0$, and the nuclear term must vanish as the nuclear potential vanishes, in which case $\delta_0 \to 0$. When we square the wave function to find the cross section, we get a term that includes both the Coulomb and nuclear scattering.) The derivation of the cross section is beyond the level of this text; for discussions of its derivation and of early work on proton–proton scattering, see J. D. Jackson and J. M. Blatt, *Rev. Mod. Phys.* 22, 77 (1950). The differential cross section is

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi\epsilon_0}\right)^2 \frac{1}{4T^2}\left\{\frac{1}{\sin^4(\theta/2)} + \frac{1}{\cos^4(\theta/2)} - \frac{\cos[\eta \ln \tan^2(\theta/2)]}{\sin^2(\theta/2)\cos^2(\theta/2)}\right.$$
$$- \frac{2}{\eta}(\sin\delta_0)\left[\frac{\cos[\delta_0 - \eta \ln \sin^2(\theta/2)]}{\sin^2(\theta/2)} + \frac{\cos[\delta_0 + \eta \ln \cos^2(\theta/2)]}{\cos^2(\theta/2)}\right]$$
$$\left. - \frac{4}{\eta^2}\sin^2\delta_0\right\} \tag{4.43}$$

Here T is the *laboratory* kinetic energy of the incident proton (assuming the target proton to be at rest), θ is the scattering angle in the center-of-mass system, δ_0 the $\ell = 0$ phase shift for pure nuclear scattering, and $\eta = (e^2/4\pi\epsilon_0 \hbar c)\beta^{-1} = \alpha/\beta$, where α is the fine-structure constant (with a value of nearly $\frac{1}{137}$) and $\beta = v/c$ is the (dimensionless) relative velocity of the protons. The six terms in brackets in Equation 4.43 can be readily identified: (1) The $\sin^{-4}(\theta/2)$ is characteristic of Coulomb scattering, also known as Rutherford scattering. We discuss this further in Chapter 11. (2) Since the two protons are identical, we cannot tell the case in which the incident proton comes out at θ and the target proton at $\pi - \theta$ (in the center-of-mass system) from the case in which the incident proton comes out at $\pi - \theta$ and the target proton at θ. Thus the scattering cross section must include a characteristic Coulomb (Rutherford) term $\sin^{-4}(\pi - \theta)/2 = \cos^{-4}(\theta/2)$. (3) This term describes the interference between Coulomb scattering at θ and at $\pi - \theta$. (4 and 5) These two terms result from the interference between Coulomb and nuclear scattering. (6) The last term is the pure nuclear scattering term. In the limit $e \to 0$ (pure nuclear scattering), only this term survives and Equation 4.43 reduces to Equation 4.28, as it should.

Although it may be complicated in practice, the procedure for studying the proton–proton interaction is simple in concept: since δ_0 is the only unknown in Equation 4.43, we can measure the differential scattering cross section as a function of angle (for a specific incident kinetic energy) and extract δ_0 from the

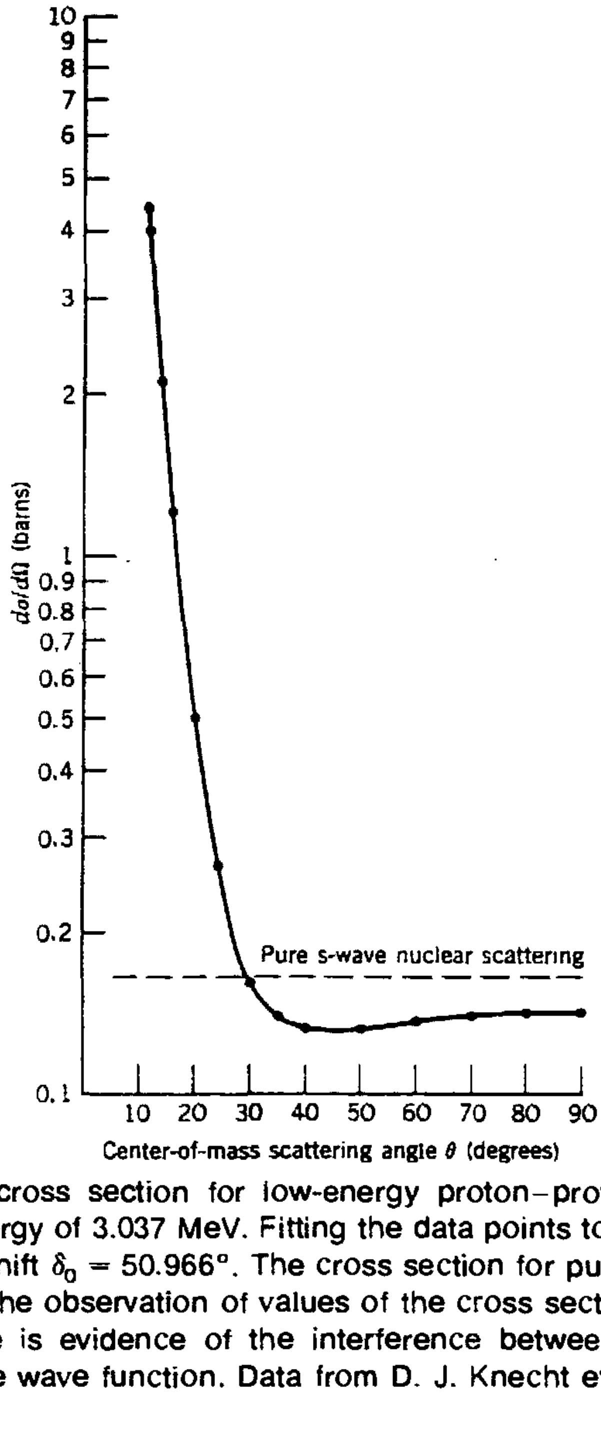

Figure 4.9 The cross section for low-energy proton–proton scattering at an incident proton energy of 3.037 MeV. Fitting the data points to Equation 4.43 gives the s-wave phase shift $\delta_0 = 50.966°$. The cross section for pure nuclear scattering would be 0.165 b; the observation of values of the cross section *smaller* than the pure nuclear value is evidence of the interference between the Coulomb and nuclear parts of the wave function. Data from D. J. Knecht et al., *Phys. Rev.* **148**, 1031 (1966).

best fit of the results to Equation 4.43. Figure 4.9 shows an example of such data, from which it is deduced that $\delta_0 = 50.966°$ at $T = 3.037$ MeV. From many such experiments we can observe the dependence of δ_0 on energy, as shown in Figure 4.10.

The next step in the interpretation of these data is to represent the scattering in terms of energy-independent quantities such as the scattering length and effective range, as we did in Equation 4.42. Unfortunately, this cannot easily be done because the Coulomb interaction has infinite range and even in the $k \to 0$ limit we cannot neglect the higher-order terms of Equation 4.42. With certain modifications, however, it is possible to obtain an expression incorporating the effects of Coulomb and nuclear scattering in a form similar to Equation 4.42 and thus to

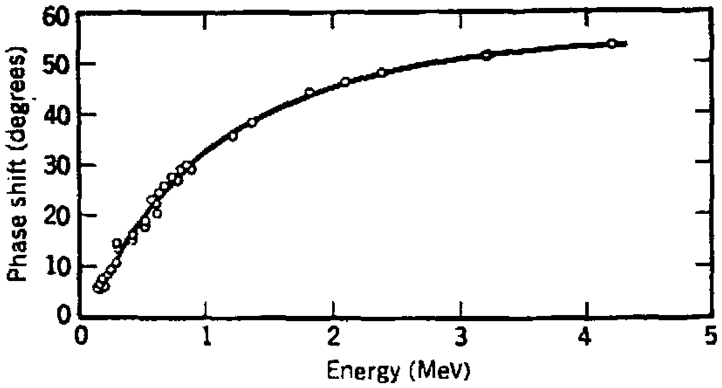

Figure 4.10 The s-wave phase shift for pp scattering as deduced from the experimental results of several workers.

obtain values for the proton–proton scattering length and effective range:

$$a = -7.82 \pm 0.01 \text{ fm}$$
$$r_0 = 2.79 \pm 0.02 \text{ fm}$$

The effective range is entirely consistent with the *singlet* np values deduced in the previous section. The scattering length, which measures the strength of the interaction, includes Coulomb as well as nuclear effects and thus cannot be compared directly with the corresponding np value. (It is, however, important to note that a is negative, suggesting that there is no pp bound state: that is, the nucleus ^{2}He does not exist.) The comparison of the pp and np scattering lengths will be discussed further in the next section.

The study of neutron–neutron scattering should be free of the effects of the Coulomb interaction that made the analysis of proton–proton scattering so complicated. Here the difficulty is an experimental one—although beams of neutrons are readily available, targets of free neutrons are not. Measurement of neutron–neutron scattering parameters therefore requires that we use a nuclear reaction to create two neutrons in relative motion within the range of each other's nuclear force. As the two neutrons separate, we have in effect a scattering experiment. Unfortunately, such reactions must also create a third particle, which will have interactions with both of the neutrons (individually and collectively), but the necessary corrections can be calculated with sufficient precision to enable values to be extracted for the neutron–neutron scattering length and effective range. The experiments that have been reported include the breakup of a deuteron following capture of a negative π meson ($\pi^- + {}^2\text{H} \rightarrow 2\text{n} + \gamma$) and following neutron scattering ($\text{n} + {}^2\text{H} \rightarrow 2\text{n} + \text{p}$). It is also possible to deduce the nn parameters from comparison of mirror reactions such as $^3\text{He} + {}^2\text{H} \rightarrow {}^3\text{H} + 2\text{p}$ and $^3\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + 2\text{n}$, using known pp parameters as an aid in calculating the final-state effects of the three particles. The analysis of these (and other) experiments gives the neutron–neutron parameters

$$a = -16.6 \pm 0.5 \text{ fm}$$
$$r_0 = 2.66 \pm 0.15 \text{ fm}$$

As with the proton–proton interaction, the negative scattering length shows that the two neutrons do not form a stable bound state. (It is tempting, but incorrect, to explain the nonexistence of the di-proton as arising from Coulomb repulsion. No such temptation exists for the di-neutron, the nonexistence of which must arise from the spin dependence of the nuclear interaction. Reviewing the evidence, we first learned that the deuteron ground state is a spin triplet and that no bound spin singlet state exists. We then argued that, because identical fermions must have total antisymmetric wave functions and because the lowest state is expected to be a spatially symmetric $\ell = 0$ state, the di-proton and di-neutron systems must have antisymmetric, or singlet, spin states which are unbound.)

4.4 PROPERTIES OF THE NUCLEAR FORCE

Based on the low-energy properties described in the previous sections, we can learn many details about the nuclear force. When we include results from higher energy experiments, still more details emerge. In this section we summarize the main features of the internucleon force and in the next section we discuss a particular representation for the force that reproduces many of these details.

The Interaction between Two Nucleons Consists to Lowest Order of an Attractive Central Potential

In this chapter we have used for this potential a square-well form, which simplifies the calculations and reproduces the observed data fairly well. Other more realistic forms could just as well have been chosen, but the essential conclusions would not change (in fact, the effective range approximation is virtually independent of the shape assumed for the potential). The common characteristic of these potentials is that they depend only on the internucleon distance r. We therefore represent this central term as $V_c(r)$. The experimental program to study $V_c(r)$ would be to measure the energy dependence of nucleon-nucleon parameters such as scattering phase shifts, and then to try to choose the form for $V_c(r)$ that best reproduces those parameters.

The Nucleon–Nucleon Interaction Is Strongly Spin Dependent

This observation follows from the failure to observe a singlet bound state of the deuteron and also from the measured differences between the singlet and triplet cross sections. What is the form of an additional term that must be added to the potential to account for this effect? Obviously the term must depend on the spins of the two nucleons, s_1 and s_2, but not all possible combinations of s_1 and s_2 are permitted. The nuclear force must satisfy certain symmetries, which restrict the possible forms that the potential could have. Examples of these symmetries are *parity* ($r \to -r$) and *time reversal* ($t \to -t$). Experiments indicate that, to a high degree of precision (one part in 10^7 for parity and one part in 10^3 for time reversal), the internucleon potential is invariant with respect to these operations. Under the parity operator, which involves spatial reflection, angular momentum vectors are unchanged. This statement may seem somewhat surprising, because upon inverting a coordinate system we would naturally expect all vectors defined in that coordinate system to invert. However, angular momentum is not a true or polar vector; it is a pseudo- or axial vector that does not invert when $r \to -r$. This follows directly from the definition $r \times p$ or can be inferred from a diagram of a spinning object. Under the time-reversal operation, all motions (including linear and angular momentum) are reversed. Thus terms such as s_1 or s_2 or a linear combination $As_1 + Bs_2$ in the potential would violate time-reversal invariance and cannot be part of the nuclear potential: terms such as s_1^2, s_2^2, or $s_1 \cdot s_2$ are invariant with respect to time reversal and are therefore allowed. (All of these terms are also invariant with respect to parity.) The simplest term involving both nucleon spins is $s_1 \cdot s_2$. Let's consider the value of $s_1 \cdot s_2$ for singlet and triplet states. To do this we evaluate the total spin $S = s_1 + s_2$

$$S^2 = S \cdot S = (s_1 + s_2) \cdot (s_1 + s_2)$$
$$= s_1^2 + s_2^2 + 2s_1 \cdot s_2$$

Thus

$$s_1 \cdot s_2 = \tfrac{1}{2}(S^2 - s_1^2 - s_2^2) \tag{4.44}$$

To evaluate this expression, we must remember that in quantum mechanics all squared angular momenta evaluate as $s^2 = \hbar^2 s(s+1)$: see Section 2.5 and Equation 2.69,

$$\langle s_1 \cdot s_2 \rangle = \tfrac{1}{2}\left[S(S+1) - s_1(s_1+1) - s_2(s_2+1)\right]\hbar^2 \tag{4.45}$$

With nucleon spins s_1 and s_2 of $\frac{1}{2}$, the value of $s_1 \cdot s_2$ is, for triplet ($S = 1$) states:

$$\langle s_1 \cdot s_2 \rangle = \tfrac{1}{2}\left[1(1+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right]\hbar^2 = \tfrac{1}{4}\hbar^2 \tag{4.46}$$

and for singlet ($S = 0$) states:

$$\langle s_1 \cdot s_2 \rangle = \tfrac{1}{2}\left[0(0+1) - \tfrac{1}{2}(\tfrac{1}{2}+1) - \tfrac{1}{2}(\tfrac{1}{2}+1)\right]\hbar^2 = -\tfrac{3}{4}\hbar^2 \tag{4.47}$$

Thus a spin-dependent expression of the form $s_1 \cdot s_2 V_s(r)$ can be included in the potential and will have the effect of giving different calculated cross sections for singlet and triplet states. The magnitude of V_s can be adjusted to give the correct differences between the singlet and triplet cross sections and the radial dependence can be adjusted to give the proper dependence on energy.

We could also write the potential including V_c and V_s as

$$V(r) = -\left(\frac{s_1 \cdot s_2}{\hbar^2} - \frac{1}{4}\right)V_1(r) + \left(\frac{s_1 \cdot s_2}{\hbar^2} + \frac{3}{4}\right)V_3(r) \tag{4.48}$$

where $V_1(r)$ and $V_3(r)$ are potentials that separately give the proper singlet and triplet behaviors.

The Internucleon Potential Includes a Noncentral Term, Known as a Tensor Potential

Evidence for the tensor force comes primarily from the observed quadrupole moment of the ground state of the deuteron. An s-state ($\ell = 0$) wave function is spherically symmetric; the electric quadrupole moment vanishes. Wave functions with mixed ℓ states must result from noncentral potentials. This tensor force must be of the form $V(\mathbf{r})$, instead of $V(r)$. For a single nucleon, the choice of a certain direction in space is obviously arbitrary; nucleons do not distinguish north from south or east from west. The only reference direction for a nucleon is its spin, and thus only terms of the form $\mathbf{s} \cdot \mathbf{r}$ or $\mathbf{s} \times \mathbf{r}$, which relate $\mathbf{r}$ to the direction of $\mathbf{s}$, can contribute. To satisfy the requirements of parity invariance, there must be an even number of factors of $\mathbf{r}$, and so for two nucleons the potential must depend on terms such as $(\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})$ or $(\mathbf{s}_1 \times \mathbf{r}) \cdot (\mathbf{s}_2 \times \mathbf{r})$. Using vector identities we can show that the second form can be written in terms of the first and the additional term $\mathbf{s}_1 \cdot \mathbf{s}_2$, which we already included in $V(r)$. Thus without loss of generality we can choose the tensor contribution to the internucleon potential to be of the form $V_T(r)S_{12}$, where $V_T(r)$ gives the force the proper radial dependence and magnitude, and

$$S_{12} = 3(\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r})/r^2 - \mathbf{s}_1 \cdot \mathbf{s}_2 \tag{4.49}$$

which gives the force its proper tensor character and also averages to zero over all angles.

The Nucleon–Nucleon Force Is Charge Symmetric

This means that the proton–proton interaction is identical to the neutron–neutron interaction, after we correct for the Coulomb force in the proton–proton system. Here "charge" refers to the character of the nucleon (proton or neutron) and not to electric charge. Evidence in support of this assertion comes from the equality of the pp and nn scattering lengths and effective ranges. Of course, the pp parameters must first be corrected for the Coulomb interaction. When this is done, the resulting singlet pp parameters are

$$a = -17.1 \pm 0.2 \text{ fm}$$
$$r_0 = 2.84 \pm 0.03 \text{ fm}$$

These are in very good agreement with the measured nn parameters ($a = -16.6 \pm 0.5$ fm, $r_0 = 2.66 \pm 0.15$ fm), which strongly supports the notion of charge symmetry.

The Nucleon–Nucleon Force Is Nearly Charge Independent

This means that (in analogous spin states) the three *nuclear* forces nn, pp, *and* pn are identical, again correcting for the pp Coulomb force. Charge independence is thus a stronger requirement than charge symmetry. Here the evidence is not so conclusive; in fact, the singlet np scattering length (-23.7 fm) seems to differ substantially from the pp and nn scattering lengths (-17 fm). However, we see from Figure 4.11 that large negative scattering lengths are extraordinarily sensitive to the nuclear wave function near $r = R$, and a very small change in ψ can give a large change in the scattering length. Thus the large difference between the scattering lengths may correspond to a very small difference (of order 1%) between the potentials, which (as we see in the next section) is easily explained by the exchange force model.

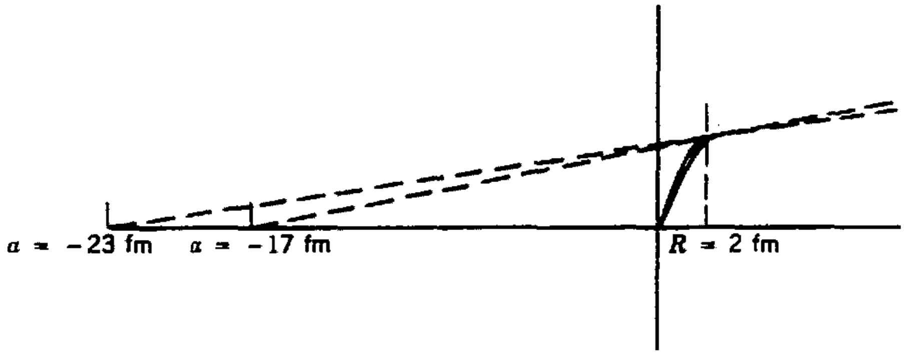

Figure 4.11 Very small changes in the nucleon−nucleon wave function near $r = R$ can lead to substantial differences in the scattering length when the extrapolation is made (compare Figure 4.7*b*).

The Nucleon−Nucleon Interaction Becomes Repulsive at Short Distances

This conclusion follows from qualitative considerations of the nuclear density: as we add more nucleons, the nucleus grows in such a way that its central density remains roughly constant, and thus something is keeping the nucleons from crowding too closely together. More quantitatively, we can study nucleon-nucleon scattering at higher energies. Figure 4.12 shows the deduced singlet s-wave phase shifts for nucleon-nucleon scattering up to 500 MeV. (At these energies, phase shifts from higher partial waves, p and d for example, also contribute to the cross

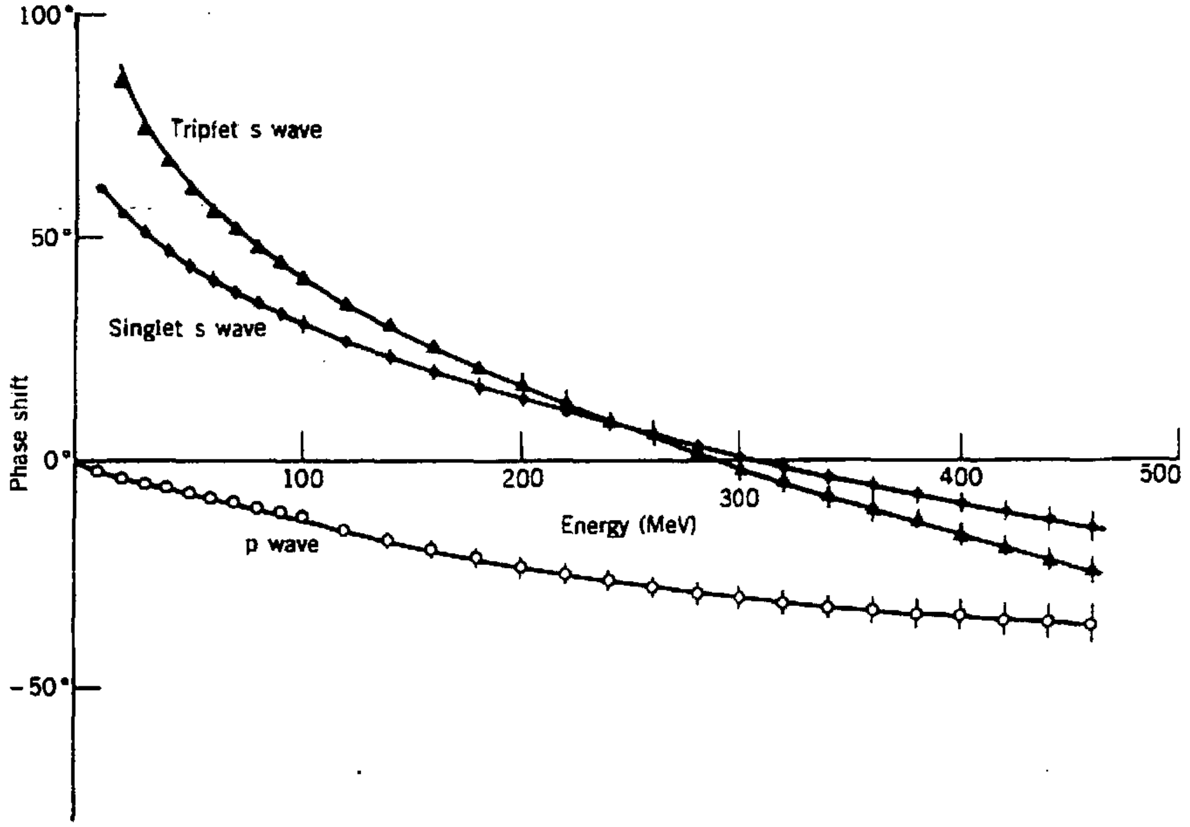

Figure 4.12 The phase shifts from neutron−proton scattering at medium energies. The change in the s-wave phase shift from positive to negative at about 300 MeV shows that at these energies the incident nucleon is probing a repulsive core in the nucleon−nucleon interaction. ▲, $^3S_1$; ●, $^1S_0$; O, $^1P_1$. Data from M. MacGregor et al., *Phys. Rev.* **182**, 1714 (1969).

sections. The s-wave phase shifts can be easily extracted from the differential scattering measurements of $d\sigma/d\Omega$ vs θ because they do not depend on θ.) At about 300 MeV, the s-wave phase shift becomes *negative*, corresponding to a change from an attractive to a repulsive force. To account for the repulsive core, we must modify the potentials we use in our calculations. For example, again choosing a square-well form to simplify the calculation, we might try

$$\begin{aligned} V(r) &= +\infty & r &< R_{\text{core}} \\ &= -V_0 & R_{\text{core}} &\le r \le R \\ &= 0 & r &> R \end{aligned} \tag{4.50}$$

and we can adjust R_{core} until we get satisfactory agreement with the observed s-wave phase shifts. The value $R_{\text{core}} = 0.5$ fm gives agreement with the observed phase shifts.

The Nucleon–Nucleon Interaction May Also Depend on the Relative Velocity or Momentum of the Nucleons

Forces depending on velocity or momentum cannot be represented by a scalar potential, but we can include these forces in a reasonable manner by introducing terms linear in $\boldsymbol{p}$, quadratic in $\boldsymbol{p}$, and so on, with each term including a characteristic $V(r)$. Under the parity operation, $\boldsymbol{p} \to -\boldsymbol{p}$, and also under time reversal $\boldsymbol{p} \to -\boldsymbol{p}$. Thus any term simply linear in $\boldsymbol{p}$ is unacceptable because it violates both parity and time-reversal invariance. Terms of the form $\boldsymbol{r}\cdot\boldsymbol{p}$ or $\boldsymbol{r}\times\boldsymbol{p}$ are invariant with respect to parity, but still violate time reversal. A possible structure for this term that is first order in $\boldsymbol{p}$ and invariant with respect to both parity and time reversal is $V(r)(\boldsymbol{r}\times\boldsymbol{p})\cdot\boldsymbol{S}$, where $\boldsymbol{S} = \boldsymbol{s}_1 + \boldsymbol{s}_2$ is the total spin of the two nucleons. The relative angular momentum of the nucleons is $\boldsymbol{\ell} = \boldsymbol{r}\times\boldsymbol{p}$, and therefore this term, known as the *spin-orbit term* in analogy with atomic physics, is written $V_{so}(r)\boldsymbol{\ell}\cdot\boldsymbol{S}$. Although higher-order terms may be present, this is the only first-order term in $\boldsymbol{p}$ that satisfies the symmetries of both parity and time reversal.

The experimental evidence in support of the spin-orbit interaction comes from the observation that scattered nucleons can have their spins aligned, or *polarized*, in certain directions. The polarization of the nucleons in a beam (or in a target) is defined as

$$P = \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)} \tag{4.51}$$

where $N(\uparrow)$ and $N(\downarrow)$ refer to the number of nucleons with their spins pointed up and down, respectively. Values of P range from $+1$, for a 100% spin-up polarized beam, to -1, for a 100% spin-down polarized beam. An unpolarized beam, with $P = 0$, has equal numbers of nucleons with spins pointing up and down.

Consider the scattering experiment shown in Figure 4.13*a*, in which an unpolarized beam (shown as a mixture of spin-up and spin-down nucleons) is incident on a spin-up target nucleon. Let's suppose the nucleon–nucleon interaction causes the incident spin-up nucleons to be scattered to the left at angle θ

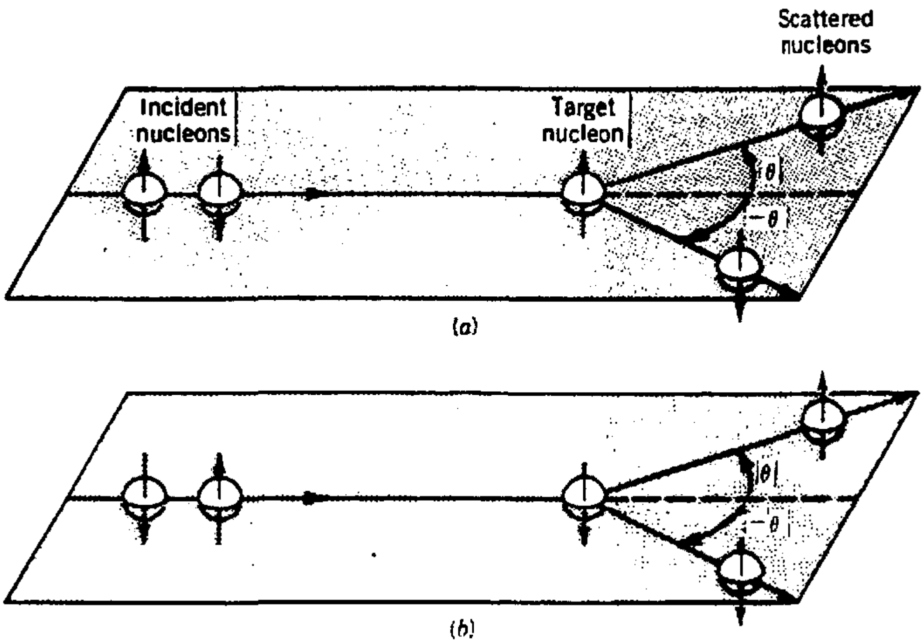

Figure 4.13 An unpolarized beam (shown as a mixture of spin-up and spin-down nucleons) is scattered from a target that can have either spin up or spin down. In part *a*, the incident nucleons with spin up are scattered to the left at angle θ, while those with spin down are scattered to the right at $-\theta$. Part *b* can be obtained from part *a* by viewing from below or by rotating 180° about the beam direction; it shows that the same conclusions follow in scattering from a spin-down polarized target

and the incident spin-down nucleons to be scattered to the right at angle $-\theta$. Part b of the figure shows the *same experiment* viewed from below or else rotated 180° about the direction of the incident beam. We can also interpret Figure 4.13b as the scattering of an unpolarized beam from a spin-down target nucleon, and once again the spin-up incident nucleons scatter to the left and the spin-down nucleons scatter to the right. The results would be the same, even in an unpolarized target, which would contain a mixture of spin-up and spin-down nucleons: when an unpolarized beam is scattered from an unpolarized target, the spin-up scattered nucleons appear preferentially at θ and the spin-down scattered nucleons at $-\theta$.

Although this situation may appear superficially to violate reflection symmetry (parity), you can convince yourself that this is not so by sketching the experiment and its mirror image. Parity is conserved if at angle θ we observe a net polarization P, while at angle $-\theta$ we observe a net polarization of $-P$.

Let's now see how the spin-orbit interaction can give rise to this type of scattering with polarization. Figure 4.14 shows two nucleons with spin up incident on a spin up target, so that $S = 1$. (Scattering that includes only s waves must be spherically symmetric, and therefore there can be no polarizations. The p-wave ($\ell = 1$) scattering of identical nucleons has an antisymmetric spatial wave function and therefore a symmetric spin wave function.) Let's assume that $V_{so}(r)$ is negative. For incident nucleon 1, $\boldsymbol{\ell} = \boldsymbol{r} \times \boldsymbol{p}$ is down (into the page), and therefore $\boldsymbol{\ell} \cdot \boldsymbol{S}$ is negative because $\boldsymbol{\ell}$ and $\boldsymbol{S}$ point in opposite directions. The combination $V_{so}(r)\boldsymbol{\ell} \cdot \boldsymbol{S}$ is positive and so there is a repulsive force between the

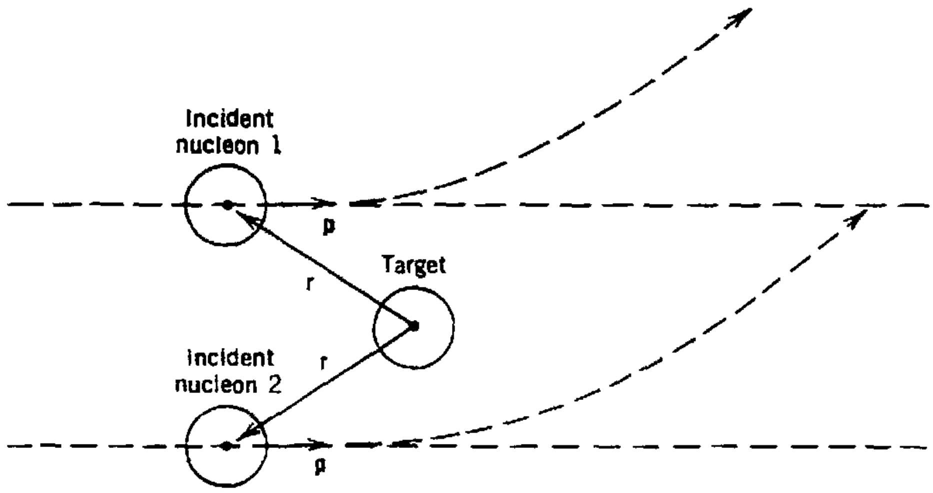

Figure 4.14 Top view of nucleon–nucleon scattering experiment. All spins point up (out of the paper). Incident nucleon 1 has $\boldsymbol{r} \times \boldsymbol{p}$ into the paper, and thus $\boldsymbol{\ell} \cdot \boldsymbol{S}$ is negative, giving a repulsive force and scattering to the left. Incident nucleon 2 has $\boldsymbol{r} \times \boldsymbol{p}$ out of the paper, resulting in an attractive force and again scattering to the left.

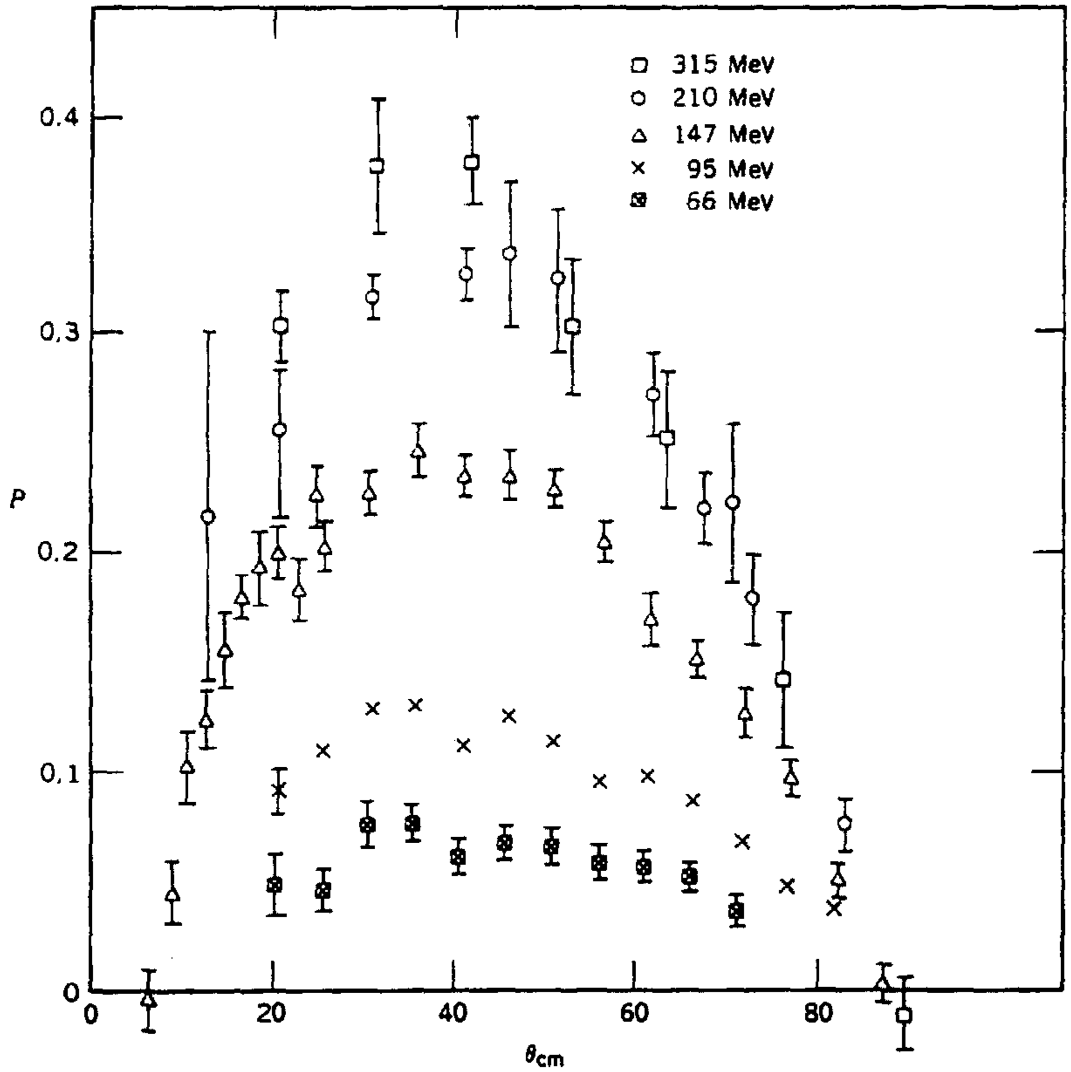

Figure 4.15 As the incident energy in proton–proton scattering increases, the maximum polarization increases. From R. Wilson, *The Nucleon–Nucleon Interaction* (New York: Wiley-Interscience, 1963).

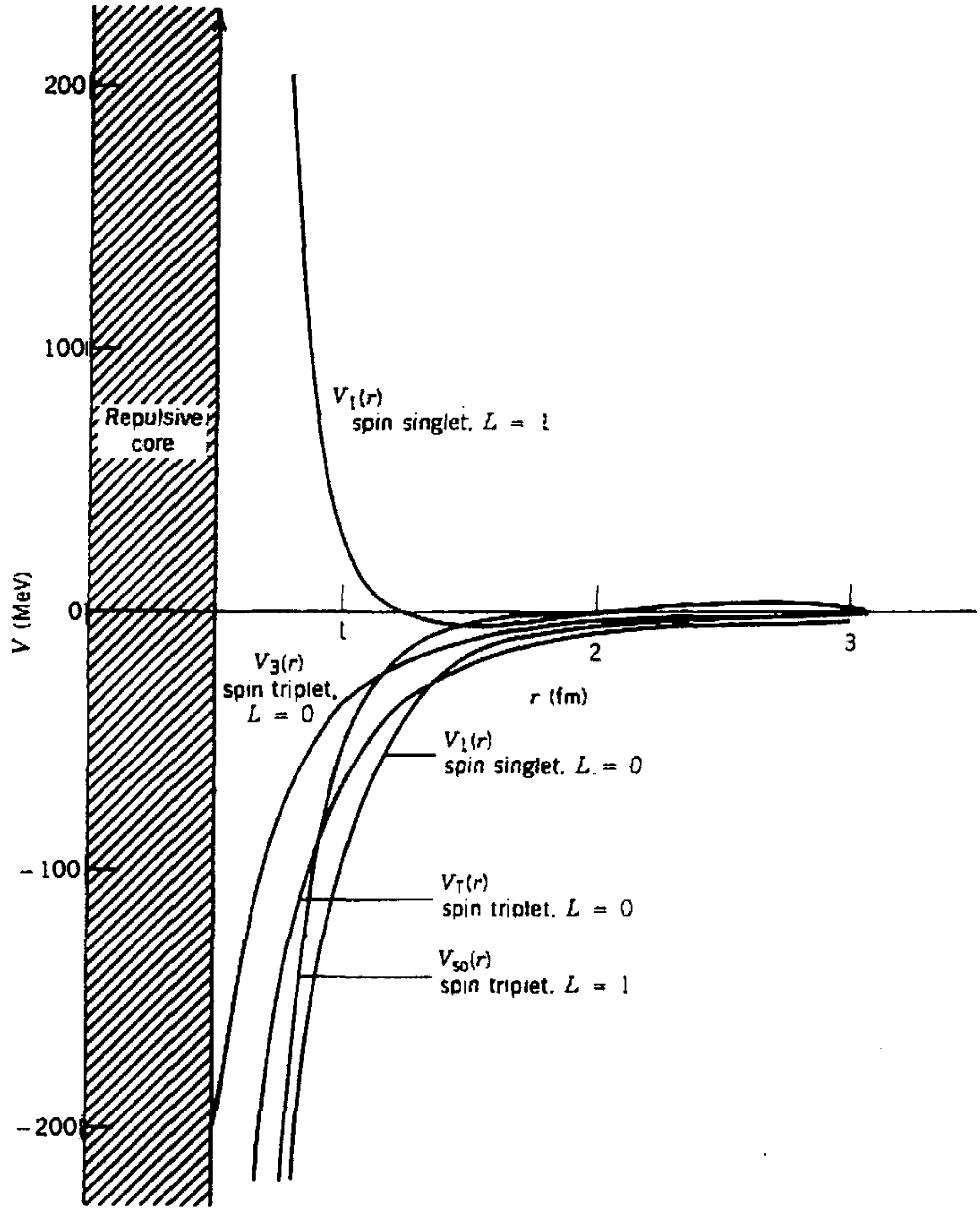

Figure 4.16 Some representative nucleon–nucleon potentials. Those shown include the attractive singlet and triplet terms that contribute to s-wave scattering, the repulsive term that gives one type of p-wave ($L = 1$) scattering, and the attractive tensor and spin-orbit terms. All potentials have a repulsive core at $r = 0.49$ fm. These curves are based on an early set of functional forms proposed by T. Hamada and I. D. Johnston, *Nucl. Phys.* **34**, 382 (1962); other relatively similar forms are in current use.

target and incident nucleon 1, which is pushed to the left. For nucleon 2, ℓ points up, $\ell \cdot S$ is positive, and the interaction is attractive; incident nucleon 2 is pulled toward the target and also appears on the left side. Spin-up incident nucleons are therefore preferentially scattered to the left and (by a similar argument) spin-down nucleons to the right. Thus the spin-orbit force can produce polarized scattered beams when unpolarized particles are incident on a target.

At low energy, where s-wave scattering dominates, we expect no polarization. As the incident energy increases, the contribution of p-wave scattering increases and there should be a corresponding increase in the polarization. Figure 4.15

shows that this expectation is correct. From the variation of P with θ and with energy, we can make deductions about the form of $V_{so}(r)$.

The general topic of polarization in nuclear reactions is far more complicated than we have indicated in this brief discussion. We should also consider the effect on the measured cross sections of using polarized beams and polarized targets, which we do in Chapter 11.

From this enormous set of experimental information (total and differential cross sections, spin dependence, polarizations), it is possible to propose a set of phenomenological potentials $V(r)$ that give reasonable agreement with the observed nucleon–nucleon data. These potentials can then be used in calculations for more complicated nuclei. As an example, Figure 4.16 illustrates one such set of potentials. As is usually the case, negative potentials give an attractive force and positive potentials give a repulsive force. Notice how the potentials incorporate such features as the range of the interactions, the repulsive core, the strong attractive s-wave phase shifts, the repulsive p-wave phase shifts, and charge independence (since no distinctions are made for the characters of the nucleons).

4.5 THE EXCHANGE FORCE MODEL

The phenomenological potentials discussed in the previous section have been fairly successful in accounting for a variety of measured properties of the nucleon–nucleon interaction. Of course, the ability of these potentials to give accurate predictions would be improved if we added more terms to the interaction. For example, we could have included a term that is second-order in the momentum dependence (proportional to ℓ^2), we could write potentials that are different for each ℓ value, and so on. Each new term in the potential may improve the calculation, but it may be at the expense of simplicity. It also serves to make us lose sight of our main objective: to understand the nucleon–nucleon interaction. Simply because we have included enough potentials to do accurate calculations does not mean we have improved our understanding of the fundamental character of the nucleon–nucleon interaction. We therefore try to postulate a physical mechanism for the nucleon–nucleon force that will yield potentials similar to those that have already proven to be successful in calculations.

A successful mechanism is that of the *exchange force*. There are two principal arguments in support of the presence of exchange forces in nuclei. The first comes from the saturation of nuclear forces. The experimental support for saturation comes from the relatively constant nuclear density and binding energy per nucleon as we go to heavier nuclei. A given nucleon seems to attract only a small number of near neighbors, but it also repels at small distances to keep those neighbors from getting too close. (We explained this behavior in the previous section by choosing a central potential that was of finite range and had a repulsive core.) We encounter exactly the same sort of behavior in molecules. When we bring two atoms together to form a diatomic molecule, such as one with covalent bonding, electrons are shared or exchanged between the two atoms, and a stable molecule forms with the atoms in equilibrium separated by a certain

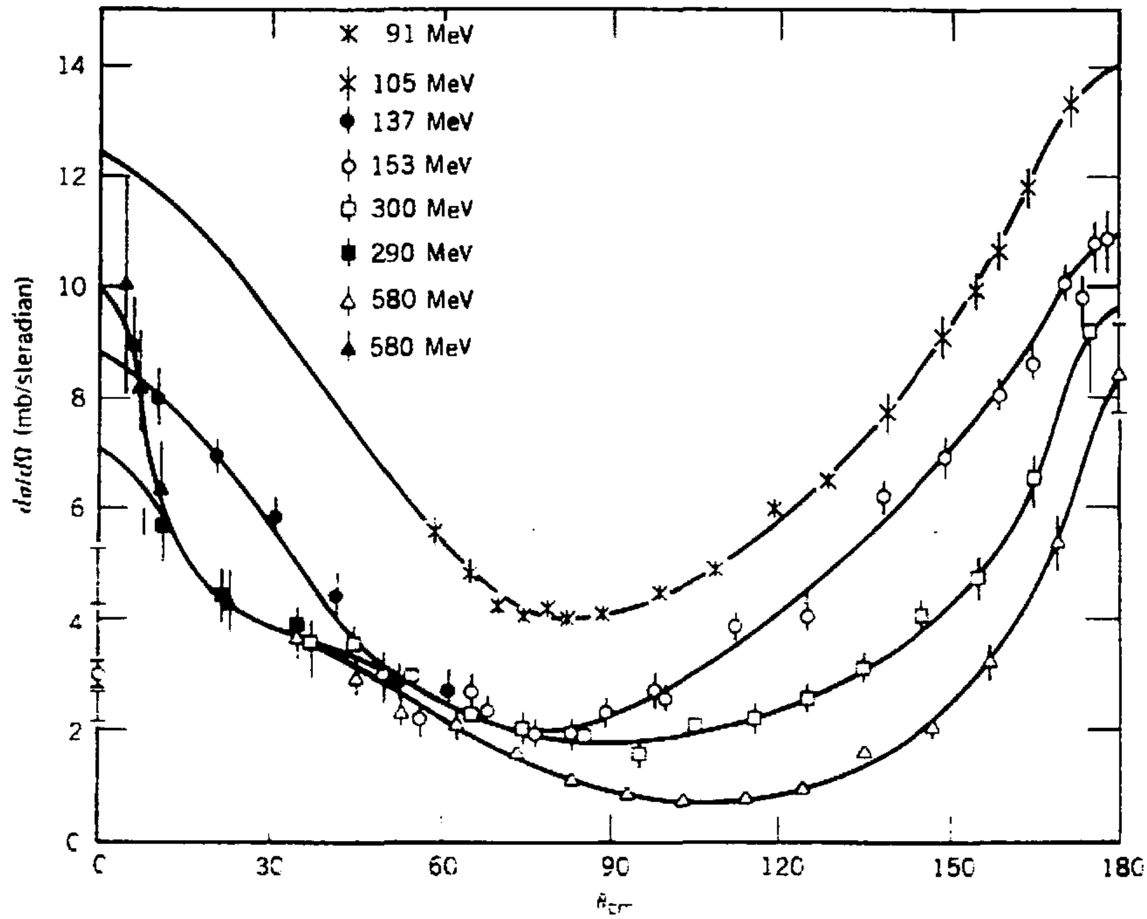

Figure 4.17 The neutron–proton differential cross section at medium energies. The strong forward-scattering peak (near 0°) is expected; the equally strong backward peak (near 180°) is evidence for the exchange force. From R. Wilson. *The Nucleon–Nucleon Interaction* (New York: Wiley-Interscience, 1963).

distance. If we try to force the atoms closer together, the overlap of the filled electronic shells causes a strong repulsive force. Furthermore, approaching the molecule with a third atom may result only in very weak forces between the first two atoms and the third; if all of the valence electrons are occupied in the first set of bonds, none are available to form new bonds. Nuclear forces show a similar saturation character.

Another argument in favor of the exchange force model comes from the study of np scattering at high energies. Figure 4.17 shows the np differential cross section. There is a strong peak in the cross section at forward angles near 0°, corresponding to a small momentum transfer between the projectile and the target. We can estimate the extent of this forward peak by studying the maximum momentum transfer in the following way: For small deflection angles, $\sin\theta \cong \theta = \Delta p/p$ where p is the momentum of the incident particle and Δp is the transverse momentum added during the collision. If F is the average force that acts during the collision time Δt, then $\Delta p = F\Delta t$. The force F is $-dV/dr$, and thus the average force should be of the order of V_0/R, where V_0 is the depth of the nucleon–nucleon square-well potential and R is its range. (Even if the actual potential is not at all constant, such as the central term of Figure 4.16, the average value of dV/dr should be of the order of V_0/R.) The collision time Δt

should be of the order of R/v, where v is the projectile velocity. Thus

$$\theta \simeq \frac{\Delta p}{p} = \frac{F\Delta t}{p} = \frac{1}{p}\frac{V_0}{R}\frac{R}{v} = \frac{V_0}{pv} = \frac{V_0}{2T} \tag{4.52}$$

where T is the projectile kinetic energy. For the energies shown in Figure 4.17, this gives values of θ in the range of 10° or smaller. We certainly do not expect to see a peak at 180°! Although it is tempting to regard this "backward" peak in the center of mass frame as the result of a head-on collision in which the incident particle has its motion reversed, our estimate above indicates such an explanation is not likely to be correct.

A more successful explanation can be found in the exchange model if, during the collision, the neutron and proton exchange places. That is, the forward-moving neutron becomes a proton and the backward-moving (in the center-of-mass system) proton becomes a neutron. The incident nucleon then reappears in the laboratory as a forward-moving nucleon (now a proton), consistent with our estimate of the small deflection angle in nucleon–nucleon scattering.

In summary, both the saturation of nuclear forces and the strong backward peak in np scattering are explained by exchange forces. In the former case, "something" is exchanged between nucleons to produce a sort of saturated bond. In the second case, "something" is exchanged between nucleons and actually changes their character.

In the early development of classical physics, objects were said to interact by means of "action at a distance." Somehow one object mysteriously transmitted through space its force on the other object. The great development in nineteenth-century theoretical physics was the introduction of the concept of *fields*, according to which one object establishes throughout space a force field (electromagnetic and gravitational fields are examples) and the second object interacts only with the field, not directly with the first object. Maxwell showed in the case of electromagnetism how the fields were transmitted through space. The major development of twentieth-century physics is quantum mechanics, according to which all exchanges of energy must occur in bundles of a discrete size. The classical field is smooth and continuous, and to bring classical field theory into agreement with quantum theory, the field itself must be quantized. That is, according to quantum field theory, the first object does not set up a classical field throughout space but instead emits field quanta. The second object can then absorb those field quanta (and reemit them back to the first object). The two objects interact directly with the exchanged field quanta and therefore indirectly with each other.

In view of the preceding discussion, it is natural to associate the "something" that is exchanged in the nucleon–nucleon interaction with quanta of the nuclear field. For a spin-$\frac{1}{2}$ neutron to turn into a spin-$\frac{1}{2}$ proton, it is clear that the exchanged particle must have integral spin (0 or 1) and must carry electric charge. In addition, if we wish to apply the same exchange-force concepts to nn and pp interactions, there must also be an uncharged variety of exchanged particle. Based on the observed range of the nuclear force, we can estimate the mass of the exchanged particle. Let us assume that a nucleon (which we denote by N, to include both neutrons and protons) emits a particle x. A second nucleon absorbs the particle x:

$$\mathrm{N}_1 \to \mathrm{N}_1 + \mathrm{x}$$
$$\mathrm{x} + \mathrm{N}_2 \to \mathrm{N}_2$$

How is it possible for a nucleon to emit a particle of mass energy $m_x c^2$ and still remain a nucleon, without violating conservation of energy? It is not possible, unless the emission and reabsorption take place within a short enough time Δt that we are unaware energy conservation has been violated. Since the limits of our ability to measure an energy (and therefore to determine whether energy is conserved) are restricted by the uncertainty principle, if $\Delta t < \hbar/(m_x c^2)$, we will be unaware that energy conservation has been violated by an amount $\Delta E = m_x c^2$. The maximum range of the force is determined by the maximum distance that the particle x can travel in the time Δt. If it moves at speeds of the order of c, then the range R can be at most

$$R = c\,\Delta t = \frac{\hbar c}{m_x c^2} = \frac{200\ \mathrm{MeV} \cdot \mathrm{fm}}{m_x c^2} \tag{4.53}$$

where we have used a convenient approximation for $\hbar c$. Equation 4.53 gives a useful relationship between the mass energy of the exchanged particles and the range of the force. For nuclear forces with a range of about 1 fm, it is clear that we must have an exchanged particle with a mass energy of the order of 200 MeV.

Such particles that exist only for fleeting instants and allow us to violate conservation of energy (and momentum—the emitting and absorbing nucleons do not recoil) are known as *virtual* particles. We can observe the force that results from the exchange of virtual particles, but we cannot observe the particles themselves during the exchange. (Exchanged virtual particles can be identical with ordinary particles, however. According to field theory, the Coulomb interaction between electric charges can be regarded as the exchange of virtual photons, which have properties in common with ordinary real photons.)

The exchanged particles that carry the nuclear force are called *mesons* (from the Greek "meso" meaning middle, because the predicted mass was between the masses of the electron and the nucleon). The lightest of the mesons, the π-meson or simply *pion*, is responsible for the major portion of the longer range (1.0 to 1.5 fm) part of the nucleon–nucleon potential. To satisfy all the varieties of the exchanges needed in the two-nucleon system, there must be three pions, with electric charges of $+1$, 0, and -1. The pions have spin 0 and rest energies of 139.6 MeV (for $\pi^{\pm}$) and 135.0 MeV (for π^0). At shorter ranges (0.5–1.0 fm), two-pion exchange is probably responsible for the nuclear binding; at much shorter ranges (0.25 fm) the exchange of ω mesons ($mc^2 = 783$ MeV) may contribute to the repulsive core whereas the exchange of ρ mesons ($mc^2 = 769$ MeV) may provide the spin-orbit part of the interaction. Further properties of these mesons are discussed in Chapter 17.

The differing masses for the charged and neutral pions may explain the possible small violation of charge independence we discussed previously. The single pion that is exchanged between two identical nucleons must be a π^0:

$$\mathrm{n}_1 \to \mathrm{n}_1 + \pi^0 \qquad \pi^0 + \mathrm{n}_2 \to \mathrm{n}_2$$

or

$$\mathrm{p}_1 \to \mathrm{p}_1 + \pi^0 \qquad \pi^0 + \mathrm{p}_2 \to \mathrm{p}_2$$

Charged pion exchange will not work:

$$n_1 \to p_1 + \pi^- \qquad \text{but } \pi^- + n_2 \to ?$$

$$p_1 \to n_1 + \pi^+ \qquad \text{but } \pi^+ + p_2 \to ?$$

because there are no nucleons with charges -1 or $+2$. (There are *excited states* of the nucleon with these charges, as we discuss in Chapters 17 and 18, but these high-energy states are unlikely to contribute substantially to the low-energy experiments we have discussed in this chapter.) However, the neutron-proton interaction can be carried by charged as well as neutral pions:

$$n_1 \to n_1 + \pi^0 \qquad \pi^0 + p_2 \to p_2$$

$$n_1 \to p_1 + \pi^- \qquad \pi^- + p_2 \to n_2$$

This additional term in the np interaction (and the difference in mass between the charged and neutral pions) may be responsible for the small difference in the potential that produces the observed difference in the scattering lengths.

The meson-exchange theory of nuclear forces was first worked out by Yukawa in 1935; some details of his work are summarized in Chapter 17. Meson exchange can be represented by a potential in the basic form of $r^{-1}e^{-r/R}$, where R is the range of the force ($R = \hbar/m_\pi c = 1.5$ fm for pions). A more detailed form for the one-pion exchange potential (called OPEP in the literature) is

$$V(r) = \frac{g_\pi^2 (m_\pi c^2)^3}{3(Mc^2)^2\hbar^2}\left[s_1 \cdot s_2 + S_{12}\left(1 + \frac{3R}{r} + \frac{3R^2}{r^2}\right)\right]\frac{e^{-r/R}}{r/R} \tag{4.54}$$

Here g_π^2 is a dimensionless coupling constant that gives the strength of the interaction (just as e^2 gives the strength of the electromagnetic interaction) and M is the nucleon mass. This particular potential describes only the long-range part of the nucleon–nucleon interaction; other aspects of the interaction are described by other potentials.

The exchange-force model enjoyed a remarkable success in accounting for the properties of the nucleon–nucleon system. The forces are based on the exchange of virtual mesons, all of which can be produced in the laboratory and studied directly. The pion is the lightest of the mesons and therefore has the longest range. Exploring the nucleus with higher energy probes (with shorter de Broglie wavelengths) allows us to study phenomena that are responsible for the finer details of the nuclear structure, such as those that occur only over very short distances. These phenomena are interpreted as arising from the exchange of heavier mesons. Studying the spatial and spin dependence of these detailed interactions allows us to deduce the properties of the hypothetical exchanged meson. On the other hand, particle physicists are able to observe a large variety of mesons from high-energy collisions done with large accelerators. Among the debris from those collisions they can observe many varieties of new particles and catalog their properties. Nuclear physicists are then able to choose from this list candidates for the mesons exchanged in various details of the nucleon–nucleon interaction. This slightly oversimplified view nevertheless emphasizes the close historical relationship between nuclear physics and elementary particle physics.

REFERENCES FOR ADDITIONAL READING

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PROBLEMS

1. What is the minimum photon energy necessary to dissociate ^{2}H? Take the binding energy to be 2.224589 MeV.
2. (a) Use the continuity and normalization conditions to evaluate the coefficients A and C in the deuteron wave functions, Equations 4.3 and 4.4.
   (b) From the resulting wave function, evaluate the root-mean-square radius of the deuteron.
3. The condition for the existence of a bound state in the square-well potential can be determined through the following steps:
   (a) Using the complete normalized wave function, Equations 4.3 and 4.4, show that the expectation value of the potential energy is

$$\langle V \rangle = \int \psi^* V \psi \, dv = -V_0 A^2 \left[\tfrac{1}{2} R - \frac{1}{4k_1} \sin 2k_1 R \right]$$

   (b) Show that the expectation value of the kinetic energy is

$$\begin{aligned} \langle T \rangle &= \frac{\hbar^2}{2m} \int_0^\infty \left| \frac{\partial \psi}{\partial r} \right|^2 dv \\ &= \frac{\hbar^2}{2m} A^2 \left[\tfrac{1}{2} k_1^2 R + \tfrac{1}{4} k_1 \sin 2k_1 R + \frac{k_2}{2} \sin^2 k_1 R \right] \end{aligned}$$

   (c) Show that, for a bound state to exist, it must be true that $\langle T \rangle < -\langle V \rangle$.
   (d) Finally, show that a bound state will exist only for $V_0 \geq \pi^2 \hbar^2 / 8mR^2$ and evaluate the minimum depth of the potential that gives a bound state of the deuteron.

(*Note:* This calculation is valid only in three-dimensional problems. In the one-dimensional square well (indeed, in all reasonably well-behaved