Update on Spectrometer Offsets Determination Using H(e,e'p) Elastics

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H(e,e’p) DATA SETS ANALYZED

H(e,e’p) Coincidence (April 2018) - After SHMS Q3 issue was fixed

Run[i,1]/	hmsP[f,4]/	hmsAngle[f,5]/	shms_P[f,6]/	shms_Angle[f,7]/	beam_e[f,8]/
3288	2.935545	37.338	8.5342	12.194	10.6005
3371	3.47588	33.545	8.5342	13.93	10.6005
3374	2.310387	42.9	8.5342	9.928	10.6005
6595	3.007	21.12	0.0	0.0	3.83350
6601	2.834	23.980	0.0	0.0	3.83350
6602	2.713	25.970	0.0	0.0	3.83350
6609	2.583	29.185	0.0	0.0	3.83350
6611	2.478	29.985	0.0	0.0	3.83350
6634	2.026	38.60	0.0	0.0	3.8335
6871	4.371	12.710	0.0	0.0	4.93090
6875	4.184	15.00	0.0	0.0	4.93090
6876	3.923	18.02	0.0	0.0	4.93090
6879	2.583	34.23	0.0	0.0	4.93090
6881	3.124	27.17	0.0	0.0	4.93090

Elastic H(e,e’p) HMS Singles (Fall 2018 - Kaon LT)

- **ONLY 3 Coincidence Runs, however, W, Em, Pmx, and Pmy are measure per run for a total of 12 measurements**

- **11 HMS Singles run (only W is measured) at 2 different beam energies and a range of momenta and angles.**

Coincidence Data Minimization Procedure

$$dW^{model} = \frac{\partial W}{\partial E_b} E_b a_0 + \frac{\partial W}{\partial E_f} E_f a_1 + \frac{\partial W}{\partial \theta_e} a_2 + \frac{\partial W}{\partial \theta_p} a_3 = dW_{meas} + \epsilon_{dW}$$

$$dE_m^{model} = \frac{\partial E_m}{\partial E_b} E_b a_0 + \frac{\partial E_m}{\partial E_f} E_f a_1 + \frac{\partial E_m}{\partial \theta_e} a_2 + \frac{\partial E_m}{\partial \theta_p} a_3 = dE_{m_{meas}} + \epsilon_{dE_m}$$

$$dP_{mx}^{model} = \frac{\partial P_{mx}}{\partial E_b} E_b a_0 + \frac{\partial P_{mx}}{\partial E_f} E_f a_1 + \frac{\partial P_{mx}}{\partial \theta_e} a_2 + \frac{\partial P_{mx}}{\partial \theta_p} a_3 = dP_{mx_{meas}} + \epsilon_{dP_{mx}}$$

$$dP_{mz}^{model} = \frac{\partial P_{mz}}{\partial E_b} E_b a_0 + \frac{\partial P_{mz}}{\partial E_f} E_f a_1 + \frac{\partial P_{mz}}{\partial \theta_e} a_2 + \frac{\partial P_{mz}}{\partial \theta_p} a_3 = dP_{mz_{meas}} + \epsilon_{dP_{mz}}$$

$$dW_{meas} = W_{data} - W_{simc}$$
$$dE_{m_{meas}} = E_{m_{data}} - E_{m_{simc}}$$
$$dP_{mx_{meas}} = P_{mx_{data}} - P_{mx_{simc}}$$
$$dP_{mz_{meas}} = P_{mz_{data}} - P_{mz_{simc}}$$

$$a_0 = \frac{dE_b}{E_b} \qquad a_2 = d\theta_e$$
$$a_1 = \frac{dE_f}{E_f} \qquad a_3 = d\theta_p$$

4 equations / run
3 coincidence runs

12 x 4 Matrix

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdot & \cdot & & & \\ \cdot & & & & \\ \cdot & & & \cdot & \\ \cdot & & & & \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} dW_{meas} \\ dE_{m_{meas}} \\ dP_{mx_{meas}} \\ dP_{mz_{meas}} \end{bmatrix} + \begin{bmatrix} \epsilon_{dW} \\ \epsilon_{dE_m} \\ \epsilon_{dP_{mx}} \\ \epsilon_{dP_{mz}} \end{bmatrix}$$

Model Matrix

Residuals

Measured Variables

$$\mathbb{M}\vec{a} = \vec{b} + \hat{\epsilon}$$

Solve for: $\mathbb{M}\vec{a} - \vec{b} = 0$

... to find the parameters $\vec{a}$ that minimize the equation.

Residuals and Chi-2 Definitions

$$\hat{\epsilon} = \mathbb{M}\vec{a} - \vec{b}$$

Input minimized parameters into residual formula to determine difference between model and data

$$\chi^2 \equiv \hat{\epsilon}^T \mathbb{N}^{-1} \hat{\epsilon}$$

Chi-2 in Matrix Form

Inverse Covariance Matrix of Measured Errors
*** We assume un-correlated measured errors**

$$\mathbb{N}^{-1} = \begin{bmatrix} 1/\sigma^2_{dW_{obs}} & 0 & 0 & 0 & \dots & \dots \\ 0 & 1/\sigma^2_{dE_{m_{obs}}} & 0 & 0 & & \\ 0 & 0 & 1/\sigma^2_{dP_{mx_{obs}}} & 0 & & \\ 0 & 0 & 0 & 1/\sigma^2_{dP_{mz_{obs}}} & & \\ \vdots & & & & & \end{bmatrix}$$

Expand Chi2 for all coincidence runs

$$\chi^2 = \underbrace{\frac{\epsilon^2_{dW}}{\sigma^2_{dW_{obs}}} + \frac{\epsilon^2_{dE_m}}{\sigma^2_{dE_{m_{obs}}}} + \frac{\epsilon^2_{dP_{mx}}}{\sigma^2_{dP_{mx_{obs}}}} + \frac{\epsilon^2_{dP_{mz}}}{\sigma^2_{dP_{mz_{obs}}}}}_{\text{4 measurements / coincidence run}} + \dots$$

Variance-Covariance Matrix Definition

- Matrix describes the errors on the minimized parameters (diagonal) and any possible correlations between those parameters (off-diagonal)

$$\mathbf{COV}_{\mathbf{M}} = (\mathbb{M}^T \mathbb{N}^{-1} \mathbb{M})^{-1}$$

The variance-covariance matrix ONLY depends on:

- **Model matrix** $\mathbb{M}$
- **Inverse matrix of measured errors** $\mathbb{N}^{-1}$

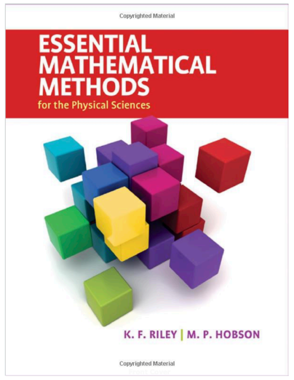
Example of how covariance matrix looks like

$$\begin{bmatrix} \sigma_1^2 & cov_{12} & \cdots & cov_{1n} \\ cov_{12} & \sigma_2^2 & \cdots & cov_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ cov_{1n} & cov_{2n} & \cdots & \sigma_n^2 \end{bmatrix}$$

Correlated errors

Variance of parameters

Source: Essential Mathematical Methods for the Physical Sciences
1st Edition by Riley & Hobson
(See Chapter 17.5 - Data Modeling)



Correlation Matrix

- **Matrix describes the strength of correlations between (i,j), and is related to the covariance matrix as follows:**

$$\mathrm{Cor}_{ij} = \frac{\mathrm{Cov}_{ij}}{\sqrt{\mathrm{Cov}_{ii}\mathrm{Cov}_{jj}}}$$

- **Matrix elements range from (-1, 1)**
- **+1 means variables are 100 % correlated**
- **-1 means variables are 100 % anti-correlated**
- **0 means variables are 0% correlated**

Example of how correlation matrix looks like

$$\mathbf{R} = \begin{pmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1p} \\ r_{21} & 1 & r_{23} & \cdots & r_{2p} \\ r_{31} & r_{32} & 1 & \cdots & r_{3p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{p1} & r_{p2} & r_{p3} & \cdots & 1 \end{pmatrix}$$

Coincidence Data Minimization Results

- The data was fitted with various models
- The model is based on which derivatives were taken with respect to the four measured variables: W, Em, Pmx Pmz (See Slide 3)
- Any difference observed in the measured quantities, dW, dEm, dPmz, dPmz is assumed to be originating from variable specified by the model

$$\text{MODEL 0:}\ (E_f, \theta_e)$$

$$\text{MODEL 1:}\ (E_b, E_f, \theta_e)$$

$$\text{MODEL 2:}\ (E_b, E_f, \theta_e, \theta_p)$$

$$\text{MODEL 3:}\ (E_f, \theta_e, \theta_p)$$

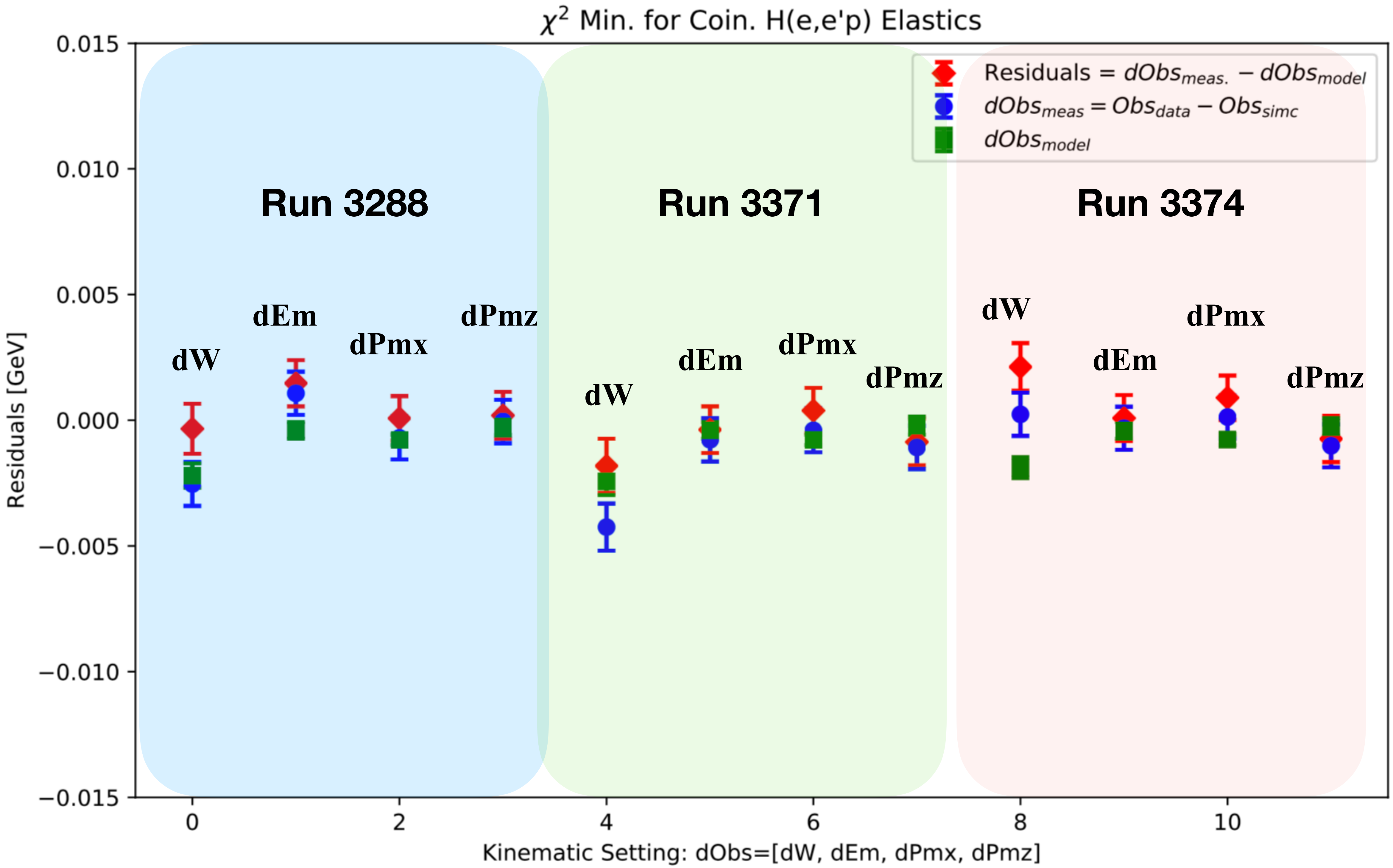
- A systematic relative error of 1e-4 was added to all measured errors in quadrature due to magnet cycling procedure (via NMR) errors

MODEL 0: (E_f, θ_e)

```
****COVARIANCE MATRIX*****
2x2 matrix is as follows

     |      0    |      1    |
-------------------------------
   0 |  1.499e-09  -6.576e-10
   1 | -6.576e-10   8.333e-10

=== Optimized Parameters ===
total equations x total runs = 4x3 = 12 observations, # parameters = 2, dof = 10
initial chi2 = 35.6321 initial chi2/dof = 3.56321
chi2 = 16.2938  chi2/dof = 1.62938
dEf / Ef = 4.70511e-05
dth_e = 8.34063e-05

=== Uncertainty in Parameters (Diagonal Elements) ===
sqrt[cov(0,0)] = dEf / Ef = 3.87172e-05
sqrt[cov(1,1)] = dth_e [rad] = 2.88668e-05

=== Uncertainty in Parameters (Off-Diagonal Elements) ===
cov(0,1) = dEf / Ef, dth_e = -6.57601e-10

****CORRELATION MATRIX*****
2x2 matrix is as follows

     |      0    |      1    |
-------------------------------
   0 |          1    -0.5884
   1 |    -0.5884          1
```

MODEL 0: (E_f, θ_e)

- **All residuals are < 5 MeV**



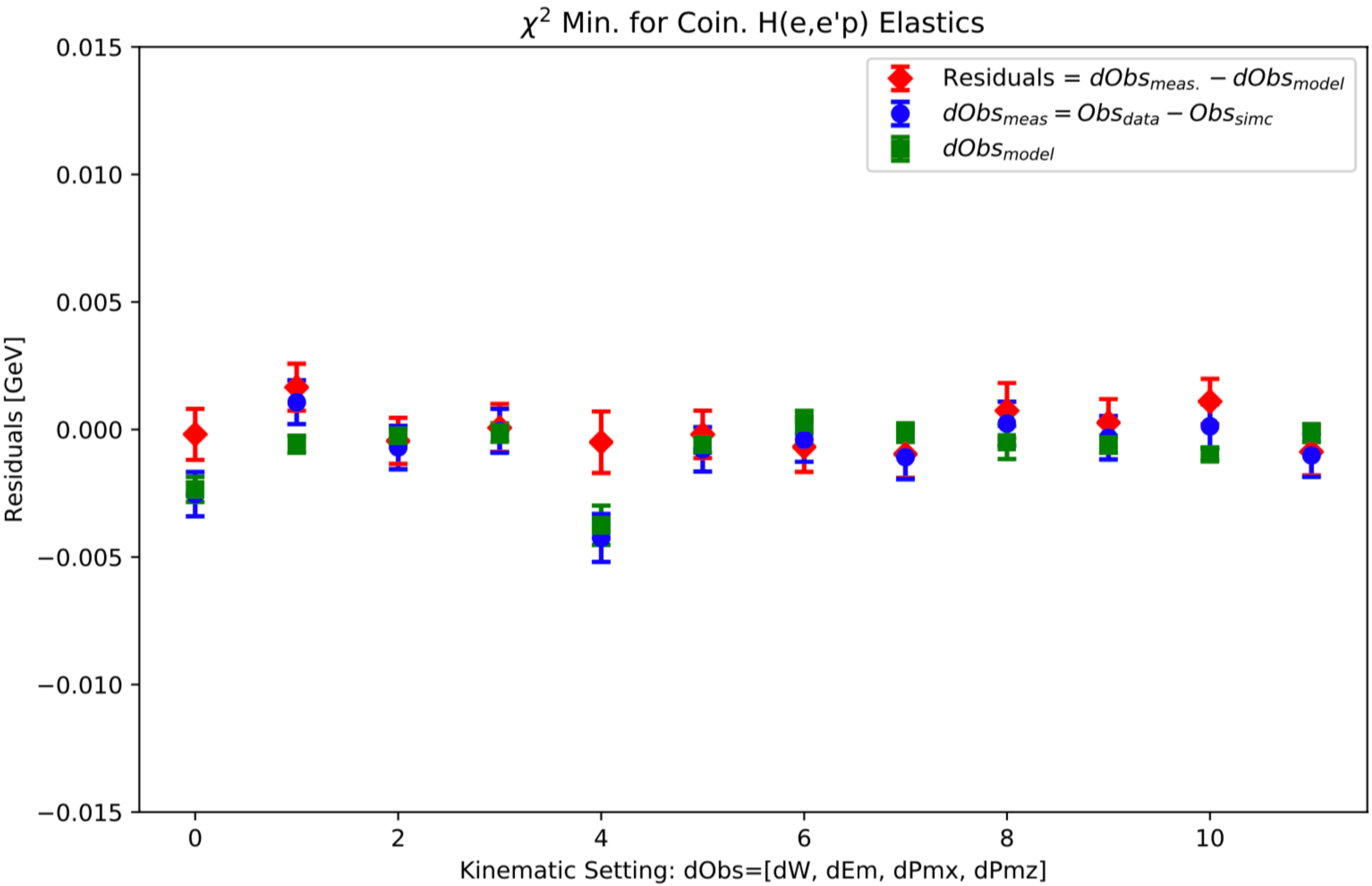
- **The residuals for this plot (and subsequent plots) are grouped based on run**

MODEL 1: (E_b, E_f, θ_e)

```
****COVARIANCE MATRIX*****
3x3 matrix is as follows

     |      0    |      1    |      2    |
------------------------------------------------
   0 |  4.923e-07   6.056e-07  -1.119e-07
   1 |  6.056e-07   7.464e-07  -1.383e-07
   2 | -1.119e-07  -1.383e-07   2.627e-08

=== Optimized Parameters ===
total equations x total runs = 4x3 = 12 observations, # parameters = 3, dof = 9
initial chi2 = 35.6321 initial chi2/dof = 3.95912
chi2 = 9.77274  chi2/dof = 1.08586
dEb / Eb = -0.00181842
dEf / Ef = -0.00219008
dth_e = 0.00049339

=== Uncertainty in Parameters (Diagonal Elements) ===
sqrt[cov(0,0)] = dEb / Eb = 0.000701658
sqrt[cov(1,1)] = dEf / Ef = 0.000863942
sqrt[cov(2,2)] = dth_e [rad] = 0.000162091

=== Uncertainty in Parameters (Off-Diagonal Elements) ===
cov(0,1) = dEb_Eb * dEf_Ef = 6.05582e-07
cov(0,2) = dEb_Eb * dth_e = -1.11914e-07
cov(1,2) = dEf_Ef * dth_e = -1.38318e-07

****CORRELATION MATRIX*****
3x3 matrix is as follows

     |      0    |      1    |      2    |
------------------------------------------------
   0 |          1       0.999      -0.984
   1 |      0.999           1     -0.9877
   2 |     -0.984     -0.9877           1
```

MODEL 1: (E_b, E_f, θ_e)

- **All residuals are < 5 MeV**

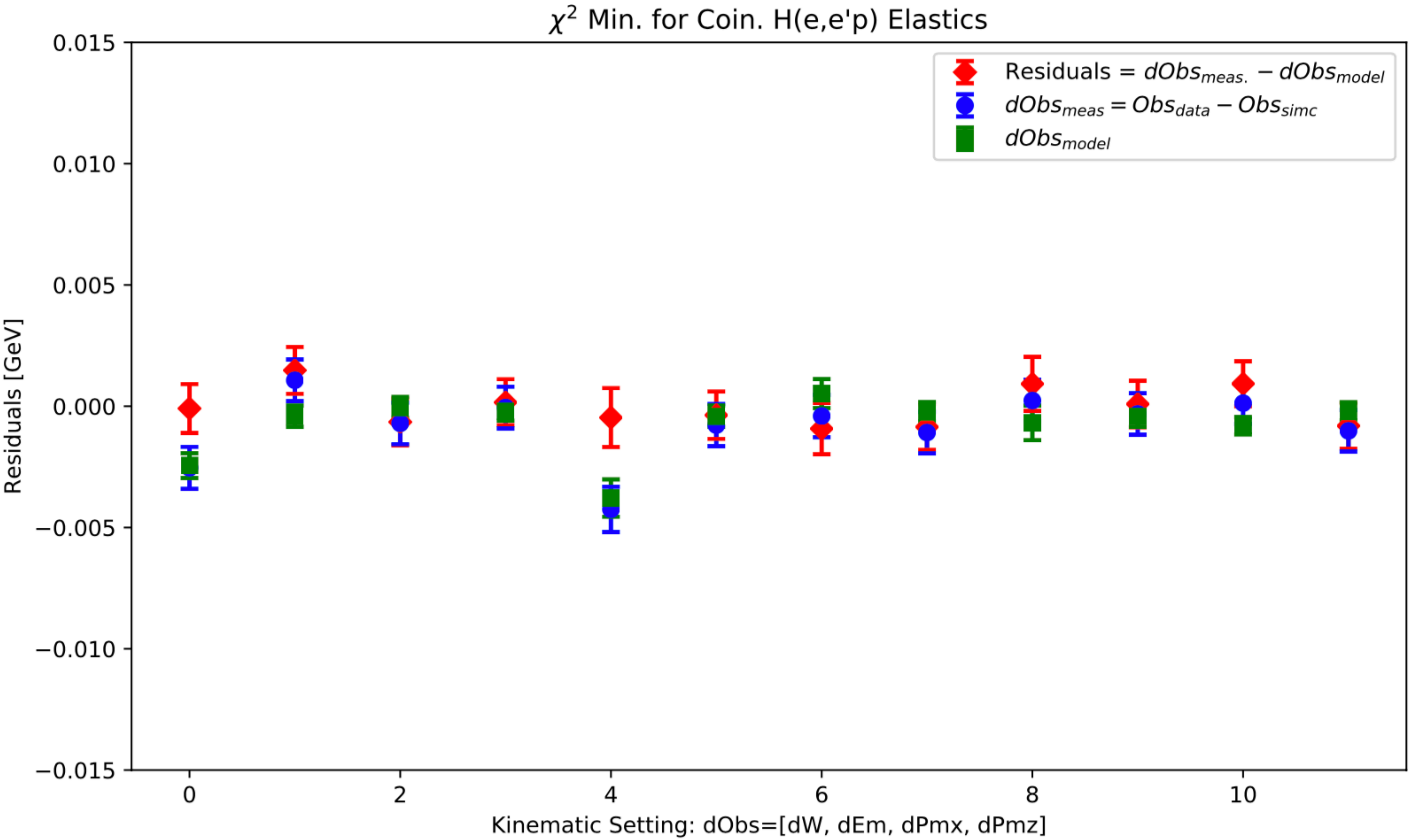


MODEL 2: $(E_b, E_f, \theta_e, \theta_p)$


```
****COVARIANCE MATRIX*****
4x4 matrix is as follows

     |      0    |      1    |      2    |      3    |
----------------------------------------------------------
   0 |  5.623e-07   6.839e-07  -1.213e-07  -6.267e-08
   1 |  6.839e-07    8.34e-07  -1.488e-07  -7.014e-08
   2 | -1.213e-07  -1.488e-07   2.754e-08   8.433e-09
   3 | -6.267e-08  -7.014e-08   8.433e-09   5.617e-08

=== Optimized Parameters ===
total equations x total runs = 4x3 = 12 observations, # parameters = 4, dof = 8
initial chi2 = 35.6321 initial chi2/dof = 4.45401
chi2 = 9.36812  chi2/dof = 1.17102
dEb / Eb = -0.00164769
dEf / Ef = -0.00199839
dth_e = 0.000469832
dth_p = -0.000147982

=== Uncertainty in Parameters (Diagonal Elements) ===
sqrt[cov(0,0)] = dEb / Eb = 0.000749839
sqrt[cov(1,1)] = dEf / Ef = 0.000913231
sqrt[cov(2,2)] = dth_e [rad] = 0.000165951
sqrt[cov(3,3)] = dth_p [rad] = 0.000236996

=== Uncertainty in Parameters (Off-Diagonal Elements) ===
cov(0,1) = dEb_Eb * dEf_Ef = 6.83851e-07
cov(0,2) = dEb_Eb * dth_e = -1.21324e-07
cov(0,3) = dEb_Eb * dth_p = -6.26743e-08
cov(1,2) = dEf_Ef * dth_e = -1.48849e-07
cov(1,3) = dEf_Ef * dth_p = -7.01424e-08
cov(2,3) = dth_e * dth_p = 8.43255e-09

****CORRELATION MATRIX*****
4x4 matrix is as follows

     |      0    |      1    |      2    |      3    |
----------------------------------------------------------
   0 |          1      0.9986      -0.975     -0.3527
   1 |     0.9986           1     -0.9822     -0.3241
   2 |     -0.975     -0.9822           1      0.2144
   3 |    -0.3527     -0.3241      0.2144           1
```

MODEL 2: $(E_b, E_f, \theta_e, \theta_p)$

- **All residuals are < 5 MeV**

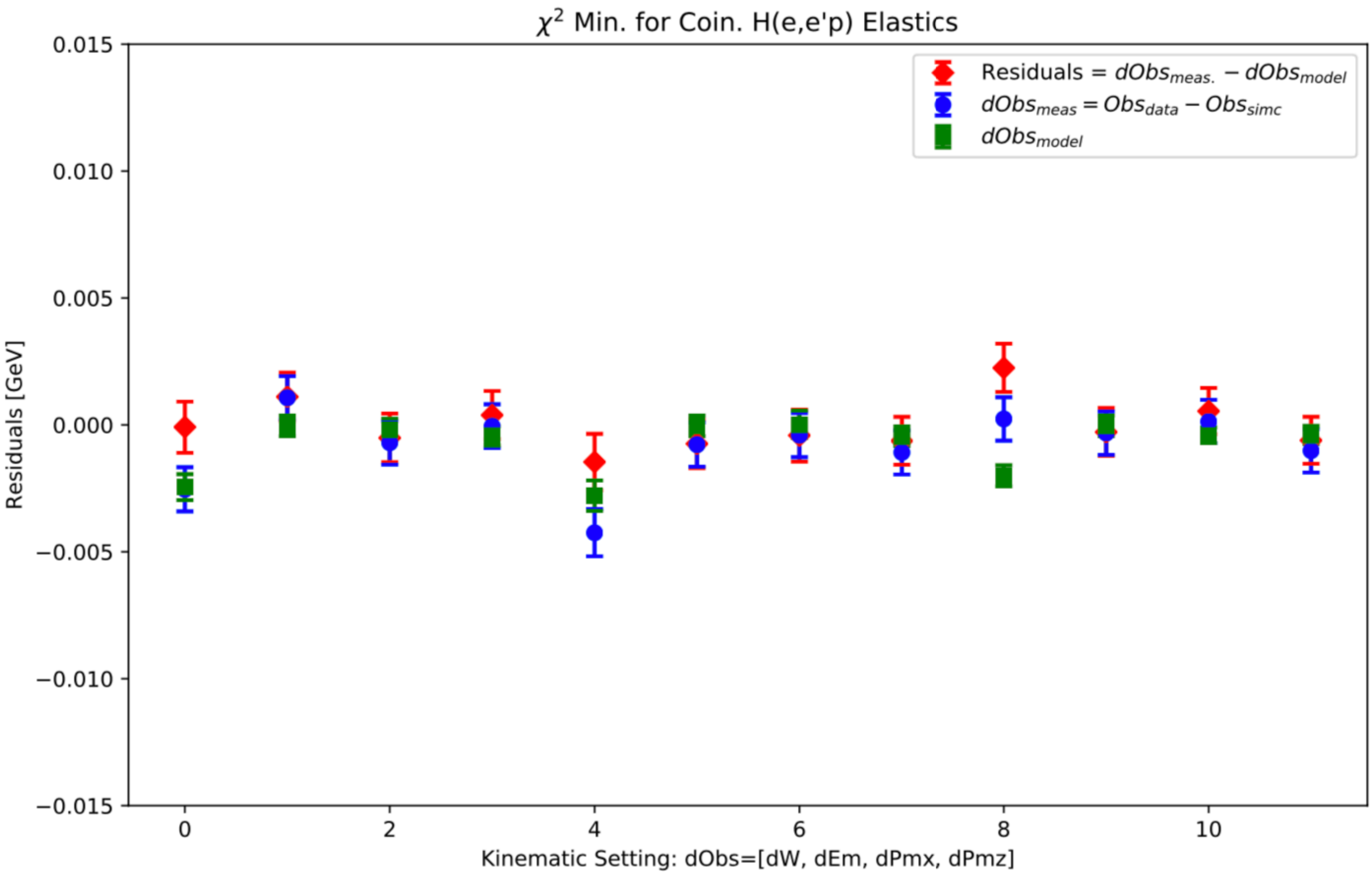


MODEL 3: $(E_f, \theta_e, \theta_p)$


```
****COVARIANCE MATRIX*****
3x3 matrix is as follows

     |      0    |      1    |      2    |
--------------------------------------------
   0 |  2.252e-09  -1.288e-09   6.086e-09
   1 | -1.288e-09    1.36e-09  -5.091e-09
   2 |  6.086e-09  -5.091e-09   4.918e-08

=== Optimized Parameters ===
total equations x total runs = 4x3 = 12 observations, # parameters = 3, dof = 9
initial chi2 = 35.6321 initial chi2/dof = 3.95912
chi2 = 14.0551  chi2/dof = 1.56168
dEf / Ef = 4.22145e-06
dth_e = 0.000118262
dth_p = -0.000345369

=== Uncertainty in Parameters (Diagonal Elements) ===
sqrt[cov(0,0)] = dEf / Ef = 4.74559e-05
sqrt[cov(1,1)] = dth_e [rad] = 3.68829e-05
sqrt[cov(2,2)] = dth_p [rad] = 0.000221768

=== Uncertainty in Parameters (Off-Diagonal Elements) ===
cov(0,1) = dEf_Ef * dth_e = -1.2876e-09
cov(0,2) = dEf_Ef * dth_p = 6.08565e-09
cov(1,2) = dth_e * dth_e = -5.09129e-09

****CORRELATION MATRIX*****
3x3 matrix is as follows

     |      0    |      1    |      2    |
--------------------------------------------
   0 |          1     -0.7356      0.5783
   1 |    -0.7356           1     -0.6224
   2 |     0.5783     -0.6224           1
```

MODEL 3: $(E_f, \theta_e, \theta_p)$

- **All residuals are < 5 MeV**



HMS Singles Data Minimization Procedure

$$dW^{model} = \frac{\partial W}{\partial E_b} E_b \color{red}{a_0} + \frac{\partial W}{\partial E_f} E_f \color{red}{a_1} + \frac{\partial W}{\partial \theta_e} \color{red}{a_2} = dW_{meas} + \epsilon_{dW}$$

11 elastic runs
(11 equations or rows)

$$\begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdot & \cdot & & & \\ \cdot & & \cdot & & \\ \cdot & & & \cdot & \\ \cdot & & & & \end{bmatrix} \begin{bmatrix} \color{red}{a_0} \\ \color{red}{a_1} \\ \color{red}{a_2} \end{bmatrix} = \begin{bmatrix} dW_{meas} \\ \vdots \\ \vdots \end{bmatrix} + \begin{bmatrix} \epsilon_{dW} \\ \vdots \\ \vdots \end{bmatrix}$$

. . . Same Procedure as Coincidence Data
(SEE SLIDES 3-6)

HMS Singles Data Minimization Results

- The data was fitted with various models
- The model is based on which derivatives were taken with respect to W.
  - Any difference observed in the measured quantities, dW = data-simc, is assumed to be originating from the derivative of the variables specified by the model below

$$\text{MODEL 0:}\ (E_f, \theta_e)$$

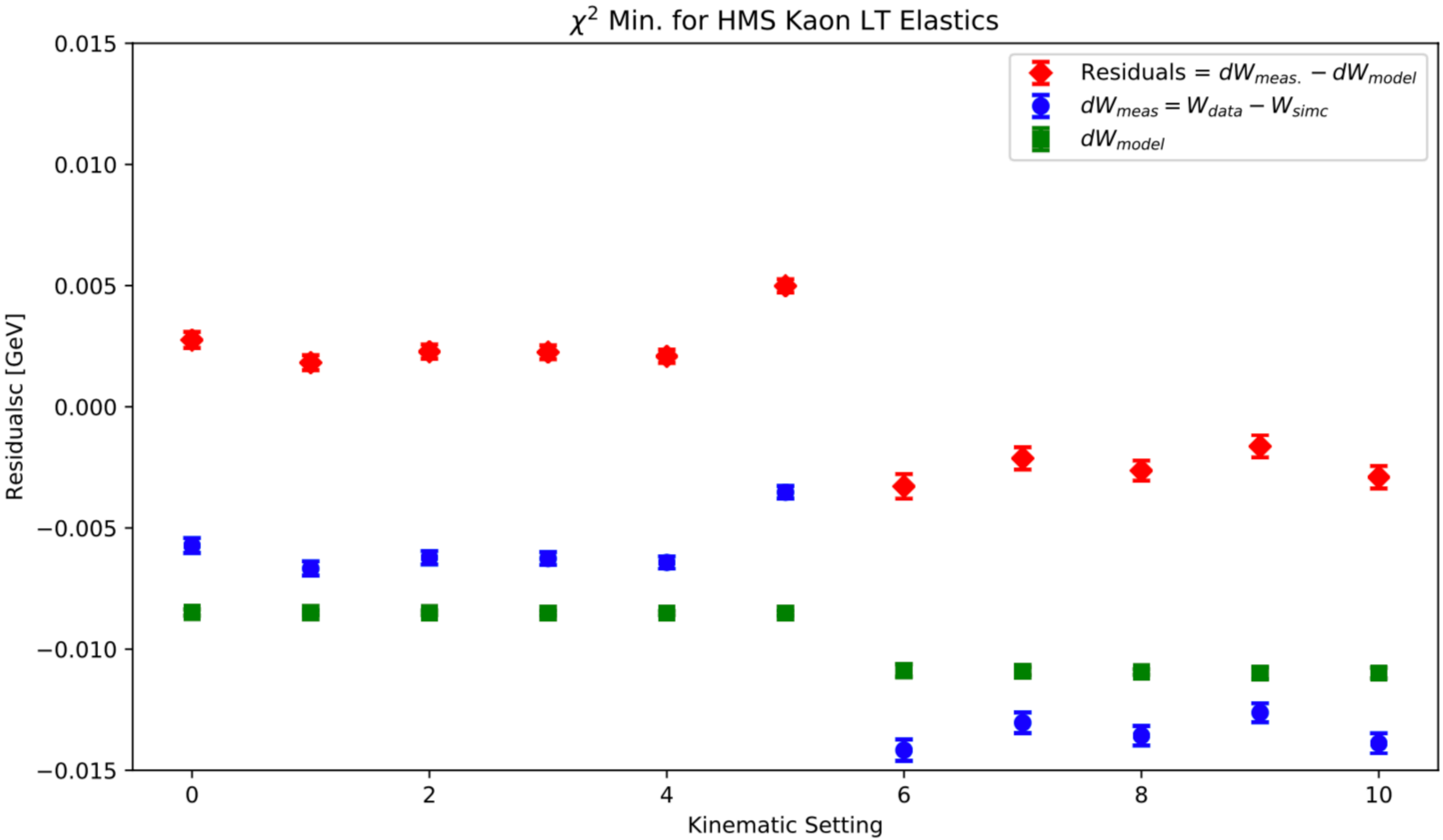
$$\text{MODEL 1:}\ (E_b, E_f, \theta_e)$$

- A systematic relative error of 1e-4 was added to all measured errors in quadrature due to magnet cycling procedure (via NMR) errors

MODEL 0: (E_f, θ_e)


```
****COVARIANCE MATRIX*****
2x2 matrix is as follows

     |      0    |      1    |
-------------------------------
   0 |  4.961e-08  -3.778e-08
   1 | -3.778e-08   2.908e-08

=== Optimized Parameters ===
total equations = 11 observations, # parameters = 2, dof = 9
initial chi2 = 7995.37 initial chi2/dof = 888.375
chi2 = 882.309  chi2/dof = 98.0343
dEf / Ef = 0.00216717
dth_e [rad] = 3.95164e-05

=== Uncertainty in Parameters (Diagonal Elements) ===
dEf / Ef = 0.00022273
dth_e [rad] = 0.000170543

=== Uncertainty in Parameters (Off-Diagonal Elements) ===
cov[0,1] (Eb,Ef) = -3.77798e-08

****CORRELATION MATRIX*****
2x2 matrix is as follows

     |      0    |      1    |
-------------------------------
   0 |          1     -0.9946
   1 |    -0.9946           1
```

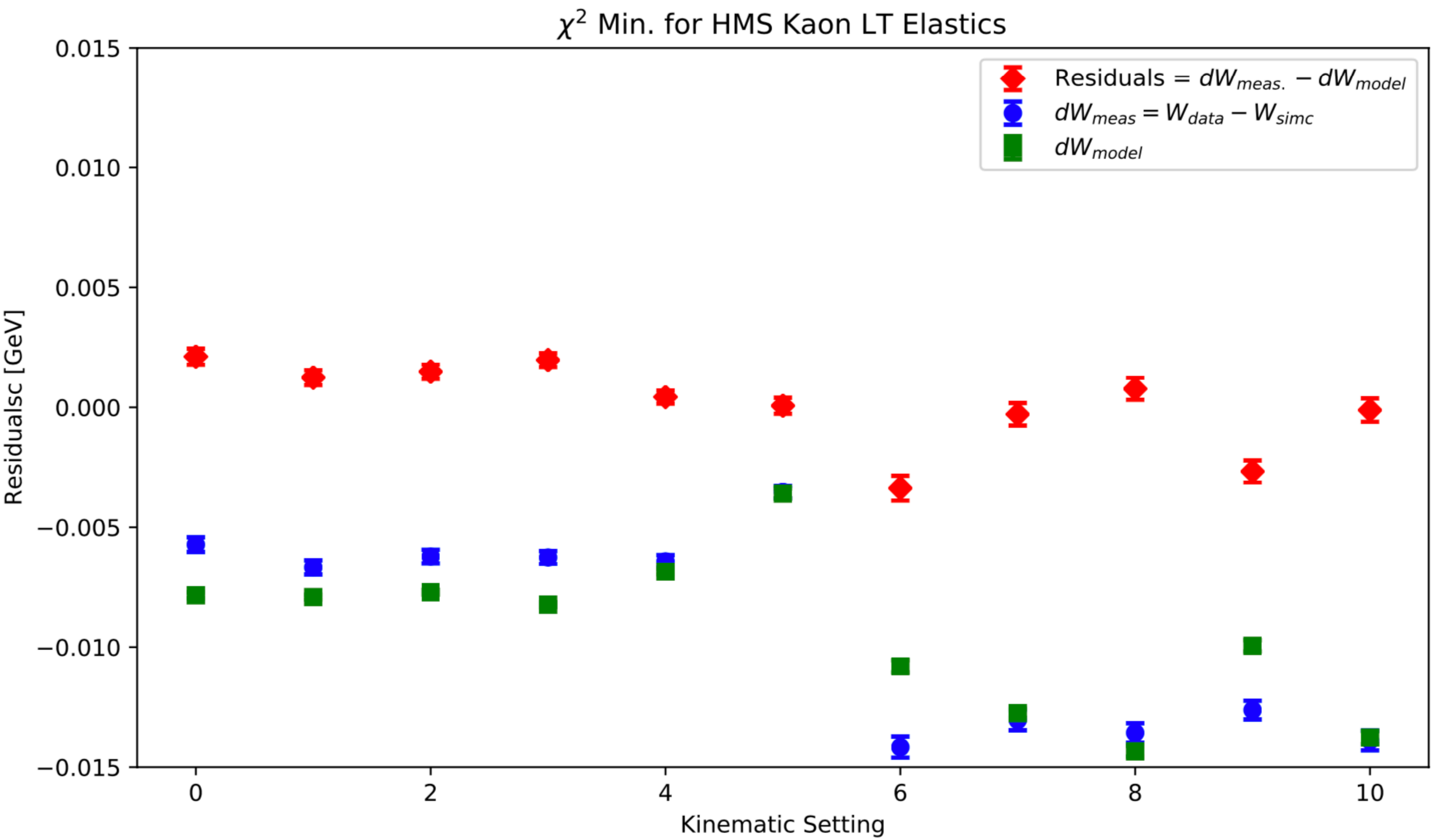
MODEL 0: (E_f, θ_e)



MODEL 1: (E_b, E_f, θ_e)


```
****COVARIANCE MATRIX*****
3x3 matrix is as follows

     |      0      |      1      |      2      |
-------------------------------------------------------
   0 |  1.393e-07    2.181e-07    -9.36e-08
   1 |  2.181e-07    3.911e-07   -1.843e-07
   2 |  -9.36e-08   -1.843e-07    9.196e-08

=== Optimized Parameters ===
total equations = 11 observations, # parameters = 3, dof = 8
initial chi2 = 7995.37 initial chi2/dof = 999.422
chi2 = 259.005  chi2/dof = 32.3756
dEb / Eb = -0.00849893
dEf / Ef = -0.0110374
dth_e = 0.00555486

=== Uncertainty in Parameters (Diagonal Elements) ===
dEb / Eb = 0.000373287
dEf / Ef = 0.000625386
dth_e [rad] = 0.000303253

=== Uncertainty in Parameters (Off-Diagonal Elements) ===
cov[0,1] (Eb,Ef) = 2.18141e-07
cov[0,2] (Eb, th_e) = -9.36033e-08
cov[1,2] (Ef, th_e) = -1.84316e-07

****CORRELATION MATRIX*****
3x3 matrix is as follows

     |      0      |      1      |      2      |
-------------------------------------------------------
   0 |          1       0.9344      -0.8269
   1 |     0.9344            1      -0.9719
   2 |    -0.8269      -0.9719            1
```

MODEL 1: (E_b, E_f, θ_e)



SUMMARY

- **Summary of the kinematics parameter offsets and their uncertainties based on each model**

NOTE : The chi2 in Coin. Data is small because the parameters had already been optimized by 'eye' in an attempt to align the missing energy and W peaks as best as we could.**

Coin. SHMS (e-)	χ^2/dof	$\Delta E_b/E_b$	$\sigma_{\Delta E_b/E_b}$	$\Delta E_f/E_f$	$\sigma_{\Delta E_f/E_f}$	$\Delta\theta_e$ [rad]	$\sigma_{\Delta\theta_e}$ [rad]	$\Delta\theta_p$ [rad]	$\sigma_{\Delta\theta_p}$ [rad]
MODEL 0	1.62938	- -	- -	4.705E-05	3.871E-05	8.341E-05	2.887E-05	- -	- -
MODEL 1	1.08586	-1.818E-03	7.016E-04	-2.190E-03	8.639E-04	4.934E-04	1.621E-04	- -	- -
MODEL 2	1.17102	-1.648E-03	7.498E-04	-1.998E-03	9.132E-04	4.698E-04	1.659E-04	-1.479E-04	2.369E-04
MODEL 3	1.56168	- -	- -	4.221E-06	4.745E-05	1.182E-04	3.688E-05	-3.453E-04	2.217E-04
Singles HMS (e-)									
MODEL 0	98.0343	- -	- -	2.167E-03	2.227E-04	3.951E-05	1.705E-04	- -	- -
MODEL 1	32.3756	-8.498E-03	3.732E-04	-1.103E-02	6.253E-04	5.554E-03	3.032E-04	- -	- -