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The Meson Theory of Nuclear Forces, II*

— High Energy Nucleon-nulceon Scattering —

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Using the potentials derived from $Ps(pv)$ meson theory of the second and fourth order, neutron-proton and proton-proton scattering at 40 Mev and 90 Mev in the laboratory system are examined. Calculation is performed by numerical integrations. The characteristic features of high energy nucleon-nucleon scattering are fairly well explained by our pseudoscalar meson potential.

§ 1. Introduction and summary

In the first part of this paper[2]** the deuteron ground state and low energy neutron-proton scattering have been investigated according to the method for treating the problems of nuclear forces proposed by one of us (M. T.) and others[3]. In this paper second and fourth order potentials derived from the pseudoscalar (pv)*** meson theory in the static approximation[4] were adopted near and outside the meson Compton wave length, i.e., in the outside region. In the inside region where the static meson potential becomes meaningless, phenomenological potential represented by the square well or hard core was adopted. Taking the coupling constant $g^2/4\pi$ between π-meson and nucleon as large as about 0.08, experimental data were fairly well accounted for.

The high energy nucleon-nucleon scattering has been treated by several authors[5]. However, these investigations are almost phenomenological, and their main object is the reproduction of the experimental data on the basis of the assumed potential. So far, the explanation of the high energy nucleon-nucleon scattering data according to the meson theory has not yet been examined sufficiently. Reasons why the sufficient treatment has not been made may be as follows: First, nucleon-nucleon potentials already derived from the meson theory are mainly of the second and fourth order. But in the course of derivation of these potentials, many approximations were unavoidably made and their effects

* Preliminary report has been published in this journal[1].

** Hereinafter referred to as I.

*** In this paper, (ps) means "with the pseudoscalar coupling" and (pv) means "with the pseudovector coupling".

have not been fully estimated. Moreover, we know little even about the main character of the sixth or eighth order term. Therefore, the meson potential does not seem to have been finally determined. Second, when we treat high energy problems, many complicated effects, e.g. the velocity dependence of potentials or the relativistic effects, become important. These effects are also scarcely examined.

Recent calculation by Machida and Semba[5] seems to show that the higher order terms do not alter the main characters of the pseudoscalar (pv) potential of the second and fourth order so severely as far as the outside region is concerned. According to their results, the main terms of the sixth order potential are of the same order of magnitude as the second and fourth order potential where the relative distance between two nucleons is about 0.6 times the meson Compton wave length and are rapidly decreasing in the outside region. In this respect, the recent analysis of the contribution of higher order effects performed by Brueckner* and Watson[7] is also to be noticed. They have estimated the correction due to the multiple scattering effects to be less than 30% of the results without taking it into account at the distance 0.6 times the meson Compton wave length and this correction is rapidly decreasing outside.

It is true that the value of the coupling constant is not finally determined and there is a room left for the relative weight of the second and fourth order potential to be varied, so that one cannot say that he has a definite answer concerning the problem of the outside potential. Moreover, the damping effect caused by the terms of higher order in the coupling constant is not thoroughly explored in the pseudoscalar meson theory, and some change may be possible in the conclusion already obtained. However, one can suppose that he has some pictures about the outside potential of nuclear force which is derived from the pseudoscalar meson theory. Recent analyses seem to agree in showing similar conclusion about the outside potential of the fourth order both by (ps) theory and by (pv) theory, owing to some damping effects of core terms in (ps) theory. And, as already mentioned, the (pv) static potential (of the second plus fourth order with $g^2/4\pi=0.08$) fits well to the low energy data if suitable procedure of cutting off the inside potential is adopted.

Here the results of the investigation on the nucleon-nucleon scattering by several different phenomenological potential[8] (performed by one of the authors (W. W.) and his collaborator) are noticable. According to them one can safely say that in the case of the collision energy lower than 100 Mev in the laboratory system, S-wave (and sometimes P-wave in the case of very strongly attractive potential) is affected by the change of the inside region of the potential, whereas P-wave and waves with higher angular momentum are mainly scattered by the outside region of the potential. Then, for the scattering problems up to 100 Mev, the behaviors of P- and higher waves are substantially determined by the outside region of the potential which is given from the meson theory, and these waves play an important role, especially, to determine the angular distribution. Therefore,

* We are indebted to Professor Brueckner for the discussion on this point while he stayed at Kyoto in Sept. 1953.

we think that it is very important at the present stage of meson theory to compare the nucleon-nucleon scattering calculated using the meson theoretical potential with the experimental results up to energy of 100 Mev. For higher energy than 100 Mev, even D-wave is affected by the inside phenomenological potential about which we can say nothing definitely. Therefore, the problems at this energy region are to be treated from different points of view, for example, taking into account the isobar effect or energy dependence of the inside potential.

In this paper, using the potential given in I, the high energy (actually 40 Mev and 90 Mev in the laboratory system) neutron-proton and proton-proton scattering are treated. This potential reproduces the characteristic features of these scattering fairly well. In section 2 the values of phase shifts are given in the Tables, and the angular distributions using these phase shifts are shown. Some corrections to the results of I are made for the parameters of the inside potential of the triplet even state. Discussions about the characteristics of the pseudoscalar meson potential are made in section 3. In section 4 speculation to the future problems is given.

§ 2. Results

In this paper, x is the distance between two nucleons in the unit of meson Compton wave length.

The values of the eigen phase shifts are shown in Tables I and II. For the notations and their meanings, see reference 9. Calculations are performed by the direct numerical integration with the interval Δx and Δy, where the transformation $y = \log x$ is used in the region $x \leq 1$, (see I). Low energy parameters given by this potential are as follows :**

Triplet n-p scattering length : ${}^3a = 5.43 \times 10^{-13}$cm.

Triplet n-p effective range : ${}^3r = 1.47 \times 10^{-13}$cm.

Binding energy of the deuteron : $|\epsilon| = 2.23$ Mev.

Quadrupole moment of the deuteron : $Q = 2.3 \times 10^{-27}\text{cm}^2$.

D-state probability of the deuteron : $p_D = 0.076$.

Singlet n-p scattering length : ${}^1a = -23.7 \times 10^{-13}$cm.

Singlet n-p effective range : ${}^1r = 2.26 \times 10^{-13}$cm.*

Some of the values in the Table 3 of I are not correct and our phenomenological inside potential in the triplet even state is different from that adopted in I. The central potential in the triplet even state by the pseudoscalar (pv) meson theory of the second plus fourth order is strongly repulsive, and it exceeds the tensor potential at $x \sim 0.68$ in its absolute value, so that for the deuteron to be bound we have to cut off this central potential at $x \lesssim 0.5$. One example of such phenomenological inside potentials

* This value is taken from reference I by interpolation.

** For the experimental values, see I.

is given in Table I. But this may not be the best one and another choice of parameters might fit better to the experiments. We notice that the main part of the sixth order pseudoscalar (pv) meson potential calculated by Machida and Semba[6] is similar to our phenomenological inside potential in the features to decrease the strong repulsive central potential of the second and fourth order. After we finished the above calculation, we were informed that Brueckner and Watson[7] treated the nonadiabatic effect in somewhat different way from the usual perturbation method and obtained another kind of potential in the triplet even state. This potential is similar to that of I but the attractive central force. This fact seems to be favourable to our results in the following sense. In the above calculation we needed to add the correction term in the inside region in order to cancel the effect of the repulsive central force. Concerning this central force Sawada pointed out that the repulsive force comes from inadequacy of the usual perturbation method and the attractive force can be derived not only by Brueckner and Watson's method but also in somewhat different way.[10]

Table I. Triplet eigen phase shift.

State ^3L_J	90 Mev	40 Mev	Computational method	Inside potential
$J=1$ { α-wave ($\rightarrow {}^3S_1$ in the absence of tensor force.)	$62.0°$		numerical integration(N. I.) with $\Delta x=0.1$, $\Delta y=0.05$.	at $x \lesssim 0.606$, for central potential attractive square well cut off of the depth of 124.6 Mev, for tensor potential zero cut off.
γ-wave ($\rightarrow {}^3D_1$)	$-2.1°$			
amount of admixture η_1	0.136			
$^3D_2$	$15.7°$		N.I. $\Delta x=0.2$, $\Delta y=0.1$	
$^3D_3$	$-1.0°$*		N.I. $\Delta x=0.2$, $\Delta y=0.1$.	
$^3P_0$	$21.6°$	$17.4°$	N.I. $\Delta x=0.2$, $\Delta y=0.1$.	hard core cut off at $x \lesssim 0.333$.
$^3P_1$	$-10.7°$	$-7.0°$	N.I. $\Delta x=0.2$, $\Delta y=0.1$.	
$J=2$ { α-wave ($\rightarrow {}^3P_2$)	$3.7°$	$1.7°$*	N.I. $\Delta x=0.2$, $\Delta y=0.1$.	
γ-wave ($\rightarrow {}^3F_2$)	$-2.0°$			
amount of admixture η_2	0.850			
$^3F_3$	$-1.5°$		Born approximation.	
$^3F_4$	$0.2°$		Born approximation.	

* This phase shift is calculated using equivalent potential.[4a)]

Table II. Singlet phase shift.

State ^1L	90 Mev	40 Mev	Computational method	Inside potential
^1S	$47.3°$	$64.8°$	numerical integration (N.I.) with $\Delta x=0.1$, $\Delta y=0.05$.	hard core at $x \lesssim 0.333$.
^1D	$2.7°$	$1.0°$	N.I. $\Delta x=0.2$, $\Delta y=0.1$.	
^1P	$-14.5°$	$-12.6°$	N.I. $\Delta x=0.2$, $\Delta y=0.1$.	hard core at $x \lesssim 0.333$.
^1F	$-2.2°$		Born approximation.	

The angular distributions of n-p and p-p scattering in the centre of mass system are shown in Figs. 1, 2, and 3. The total cross section of n-p scattering at 90 Mev is 10.7

$\times 10^{-26}\text{cm}^2$ which is larger compared with the experimental one. As to this point we shall discuss later. The p-p scattering due to Coulomb force is also included but the Coulomb modification of the nuclear phase shift is made only for singlet S-wave. For other waves this modification is negligibly small compared with the nuclear phase shifts.

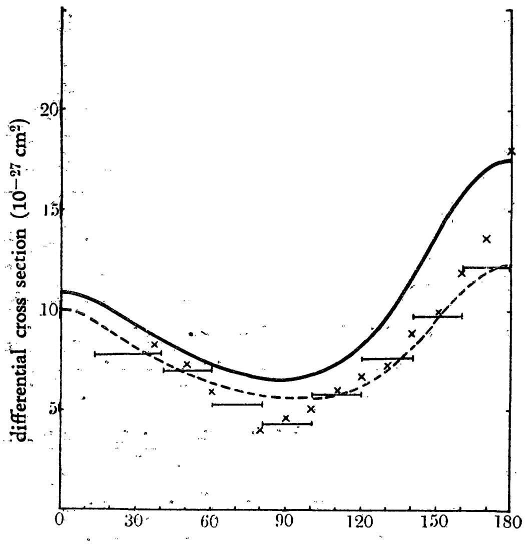

Fig. 1. n-p scattering at 90 Mev.

——theoretical differential cross section
······triplet part of the theoretical differential cross section
× experimental data[11]
⊢⊣ experimental data[12]
These two experimental data are normalized to the experimental total cross section of $(7.9\pm0.7)\times 10^{-26}$ cm^2.

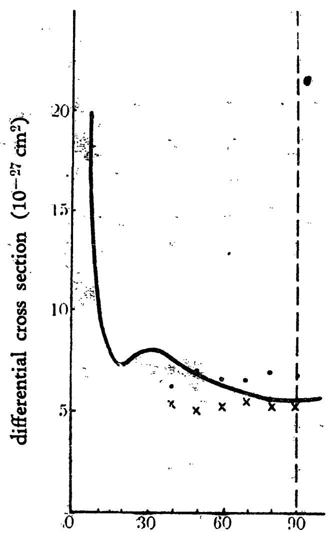

Fig. 2. p-p scattering at 90 Mev.
. experimental data at 75 Mev[13]
× experimental data at 105 Mev[13]

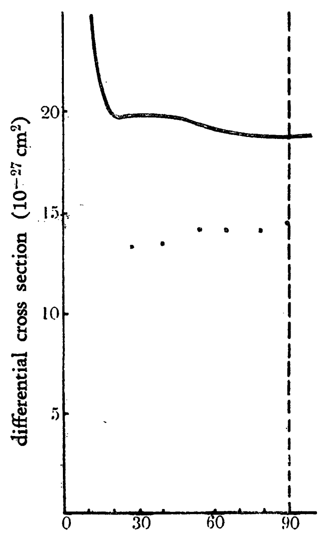

Fig. 3. p-p scattering at 40 Mev.
experimental data at 31.8 Mev[14]

§ 3. Discussions

From the results given in the last section, one can say that the potential given by the pseudoscalar meson theory can give the characteristic features of neutron-proton and proton-proton scattering up to energy of 100 Mev. The reasons for this statement are as follows :

i) As has already been shown in I and by Brueckner and Watson[7], it is quite possible for pseudoscalar meson potential to reproduce the low energy scattering data. This is very satisfactory, because with the appropriate inside potential, S-wave phase shifts at high energies are possibly not as much different from those due to many phenomenological potentials that can give the explanation of high energy nucleon-nucleon scattering data.

As to our results, the total cross section of n-p scattering at 90 Mev is 10.7×10^{-26} cm^2, which is larger than the experimental cross section of $(7.9\pm 0.7)\times 10^{-26}$cm^2.[11] Also the p-p differential cross section at 40 Mev is too large. These two facts are due respectively to the large phase shift of $^3S_1$-and ^1S-wave. Here, it is interesting to note that these phase shifts depend on the inside potential sensitively and have some phenomenological character[8]. So that they may be more or less reduced by taking another way of cutting off.

ii) For the neutron-proton scattering, the meson potential in the triplet odd state is, near and outside its range, not large. Moreover, tensor potential is stronger than central one in this region. Therefore, the effects of $^3P_0$-, $^3P_1$-, and $^3P_2$-phase shifts cancel out as a whole, consequently the angular distribution by the triplet potential is almost symmetric about 90°. Quantitatively, if we expand the triplet differential cross section into the power series of cosine of scattering angle θ as follows[9] :

$$^3\sigma(\theta)=4/k^2\cdot\sum c_n \cos^n\theta,$$

where k is the wave number, then the coefficient c_1 is due to the interference of P-waves and other waves. The ratio $c_1/c_0=0.116$ at 90 Mev is very small compaired with $c_2/c_0=0.315$. In Fig. 4 the potentials effective for $^3P_0$-, $^3P_1$- and $^3P_2$-wave and the wave functions are plotted. If we denote the central potential in the triplet odd state as V_c and tensor potential as V_t (in the unit of (meson mass)$^2\times$(light velocity)^2 / (nucleon mass)), the potential effective for $^3P_0$-state is

$$V_c-4V_t,$$

and for $^3P_1$-state is

$$V_c+2V_t.$$

For $^3P_2$-state, the equivalent potential is (without centrifugal force)

$$V_c-(4/5)V_t+5/x^2-\sqrt{(25/x^4)-(4/x^2)V_t+(44/5)V_t^2}$$
$$\sim V_c-(2/5)V_t-(22/25)x^2V_t^2 \qquad \text{for large } x,$$

and the corresponding phase shift is 1.6°. (We did not use this phase shift to determine the angular distribution, but used the exact phase shifts. See Table I.) From Fig. 4, it can be seen that for P-waves the outside potential does almost determine the sign and magnitude of the phase shift.

The large differential cross section in the backward direction is due to the repulsive potential in the singlet odd state, which is also predicted by pseudoscalar (ps) meson theory.[7] This tendency is in agreement with the recent experiment at 135 Mev reported by Snowden at Birmingham conference.*

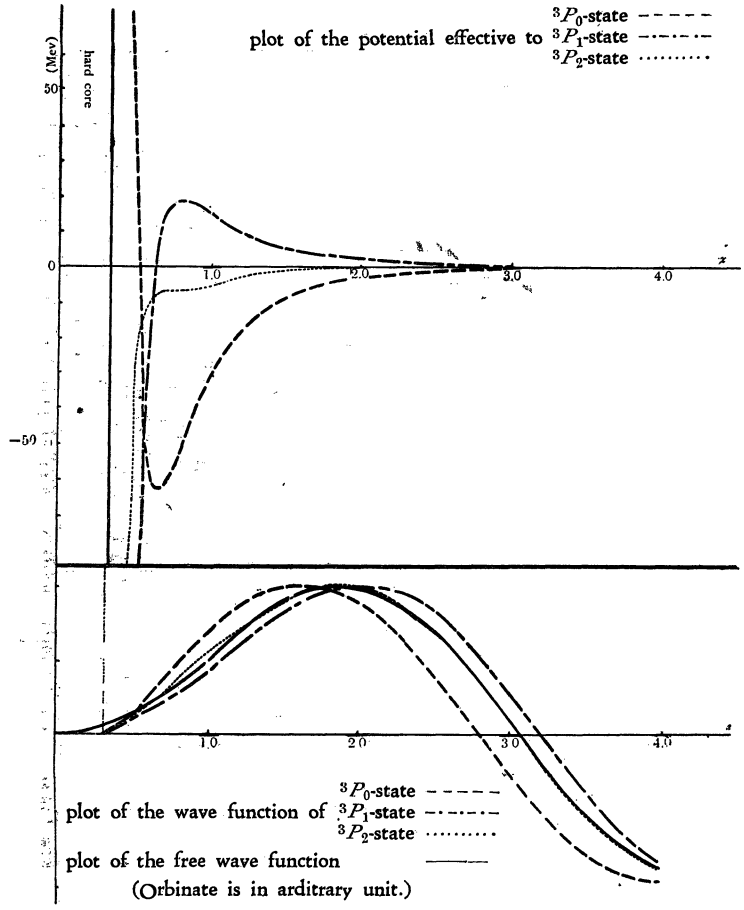

Fig. 4. plots of the potentials and wave functions in the triplet P-states.

iii) For the proton-proton scattering, the potential in the singlet even state is, near and outside its range, very small. Therefore, the phase shift of ^1D-wave is also small and consequently the destructive interference does not break the isotropy of angular distribution so severely.

The polarization $P(\theta, \varphi)$ appearing after p-p scattering by this potential is calculated at 90 Mev, where φ is the azimuthal angle, the z-axis being parallel to the incident

* held at Birmingham in July, 1953.

direction of the beam and x-axis lying in the plane determined by the incident and scattering direction. The polarization of the beam scattered with the angle $\varphi=0°$ and $\theta\sim45°$, where $P(\theta,\varphi)$ has its maximum value, is only abont 6% and so it is now probably impossible to detect it by the double scattering experiment at this energy.

§ 4. Speculation to the future problems

From the discussions above, we can conclude that pseudoscalar meson potential has a satisfactory characters to explain the experimental data in its outside region, in other words, for P- and higher waves. Of course, many but small corrections will necessarily be added to this outside potential in future from meson theoretical point of view. But at the present stage of meson theoretical approach to nuclear force, we consider it also necessary to find the aspects of the corrections required by experimental data dealing with S-wave phase shifts phenomenologically.

When the appropriate inside potentials are adopted to give the good values of low energy parameters (including deuteron parameters) together with the outside potential derived from meson theory in th region $x\gtrsim 0.6$, can we get high energy $^3S_1$- and ^1S-wave phase shifts that reduce the too large cross sections? This is the first problem. It is to be noted that usual phenomenological potentials predicted also larger n-p cross sections.

The potential in the triplet odd state is very delicate: Owing to the cancellation of many terms derived from meson theory, it is very small near the range and its tensor part changes its sign at $x\sim 0.6$. Therefore, even small correction can change the character of the potential features near its range.* For example, the potential of ours and of Brueckner and Watson's[7] agree well in gross features, but the slight difference between them may yield appreciably different values of $^3P_0$-, $^3P_1$-, and $^3P_2$-phase shifts, which are very important to determine both the n-p and p-p angular distribution and p-p polarization. This is not the case for other states. So we must reexamine triplet odd state and, if possible, find the desired potential shape from the experiments. This is the second problem.

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* This point is first suggested by Mr. S. Machida. We are very much indebted to him for his discussion about the meson theoretical potentials.

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