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PION EXCHANGE CURRENTS AND ISOBAR CONFIGURATIONS IN DEUTERON ELECTRO-DISINTEGRATION BELOW PION THRESHOLD†

WERNER FABIAN†† and HARTMUTH ARENHOVEL

Institut fur Kernphysik, Universitat Mainz, Mainz, West Germany

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Abstract: The influence of meson exchange currents (MEC) and isobar configurations (IC) on the electro-disintegration of the deuteron has been investigated for excitation energies below pion threshold and momentum transfer up to $q^2 = 14\ \text{fm}^{-2}$. For the pion exchange current a multipole expansion is given. In the calculation of the form factors and the cross section at 180° all multipoles up to $L = 6$ are taken into account with consecutive convergence acceleration with a [1, 1] Pade approximant. Near threshold MEC contribute dominantly while IC are less important. With increasing excitation energy up to 40 MeV both interaction effects become rather small. But above 40 MeV they increase again and give considerable contributions for $q^2 \leqq 5\ \text{fm}^{-2}$. Above 80 MeV the IC dominate over the MEC.

1. Introduction

During the past years, the influence of nucleonic interaction effects, i.e. virtual baryon resonance admixtures, the so-called isobar configurations (IC) and mesonic exchange currents (MEC) within the nucleus, have been the subject of numerous investigations. Hereby, special attention has been paid to electromagnetic properties of few-nucleon systems in order to avoid difficulties with unknown interaction mechanisms and problems of the "normal" nuclear structure calculations. In the work that was done up to now on electro- and photo-disintegration[1–12]), the IC have been taken into account only by effective current operators using closure approximations if at all, and in the MEC calculations, either use has been made of the long wavelength approximation or only M1 transitions have been considered, thus limiting the results to static or low-energy properties.

We have developed a method to systematically take into account the pionic MEC by means of a multipole expansion, the fair convergence of which makes calculations up to pion threshold possible. We have restricted ourselves to pionic exchange currents since they are expected to give the most important contributions for not too high momentum transfers. In fact, the extension of this formalism to heavier boson exchanges is straightforward and will be considered in future work. Furthermore, we

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treat IC not by effective current operators but explicitly as additional nuclear wave function components

In this paper, we want to present this treatment of the MEC together with results for the electro-disintegration of the deuteron. A short account of our results for elastic electron-deuteron scattering and for photo-disintegration of the deuteron has already been published elsewhere[13,14]). The procedure for computing the IC is the same as described in refs [15,16]), but ρ-meson contributions to the transition potentials leading to IC have been included

In sect 2 the multipole expansion of the MEC is given in some detail, while sect 3 deals with a Siegert correction which has to be done if the Siegert theorem has been used for the gradient part of the electric multipoles. The differential cross section for the electro-disintegration of the deuteron is derived in sect 4. Finally the results are discussed in sect 5

2. Multipole expansion of the pion exchange current

The exchange of charged virtual pions between two nucleons leads to the isospin dependence of nuclear forces and gives rise to additional two-body operators $\boldsymbol{j}_{[2]}(\boldsymbol{r}, \boldsymbol{r}_1, \boldsymbol{r}_2)$ in the nuclear current density operator, the interaction of which with a real or virtual photon of four-momentum $q = (\omega, \boldsymbol{q})$ and polarization $\boldsymbol{\varepsilon}_\mu$ is given by the operator

$$H^{\mathrm{int}}_{[2]} = -\sqrt{\frac{2\pi}{\omega\Omega_{\mathrm{N}}}}\int \mathrm{d}^3 r \boldsymbol{\varepsilon}_\mu \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}} \cdot \boldsymbol{j}_{[2]}(\boldsymbol{r}, \boldsymbol{r}_1, \boldsymbol{r}_2)$$

$$= -\sqrt{\frac{2\pi}{\omega\Omega_{\mathrm{N}}}}\boldsymbol{\varepsilon}_\mu \cdot \boldsymbol{j}_{[2]}(\boldsymbol{q}, \boldsymbol{r}_1 - \boldsymbol{r}_2), \tag{1}$$

where Ω_{N} is a normalization volume. For $\boldsymbol{\varepsilon}_\mu \cdot \boldsymbol{j}_{[2]}(\boldsymbol{q}, \boldsymbol{r}_1 - \boldsymbol{r}_2)$, one finds [see e g ref [17])] the expression

$$\boldsymbol{\varepsilon}_\mu \cdot \boldsymbol{j}_{[2]}(\boldsymbol{q}, \boldsymbol{r}_1 - \boldsymbol{r}_2) = \frac{e}{4\pi}\left(\frac{f_{\mathrm{NN}\pi}}{m_\pi}\right)^2 (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_3 \Omega_\mu(\boldsymbol{q}, \boldsymbol{r}_1 - \boldsymbol{r}_2), \tag{2}$$

with

$$\Omega_\mu(\boldsymbol{q}, \boldsymbol{r}) = -\frac{1}{\pi^2}(\boldsymbol{\sigma}_1 \cdot \boldsymbol{a}_-)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{a}_+)(\nabla \cdot \boldsymbol{\varepsilon}_\mu) I(\boldsymbol{q}, \boldsymbol{r}) + \{K(1, 2, \boldsymbol{q}) + K(2, 1, -\boldsymbol{q})\} f_1(r) \tag{3}$$

We have used

$$I(\boldsymbol{q}, \boldsymbol{r}) = \int \mathrm{d}^3 p \frac{\mathrm{e}^{i\boldsymbol{p}\cdot\boldsymbol{r}}}{[(\boldsymbol{p}+\frac{1}{2}\boldsymbol{q})^2 + m_\pi^2][(\boldsymbol{p}-\frac{1}{2}\boldsymbol{q})^2 + m_\pi^2]}, \tag{4}$$

$$K(j, l, \boldsymbol{q}) = \mathrm{e}^{\frac{1}{2}i\boldsymbol{q}\cdot\boldsymbol{r}}(\boldsymbol{\sigma}_l \cdot \hat{\boldsymbol{r}})(\boldsymbol{\sigma}_j \cdot \boldsymbol{\varepsilon}_\mu), \tag{5}$$

$$\hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}, \qquad \boldsymbol{a}_\pm = i\nabla \pm \tfrac{1}{2}\boldsymbol{q}, \qquad f_1(r) = \frac{\mathrm{d}}{\mathrm{d}r} R(r) \frac{\mathrm{e}^{-m_\pi r}}{r} \tag{6}$$

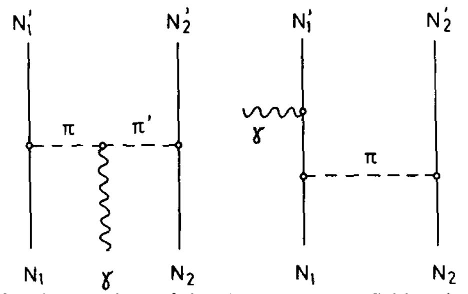

Fig 1 Feynman graphs for the coupling of the electromagnetic field with two nucleons interacting by exchange of a pion in ps coupling

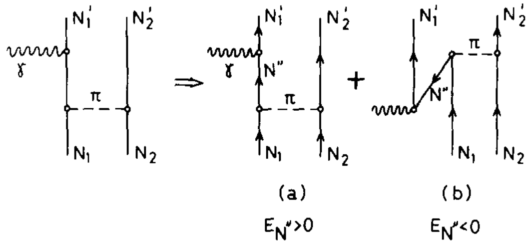

Fig 2 Time ordering with respect to nucleon lines of the second graph from fig 1 leads to nucleon-antinucleon pairs in the intermediate state N''

Here, m_π is the pion mass, m_N the nucleon mass, and $R(r)$ is a regularizing function of the OPE vertex The NNπ coupling constant is taken to be $f^2_{NN\pi}/4\pi = 0.08$ The first term in eq (3) describes the interaction of the photon with an exchanged virtual meson, while the second one is either the contact term in pv coupling or the interaction with an intermediate nucleon-antinucleon pair for ps coupling non-relativistically contracted – the so-called pair term (see figs 1 and 2)

Long wavelength limit expressions for eqs (2) and (4) may be found in ref [17]) For electron scattering with momentum transfers up to $q^2 = 14\ \mathrm{fm}^{-2}$, such approximations are expected to fail We therefore expand eq (2) into multipoles

$$\varepsilon_\mu \; J_{[2]}(\boldsymbol{q}, \boldsymbol{r}) = e\left(\frac{f_{NN\pi}}{m_\pi}\right)^2 (\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2)_3 (4\pi)^{-\frac{1}{2}} \sum_{LM} D^L_{M\mu}(0, -\theta, -\phi)\{\tilde{E}^{[L]}_M + \mu \tilde{M}^{[L]}_M\}, \tag{7}$$

where $D^L_{M\mu}$ are the rotation matrices as defined in ref [18]) The Euler angles, ϕ and θ describe the rotation of the coordinate system Σ', in which the photon momentum $\boldsymbol{q}$ coincides with the z-axis, into some arbitrary coordinate system Σ The choice of Σ is determined by the requirement that the final state nucleon wave function takes a simple form [for details see ref [19])]

To find the operators $\tilde{E}^{[L]}$ and $\tilde{M}^{[L]}$, we first expand $I(\boldsymbol{q}, \boldsymbol{r})$ Taking

$$z = \frac{1}{pq}(p^2 + \tfrac{1}{4}q^2 + m_\pi^2), \qquad x = \cos\theta_{pq}, \tag{8}$$

we have

$$I(\boldsymbol{q}, \boldsymbol{r}) = \frac{1}{q^2}\int \frac{d^3p}{p^2}\frac{e^{i\boldsymbol{p}\cdot\boldsymbol{r}}}{z^2-x^2} \tag{9}$$

Now we use the multipole expansion of the plane wave in eq (9),

$$e^{i\boldsymbol{p}\cdot\boldsymbol{r}} = 4\pi\sum_l \frac{\hat{l}}{i^l} j_l(pr)[Y^{[l]}(\hat{\boldsymbol{r}})\times Y^{[l]}(\hat{\boldsymbol{p}})]^{[0]}, \qquad \hat{l} = \sqrt{2l+1} \tag{10}$$

In p-space we choose the z-axis in the direction of $\boldsymbol{q}$ Making use of the relation

$$\int_{-1}^{+1}\frac{dx}{z^2-x^2}P_l(x) = (1+(-)^l)\frac{1}{z}Q_l(z), \tag{11}$$

and the addition theorem for spherical harmonics

$$P_l(\cos\theta_{rq}) = 4\pi\frac{(-)^l}{\hat{l}}[Y^{[l]}(\hat{\boldsymbol{r}})\times Y^{[l]}(\hat{\boldsymbol{q}})]^{[0]}, \tag{12}$$

we finally get

$$I(\boldsymbol{q}, \boldsymbol{r}) = (4\pi)^2\sum_{l=\text{even}}\frac{\hat{l}}{i^l}\phi^{(0)}_{l\,l}(r)[Y^{[l]}(\hat{\boldsymbol{r}})\times Y^{[l]}(\hat{\boldsymbol{q}})]^{[0]}, \tag{13}$$

where the function $\phi^{(\nu)}_{\sigma\,l}(r)$ is defined by

$$\phi^{(\nu)}_{\sigma\,l}(r) = \frac{1}{q^2}\int_0^\infty \frac{dp\,p^\nu}{z} j_\sigma(pr)Q_l(z) \tag{14}$$

Letting

$$\nabla^{[\rho]}_2 = [\nabla^{[1]}\times\nabla^{[1]}]^{[\rho]}, \qquad \Sigma^{[\rho]}_{12} = [\boldsymbol{\sigma}^{[1]}_1\times\boldsymbol{\sigma}^{[1]}_2]^{[\rho]}, \tag{15}$$

the first term in eq (3) becomes

$$(\boldsymbol{\sigma}_1\cdot\boldsymbol{a}_-)(\boldsymbol{\sigma}_2\cdot\boldsymbol{a}_+)(\nabla\cdot\boldsymbol{\varepsilon}_\mu)I(\boldsymbol{q}, \boldsymbol{r}) = X_1(\nabla\cdot\boldsymbol{\varepsilon}_\mu)I(\boldsymbol{q}, \boldsymbol{r}) + X_2(\nabla\cdot\boldsymbol{\varepsilon}_\mu)I(\boldsymbol{q}, \boldsymbol{r}) + (\nabla\cdot\boldsymbol{\varepsilon}_\mu)X_3 I(\boldsymbol{q}, \boldsymbol{r}), \tag{16}$$

with

$$X_1 = -\sum_{\rho=0,2}\hat{\rho}[\Sigma^{[\rho]}_{12}\times\nabla^{[\rho]}_2]^{[0]},$$
$$X_2 = -\sqrt{\tfrac{1}{4}\pi}q^2\sum_{\rho=0,2}\hat{\rho}\begin{pmatrix}1 & 1 & \rho\\ 0 & 0 & 0\end{pmatrix}[\Sigma^{[\rho]}_{12}\times Y^{[\rho]}(\hat{\boldsymbol{q}})]^{[0]}, \tag{17}$$
$$X_3 = i\sqrt{4\pi}q[\Sigma^{[1]}_{12}\times[\nabla^{[1]}\times Y^{[1]}(\hat{\boldsymbol{q}})]^{[1]}]^{[0]}$$

Application of the generalized gradient formula

$$[\nabla^{[1]} \times F(r) Y^{[l]}(\hat{r})]^{[J]} = (-)^{l+1} \hat{l} \begin{pmatrix} 1 & l & J \\ 0 & 0 & 0 \end{pmatrix} \frac{1}{r} \left\{ \frac{\mathrm{d}}{\mathrm{d}r} r + \tfrac{1}{2}[l(l+1) - J(J+1)] \right\} F(r) Y^{[J]}(\hat{r}), \tag{18}$$

yields

$$(\nabla \cdot \varepsilon_\mu) I(\boldsymbol{q}, \boldsymbol{r}) = (4\pi)^2 \sum_{l = \text{even}} \sum_{\gamma} \imath^{\gamma+1} \hat{l} \hat{\gamma} \begin{pmatrix} 1 & l & \gamma \\ 0 & 0 & 0 \end{pmatrix} \phi^{(1)}_{\gamma\ l}(r) [Y^{[\gamma]}(\hat{r}) \times \Gamma^{[\gamma]}_{\mu\ l}]^{[0]}, \tag{19}$$

with

$$\Gamma^{[\gamma]}_{\mu\ l} = [\varepsilon^{[1]}_\mu \times Y^{[l]}(\hat{\boldsymbol{q}})]^{[\gamma]} \tag{20}$$

Defining

$$\Omega^{[L]}_{(\sigma,\rho)} = [Y^{[\sigma]}(\hat{r}) \times \Sigma^{[\rho]}_{12}]^{[L]}, \tag{21}$$

the first term in eq (16) after some recoupling becomes

$$\begin{aligned} X_1(\nabla \cdot \varepsilon_\mu) I(\boldsymbol{q}, \boldsymbol{r}) = (4\pi)^2 & \sum_{\rho, L} \sum_{\sigma} \sum_{l = \text{even}} \imath^{\sigma+1} (-)^L \hat{\rho} \hat{\sigma} \hat{l} \hat{L} \begin{pmatrix} 1 & 1 & \rho \\ 0 & 0 & 0 \end{pmatrix} \\ & \times \begin{pmatrix} 1 & l & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & \sigma & \rho \\ 0 & 0 & 0 \end{pmatrix} \phi^{(3)}_{\sigma, l}(r) [\Omega^{[L]}_{(\sigma\ \rho)} \times \Gamma^{[L]}_{\mu\ l}]^{[0]} \end{aligned} \tag{22}$$

In the coordinate system chosen in p-space, we have

$$(\varepsilon^{[1]}_\mu)_m = (-)^m \delta_{m, -\mu}, \tag{23}$$

$$Y^{[l]}_m(\hat{\boldsymbol{q}}) = \frac{\hat{l}}{\sqrt{4\pi}} \delta_{m,0}, \tag{24}$$

and thus, after a rotation by (ϕ, θ),

$$(\Gamma^{[L]}_{\mu, l})_m = (-)^{l+1} \frac{\hat{l}\hat{L}}{\sqrt{4\pi}} \begin{pmatrix} 1 & l & L \\ -\mu & 0 & \mu \end{pmatrix} D^L_{-\mu M}(\phi, \theta, 0) \tag{25}$$

Inserting this into eq (22) we finally get

$$\begin{aligned} X_1(\nabla \cdot \varepsilon_\mu) I(\boldsymbol{q}, \boldsymbol{r}) = (4\pi)^{\frac{3}{2}} & \sum_{\rho} \sum_{\sigma} \sum_{L} \sum_{M, l = \text{even}} \imath^{\sigma+1} \hat{\rho} \hat{\sigma} \hat{l}^2 \hat{L} \begin{pmatrix} 1 & 1 & \rho \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & \sigma & \rho \\ 0 & 0 & 0 \end{pmatrix} \\ & \times \begin{pmatrix} 1 & l & L \\ -\mu & 0 & \mu \end{pmatrix} \begin{pmatrix} 1 & l & L \\ 0 & 0 & 0 \end{pmatrix} \phi^{(3)}_{\sigma\ l}(r) D^L_{M\mu}(0, -\theta, -\phi) \Omega^{[L]}_{(\sigma\ \rho)} \end{aligned} \tag{26}$$

A similar treatment of the remaining terms in eq (16) and the pair terms yields the final result for the multipole operators

$$
\begin{aligned}
\frac{1}{\hat{L}}\tilde{E}^{[L]} = {} & (-)^{L+1}\sum_{L'=L\pm 1} i^{L'}\hat{L}'^{2}\begin{pmatrix} 1 & L' & L \\ -1 & 0 & 1\end{pmatrix}\Bigg[\sum_{\rho\,\sigma}(1+(-)^{\rho+L})\hat{\sigma}\hat{\rho} \\
& \times\begin{pmatrix} L' & 1 & \sigma \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} \rho & 1 & 1 \\ L' & L & \sigma\end{Bmatrix}\Omega^{[L]}_{(\sigma,\rho)}\Bigg] j_{L'}(\tfrac{1}{2}qr)f_1(r) \\
& -\tfrac{1}{2}(1-(-)^{L})\frac{q^2}{\pi}\sum_{\rho\ l=\mathrm{even}\ \sigma=\mathrm{odd}} i^{\sigma+1}\hat{l}^{2}\hat{\rho}\hat{\sigma} \\
& \times\begin{pmatrix} 1 & 1 & \rho \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & l & \sigma \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & l & \sigma \\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix} \rho & L & \sigma \\ 0 & 1 & -1\end{pmatrix}\Omega^{[L]}_{(\sigma,\rho)}\phi^{(1)}_{\sigma,l}(r) \\
& -\tfrac{1}{2}(1+(-)^{L})\frac{4\sqrt{3}}{\pi}q\sum_{l=\mathrm{even}\ \sigma} i^{\sigma+1}\hat{l}^{2}\hat{\sigma}\Bigg[\sum_{\lambda=\mathrm{odd}}\hat{\lambda}^{4}\begin{pmatrix} 1 & l & \lambda \\ 0 & 0 & 0\end{pmatrix}^{2}\begin{pmatrix} 1 & \lambda & \sigma \\ 0 & 0 & 0\end{pmatrix} \\
& \times\begin{pmatrix} 1 & \lambda & L \\ -1 & 0 & 1\end{pmatrix}\begin{Bmatrix} 1 & 1 & 1 \\ l & \lambda & \lambda\end{Bmatrix}\begin{Bmatrix} 1 & \lambda & \sigma \\ 1 & L & \lambda\end{Bmatrix}\Bigg]\Omega^{[L]}_{(\sigma\ 1)}\phi^{(2)}_{\sigma\ l}(r) \\
& -\frac{4}{\pi}\sum_{l=\mathrm{even}}\hat{l}^{2}\begin{pmatrix} 1 & l & L \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & l & L \\ -1 & 0 & 1\end{pmatrix}\sum_{\rho\,\sigma} i^{\sigma+1}\hat{\sigma}\hat{\rho} \\
& \times\begin{pmatrix} 1 & 1 & \rho \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} L & \rho & \sigma \\ 0 & 0 & 0\end{pmatrix}\Omega^{[L]}_{(\sigma\ \rho)}\phi^{(3)}_{\sigma\ l}(r),
\end{aligned}
\tag{27}
$$

and

$$
\begin{aligned}
\frac{1}{\hat{L}}\tilde{M}^{[L]} = {} & (-)^{L+1} i^{L}\hat{L}^{2}\begin{pmatrix} 1 & L & L \\ -1 & 0 & 1\end{pmatrix} \\
& \times\Bigg[\sum_{\rho\,\sigma}(1+(-)^{\rho+L})\hat{\sigma}\hat{\rho}\begin{pmatrix} L & 1 & \sigma \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} \rho & 1 & 1 \\ L & L & \sigma\end{Bmatrix}\Omega^{[L]}_{(\sigma,\rho)}\Bigg] j_{L}(\tfrac{1}{2}qr)f_1(r) \\
& +(-)^{L}\frac{q^2}{\pi}\sum_{\sigma-L=\mathrm{odd}\ l=\mathrm{even}} i^{\sigma+1}\hat{\rho}\hat{\sigma}\hat{l}^{2} \\
& \times\begin{pmatrix} 1 & 1 & \rho \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & l & \sigma \\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix} 1 & l & \sigma \\ -1 & 0 & 1\end{pmatrix}\begin{pmatrix} \rho & L & \sigma \\ 0 & -1 & 1\end{pmatrix}\Omega^{[L]}_{(\sigma\ \rho)}\phi^{(1)}_{\sigma\ l}(r) \\
& -\tfrac{1}{2}(1-(-)^{L})\frac{4\sqrt{3}}{\pi}q\hat{L}^{4}\begin{pmatrix} 1 & L & L \\ -1 & 0 & 1\end{pmatrix}\sum_{l=\mathrm{even}\ \sigma} i^{\sigma+1}\hat{\sigma}\hat{l}^{2} \\
& \times\begin{pmatrix} 1 & l & L \\ 0 & 0 & 0\end{pmatrix}^{2}\begin{pmatrix} 1 & L & \sigma \\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix} 1 & 1 & 1 \\ l & L & L\end{Bmatrix}\begin{Bmatrix} 1 & L & \sigma \\ 1 & L & L\end{Bmatrix}\Omega^{[L]}_{(\sigma\ 1)}\phi^{(2)}_{\sigma,l}(r)
\end{aligned}
\tag{28}
$$

3. Siegert corrections

In the evaluation of the transition matrix elements of the "normal" one-body current

$$H_{\mathrm{fi}} = \langle \mathrm{f} | \int \varepsilon_\mu \mathrm{e}^{iq \cdot r} \cdot j_{[1]}(r) \mathrm{d}\tau | \mathrm{i} \rangle, \tag{29}$$

one usually starts from the following multipole expansion of the plane wave,

$$\varepsilon_\mu \mathrm{e}^{iq \cdot r} = -\sqrt{2\pi} \sum_{LM} D^L_{M\mu}(0, -\theta, -\phi) \frac{i^L \hat{L}}{\sqrt{L(L+1)}}$$
$$\times \left\{ i \left(\frac{1}{q} \nabla \Phi^{[L]}_M(r) + qrj_L(qr) Y^{[L]}_M(\hat{r}) \right) + \mu j_L(qr) [\mathbb{L}\, Y^{[L]}_M(\hat{r})] \right\}, \tag{30}$$

where

$$\Phi^{[L]}(r) = \left(1 + r\frac{\mathrm{d}}{\mathrm{d}r}\right) j_L(qr) Y^{[L]}(\hat{r}). \tag{31}$$

For the calculation of the electric multipoles in eq. (29), the gradient usually operates upon the current density operator after partial integration

$$\int j_{[1]}(r) \cdot \nabla \Phi^{[L]}(r) \mathrm{d}\tau = - \int \Phi^{[L]}(r) \nabla \cdot j_{[1]}(r) \mathrm{d}\tau, \tag{32}$$

and then one makes use of the continuity equation

$$\nabla \cdot j = -i[H, \rho]. \tag{33}$$

This is the content of Siegert's theorem[21]). Hereby, it must be emphasized that eq. (33) is valid only if $j = j_{[1]} + j_{[2]}$ is the *total* nuclear current density (including MEC). Since (in the non-relativistic limit) the electric charge density remains unchanged by MEC effects [see e.g. ref. [20])] as has been conjectured long ago by Siegert[21]), MEC are taken into account already by a part of the "normal" electric transitions. Thus, if MEC are calculated in addition, these terms must be subtracted again to avoid double counting of the mesonic effects. It seems that this has not been done in ref. [22]). Thus, we have to subtract from the electric multipoles in eq. (7) the term $\tilde{\tilde{E}}^{[L]}$ which is defined by

$$\frac{1}{\hat{L}} \tilde{\tilde{E}}^{[L]} = \frac{i^{L-1}}{\sqrt{2L(L+1)}} \frac{1-(-)^L}{q} \tilde{V}^{\mathrm{OPE}} \Phi^{[L]}(r). \tag{34}$$

In the evaluation of the commutator in eq. (33), we have used the OPE potential

$$V = \left(\frac{f_{\mathrm{NN}\pi}}{m_\pi}\right)^2 \tau_1 \cdot \tau_2 \tilde{V}^{\mathrm{OPE}},$$

$$\tilde{V}^{\mathrm{OPE}} = (\sigma_1 \cdot \nabla)(\sigma_2 \cdot \nabla) R(r) \mathrm{e}^{-m_\pi r}/r, \tag{35}$$

and the charge density ρ of point nucleons Some recoupling finally yields

$$\frac{1}{\hat{L}}\tilde{\tilde{E}}^{[L]} = \frac{(1-(-)^L)i^{L-1}}{\sqrt{2L(L+1)}}\sum_{\rho\,\sigma} i^\rho(-)^\sigma\hat{\rho}\hat{\sigma}\begin{pmatrix}1 & 1 & \rho\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\rho & L & \sigma\\ 0 & 0 & 0\end{pmatrix}m_\pi^3$$

$$\times\left[f_\rho(m_\pi r)\left(1+r\frac{\mathrm{d}}{\mathrm{d}r}\right)j_L(\tfrac{1}{2}qr)\right]\Omega^{[L]}_{(\sigma\,\rho)}, \tag{36}$$

with

$$f_0(m_\pi r) = \frac{1}{m_\pi}R(r)\frac{\mathrm{e}^{-m_\pi r}}{r}, \tag{37}$$

$$f_2(x) = x\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x}\frac{\mathrm{d}}{\mathrm{d}x}f_0(x) \tag{37}$$

4. The electro-disintegration cross section

It has been shown that electro-excitation is formally equivalent to photo-excitation with polarized photons[23]) That means, we can essentially use the most complete and elaborate expression of Partovi for the photo-disintegration of the deuteron[19]) to derive the corresponding expression of the differential cross section for the electro-disintegration of the deuteron

The electro-disintegration cross section is given by

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_\mathrm{e}\mathrm{d}\Omega_\mathrm{np}} = \mathrm{Tr}\,\zeta_\mathrm{f}, \tag{38}$$

where Ω_e and Ω_np are the solid angles of the scattered electron and the detected nucleon, respectively The final state density matrix element

$$\zeta_\mathrm{f} = T_{\mathrm{fi}}\zeta_\mathrm{i}T_{\mathrm{fi}}^+, \tag{39}$$

is given in terms of the initial state density matrix for deuteron and virtual photon states ζ_i and the transition matrix element

$$T_{\mathrm{fi}} = \Omega_\mathrm{N}\sqrt{\frac{km_\mathrm{N}}{8\pi^2}}H^{\mathrm{int}}_{\mathrm{fi}}$$

$$= -\sqrt{\frac{\Omega_\mathrm{N}km_\mathrm{N}}{4\pi\omega}}(\varepsilon_\mu\; J)_{\mathrm{fi}}, \tag{40}$$

where k is the momentum in the final np state c m system Allowing for virtual (transverse and longitudinal) photons, the current density matrix element in eq (40) takes the form [see ref [23])]

$$(\varepsilon_\mu\; J)_{\mathrm{fi}} = (-)^\mu\sqrt{2\pi(1+\delta_{\mu 0})}\sum_{LM} i^L\hat{L}D^L_{M\mu}(0, -\theta, -\phi)\langle\mathrm{f}|Q^{[L]}_{M\mu}|\mathrm{i}\rangle, \tag{41}$$

with

$$Q_{\mu}^{[L]} = \delta_{|\mu|1}\{T_{\mathrm{el}}^{[L]}+\mu T_{\mathrm{mag}}^{[L]}\}+\delta_{\mu 0}M^{[L]}, \tag{42}$$

where $T_{\mathrm{el}}^{[L]}$, $T_{\mathrm{mag}}^{[L]}$, and $M^{[L]}$ are defined in the usual way[23]) Making the following ansatz for the wave functions[19])

$$\begin{aligned}|\mathrm{f}\rangle &= \frac{1}{k}\sqrt{\frac{4\pi}{\Omega_{\mathrm{N}}}}\sum_{\substack{n\,ll\\ s'\lambda j}}\hat{l}(l0sm^{s}|(ls)jm^{s})\mathrm{e}^{i\delta_{\lambda}^{j}}U_{ls\lambda}^{j}\,U_{l\,s'\lambda}^{j}|n',(l's')jm^{s}\rangle,\\ |\mathrm{i}\rangle &= \sum_{n''l\,s''}|n'',(l''s'')1m\rangle,\end{aligned} \tag{43}$$

(n' and n'' stand for the different isobar configurations in the final and initial states), we finally arrive at the following expressions

$$T_{\mathrm{fi}} = \mathrm{e}^{i\mu\phi}\sqrt{1+\delta_{\mu 0}}\sum_{L}d_{\mu\ m^{s}-m}^{L}(\theta)\langle sm^{s}|O_{\mu}^{[L]}|\mu m\rangle, \tag{44}$$

with

$$\langle sm^{s}|O_{\mu}^{[L]}|\mu m\rangle = \sqrt{4\pi}\sum_{\lambda jl}\frac{\hat{l}}{\hat{j}}\mathrm{e}^{i\delta_{\lambda}^{j}}U_{ls\lambda}^{j}(l0sm^{s}|(ls)jm^{s})(1mLm^{s}-m|(1L)jm^{s})N_{\mu}^{L}(\lambda j), \tag{45}$$

where

$$\begin{aligned}N_{\mu}^{L}(\lambda j) &= i^{L}\hat{L}\sqrt{\frac{m_{\mathrm{N}}}{2\omega k}}\sum_{\substack{n'l's\\ n'\,l\,s'}}U_{l\,s'\lambda}^{j}\langle n',l's'\lambda j||Q_{\mu}^{[L]}||n'',(l''s'')1\rangle\\ &\equiv \delta_{|\mu|1}\{E^{L}(\lambda j)+\mu M^{L}(\lambda j)\}+\delta_{\mu 0}C^{L}(\lambda j)\end{aligned} \tag{46}$$

Explicit expressions for $E^{L}(\lambda j)$ and $M^{L}(\lambda j)$ are given in ref [19]) for the normal part and in ref [29]) for the IC for real photons They easily can be generalized to the case of virtual photons These expressions, together with the Coulomb matrix elements $C^{L}(\lambda j)$, may be found in ref [25]) According to ref [23]), the initial state density matrix is

$$\zeta_{i} = \tfrac{1}{3}\delta_{m'm}\sigma_{\mu'\mu}^{(i)}, \tag{47}$$

with the density matrix for virtual photons

$$\sigma_{\mu'\mu}^{(i)} = \frac{\alpha\omega}{2\pi^{2}(q_{\mu}^{2})^{2}}\frac{k_{2}}{k_{1}}\rho_{\mu'\mu}^{(i)}\delta(\omega-\omega_{\mathrm{fi}})\mathrm{d}\omega \tag{48}$$

The expressions for $\rho^{(i)}_{\lambda'\lambda}$ are given in ref [23])

$$\rho^{(i)}_{00} \equiv V_{\mathrm{long}}(\theta_e) = \frac{1}{2}\frac{(q_\mu^2)^2}{q^4}[(\varepsilon_1+\varepsilon_2)^2-q^2],$$

$$\rho^{(i)}_{11} \equiv V_{\mathrm{trans}}(\theta_e) = \boldsymbol{Q}^2 - \left(\frac{\boldsymbol{Q}\cdot\boldsymbol{q}}{q}\right)^2 + \tfrac{1}{2}q_\mu^2,$$

$$\rho^{(i)}_{01} = \frac{1}{\sqrt{2}}\frac{q_\mu^2\kappa}{q^3}(\varepsilon_1+\varepsilon_2), \tag{49}$$

$$\rho^{(i)}_{1-1} = -\frac{\kappa^2}{q^2},$$

$$\rho^{(i)}_{\lambda\lambda'} = \rho^{(i)}_{\lambda'\lambda} = (-)^{\lambda-\lambda'}\rho^{(i)}_{-\lambda-\lambda'},$$

where $(\varepsilon_1, \boldsymbol{k}_1)$ and $(\varepsilon_2, \boldsymbol{k}_2)$ are the four-momenta of the electron in the initial and final state, respectively, and

$$\boldsymbol{Q} = \tfrac{1}{2}(\boldsymbol{k}_1+\boldsymbol{k}_2), \qquad \boldsymbol{q} = \boldsymbol{k}_2-\boldsymbol{k}_1,$$
$$q = |\boldsymbol{q}|, \qquad q_\mu^2 = q^2-\omega^2, \qquad \boldsymbol{\kappa} = \boldsymbol{k}_1\times\boldsymbol{k}_2 \tag{50}$$

Inserting eqs (47) into eq (40) we find

$$\frac{d^2\sigma}{d\Omega_e d\Omega_{np}} = \mathrm{Tr}\, T_{f_1}\zeta_1 T_{f_1}^+ = \frac{\alpha\omega}{6\pi^2(q_\mu^2)^2}\frac{k_2}{k_1} \times \sum_{\mu'\mu}\rho^{(i)}_{\mu'\mu}\sqrt{(1+\delta_{\mu 0})(1+\delta_{\mu' 0})}\,\tilde{C}(\Omega_{np})\delta(\omega-\omega_{f_1})d\omega \tag{51}$$

where

$$\tilde{C}(\Omega_{np}) = e^{i(\mu'-\mu)\phi}\sum_{sm^sm} d^{L'}_{\mu'm^s-m}(\theta)d^{L}_{\mu m^s-m}(\theta)\langle sm^s|O^{[L']}_{\mu'}|\mu'm\rangle\langle sm^s|O^{[L]}_{\mu}|\mu m\rangle^* \tag{52}$$

This is the complete differential cross section if one nucleon is detected in coincidence with the scattered electron If only the scattered electron is detected, one has to integrate over Ω_{np} and ω_{f_1} Then the sums over m and m^s collapse because of the orthogonality of the Clebsch-Gordan coefficients, and the sum over s collapses because of the orthogonality of the coefficients $U^J_{ls\lambda}$ [see ref [19])] One finds

$$\frac{d^2\sigma}{d\Omega_e d\omega_e} = \frac{1}{\pi^2}\frac{\alpha\omega}{(q_\mu^2)^2}\frac{k^2}{k_1}\{V_{\mathrm{trans}}(\theta_e)f_{\mathrm{trans}}(q^2)+V_{\mathrm{long}}(\theta_e)f_{\mathrm{long}}(q^2)\} \tag{53}$$

The form factors

$$f_{\text{trans}}(q^2) = \tfrac{16}{3}\pi^2 \sum_{Lj\lambda} \frac{1}{2L+1} \{|E^L(\lambda j)|^2 + |M^L(\lambda j)|^2\},$$
$$f_{\text{long}}(q^2) = \tfrac{16}{3}\pi^2 \sum_{Lj\lambda} \frac{1}{2L+1} |C^L(\lambda j)|^2, \tag{54}$$

are normalized so that for real photons ($q = \omega$) the transverse form factor $f_{\text{trans}}(q^2)$ equals the total photo-disintegration cross section $\sigma_{\text{tot}}(\omega)$ Neglecting the electron mass in eq (49) as usual, we find for the special case of backward angle scattering

$$k_1^2 \frac{d^2\sigma}{d\Omega_e d\omega_e}\bigg|_{180} = \frac{\alpha\omega}{8\pi^2} f_{\text{trans}}(q^2) = \tfrac{2}{3}\alpha\omega \sum_{Lj\lambda} \frac{1}{2L+1} \{|E^L(\lambda j)|^2 + |M^2(\lambda j)|^2\} \tag{55}$$

5. Results and discussion

For the description of the normal deuteron and the np scattering states we have used the phenomenological Hamada-Johnston potential[24]) As IC we have included the NΔ(1232), NN′(1470) and Δ(1232)Δ(1232) configurations obtained in the impulse approximation using π- and ρ-exchange for the appropriate transition potentials For regularization we have included a dipole form factor of range $\Lambda = 50\ \text{fm}^{-1}$, which actually is not necessary because of the hard core of the HJ potential The total IC probability in the deuteron is 1 4% (1 2% for the double Δ probability)

The use of the phenomenological Hamada-Johnston potential is not quite consistent if one includes pionic MEC only and current conservation is not completely fulfilled However, since the long-range part of the Hamada-Johnston potential is given by the OPE, this violation is not expected to be very serious Only for very high momentum transfers, where exchange of heavier bosons becomes important, one should use an appropriate OBE potential with corresponding MEC contributions However, then also higher order terms of the exchange current and relativistic corrections in the wave functions may become important

We have calculated the transverse and longitudinal form factors and the differential cross section at 180° for various excitation energies $E_{\text{np}}^{(\text{rel})}$ of the final np system as a function of momentum transfer q As nucleon form factors we have taken the phenomenologic dipole form factors

$$G_{\text{E/M}}^{\text{N/N*}}(q^2) = G_{\text{E/M}}^{\text{N/N*}}(q^2 = 0)(1 + q^2[\text{GeV}^2/c^2]/0\ 71)^{-2}$$

To insure gauge invariance, the same momentum dependence had to be used for the pion form factor

Since we consider energies up to meson threshold and momentum transfers up to $q^2 = 14\ \text{fm}^{-2}$, we have included multipoles up to $L = 6$ in the calculation and have improved upon the convergence in L by means of [1 1] Padé approximants of type I

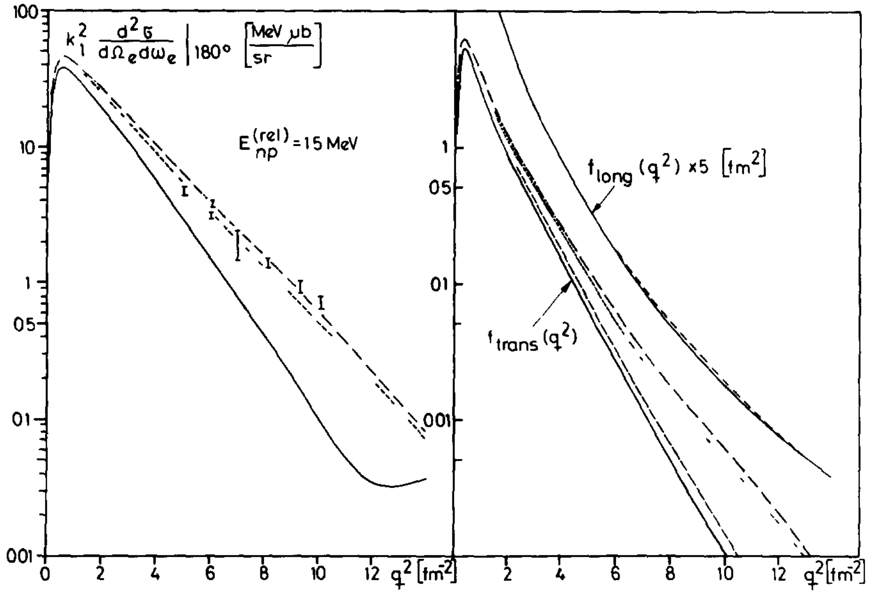

Fig 3 Inelastic transverse and longitudinal form factors of the deuteron and 180 cross section for excitation energy $E_{np}^{(rel)} = 1.5$ MeV Full curve normal part, dashed curve normal+IC, dotted curve normal+MEC, dashed-dotted curve normal+IC+MEC

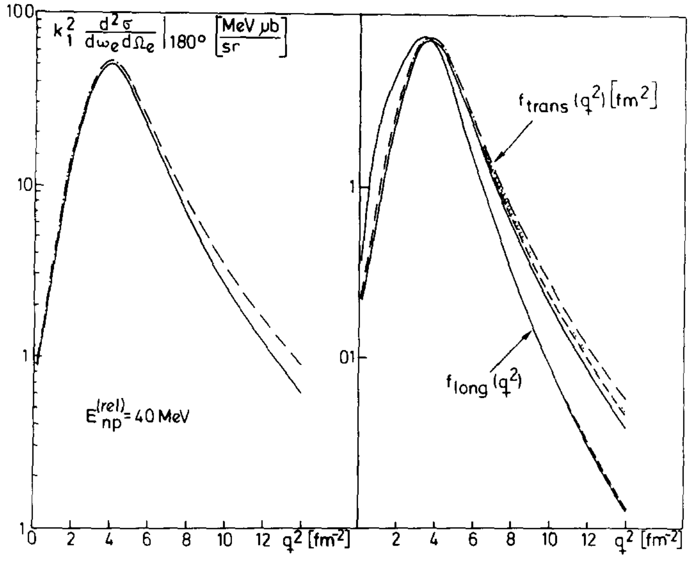

Fig 4 Same as fig 3, but for $E_{np}^{(rel)} = 40$ MeV

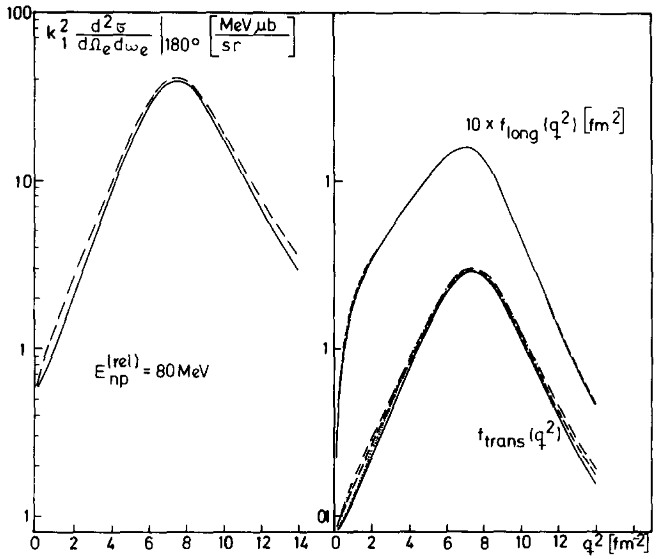

Fig 5 Same as fig 3, but for $E_{np}^{(rel)} = 80$ MeV

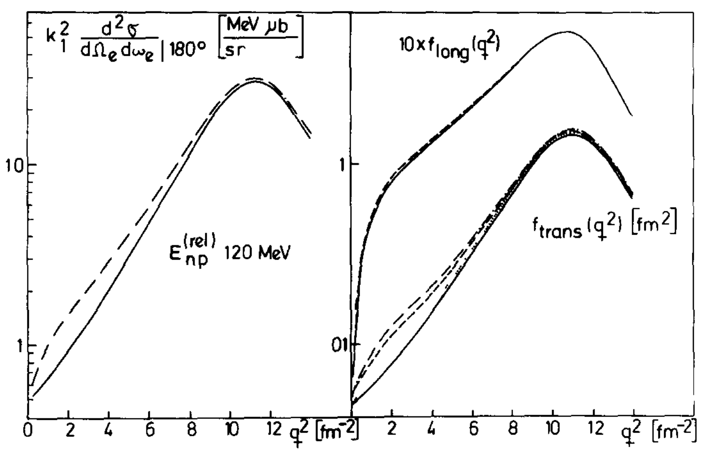

Fig 6 Same as fig 3, but for $E_{np}^{(rel)} = 120$ MeV

[For further details on the convergence with respect to the multipole order, see appendix B and ref [25])] The results for $E_{np}^{(rel)} = 1\ 5$, 40, 80 and 120 MeV are shown in figs 3–6 The left-hand sides of the figures show the differential cross sections for backward angles of the outgoing electron, the right-hand sides give the form factors neglecting or including IC or MEC or both A comparison with experiment is possible only for energies $E_{np}^{(rel)}$ near disintegration threshold The experimental points[26]) were converted into the 180 cross section in ref [2]) and taken from there Taking IC and MEC into account, our results are in good agreement with experiment as well as with previous calculations[2]) (see fig 3) The main contribution to the 180° cross section comes from MEC (about 80%) while IC lead to a small correction only This is seen most clearly in the transverse form factor The longitudinal form factor (which depends on the charge density only and thus cannot be influenced by MEC) remains – for any value of $E_{np}^{(rel)}$ – nearly unchanged by IC

Above threshold, for increasing $E_{np}^{(rel)}$ (figs 4–6), the influence of the interaction effects at higher momentum transfers – say above 5 fm^{-2} – diminishes drastically and the dependence on q^2 becomes less pronounced This may be explained by the following argument Near threshold, the "normal" part of the form factor has a minimum at $q^2 \approx 12.5$ fm^{-2} and thus the interaction effects dominate For increasing $E_{np}^{(rel)}$, this minimum is shifted towards higher values of q^2 and thus is of less importance in the momentum transfer region which we have considered However, IC and MEC now become stronger and rather important for small q^2 ($\leqq 5$ fm^{-2}) Furthermore, the MEC contributions no longer dominate IC at $E_{np}^{(rel)} \approx 40$ MeV, both are approximately of the same magnitude, and for increasing energy, the MEC corrections become distinctly smaller than the IC contributions, an observation we already have made in the photo-disintegration[14])

The numerical calculations have been performed at the Rechenzentrum der Universitat Mainz

Appendix A

CALCULATION OF THE FUNCTIONS $\phi_{\sigma}^{(\nu)}{}_L(r)$

Expressing $j_\sigma(pr)$ in eq (14) by spherical Hankel functions, one finds

$$\phi_{\sigma,L}^{(\nu)}(r) = \frac{1}{2q^2}\int_{-\infty}^{+\infty} \frac{\mathrm{d}pp^\nu}{z} h_\sigma^{(1)}(pr)Q_L(z) \tag{56}$$

With the representation[27])

$$Q_L(z) = \tfrac{1}{2}\int_{-1}^{+1} \frac{\mathrm{d}x}{z-x} P_L(x), \tag{57}$$

this becomes

$$\phi_{\sigma}^{(\nu)}{}_L(r) = \frac{1}{4q}\int_{-1}^{+1} \mathrm{d}x G_\sigma^{(\nu)}(x) P_L(x), \tag{58}$$

with

$$G_\sigma^{(\nu)}(x) = \frac{1}{q}\int_{-\infty}^{+\infty} \frac{\mathrm{d}pp^\nu}{z(z-x)} h_\sigma(pr) \tag{59}$$

Eq (59) may simply be evaluated by complex contour integration to yield

$$G_\sigma^{(\nu)}(x) = \frac{i\pi}{x}\left\{\frac{s_2^{\nu+1}}{s_1} h_\sigma(s_2 r) - s_0^\nu h_\sigma(s_0 r)\right\}, \tag{60}$$

where

$$\begin{aligned} s_0 &= i\sqrt{\tfrac{1}{4}q^2 + m_\pi^2}, \\ s_1 &= i\sqrt{\tfrac{1}{4}q^2(1-x^2) + m_\pi^2}, \\ s_2 &= \tfrac{1}{2}qx + s_1 \end{aligned} \tag{61}$$

To calculate $\phi_{\sigma,L}^{(\nu)}(r)$ numerically, one makes a Taylor expansion of eq (60) so that the coefficients do no longer depend on r

$$G_\sigma^{(\nu)}(x) = \sum_l \sum_\kappa r^\kappa h_l(s_0 r) \sum_n \mathscr{G}_{nl\kappa}^{\nu\sigma} x^n \tag{62}$$

The power series in x is now expanded in Legendre polynomials Using the orthogonality of the Legendre polynomials, one finally arrives at

$$\phi_{\sigma\, L}^{(\nu)}(r) = \frac{1}{q}\sum_l \sum_\kappa \gamma_{Ll\kappa}^{\nu\sigma} r^\kappa h_l(s_0 r) \tag{63}$$

This procedure has the advantage that for fixed photon momentum the set of coefficients $\gamma_{Ll\kappa}^{\nu\sigma}$ must be calculated only once for all r In the present calculation, we have used a Taylor expansion up to tenth order to assure good convergence

An alternative (maybe better) method to calculate $\phi_\sigma^{(\nu)}{}_l(r)$ might be the following one in eq (14) one can use[27]) for even l,

$$Q_l(z) = \sum_{\lambda=\frac{1}{2}l}^{\infty} a_\lambda^{(l)} z^{-2\lambda-1}, \tag{64}$$

$$j_\sigma(pr) = q_1^{(\sigma)}\left(\frac{1}{pr}\right)\sin pr + q_2^{(\sigma)}\left(\frac{1}{pr}\right)\cos pr, \tag{65}$$

with appropriate coefficients $a_\lambda^{(l)}$ and polynomials

$$q_1^{(\sigma)}(x) = \sum_{\nu=0}^{[\frac{1}{2}\sigma]} b_\nu^{(l)} x^{2\nu+1}, \qquad q_2^{(\sigma)}(x) = \sum_{\nu=0}^{[\frac{1}{2}(\sigma-1)]} c_\nu^{(l)} x^{2\nu+2} \tag{66}$$

Then one has (note that l always is even)

$$\phi_l^{(0)}{}_l(r) = \sum_{\lambda=L+1}^{\infty} a_\lambda^{(l)} \sum_{\nu=0}^{\frac{1}{2}l} \{b_\nu^{(l)} f_s(\nu, \lambda, r) + c_\nu^{(l)} f_c(\nu, \lambda, r)\}, \tag{67}$$

where

$$f_s(\nu, \lambda, r) = \int_0^\infty \frac{dp}{(pr)^{2\nu+1}} \frac{\sin pr}{z^{2\lambda+2}}$$

$$= (-)^{\lambda-\nu} \frac{q^{2\lambda+2}}{r^{2\nu+1}} \frac{d^{2\lambda-2\nu}}{dr^{2\lambda-2\nu}} I_s(\lambda, r), \tag{68}$$

$$f_c(\nu, \lambda, r) = \int_0^\infty \frac{dp}{(pr)^{2\nu+2}} \frac{\cos pr}{z^{2\lambda+2}}$$

$$= (-)^{\lambda-\nu} \frac{q^{2\lambda+2}}{r^{2\nu+2}} \frac{d^{2\lambda-2\nu}}{dr^{2\lambda-2\nu}} I_c(\lambda, r) \tag{69}$$

We have used the notation

$$I_s(n, r) = \int_0^\infty \frac{dp p \sin pr}{(p^2 + \frac{1}{4}q^2 + m_\pi^2)^{2(n+1)}}, \tag{70}$$

$$I_c(n, r) = \int_0^\infty \frac{dp \cos pr}{(p^2 + \frac{1}{4}q^2 + m_\pi^2)^{2(n+1)}} \tag{71}$$

The latter two integrals can be solved analytically[28]) Thus, keeping in mind that

$$\frac{d^k}{dx^k}(f(x)g(x)) = \sum_{m=0}^{k} \binom{k}{m} \left(\frac{d^m}{dx^m} f(x)\right)\left(\frac{d^{k-m}}{dx^{k-m}} g(x)\right), \tag{72}$$

the evaluation of $\phi_\sigma^{(\nu)}{}_L(r)$ becomes straightforward if one uses the recurrence relations

$$\left(-\frac{d}{dr} + \frac{\sigma}{l}\right) \phi_\sigma^{(\nu)}{}_l(r) = \phi_{\sigma+1}^{(\nu+1)}{}_l(r), \tag{73}$$

$$\left(\frac{d}{dr} - \frac{\sigma+1}{l}\right) \phi_\sigma^{(\nu)}{}_l(r) = \phi_{\sigma-1}^{(\nu+1)}{}_l(r) \tag{74}$$

Appendix B

CONVERGENCE OF THE MULTIPOLE EXPANSION

In order to obtain a better insight into the convergence behaviour of the multipole expansion, in particular, the relative contributions of the different multipole orders L to the electric, magnetic and Coulomb transitions, respectively, we define the following quantities

$$\eta_L(X) = \frac{\frac{1}{2L+1}\sum_{\lambda J} |X^L(\lambda J)|^2}{\sum_{L'} \frac{1}{2L'+1} \sum_{\lambda J} |X^{L'}(\lambda J)|^2}, \tag{75}$$

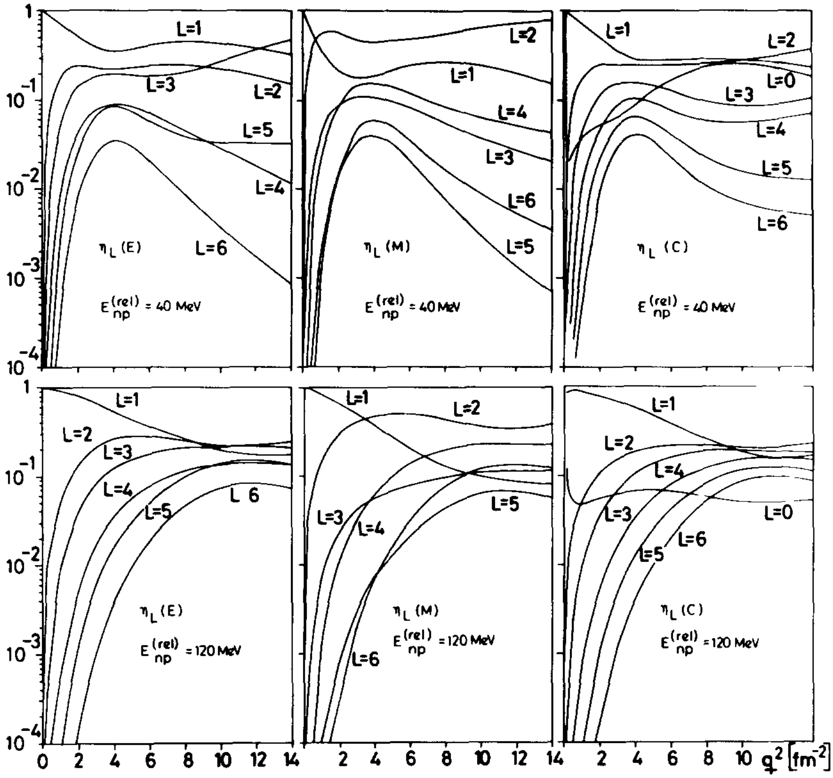

Fig 7 Relative contributions of different multipoles to the inelastic deuteron form factors for $E_{np}^{(rel)} =$ 40 and 120 MeV

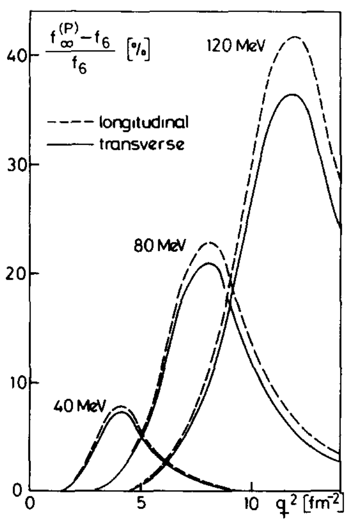

Fig 8 Relative increase of the inelastic deuteron form factors after convergence acceleration with Pade approximation

for $X = E, M, C$ as defined in eq (46) They are shown in fig 7 as function of the momentum transfer for two typical energies $E_{\mathrm{np}}^{(\mathrm{rel})} = 40$ and 120 MeV At low momentum transfer, there is a small region in which the dipole dominates With increasing momentum transfer, higher multipole orders (and the monopole in case of Coulomb transitions) become more and more important until a "point of slowest convergence" is reached near the maximum of the form factor (compare figs 4 and 6) At $E_{\mathrm{np}}^{(\mathrm{rel})} =$ 120 MeV this region of slow convergence is broader and more critical for $q^2 \geqq 7\ \mathrm{fm}^{-2}$ Beyond this region at higher momentum transfers, the main contributions come from a smaller group of relatively low multipole orders (e g , the dipole and quadrupole for the 40 MeV magnetic transitions), while the higher orders are much less important

To obtain a better convergence of the multipole expansion, one may use a Padé approximation [see e g ref [30]] Let f_L denote the form factor as defined in eq (54), where only multipoles up to L have been taken into account in the calculation Taking f_2, f_4 and f_6 as successive approximations, we find a better approximation for the form factor by means of a [1, 1] Padé approximant of type I by

$$f_{\alpha}^{(p)} = f_2 + \frac{f_4 - f_2}{1-(f_6 - f_4)/(f_4 - f_2)} \tag{76}$$

For $E_{\mathrm{np}}^{(\mathrm{rel})}$ near threshold, this does not tell anything new, but for energies $E_{\mathrm{np}}^{(\mathrm{rel})}$ above 40 MeV, eq (76) leads in the region of weak convergence to corrections of some 10% (see fig 8)

Finally, we want to point out a spurious effect for monopole Coulomb transitions which shows up in $\eta_0(C)$ going to one and in $\eta_1(C)$ going to zero for $q \to 0$ (see fig 7) It appears, because with inclusion of IC the transition monopole moment does not vanish if the momentum transfer approaches zero The reason for this is as follows We calculate the IC in impulse approximation [see ref [15]] because with up to forty different IC a coupled channel calculation is impossible This introduces small non-orthogonalities into the IC wave functions and in turn leads to the fact that $C^0(\lambda j)$ does not vanish at $q^2 = 0$ as the remaining $C^L(\lambda j)$, $L \neq 0$ do Consequently, if IC are excluded from the calculation, the aforementioned spurious effect vanishes However, for the lowest momentum transfer which we have considered, the Coulomb monopole stays distinctly below the dipole and thus does not introduce any serious errors

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