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D-STATE ADMIXTURE AND TENSOR FORCES IN LIGHT NUCLEI

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1. - INTRODUCTION

Early evidence for tensor interaction

Already in 1939 Schwinger¹⁾ realized that the quadrupole moment of the deuteron²⁾ provided decisive evidence for a tensor component in the nuclear force: the bound state of the n-p system is not spherically symmetric as required by a central potential interaction, but has a D-state admixture in the wave function so that the deuteron is prolate with respect to its spin direction. Even in those early days of nuclear physics, the phenomenological analysis of the low energy n-p scattering and the deuteron properties revealed that the tensor force was crucial, accounting for 70% of the binding interaction in spite of the small D-state admixture^{3),4)}.

The close relation between the deuteron quadrupole moment and the long-range tensor potential generated by vector or pseudoscalar meson exchange was realized by Bethe³⁾ years before the discovery of the π meson⁵⁾. Pauli⁶⁾ used this fact as evidence for the exchange of a light pseudoscalar isovector meson.

Stimulated by these results, Gerjuoy and Schwinger⁷⁾ explored the consequences of the phenomenological tensor force for the three- and four-nucleon systems using variational techniques. The conclusion was that the tensor force was also crucial to the binding of these systems, accounting for 50% and 40% of the binding interaction respectively. The wave functions were found to contain D-state components generated by the tensor force with probability similar to that in the deuteron. Unlike the deuteron, however, the ^3H , ^3He and ^4He systems have spins $\frac{1}{2}$ and 0 so that there is no longer any qualitative signature of the D-state components in the ground state observables (e.g., quadrupole moment).

Qualitative importance of pion tensor interaction

The next major advance was the systematic introduction in the mid 50's⁸⁾ of the static one-pion exchange (OPE) potential in the description of the long-range NN potential: the pion exchange gives a well-defined and natural dynamical origin to the strong tensor force in the long and intermediate range region. Although the uncertainties in the description could not be well assessed, it became clear in numerous descriptions using NN potentials with OPE that the deuteron quadrupole moment is given to a precision of 10%⁹⁾. These results were supported

by dispersion relations¹⁰⁾. The importance of OPE for the deuteron is also clear from the fact that such OPE models consistently give an effective range close to the experimental value. An additional quantity that could give further information was the asymptotic D/S ratio η in the deuteron, but lack of data made comparison with theory impossible^{11),12)}.

A major experimental advance occurred with the introduction of tensor polarized deuteron beams. This technique has permitted precise measurements of the deuteron D/S ratio by a variety of methods¹³⁾⁻¹⁵⁾. In addition, direct measurements of the asymptotic D-state components in both three- and four-nucleon systems have become available¹⁶⁾⁻¹⁹⁾, leaving it beyond doubt that these components exist and have qualitatively the expected magnitudes.

For the deuteron these experimental advances were matched and even surpassed on the theoretical level with the recent development of an approach in which the theoretical uncertainty can be explicitly assessed and in which the values of Q and η can be directly linked to details of the NN interaction, notably OPE^{20),21)}.

2. - THE ONE-PION EXCHANGE TENSOR POTENTIAL

The tensor interaction, responsible for D-state phenomena in light nuclei, originates mainly in the OPE interaction. Consider the limit of two nucleons 1 and 2 with large mass M at a relative distance $\vec{r} = r\hat{r} = \vec{r}_1 - \vec{r}_2$. The pion exchange gives rise to a potential that has the characteristic form of a dipole-dipole interaction:

$$V_{\pi}(\vec{r}) = f^2 (\vec{\tau}_1 \cdot \vec{\tau}_2) (\vec{\sigma}_1 \cdot \vec{v})(\vec{\sigma}_2 \cdot \vec{v}) \frac{e^{-m_{\pi} r}}{r}. \quad (1)$$

Here $f^2 \approx 0.08$, $\vec{\tau}_i$ and $\vec{\sigma}_i$ denote the nucleon isospin and spin operators and m_{π} is the pion mass. The structure of this potential is identical to the interaction between two classical electric dipoles \vec{d}_1 and \vec{d}_2 with the replacements $\vec{d}_i \rightarrow \vec{\tau}_i f \vec{\sigma}_i$, where $i = 1, 2$ and $r^{-1} \rightarrow \exp(-m_{\pi} r)/r$.

The well-known tensor interaction resulting from two electric (or two magnetic) dipoles therefore has an analogy in the pionic tensor potential $(\vec{\tau}_1 \cdot \vec{\tau}_2) S_{12}(r) V_T(r)$ with the tensor operator $S_{12}(r) = 3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$ and

$$V_T(r) = f^2 \left[1 + m_\pi r + \frac{1}{3}(m_\pi r)^2 \right] \frac{e^{-m_\pi r}}{m_\pi^2 r^3} . \quad (2)$$

This form of the tensor potential results both from a pseudoscalar and a pseudovector πNN coupling in the static limit with the well-known relation²²⁾ between the corresponding coupling constants $f = g_{\pi}^2 / 2M$.

The basic property of the tensor interaction is that it mixes orbital angular momentum states. In its absence the deuteron would be in a pure S-state and it would have no quadrupole moment. The OPE tensor interaction is very strong and it is mainly responsible for the deuteron binding. In fact the central OPE potential in the deuteron channel ($S=1$, $T=0$) is three times too weak to bind the system in the absence of the tensor force.

3. - DEUTERON EQUATIONS AND D-STATE OBSERVABLES

Asymptotic amplitudes and effective range

The normalized deuteron wave function can be written, using the S- and D-state radial wave functions $u(r)$ and $w(r)$, as

$$\psi_{JM} = \frac{1}{\sqrt{4\pi}} \left[\frac{u(r)}{r} + \frac{w(r)}{r} \frac{1}{\sqrt{2}} S_{12}(\hat{r}) \right] \chi_{1M} , \quad (3)$$

where χ_{1M} is the normalized spin wave function. Outside the range of the nuclear interaction the radial wave functions are completely determined by the deuteron binding energy ϵ and the asymptotic normalization constants A_S and $A_D = \eta A_S$:

$$u(r) \equiv A_S \tilde{u}(r) \xrightarrow{r \rightarrow \infty} A_S e^{-\alpha r} , \quad (4a)$$

$$w(r) \equiv A_S \tilde{w}(r) \xrightarrow{r \rightarrow \infty} \eta A_S \left(1 + \frac{3}{\alpha r} + \frac{3}{\alpha^2 r^2} \right) e^{-\alpha r} , \quad (4b)$$

where $\alpha^2 = M\epsilon$ and $M_R = M$ is the reduced mass of the nucleons. Both A_S and η are external properties of the deuteron like the quadrupole moment and they can be

determined experimentally. The asymptotic S-wave amplitude A_S is deduced to high precision both from the deuteron radius²³⁾ and from effective range theory²⁴⁾:

$$(1+\eta^2)A_S^2 = \frac{2\alpha}{[1 - \alpha \rho(-\epsilon, -\epsilon)]} , \quad (5)$$

where $\rho(-\epsilon, -\epsilon)$ is the diagonal effective range parameter defined by

$$\rho(-\epsilon, -\epsilon) = 2 \int_0^\infty dr [e^{-2\epsilon r} - \tilde{u}(r)^2 - \tilde{w}(r)^2] . \quad (6)$$

The D/S asymptotic ratio η has been measured only recently to high precision. The experimental values are given in the Table (Section 6).

The coupled channel equations

Consider now the deuteron in a potential description. The S and D wave functions then satisfy the coupled equations:

$$u''(r) = [\alpha^2 + U_{00}(r)]u(r) + U_{02} w(r) , \quad (7a)$$

$$w''(r) = [\alpha^2 + \frac{6}{r^2} + U_{22}(r)]w(r) + U_{20}(r)u(r) , \quad (7b)$$

where the terms $U_{ij} = MV_{ij}$ are linear combinations of the central, tensor, spin-orbit and quadratic spin-orbit potentials V_C , V_T , V_{LS} and V_{LL} : $V_{00} = V_C$; $V_{02} = V_{20} = \sqrt{8}V_T$; $V_{22} = V_C - 2V_T - 3V_{LS} - 3V_{LL}$. One notes from Eqs. (7a) and (7b) that the S-D coupling is produced only by the non-diagonal tensor interaction. The D-state admixture in the deuteron is therefore generated from the S-state by the tensor term $U_{02}(r)u(r)$, which acts as a D-wave source in Eq. (7b).

The interaction in the D-state itself is otherwise repulsive because of both the centrifugal barrier and the interaction potential V_{22} . This repulsion reduces the contributions to the asymptotic D-wave parameter η from the short-range interaction region.

The quadrupole moment

In a potential description the quadrupole moment is usually broken into two terms $Q = Q_1 + Q_2$:

$$Q_1 = \frac{1}{\sqrt{50}} \int_0^\infty dr r^2 u(r) w(r), \quad (8a)$$

$$Q_2 = -\frac{1}{20} \int_0^\infty dr r^2 w(r)^2. \quad (8b)$$

The main term Q_1 is linear in the D-wave function $w(r)$, while the term Q_2 , which is quadratic in $w(r)$, gives only a -6% correction. Since the integrand of Q_1 is weighted by r^2 , its main contributions are from the asymptotic region of the D-wave function. Therefore, the quadrupole moment is closely related to the asymptotic D/S parameter η . In addition, meson exchange currents give small corrections of a few per cent originating at rather short range²⁵⁾⁻²⁸⁾.

4. - THE DEUTERON WAVE FUNCTION AND OPE

The pure OPE wave functions, Q and η

All modern NN potentials²⁹⁾⁻³³⁾ contain the OPE as the long-range interaction. They all agree that the tensor component is responsible for about 70% of the deuteron binding interaction, the bulk of it coming from OPE. On the other hand, the binding energy ϵ is quite small, and results from a delicate cancellation between potential attraction and kinetic energy repulsion. As a consequence, the precise value of ϵ is very sensitive to details of the short-range interaction, since these provide the fine tuning of ϵ to the observed value. Since the binding energy also sets the scale of the deuteron size, it is extremely important to discuss other observables in the deuteron within a description that reproduces it correctly. This was done by Glendenning and Kramers⁹⁾ for a variety of models with the correct long-range OPE potential. The remarkable conclusion was that observables such as Q , A_S and η are insensitive to the short-range features and in good agreement with experiments. Even a pure OPE model with a short-range cut-off radius $r_0 = 0.48$ fm so as to reproduce the correct binding energy gives values for these quantities similar to more sophisticated approaches. The main differences occur in the effective range parameter and in the D-state probability. The stability of the results is such that we can conclude with the authors: "it just happens to be one of the quirks of Nature that the force that binds the deuteron does so in such a way as not to reveal itself too intimately"⁹⁾.

This conclusion can be better understood and improved upon by studying the solutions to the deuteron equations (7a) and (7b) with a pure OPE interaction and using the correct binding energy ϵ ³⁴⁾. The coupled Schrödinger equation can then be integrated inward from large r for any given value of the D/S ratio η (see Fig. 1). The solution will of course in general be irregular near the origin both for $\tilde{u}(r)$ and $\tilde{w}(r)$, since OPE does not reproduce ϵ . It is remarkable, however, that either the S-wave function or the D-wave function can be made regular for values of η differing by only 1%, with $\eta_{\text{OPE}} = 0.0274$ giving the regular D-wave function. A second remarkable feature is that the S-wave function $\tilde{u}(r)$ is nearly identical to sophisticated modern wave functions apart from the region inside 0.6 fm. This is nearly entirely a consequence of the long-range part of the OPE interaction. On the other hand, the same comparison for the D-wave function reveals clear differences in the inner region in spite of the excellent asymptotic behaviour.

These OPE S- and D-"wave functions" can be normalized. Although they do not correspond to solutions of the Schrödinger equation, they can then be used to calculate an OPE quadrupole moment using Eqs. (8a) and (8b) with the result $Q_{\text{OPE}} = 0.284 \text{ fm}^2$ for $f^2 = 0.078$. The corresponding OPE diagonal effective range parameter from Eq. (6) gives $\rho_{\text{OPE}}(-\epsilon, -\epsilon) \approx 1.74 \text{ fm}$. These OPE results are close to the experimental values $\eta = 0.0271$, $Q = 0.2859 \text{ fm}^2$ and $\rho(-\epsilon, -\epsilon) = 1.767 \text{ fm}$ ²¹⁾, emphasizing that all these quantities primarily follow from the OPE interaction. The main deviation occurs in the effective range parameter $\rho(-\epsilon, -\epsilon)$ which experimentally is 2% larger than the OPE value.

The empirical deuteron wave function

The experimental invariant structure functions $A(q^2)$ and $B(q^2)$ in elastic electron-deuteron scattering are consistent with these results. Both $A(q^2)$ and $B(q^2)$ are dominated by D-state contributions for squared momentum transfers $q^2 \gtrsim 15 \text{ fm}^2$, with significant additional contributions from meson exchange currents in $B(q^2)$ ³⁵⁾. Locher and Švarc³⁶⁾ have parametrized the corresponding deuteron S and D vertex functions and deduced their empirical form using the static deuteron observables $[Q, \eta, \rho(-\epsilon, -\epsilon), \epsilon]$ as additional constraints, as well as the condition of deuteron wave functions which are regular at the origin. The corresponding empirical D-wave function (see Fig. 1) has a D-state probability $P_D = (6.3 \pm 0.5)\%$, closely similar to the previous value. It is, however, difficult at present to evaluate the systematic uncertainty in this empirical wave function.

OPE effective range and D-state probability

Since the S-wave function $\tilde{u}(r)$ is very stable, we can assume it to be known with sufficient accuracy for an approximate determination of the D-state probability. Define the effective range discrepancy $\delta\rho$ as

$$\delta\rho = \rho_{\text{exp}}(-\epsilon, -\epsilon) - 2 \int_0^{\infty} dr [e^{-2\alpha r} - \tilde{u}(r)^2] \quad (9)$$

The D-state probability P_D is then³⁷⁾

$$P_D = \frac{A_S^2 \delta\rho}{2} \quad (10)$$

Using the Paris wave function²⁹⁾ and the OPE wave function as two limits, one obtains from the experimental effective range an 8% discrepancy $\delta\rho = (1.47 \pm 0.20) \cdot 10^{-2}$ fm, where the uncertainty is mainly due to the model dependence of $\tilde{u}(r)$ in the region 1 to 1.5 fm. From this value the D-state probability is obtained very directly (but only approximately) as $P_D = (5.8 \pm 0.8)\%$, while $P_D^{\text{OPE}} = 7.58\%$ using the OPE D-"wave function".

This determination is far superior to the historical extraction of P_D from the orbital contribution to the small (-2.6%) discrepancy between the deuteron magnetic moment and the sum of the proton and neutron ones, which is very model-dependent with many uncontrollable short-range corrections²³⁾.

The D-state probability is not an observable as was emphasized by Amado³⁸⁾ and Friar³⁹⁾, but depends on the theoretical description. In fact, unitary transformations can be applied to the wave functions, thereby preserving phase shifts and observables, while changing the D-state probability. Such results are also obtained from equivalent descriptions using energy dependent potentials. These limitations do not negate the usefulness of the concept of P_D for practical purposes, but indicate that accurate theoretical discussions should concentrate on direct observables like Q and η .

5. - MODERN THEORY OF THE DEUTERON D-STATE OBSERVABLES

The previous discussion emphasizes the intimate relation between the D-state observables and the OPE tensor interaction. The standard evaluations from the NN interaction solving the Schrödinger equation do not permit a clear estimate of the systematic uncertainties in the theory, nor do they clearly exhibit the nature and reliability of the correction terms, although it is clear that the agreement between theory and experiment is excellent.

Recent developments have remedied these defects, so that the precisely-known D-state observables become powerful tests of models of the NN force in the intermediate and long-range region. In particular, Ericson and Rosa-Clot^{(20), (21), (40)} have developed a rigorous constructive approach to the D/S ratio and quadrupole moment, both of which are expressed directly in terms of the underlying potential. This permits one to visualize the finer details in the NN force directly such as the intermediate-range region contributions and the strong suppression of the short-range contribution.

5.1. The integral formulation of η

The D-state wave function of Eq. (7b) can be formally obtained using a Green function technique with the tensor coupling $U_{20}(r)\tilde{u}(r)$ to the S-state viewed as a source term⁽⁴¹⁾. Ericson and Rosa-Clot^{(20), (21)} realized that the asymptotic D/S ratio η is given by the following simple and exact expression

$$\eta \equiv \int_0^\infty dr \, \eta(r) = \sqrt{8} M \int_0^\infty dr \, r \mathcal{J}_2(r) V_T(r) \tilde{u}(r). \quad (11)$$

Here $\mathcal{J}_2(r)$ is the regular solution of the homogeneous D-wave equation normalized to the corresponding spherical Bessel function at large r : $\mathcal{J}_2(r) \xrightarrow{r \rightarrow \infty} j_2(iar)$.

Two features of this formula are important for further discussion and make it particularly convenient. First, as already mentioned, the S-wave function $\tilde{u}(r)$ in the integrand is only very weakly dependent on the detailed description of the NN interaction. Second, the diagonal D-state potential V_{22} is strongly repulsive as a result of the OPE tensor potential. Consequently, $\mathcal{J}_2(r)$ is suppressed for small r , since it represents an exponentially decreasing tunnelling solution towards the origin produced by the repulsive barrier of the centrifugal and NN

forces. The contribution from the short-range region to the integrand in Eq. (11) is controllably suppressed by this mechanism.

5.2. Discussion of corrections and uncertainties

The characteristic shape of the D/S density curve $\eta(r)$ is shown in Fig. 2 for the OPE interaction and for a typical modern NN potential²⁹⁾ using the same π NN coupling constant $f^2 = 0.078$ and the same S-wave function $\tilde{u}_{\text{Paris}}(r)$ in both cases. The curves are strikingly similar, differing only by 4.6% with $\eta_{\text{OPE}} = 0.02762$ and $\eta_{\text{Paris}} = 0.02633$. Only a few per cent of the contribution to η originates inside 1 fm and less than 12% inside 1.5 fm. The entire sophistication of the modern NN potential including theoretical 2π exchange amounts therefore to only a 4-5% correction arising nearly entirely from the intermediate-range region. We now discuss how to control the leading corrections and uncertainties without appealing to a particular model²¹⁾.

The S-wave function $\tilde{u}(r)$

Different wave functions have little effect on η with a variation that is approximately proportional to the variation in the D-state probability: $\delta\eta/\eta \approx -1/3\delta P_D$. Since a variation of about 1% is the largest one that can be envisioned in P_D , the uncertainties in η of this origin are well below 1%.

We note at this point that the small difference between $\eta_{\text{OPE}} = 0.0275$ of the previous section and $\eta = 0.02762$ from the integral relation (11) is due to the different S-wave functions.

The π NN coupling constant

In the OPE potential it is important to account consistently for the masses and coupling constants of neutral (π^0) and charged (π^\pm) pions. To a precision of a few parts per thousand, this is achieved simply by using the averaged pion mass $m_\pi = 1/3(m_{\pi^0} + 2m_{\pi^\pm}) = 138.032$ MeV and $f^2 = 1/3(f_{\pi^0}^2 + 2f_{\pi^\pm}^2)$ ²¹⁾. The most recent compilation of π NN coupling constants⁴²⁾ gives an experimental value $f^2 = 0.0776$ ⁹⁾. This number includes two new determinations of $f_{\pi^0}^2$ from pp-scattering^{42), 43)} that substantially improve the precision in this quantity.

The D/S parameter η is, to leading order, proportional to f^2 , which therefore contributes $\pm 1.2\%$ to its theoretical uncertainty.

The 2π exchange potential

The principal correction to the OPE result arises from the 2π exchange potential^{44),30)-33)}, which is a theoretical ingredient in modern NN potentials. The two pions have a continuous mass distribution. The weight factor for any given mass is closely linked to the πN scattering amplitude, as well as to the $\pi\pi$ scattering amplitude in S and P states. The mass distribution can be quantitatively evaluated using a dispersion approach for masses up to about $7m_\pi$ using the experimental information on πN and $\pi\pi$ scattering. Its shape is quite well known, but it has a 15% over-all normalization uncertainty⁴⁵⁾. The dominant correction to η comes from the 2π isovector tensor potential, which is strongly dominated by P-wave $\pi\pi$ exchange (the " ρ meson" channel).

These contributions to η come mainly from distances larger than 1 fm. The 2π exchange terms give a theoretically well-founded ($-4.3 \pm 0.6\%$) over-all correction²¹⁾.

The πNN form factor

In the description of the OPE potential the nucleons can be considered to be pointlike only in the idealized, long wavelength approximation. In reality the πNN vertex has a form factor which can be parametrized as:

$$K(t) = \frac{(\Lambda^2 - m_\pi^2)}{(\Lambda^2 + t)} \quad (12)$$

corresponding to an equivalent uniform radius $R = \sqrt{10}/\Lambda$. The πNN form factor is associated with at least a 3π exchange because of quantum numbers, so that the previous investigation of 2π contributions gives no indication about its importance.

Empirically, the consequence of form factors with varying cut-off parameters Λ for the tensor potential and for $\eta(r)$ is displayed in Figs. 3 and 4. In comparison with the corresponding contributions from a modern NN potential²¹⁾, one concludes that its shape is simulated by $\Lambda \approx 750$ MeV/c. This should not lead one to believe, however, that this provides evidence for a πNN form factor with

this Λ , since both $\eta(r)$ and $V_T(r)$ have nearly this shape using only the theoretical 2π contributions with a pointlike πNN vertex.

Direct experimental evidence of the πNN form factor is poor. Dominguez and VerWest⁴⁶⁾ deduce from nearly forward np and pp charge exchange reactions at 8 GeV/c a value $\Lambda \approx 900$ MeV/c (i.e., $R = 0.6$ fm). Holinde³⁰⁾ points out that a low value for Λ would lead to dramatic effects in the lower partial waves in NN scattering. In particular, the 3D_1 phase-shift would be profoundly modified even changing its sign for $\Lambda < 1000$ MeV/c, which indicates a value in the range $1200 < \Lambda < 1400$ MeV/c.

The strongest empirical argument for a low value of Λ is the fact that the axial form factor for the nucleon has $\Lambda \approx 700$ MeV/c⁴⁷⁾ and that pionic and axial phenomena are closely related because of their similarity in quantum numbers. Ericson and Rosa-Clot²¹⁾ point out that such a low value for Λ would lead to a large (-10%) correction to η and an even larger (-16%) correction to Q which would be very difficult to accommodate.

Theoretically there are two different approaches to the form factor leading to very different predictions. Quark bag models of the MIT⁴⁸⁾ or Cloudy Bag⁴⁹⁾ type give a characteristic form factor with $\Lambda < \Lambda_{\text{axial}}$, that is, a quite soft πNN vertex with $R > 0.88$ fm. Such models lead therefore to an important modification in η and Q . Dispersion and field theory approaches^{50),51)} give typically rather pointlike form factors with $\Lambda > 1000$ MeV/c or even larger, with small corrections to η and Q , which is also typical of the little bag quark model⁵²⁾.

The experimental values for both η and Q (see the Table in Section 6) make it difficult, not to say impossible, inside the present description, to accept a pion source distribution for the nucleon as soft as the axial one without compensating dynamical effects⁵³⁾. There is consequently a conflict between the experimental values of η and Q and the quark bag form factor, but there is no consensus on how this conflict should be resolved.

Minor effects

The magnetic moment tensor interaction between the neutron and the proton gives a well-defined -0.34% correction to η . There are small differences of 0.1% between the results from pseudoscalar and pseudovector πNN coupling using the charge averaged mass and coupling constant²¹⁾.

5.3. The integral formulation for Q

A similar analysis can be made of the quadrupole moment $Q = Q_1 + Q_2$ [Eqs. (8a) and (8b)]. The correction term $Q_2 = -0.0182 \text{ fm}^2$ can easily be evaluated to the required accuracy. The leading term Q_1 can be identically rewritten as

$$Q_1 = \frac{1}{\sqrt{50}} \frac{A_S^2}{\alpha^3} \int_0^\infty dr F(r) \eta(r). \quad (13)$$

Its integrand consists of the previous integrand $\eta(r)$ multiplied by the smooth weight function $F(r)$, which is nearly model-independent apart from the unimportant region at very short range (see Fig. 5). The corrections to Q are very similar to those to η with the 2π exchange correction (-6.3±0.9)% the most important one. Like η , the quadrupole moment is very sensitive to the πNN form factor.

5.4. Universal relation between Q and η

The integrands for Q and η have characteristically a long and intermediate range part dominated by OPE. In the intermediate and short-range regions, a modification in one of the integrands leads to a corresponding modification in the other one. There should therefore be a general linear relation between η and Q once the external proportionality factors A_S^2 and f^2 have been removed²¹⁾. Figure 6 shows that this is indeed the case for a series of standard major models where Q/A_S^2 is plotted against η in reduced units. For models with the same effective range, i.e. the same A_S , the linear relation is $\delta Q/Q \approx 1.7 \delta\eta/\eta$. The Q - η values using a πNN form factor with $\Lambda = \Lambda_{\text{axial}}$ also lies on this universal curve, but the corresponding point falls outside Fig. 6. This is a very strong indication that quark models using πNN form factor with inert nucleons are not a satisfactory approach to the NN interaction. Guichon and Miller⁵³⁾ have suggested a possible resolution of this problem: the exchange terms from πqq -couplings can

provide the compensating mechanism leading to an apparent near point-like NN interaction. This idea has not been systematically explored.

In addition, there will be small contributions to the quadrupole moment from meson exchange currents^{25),28)}. The theoretical value of this correction is $+0.0083(30) \text{ fm}^2$ [i.e., $(+3\pm 1)\%$] according to Kohno²⁴⁾: this is smaller than the correction from the π NN form factor.

The very exact universal relation between Q and η in Fig. 5 determines η more accurately at present than direct measurements of η and suggests the value $\eta = 0.0264(3)$. This is two standard deviations below the most accurately quoted experimental value for this quantity (see the Table in Section 6) and indicates a possible small inconsistency.

5.5 MATHEMATICAL CONSTRAINTS ON η

The previous discussion was based on a constructive approach to the quantities η and Q . Klarsfeld, Martorell and Sprung⁵⁴⁾⁻⁵⁶⁾ explored instead the consequences of OPE when the experimental values of Q and the deuteron RMS radius r_d are imposed as constraints. It is then possible to establish rigorous Schwarz inequalities for the contributions of Q and r_d from the region inside a radius R assuming a potential description outside. The input for the inequalities is obtained by integrating the deuteron equations from infinity inward using the correct binding energy and assuming that the long-range interaction (mainly OPE) is known. The inequalities determine η to within narrow limits independent of direct experiments with $0.0262 < \eta < 0.0264$. This approach also gives a good description of the S-wave function outside 1 fm and determines the D-wave function as well although with much larger uncertainties. These results confirm the conclusion in Section 4 that η and $\tilde{u}(r)$ are very stable with respect to different approaches. The special merit of this approach is that it makes very weak assumptions concerning the short-range region. It is no accident that its limits on η are in excellent agreement with the ones deduced from the universal Q - η relation.

6. - PRECISE DETERMINATIONS OF THE D-STATE OBSERVABLES

6.1. The deuteron quadrupole moment

The quadrupole moment is deduced from the hyperfine splitting in the HD and D₂ molecules. The experimental precision is mainly given by the D₂ (J=1) state for which the splitting is measured to an accuracy of 0.01%⁵⁷⁾. The theoretical analysis uses variational electron wave functions. The systematic uncertainty in this procedure is the present limit to the accuracy of Q. The earlier result by Reid and Vaida^{58),59)} with $Q = 0.2860(15) \text{ fm}^2$, has recently been improved by Bishop and Cheung⁶⁰⁾ using more sophisticated wave functions; their result is $Q = 0.2859(3) \text{ fm}^2$. This result should be quite reliable because the quadrupole interaction producing the splitting contributes from the region of large electron probability²¹⁾. The value includes a small correction for the non-adiabatic electron motion. The corrections due to finite deuteron size, polarizability and relativistic effects are very small²¹⁾.

6.2. The D/S ratio and tensor polarization experiments

The quantity η has been notoriously difficult to determine experimentally. In the past few years three different precise methods have become available. They are all based on the orientation of the deuteron using tensor polarization. This singles out the D-state by orienting the deuteron along the major axis of its prolate matter distribution. The experimentally determined values are summarized in the Table.

6.2.1. Pole extrapolation methods

Amado et al.^{15),61)} have pointed out that an analytical extrapolation in the angular variable $\cos\theta$ to the nucleon exchange pole permits a direct determination of η from tensor polarized angular distributions. At the pole position the process has infinite range so that it directly measures the asymptotic amplitudes^{62),63)}.

Elastic $\vec{p}d$ and $\vec{d}p$ scattering

The pole occurs in the exchange graph at a position $\cos\theta = -1.25 - 2.25 \epsilon/E$, where E is the deuteron kinetic energy.

The process measures the quantity $A_S^4 \eta$. The main problems in deducing η are:

- (i) Coulomb effects which, according to Santos and Colby^{64),65)}, and Londergan, Price and Stephenson⁶⁶⁾ can be correctly included on a level of a few per cent;
- (ii) uniqueness and stability of the extrapolation procedure; this is the main source of uncertainty and debate⁶⁷⁾⁻⁶⁹⁾.

In addition, it is important to use a sufficiently accurate value for A_S in the analysis. The value of η obtained in this way is $\eta = 0.0263(9)^{21)}$.

Stripping reactions

The analytical extrapolation in $\cos\theta$ to the deuteron pole determines the quantity η . A particular favourable case is the process $\vec{d}d \rightarrow p^3H$; Coulomb corrections are in this case less important than for the elastic case and the extrapolation procedure is more stable⁷⁰⁾. Borbély et al.⁷¹⁾ deduce the value $\eta = 0.0272(4)$ by this method.

6.2.2. SubCoulomb stripping

Stripping reactions below the Coulomb barrier provide η very accurately. The repulsive Coulomb field keeps the deuteron far from the nuclear interaction region (see Fig. 7)⁷²⁾. In a (\vec{d},p) reaction the neutron must tunnel to the nucleus, which is facilitated by orienting the deuteron with the neutron as close to the nucleus as possible. Processes with near zero Q -value are particularly interesting since the range becomes particularly large (resonance condition). Only a small number of suitable cases exist.

Such processes determine the quantity

$$D_2 = \frac{1}{15} \int_0^\infty dr r^4 w(r) / \int_0^\infty r^2 u(r) dr, \quad (14)$$

which coincides to 0.1% with the corresponding quantity $\tilde{D}_2 = \eta/\alpha^2$ using asymptotic deuteron wave functions^{13),73),74)}. Goddard et al.⁷⁵⁾ have studied the influence of the deuteron structure and the Coulomb field: the dipole polarizability and the electric quadrupole interaction give small, well-defined corrections⁷⁶⁾⁻⁷⁸⁾. The main uncertainty in the analysis is due to the deuteron-nuclear tensor interaction.

The most accurate determination of η by this method⁷⁵⁾ gives a value of $\eta = 0.0271(8)$ corresponding to $D_2 = 0.506(15)$ from the $d^{208}\text{Pb} \rightarrow p^{209}\text{Pb}$ reaction⁷⁹⁾ with a Q-value of 0.3 MeV.

7. - THEORY OF THE D-STATE ADMIXTURE FOR A = 3 AND 4

As with the deuteron, the NN tensor force also generates D-state components in the wave functions of ^3H , ^3He and ^4He . These nuclei are predominantly in a pure S-state. The $^{2S+1}D_J$ -states generated by the tensor force from this S-state is to the leading order $^4D_{1/2}$ for ^3H and ^3He , while ^4He acquires a 5D_0 component in this approximation. In addition, there are also other weaker D-state amplitudes^{80),81)}.

Magnetic moments

For A = 3 a rough indication of the D-state probability in the wave function is given by its orbital contribution to the isoscalar magnetic moment μ_s as compared to the isoscalar nucleon value $\mu_s^N = \frac{1}{2}(\mu_p + \mu_n)$ ⁸²⁾:

$$\mu_s - \mu_s^N = -2P_D (\mu_s - \frac{1}{2}) + \delta\mu_s^{\text{MEC}} = 0.851 - 0.880 = -0.025 \text{ n.m.} \quad (15)$$

where $\delta\mu_s^{\text{MEC}}$ represents the isoscalar meson exchange current, which is small and positive. From this 3% correction the impulse approximation ($\delta\mu_s^{\text{MEC}}=0$) gives $P_D = 4\%$, while more realistic values for $\delta\mu_s^{\text{MEC}}$ indicate $P_D \approx 8\%$ ⁸³⁾⁻⁸⁵⁾. This deduced D-state probability is, however, based on a small correction term. As

with the deuteron, uncontrollable additional short-range terms can easily change the result.

Asymptotic D-wave function

In contrast, the asymptotic amplitudes for the virtual processes ${}^3\text{He} \rightarrow \text{pd}$, ${}^3\text{H} \rightarrow \text{nd}$, ${}^4\text{He} \rightarrow \text{dd}$ are model-independent. Each of these can be separated into the S-wave and D-wave amplitudes A_S and A_D given by the asymptotic behaviour of the normalized wave function:

$$u(r) \rightarrow A_S e^{-\alpha r} ; w(r) \rightarrow \gamma A_S \left(1 + \frac{3}{\alpha r} + \frac{3}{(\alpha r)^2}\right) e^{-\alpha r}. \quad (16)$$

Here r is the relative distance between the centre-of-mass of the two components and $\alpha^2 = 2M_R E$ in terms of their reduced mass M_R and binding energy ϵ in the channel. In a more accurate treatment the asymptotic Coulomb wave functions should be used^{86),87)}.

The D-state amplitudes in the $A = 3$ and $A = 4$ systems are less transparently connected to the underlying tensor force than in the deuteron case owing to the many-body nature of these systems. The qualitative features appear, however, by using the following simple argument formulated by Santos^{88),89)}.

Consider the average tensor potential $\bar{V}_T(r)$ acting between the asymptotic constituents in terms of their relative distance r . If the folding correction from the extended deuteron is neglected, the tensor potential for the coupled S-D channel equation becomes $\bar{V}_T(r) = \sqrt{8} S(A) V_T(r)$, where $V_T(r)$ is the NN tensor potential in the $I = 0$ channel. The kinematical factor $S(A)$ accounts for the spin summation and takes the values 1, -1, -2 for $A = 2, 3$ and 4, respectively.

In the Born approximation, the D-state is then generated from the initially symmetric S-wave function, as in Eq. (11), by the expression

$$\gamma = \sqrt{8} M S(A) \int_0^\infty dr r j_2(\alpha r) \bar{V}_T(r) \tilde{u}(r). \quad (17)$$

This expression is typically accurate to about 30%. It emphasizes that the different D/S ratios in the light nuclei are related.

For the three-body system, it is possible to generate the asymptotic wave functions exactly from a given interaction using Faddeev equations. A systematic but simplified exploration of the asymptotic ${}^3\text{H} \rightarrow \text{nd}$ amplitude has been performed^{85),90)} using a separable interaction with the deuteron binding energy and quadrupole moment constrained to the observed values. From the results of this oversimplified interaction, one can deduce a strong linear correlation between the deuteron D/S ratio η_d and the triton one η_t as the interaction parameters vary (see Fig. 7). The likelihood of such a correlation is strongly increased by the fact that a Faddeev calculation with the Reid soft-core potential by Kim and Muslim⁹¹⁾ gave parameters which nearly fall on the correlation line. This result suggests that the relation also holds for realistic forces: the $A = 2$ and $A = 3$ D/S ratios give analogous information on the tensor force. The experimental observations are consistent with this relation (see Fig. 7).

D-state probability in recent calculations

Modern realistic calculations are in substantial agreement with the result of Gerjuoy and Schwinger⁷⁾ and predict a contribution to the binding of the order of 50% by the tensor potential. There is, however, a large variation in the predicted D-state probability which ranges from 6 to 8% in the three-body nuclei⁸⁵⁾ and from 4 to 12% in ${}^4\text{He}$ ⁹²⁾⁻⁹⁴⁾. Unfortunately these calculations with realistic forces do not quote the asymptotic value η . This deficiency in the description can and should be remedied.

8. - EXPERIMENTS ON THE D/S RATIO FOR $A = 3$ AND 4

Pick-up reactions

Direct measurements of D-wave amplitudes were first obtained using $(\vec{d}, {}^3\text{He})$ and $(\vec{d}, {}^3\text{H})$ pick-up reactions on nuclei^{16),95),96)}. The early results were obtained using rather high incident energies, which introduces large uncertainties in the theoretical analysis. More recent systematic experiments have therefore concentrated on the subCoulomb energy region, which permits D_2 -values [Eq. (14)] to be deduced to a precision of 5-10%⁹⁷⁾. The technique was reviewed by Knutson and Haeberli¹⁵⁾. Typical values for D_2 for ${}^3\text{H}$ and ${}^3\text{He}$

are $D_2 \approx 0.27 \text{ fm}^2$ with the uncertainty due mainly to systematic effects in the analysis.

Santos et al.⁸⁸⁾ and Tostevin⁹⁷⁾ have recently obtained the first clear information on the D-state in ^4He from the $(\vec{d}, ^4\text{He})$ pick-up reaction on ^{32}S and $^{36,38}\text{Ar}$ at an energy well above the Coulomb barrier⁹⁸⁾. The pick-up reaction on ^{89}Y just above the Coulomb barrier, which is more easily interpretable, was also recently analyzed with the result $D_2(^4\text{He}) \approx (-0.3 \pm 0.1) \text{ fm}^2$ ^{19),99)}.

This pick-up analysis does not permit a direct determination of the asymptotic amplitude η , but D_2 differs from $\tilde{D}_2 = \eta/\alpha^2$ by less than 10%. The D/S ratio can be obtained by the pole extrapolation method which, however, has been used much less frequently than for the deuteron. Borbély et al.^{70),91),90)} measured η for the ^3H system from the $\vec{d}d \rightarrow p^3\text{H}$ reaction. They found $\eta = -0.051(5)$ corresponding to $D_2 = -0.21(4) \text{ fm}^2$, which is consistent with the D_2 values from the pick-up reactions on heavy nuclei.

Radiative capture

A very promising method for exploring the asymptotic D-state in ^4He and also in ^3He is the determination of the E2 component in the radiative capture reactions $\vec{d}d \rightarrow \gamma ^4\text{He}$ and $\vec{d}p \rightarrow \gamma ^3\text{He}$. In the case of $\vec{d}d \rightarrow \gamma ^4\text{He}$ the two incident deuterons are identical bosons. This fact and spin-isospin selection rules lead to a very strong dominance of the E2 radiation with the principal contributions from the transitions $^0D_2 \rightarrow ^0S_0$ and $^5S_2 \rightarrow ^5D_0$. The quadrupole matrix elements depend mainly on the asymptotic wave functions, i.e. on η , so that the angular distribution becomes directly sensitive to the D-state asymptotic amplitude. With this technique Weller et al.¹⁸⁾ and Santos et al.¹⁰⁰⁾ recently obtained extremely clear evidence for the asymptotic D-state amplitude in ^4He (see Fig. 9) with $-0.5 < \eta_\alpha < -0.4$. The analogous method applied to $\vec{p}d \rightarrow \gamma ^3\text{He}$ ^{101),102)} leads to a much more complicated analysis, since M1 transitions give important contributions. The results are consistent with other approaches.

The radiative capture technique appears the best and most direct approach for determining η_α ¹⁰³⁾.

9. - OUTLOOK AND OUTSTANDING PROBLEMS

The independent development of highly precise theoretical and experimental methods for the study of the asymptotic D-state amplitudes in the lightest elements has had a powerful impact on our perception of the structure of these systems. The use of integral methods in the deuteron forcefully emphasizes the crucial role of the tensor interaction and, in particular, of the tensor OPE interaction in this system¹⁰⁴⁾. The fact that Q and η can be understood directly in these terms to an accuracy of few per cent without free parameters makes them the best understood dynamical quantities in nuclear physics. In addition the agreement with the accurate experimental data implies very strong constraints on models for exotic components in the NN interaction, and in particular on quark bag descriptions. Although the experimental precision on η is excellent, the theoretical understanding has reached a level in which improving this quantity to the 1% level becomes desirable. In particular, this would permit a detailed test of the universal Q - η correlation.

As a consequence of the highly successful description of the asymptotic deuteron quantities, it becomes natural to view the entire deuteron wave function as dominated by OPE with the exact binding energy fine-tuned by short-range interactions with an effective range that indicates a D-state probability $P_D \approx 5.8\%$. A similar situation, although on a more primitive level, has also developed in the $A = 3$ and 4 systems. The experimental data establish unambiguously the existence of D-state components not only in ^3He and ^3H , but also in ^4He , which therefore must be considered a deformed nucleus. The sign and magnitude of these components are consistent with the deuteron D-state amplitude and with OPE dominance. These observations provide strong direct evidence for the OPE dominated tensor interactions in these systems in which it theoretically contributes half the binding energy. It now becomes urgent to obtain reliable theoretical values for these D-wave asymptotic amplitudes, matching the experimental data. In this respect, an interesting linear correlation between the deuteron and triton D/S ratios has been found to hold to high precision in a theoretical model; it should be established whether this relation is model-independent or not, as well as how accurately the relation relates to experiments.

Present investigations in light elements have larger implications. It is well known that the nuclear tensor force should contribute about $3/4$ of the

binding in nuclear matter, but it is difficult to isolate its effects transparently and unambiguously¹⁰⁵⁾. In this perspective, the D-state components in the lightest nuclei vividly and directly demonstrate for the importance of tensor interaction to nuclear physics in general.

^2H		^3H	^3He	^4He
η	0.0272(4) ^(a) 0.0271(8) ^(b) 0.0263(9) ^(c)	-0.051(5) ^(a)		$-0.5 < \eta < -0.4$ ^(e)
$\bar{\eta} = 0.0271(4)$ ^(g)				
$D_2(\text{fm}^2)$	0.5055(75)	-0.279(12) ^(d)	-0.344 ^(d)	-0.3 ^(f)
$Q(\text{fm}^2)$	0.2859(3)	-	-	-
$\alpha(\text{fm}^{-1})$	0.23154	0.4186	0.4467	1.072

TABLE - Experimental values of the D-state observables

- (a) Pole extrapolation method $\vec{d}d \rightarrow p^3\text{H}$ ⁶⁸⁾
(b) SubCoulomb stripping^{72),21)}.
(c) Pole extrapolation; $\vec{d}p$ elastic scattering^{65),21)}.
(d) SubCoulomb pick-up⁹⁴⁾.
(e) Pick-up on ^{89}Y ¹⁷⁾.
(f) Radiative capture¹⁸⁾.
(g) Average value²¹⁾.

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FIGURE CAPTIONS

Fig. 1 Deuteron S- and D-wave functions for a modern NN potential²⁹⁾ compared to the OPE deuteron "wave functions" [from Ref. 34)]. The empirical wave functions are deduced from Ref. 36).

Fig. 2 Comparison of $\eta(r)$ for OPE and Paris potential as well as the difference between them. The total area of the difference is -4.3% [from Ref. 21)].

Fig. 3 Dependence of $\eta(r)$ on monopole form factors for the πNN vertex [Eq. (12)]. The integrand decreases by 3.1%, 8.5% and 19%, respectively, for the three cases. The corresponding uniform source radii are $R = 0.63$ fm, 0.84 fm and 1.26 fm [from Ref. 21)].

Fig. 4 The OPE tensor potential for the various monopole form factors compared with the Paris tensor potential. Same radii as for Fig. 3 [from Ref. 21)].

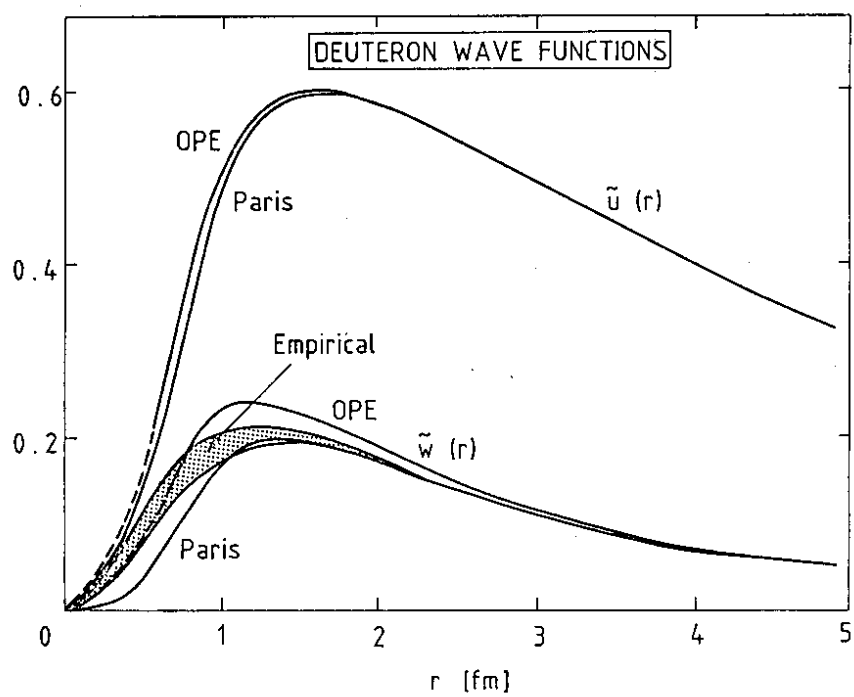
Fig. 5 Model-dependence of the quadrupole weight function $F(r)$ with no interaction and with the OPE potential. A realistic wave function $\tilde{u}(r)$ has been used. The shape of $\eta(r)$ is given for comparison [adapted from Ref. 21)].

Fig. 6 Variation of Q/A_S^2 versus η in various potential models. Both Q and η have been corrected for the proportionality to the πNN coupling constant with $\bar{Q} = (0.078/f^2)Q$ and $\bar{\eta} = (0.078/f^2)\eta$ [adapted from Refs. 21) and 56)]. The experimental values are also shown uncorrected and with meson exchange corrections²⁵⁾ as solid points.

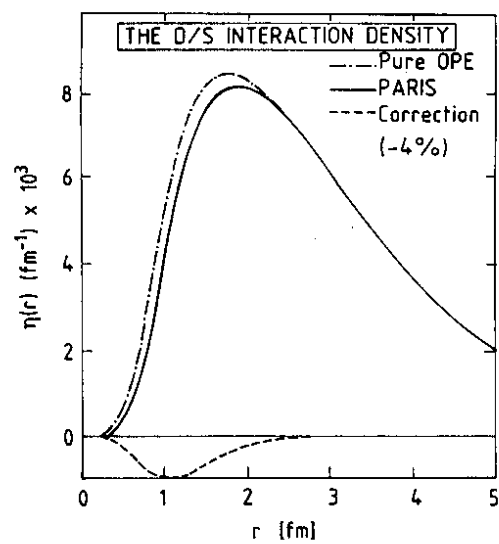
Fig. 7 Contribution to the subCoulomb stripping amplitude according to Goldfarb⁷¹⁾.

Fig. 8 Plot of the tritium and deuteron D/S ratios η_t versus η_d in a separable potential model⁸⁷⁾. The crosses correspond to different percentages of the D-state probability, while the dot results from using a Reid soft-core interaction⁹¹⁾. The experimental value is also given (solid dot).

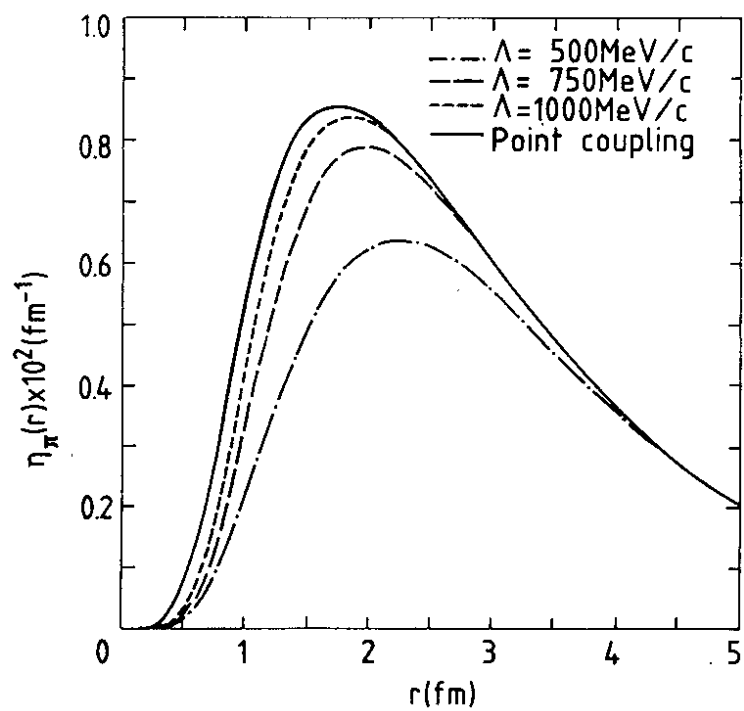
Fig. 9 Differential cross-section $\sigma(\theta)$ and tensor analyzing power for the reaction $\vec{d}d \rightarrow \gamma^4\text{He}$. The $T_{20}(\theta)$ values are manifestly non-vanishing and indicate the ^4He D-state.



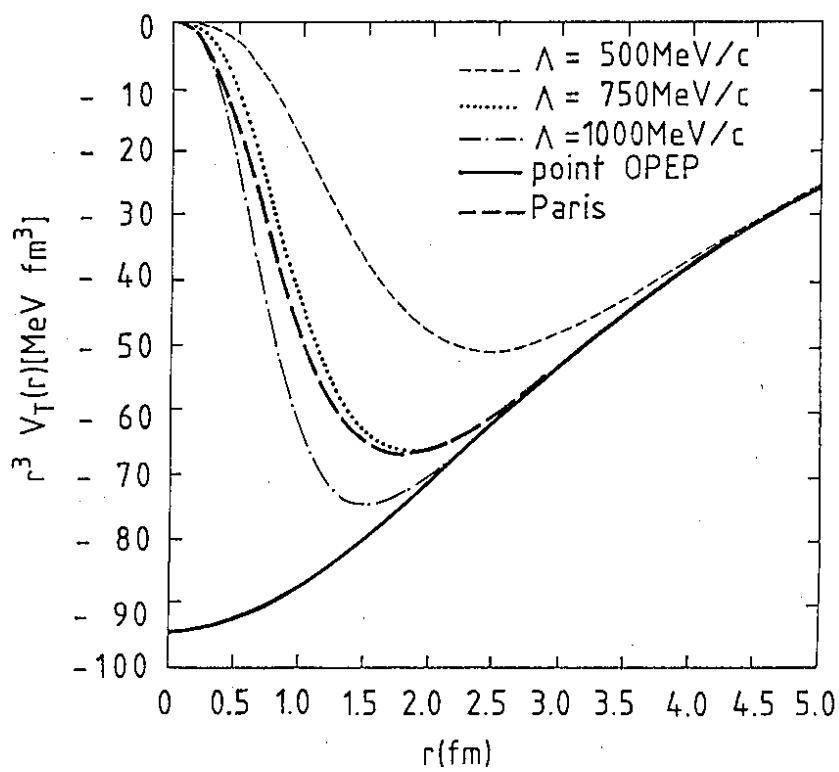
- Figure 1 -



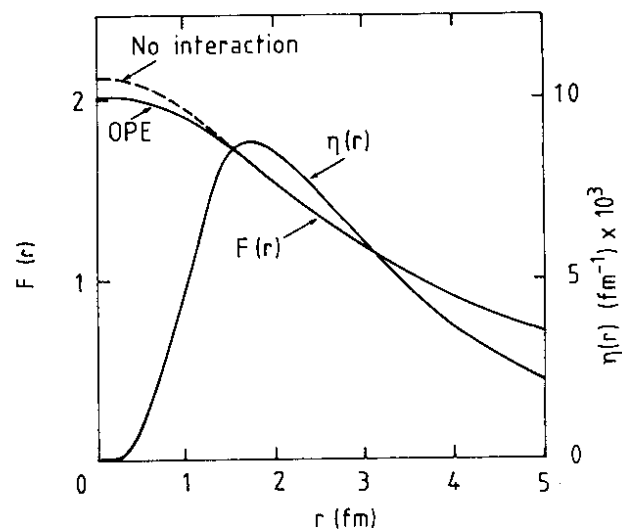
- Figure 2 -



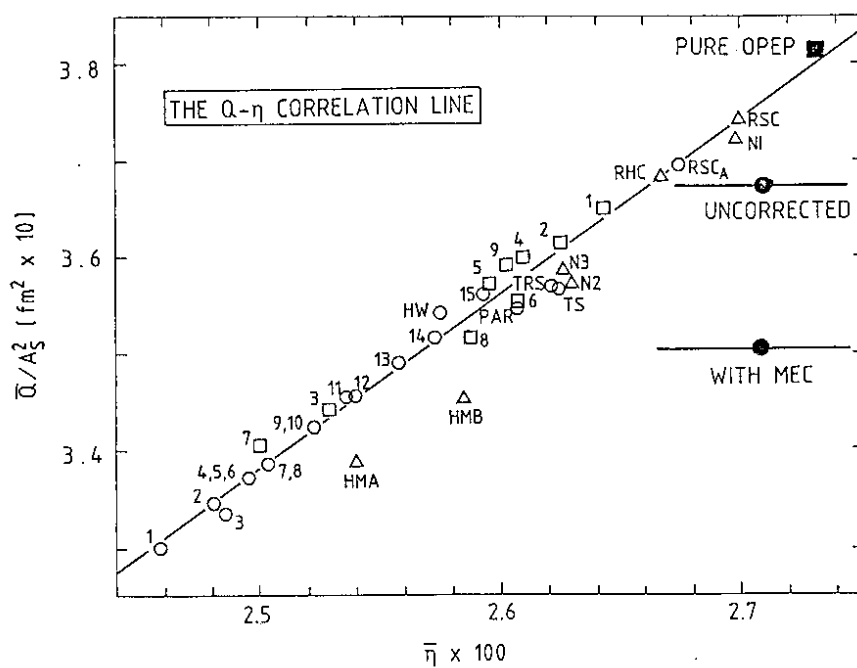
- Figure 3 -



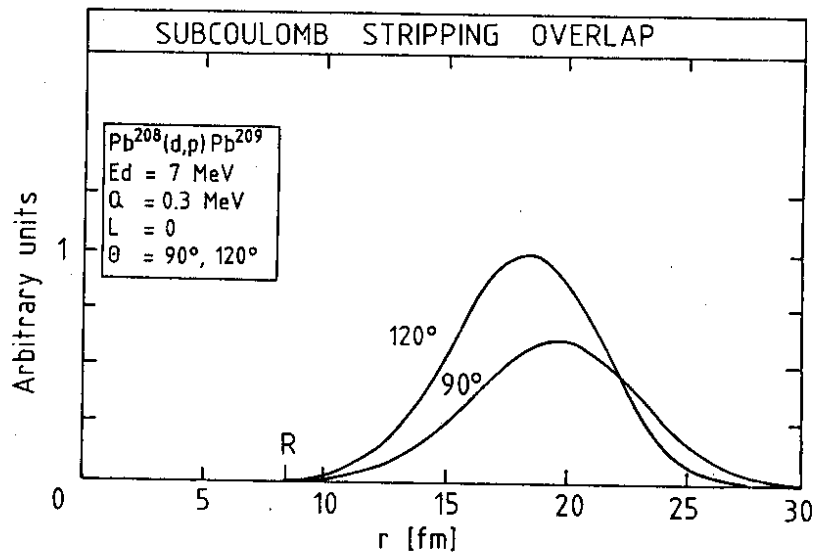
- Figure 4 -



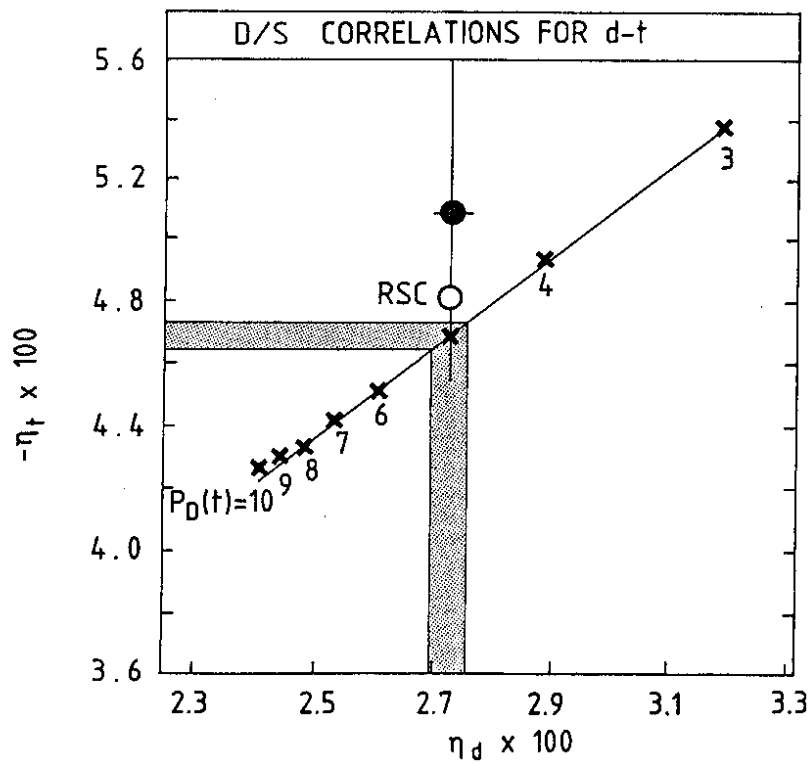
- Figure 5 -



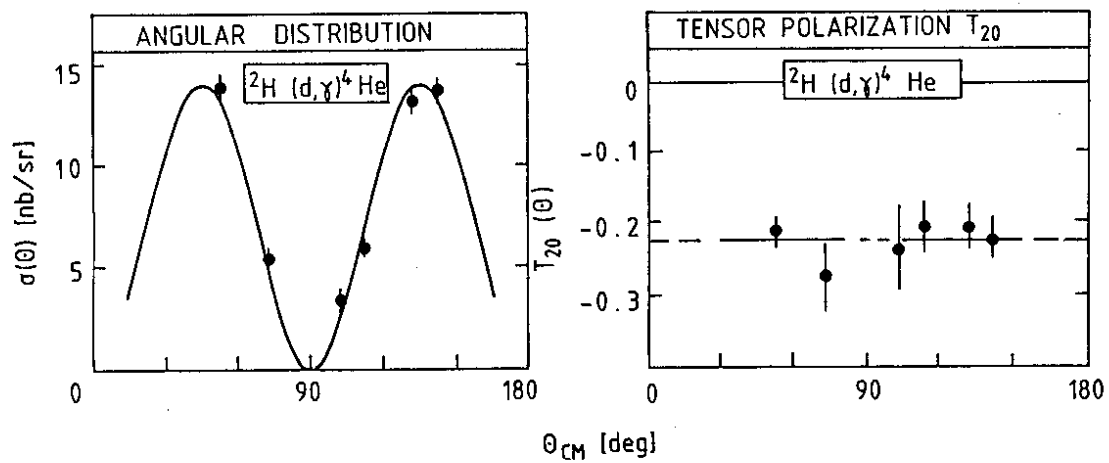
- Figure 6 -



- Figure 7 -



- Figure 8 -



- Figure 9 -