## The Non-Local Nature of the Nuclear Force and Its Impact on Nuclear Structure

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We calculate the triton binding energy with a non-local NN potential that fits the world NN data below 350 MeV with the almost perfect  $\chi^2/{\rm datum}$  of 1.03. The non-locality is derived from relativistic meson field theory. The result obtained in a 34-channel, charge-dependent Faddeev calculation is 8.00 MeV, which is 0.4 MeV above the predictions by local NN potentials. The increase in binding energy can be clearly attributed to the off-shell behavior of the non-local potential. Our result cuts in half the discrepancy between theory and experiment established from local NN potentials. Implications for other areas of microscopic nuclear structure, in which underbinding is a traditional problem, are discussed.

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One of the most fundamental traditional goals of theoretical nuclear physics is to explain the properties of atomic nuclei in terms of the elementary interactions between nucleons. For this program, the basic interaction between two nucleons is the most important ingredient. But even if one considers only two-nucleon interactions, the nuclear many-body problem does not have a unique solution. This non-uniqueness is due to the fact that, in the many-body system, the nucleon-nucleon (NN) interaction contributes also "off the energy shell" (off-shell).

By construction, NN interactions reproduce the twonucleon scattering data and the properties of the deuteron. Assuming the existence of NN scattering data of increasing quantity and quality, the NN interaction can be fixed with arbitrary accuracy—"on the energy shell" (on-shell); i. e., for processes in which the two nucleons have the same energy before and after the interaction, like in free-space NN scattering.

In a nucleus with more than two nucleons (A > 2), the energy of the A-particle system is conserved. However, that does not imply that energy is conserved in any individual interaction between two nucleons in the nucleus. Thus, in a many-body system, two nucleons may have different energies before and after they interact; i. e., their mutual interaction may be off the energy shell. Therefore, the calculation of, e. g., the binding energy of an A-particle nucleus involves the off-shell NN interaction, which is empirically undetermined; only theory can provide it.

The off-shell problem in microscopic nuclear structure has been known for several decades [1]. However, in spite of many efforts, it has not been possible, to date, to precisely pin down the off-shell effect on, e. g., the binding energy of a nucleus. Past work on this topic has suffered from two major drawbacks. In some work [2], the NN interactions used were realistic, but not exactly identical on-shell (or not exactly "phase-equivalent", where phase refers to phase shifts of free-space NN scattering). In other works [3,4], phase-equivalent potentials have been constructed by some mathematical methods, but it is doubtful whether the constructed off-shell behavior is realistic, i. e., resembles anything that would be created by mechanism underlying the nuclear force.

The problems in past studies may give us some idea of which minimal requirements should be met by a reliable investigation of the issue: The NN potentials considered should predict the NN observables identically and in accurate agreement with the data. Furthermore, the potentials should have some basis in theory.

Recent substantial progress in the field of nuclear few-body physics has finally set the stage for an investigation of off-shell effects in microscopic nuclear structure which can fulfil the above requirements. In 1993, the Nijmegen group has published a phase-shift analysis of all proton-proton and neutron-proton data below 350 MeV lab. energy with a  $\chi^2$  per datum of 0.99 for 4301 data.

Based upon these data, charge-dependent NN potentials have been constructed by the Nijmegen [6] and the Argonne [7] groups which reproduce the NN data with a  $\chi^2$ /datum of 1.03 and 1.09, respectively. This agreement between the potential predictions and the data as well as the agreement among the various potentials is, on statistical grounds, as accurate as it can be.

An appropriate sample nucleus for microscopic test calculations is the triton  $(^3H)$ . It is the smallest A>2 nucleus which does not involve the Coulomb force, and rigorous, charge-dependent calculations of this nucleus are now-a-days a routine matter. At first glance, the triton may appear too simplistic to represent a reliable test ground for some general features of microscopic nuclear structure; however, Glöckle and Kamada [8] have shown that the rigorous solutions of larger few-nucleon problems (which are very involved and expensive) have essentially the same characteristics as the three-nucleon system. Moreover, there are even obvious parallels between results for the triton, on the one hand, and predictions for nuclear matter and excited nuclei, on the other [9].

Friar et al. [12] have calculated the binding energy of the triton (in charge-dependent 34-channel Faddeev calculations) applying the new, high-quality potentials and obtained almost identical results for the various local models, namely,  $7.62 \pm 0.01$  MeV (experimental value: 8.48 MeV), where the uncertainty of  $\pm 0.01$  MeV is the variation of the predictions which occurs when different local potentials are used. The smallness of the variation is due to the fact that all the local potentials used in the study have essentially the same off-shell behavior.

All new high-quality NN potentials use the local version of the one-pion-exchange potential for the long-range part of the interaction which is, e. g., for *pp* scattering:

$$V_{\pi}^{(loc)}(\mathbf{r}) = \frac{g_{\pi}^{2}}{12\pi} \left(\frac{m_{\pi}}{2M}\right)^{2} \left[ \left(\frac{e^{-m_{\pi}r}}{r} - \frac{4\pi}{m_{\pi}^{2}} \delta^{(3)}(\mathbf{r})\right) \boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2} + \left(1 + \frac{3}{m_{\pi}r} + \frac{3}{(m_{\pi}r)^{2}}\right) \frac{e^{-m_{\pi}r}}{r} \boldsymbol{S}_{12} \right], \quad (1)$$

where  $m_{\pi}$  denotes the neutral pion mass and M the proton mass. For np scattering the appropriate combination of neutral and charged pion exchange is used which creates the charge-dependence in the models. The intermediate and short-range parts are parametrized in different ways. Here, the Argonne  $V_{18}$  potential [7] uses local functions of Woods-Saxon type, while the other potentials apply local Yukawas of either multiples of the pion mass (Reid'93 [6]) or of the empirical masses of existing mesons and meson distributions (Nijm-II [6]). All potentials are regularized at short distances by either exponential ( $V_{18}$ , Nijm-II) or dipole (Reid'93) form factors (which are also local).

Ever since NN potentials have been developed, local potentials have enjoyed great popularity because they are easy to apply in configuration-space calculations. Note, however, that numerical ease is not a proof for the lo-

cal nature of the nuclear force. In fact, any deeper insight into the reaction mechanisms underlying the nuclear force suggests a non-local character. In particular, the composite structure of hadrons should lead to large non-localities at short range [13]. But, even the conventional and well-established meson theory of nuclear forces—when derived properly and without crude approximations—creates a non-local interaction. In this paper, we will focus on this "simplest" source of non-locality.

Common Lagrangians for meson-nucleon coupling are:

$$\mathcal{L}_{ps} = -g_{ps}\bar{\psi}i\gamma^5\psi\varphi^{(ps)} \tag{2}$$

$$\mathcal{L}_s = g_s \bar{\psi} \psi \varphi^{(s)} \tag{3}$$

$$\mathcal{L}_{v} = g_{v}\bar{\psi}\gamma^{\mu}\psi\varphi_{\mu}^{(v)} + \frac{f_{v}}{4M}\bar{\psi}\sigma^{\mu\nu}\psi(\partial_{\mu}\varphi_{\nu}^{(v)} - \partial_{\nu}\varphi_{\mu}^{(v)})$$
 (4)

where ps, s, and v denote pseudoscalar, scalar, and vector couplings/fields, respectively.

The lowest order contributions to the nuclear force from the above Lagrangians are the second-order Feynman diagrams which, in the c. m. system of the two interacting nucleons, produce the amplitude:

$$\mathcal{A}_{\alpha}(q',q) = \frac{\bar{u}_1(\mathbf{q}')\Gamma_1^{(\alpha)}u_1(\mathbf{q})P_{\alpha}\bar{u}_2(-\mathbf{q}')\Gamma_2^{(\alpha)}u_2(-\mathbf{q})}{(q'-q)^2 - m_{\alpha}^2}, (5)$$

where  $\Gamma_i^{(\alpha)}$  (i=1,2) are vertices derived from the above Lagrangians,  $u_i$  Dirac spinors representing the interacting nucleons, and q and q' their relative momenta in the initial and final states, respectively;  $P_{\alpha}$  divided by the denominator is the meson propagator.

The simplest meson-exchange model for the nuclear force is the one-boson-exchange (OBE) potential [10,14] which sums over several second-order diagrams, each representing the single exchange of a different boson,  $\alpha$ :

$$V(\mathbf{q}', \mathbf{q}) = \sqrt{\frac{M}{E'}} \sqrt{\frac{M}{E}} \sum_{\alpha} i \mathcal{A}_{\alpha}(\mathbf{q}', \mathbf{q}) F_{\alpha}^{2}(\mathbf{q}', \mathbf{q}) .$$
 (6)

As customary, we included form factors,  $F_{\alpha}(\mathbf{q}',\mathbf{q})$ , applied to the meson-nucleon vertices, and a square-root factor  $M/\sqrt{E'E}$  (with  $E=\sqrt{M^2+\mathbf{q}^2}$  and  $E'=\sqrt{M^2+\mathbf{q}'^2}$ ). The form factors regularize the amplitudes for large momenta (short distances) and account for the extended structure of nucleons in a phenomenological way. The square root factors make it possible to cast the unitarizing, relativistic, three-dimensional Blankenbecler-Sugar equation for the scattering amplitude (a reduced version of the four-dimensional Bethe-Salpeter equation) into a form which is identical to the (non-relativistic) Lippmann-Schwinger equation [10,14]. Thus, Eq. (6) defines a relativistic potential which can be consistently applied in conventional, non-relativistic nuclear structure.

Clearly, the Feynman amplitudes, Eq. (5), are in general non-local expressions; i. e., Fourier transform into

configuration space will yield functions of r and r', the relative distances between the two in- and out-going nucleons, respectively. The square root factors create additional non-locality, as pointed out by Glöckle and Kamada [15].

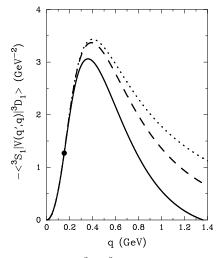


FIG. 1. Half off-shell  $^3S_1$ – $^3D_1$  amplitude for the relativistic CD-Bonn potential (solid line), Eq. (6). The dashed (dotted) curve is obtained when the local approximation, Eq. (8), is used for OPE (OPE and one- $\rho$  exchange). q'=153 MeV.

While for heavy vector-meson exchange (corresponding to short distances) non-locality appears quite plausible, we have to stress here that even the one-pion-exchange (OPE) Feynman amplitude is non-local. This is important because the pion creates the dominant part of the tensor force which plays a crucial role in nuclear structure

Applying  $\Gamma^{(\pi)} = g_{\pi} \gamma_5$  in Eq. (5), yields the Feynman amplitude for neutral pion exchange in pp scattering,

$$i\mathcal{A}_{\pi}(\mathbf{q}',\mathbf{q}) = -\frac{g_{\pi}^{2}}{4M^{2}} \frac{(E'+M)(E+M)}{(\mathbf{q}'-\mathbf{q})^{2}+m_{\pi}^{2}} \left(\frac{\boldsymbol{\sigma_{1} \cdot \mathbf{q}'}}{E'+M} - \frac{\boldsymbol{\sigma_{1} \cdot \mathbf{q}}}{E+M}\right) \times \left(\frac{\boldsymbol{\sigma_{2} \cdot \mathbf{q}'}}{E'+M} - \frac{\boldsymbol{\sigma_{2} \cdot \mathbf{q}}}{E+M}\right) . \tag{7}$$

In static approximation, i. e., for  $E' \approx E \approx M$ , this reduces to

$$V_{\pi}^{(loc)}(\mathbf{k}) = -\frac{g_{\pi}^2}{4M^2} \frac{(\boldsymbol{\sigma_1} \cdot \mathbf{k})(\boldsymbol{\sigma_2} \cdot \mathbf{k})}{\mathbf{k}^2 + m_{\pi}^2}$$
(8)

with  $\mathbf{k} = \mathbf{q}' - \mathbf{q}$ ; this is nothing but the Fourier transform of the local OPE potential,  $V_{\pi}^{(loc)}(\mathbf{r})$ , given in Eq. (1). Notice also that on-shell, i. e., for  $|\mathbf{q}'| = |\mathbf{q}|$ ,  $V_{\pi}^{(loc)}$  equals  $i\mathcal{A}_{\pi}$ . Thus, the non-locality affects the OPE potential only off-shell.

In Fig. 1, we show the half off-shell  ${}^3S_1 - {}^3D_1$  potential that can be produced only by tensor forces. The onshell momentum q' is held fixed at 153 MeV (equivalent to 50 MeV lab. energy), while the off-shell momentum q runs from zero to 1400 MeV. The on-shell point (q = 153

MeV) is marked by a solid dot. The solid curve is a relativistic OBE potential (CD-Bonn, s. below), Eq. (6). When the relativistic OPE amplitude, Eq. (7), is replaced by the static/local approximation, Eq. (8), the dashed curve is obtained. When this approximation is also used for the one- $\rho$  exchange, the dotted curve results. It is clearly seen that the static/local approximation substantially increases the tensor force off-shell.

From the discussion here, it is evident that relativity and non-locality are intimately interwoven. At this advanced stage of nuclear few-body physics, there is a need for relativistic potentials, also, for reasons other than non-locality [16]. Potentials based upon the invariant Feynman amplitudes, Eq. (5), are examples for relativistic potentials. A very quantitative model of this kind will be given below.

We believe that the non-localities created by relativistic meson exchange are "real" and deserve attention. An important question is: What is their impact on microscopic nuclear structure calculations?

To investigate this point, we have constructed a new relativistic OBE potential based upon Eqs. (5) and (6) of very high precision. The potential (dubbed "CD-Bonn") is charge-dependent due to nucleon and pion mass splitting; therefore, a pp, np, and nn potential are provided.

To meet the requirements for a reliable investigation pointed out above, we have fitted the 4301 pp and npdata below 350 MeV lab. energy with a  $\chi^2/{\rm datum}$  of 1.03 [17] (i. e., with the same accuracy as the new local high-quality potentials [6,7]). As pointed out in Refs. [5,6], this high accuracy cannot be achieved with the usual, about a dozen, parameters of the conventional OBE model. Some additional fit freedom is needed, for which we choose to adjust the fictitious  $\sigma$  boson individually in each partial wave. Physical justification for this procedure comes from the fact that—based upon the more realistic meson model for the nuclear force which includes all important multi-meson exchanges [18]—the one- $\sigma$  exchange in the OBE model stands for the sum of all higher order diagrams, and not just for the  $2\pi$ exchange (as commonly believed). Of course, this is

TABLE I. Recent high-precision NN potentials and predictions for the two- and three-nucleon system  $\,$ 

	CD-Bonn	Nijm-II	Reid'93	$V_{18}$
Character	non-local	local	local	local
$\chi^2/{ m datum}$	1.03	1.03	1.03	1.09
$g_{\pi}^2/4\pi$	13.6	13.6	13.6	13.6
Deuteron properties:				
Quadr. moment (fm <sup>2</sup> )	0.270	0.271	0.270	0.270
Asymptotic D/S state	0.0255	0.0252	0.0251	0.0250
D-state probab. (%)	4.83	5.64	5.70	5.76
Triton binding (MeV):				
non-rel. calculation	8.00	7.62	7.63	7.62
relativ. calculation	8.19	_		-

a very crude approximation and, therefore, typical discrepancies occur in various partial waves (cf. Fig. 11 of Ref. [18]), which can be removed by individual adjustments of the  $\sigma$  boson. More details concerning the new CD-Bonn potential will be published elsewhere [19].

In Table I (upper part), we summarize two-nucleon properties predicted by the new CD-Bonn potential and compare with the other recent high-quality potentials. Using the same  $\pi NN$  coupling constant, all potentials predict almost identical deuteron observables (quadrupole moment and asymptotic D/S state normalization). Note, however, that the (un-observable) deuteron D-state probability comes out significantly larger for the local potentials ( $\approx 5.7\%$ ) as compared to the non-local CD-Bonn potential (4.8%). Obviously, the deuteron D-state probability is kind of a numerical measure for the off-shell strength of the tensor force, shown graphically in Fig. 1.

We have performed a (34-channel, charge-dependent) Faddeev calculation for the triton with the new CD-Bonn potential and obtained 8.00 MeV binding energy (cf. Table I). This is 0.38 MeV more than local potentials predict. The unaquainted observer may be tempted to believe that this difference of 0.38 MeV is quite small, almost negligible. However, this is not true. The difference between the predictions by local potentials (7.62 MeV) and experiment (8.48 MeV) is 0.86 MeV. Thus, the problem with the triton binding is that 0.86 MeV cannot be explained in the simplest way, that is all. Therefore, any non-trivial contribution must be measured against the 0.86 MeV gap between experiment and simplest theory. On this scale, the non-locality considered in this investigation explains 44% of the gap; i. e., it is substantial with respect to the remaining discrepancy.

We note that there is one new Nijmegen potential (Nijm-I [6]) which has a non-local central force (but is local otherwise) and predicts 7.72 MeV for the triton binding [12], 0.10 MeV above the local benchmark. Since in our model, all components of the nuclear force (central, tensor, etc.) are non-local, this effect is included in our calculations; and one may conclude that the non-locality in the tensor force increases the binding by about 0.3 MeV.

The above three-body results were obtained by using the conventional non-relativistic Faddeev equations. However, since CD-Bonn is a relativistic potential, one can also perform a relativistic Faddeev calculation by extending the relativistic three-dimensional Blanckenbecler-Sugar formalism to the three-body system [20]. The binding energy prediction by CD-Bonn then goes up to 8.19 MeV. This further increase can be understood as an additional off-shell effect from the relativistic two-nucleon t-matrix applied in the three-nucleon system (see Ref. [20] for details).

We stress that our present calculations take only the most "primitive" source of non-locality into account.

Since meson exchange is mainly responsible for the long and intermediate range of the nuclear force, we do not expect it to be the main source for non-locality. The short-range part of the nuclear force, where the composite structure of hadrons should play an important role, may provide much larger non-localities. It is an challenging topic for future research to derive this additional non-locality [13], and test its impact on nuclear structure predictions.

Obviously, our results leave little room for contributions from three-nucleon forces (3NF). Still, this does not mean that they do not exist in nature. In fact, the meson theory of  $\pi N$  and and NN scattering (including meson resonances and isobar degrees of freedoms) implies a large variety of 3NF. However, consistent calculations which treat the 2N and 3N system on an equal footing have shown that large cancellations can occur between "genuine" 3NF contributions and medium effects on the 2N force when inserted into the three-nucleon system [21,22]. If the 3NF is weak, then because of cancellations of this kind. If it should turn out that these cancellations are almost perfect, then the challenging question will be if this is just an accident of nature (which is hard to believe) or if we are still missing some symmetries underlying nuclear structure.

In summary, a non-local NN potential based upon relativistic meson theory predicts 0.4 MeV more triton binding energy than local NN potentials. This result cuts in half the discrepancy between theory and experiment established from local potentials. Based upon nuclear matter results and earlier calculations in finite nuclei [11], one may expect that this new high-precision non-local potential could also improve predictions in other areas of microscopic nuclear struture where underbinding is a traditional problem.

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